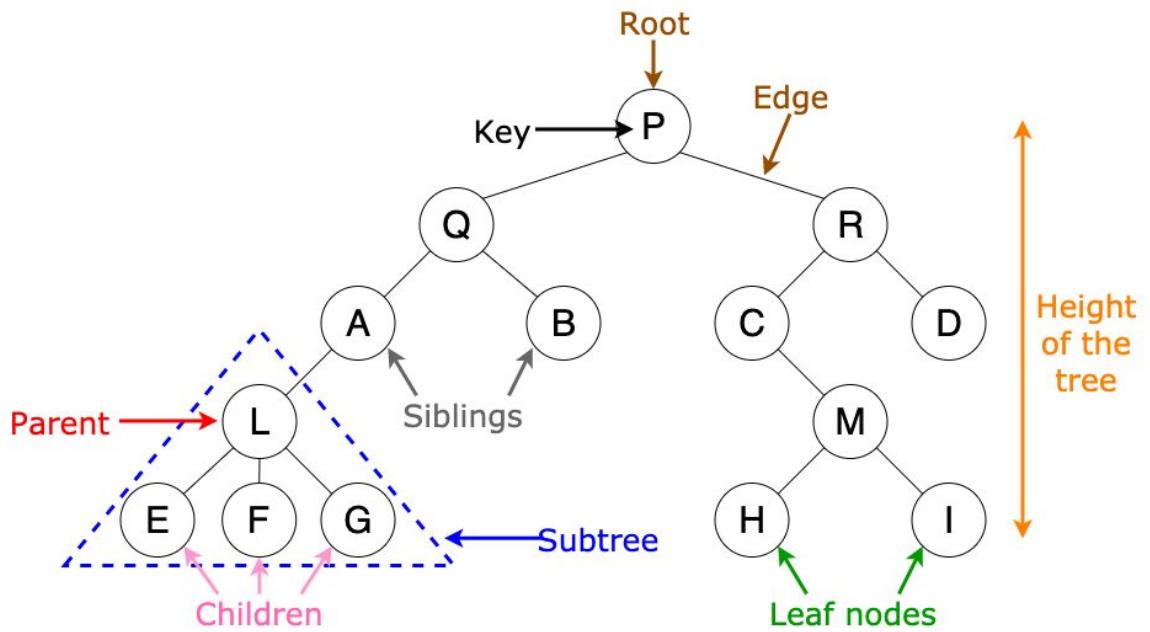


# Tree Data Structure

Abdul Ghafoor





# Trees – A Nonlinear Approach to Data Organization

- Productivity experts often emphasize "nonlinear" thinking as a key to innovation and breakthroughs.
- In computing, trees are a primary example of a nonlinear structure, offering a powerful way to organize and access data more efficiently than linear structures, such as lists.

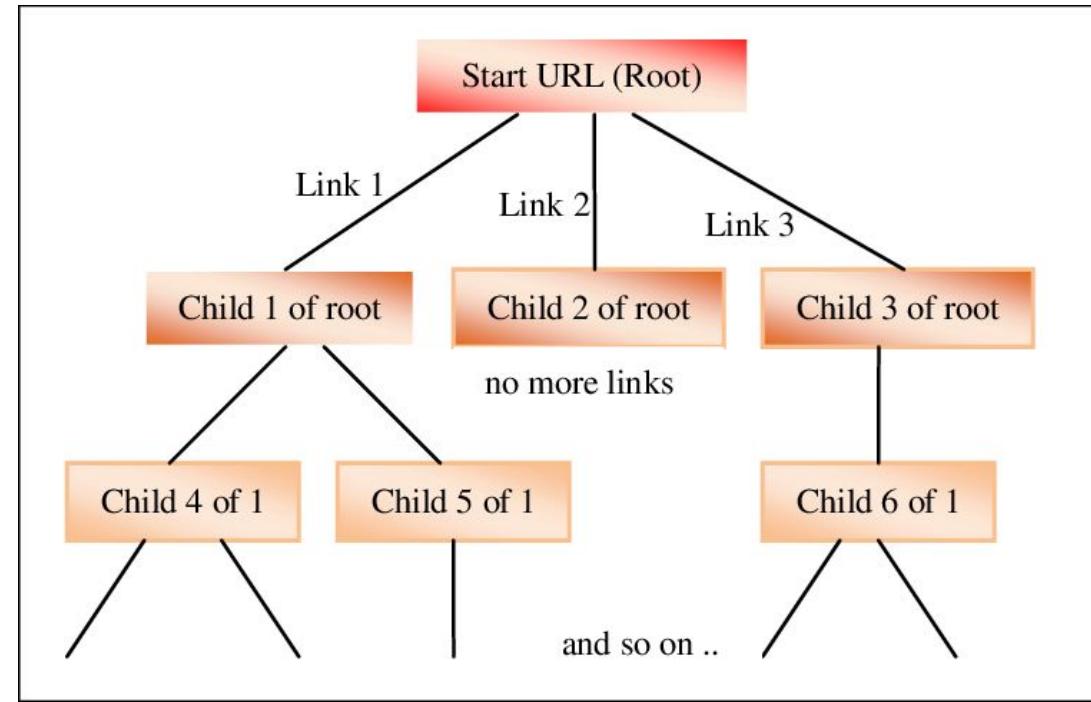
# Trees – A Nonlinear Approach to Data Organization

**Question:** Imagine you're looking for a book in a large library where all the books are arranged in a single row. How efficient would it be to find a specific book if you had to walk through every title one by one?

**Ans:** This linear arrangement would be slow and impractical. A **tree structure** would organize the library hierarchically: starting with broad categories (like Fiction and Non-Fiction), then genres (e.g., Mystery, History), and finally individual shelves for each genre. With this structure, you can go directly to the relevant section, saving time and making it easy to find specific books.

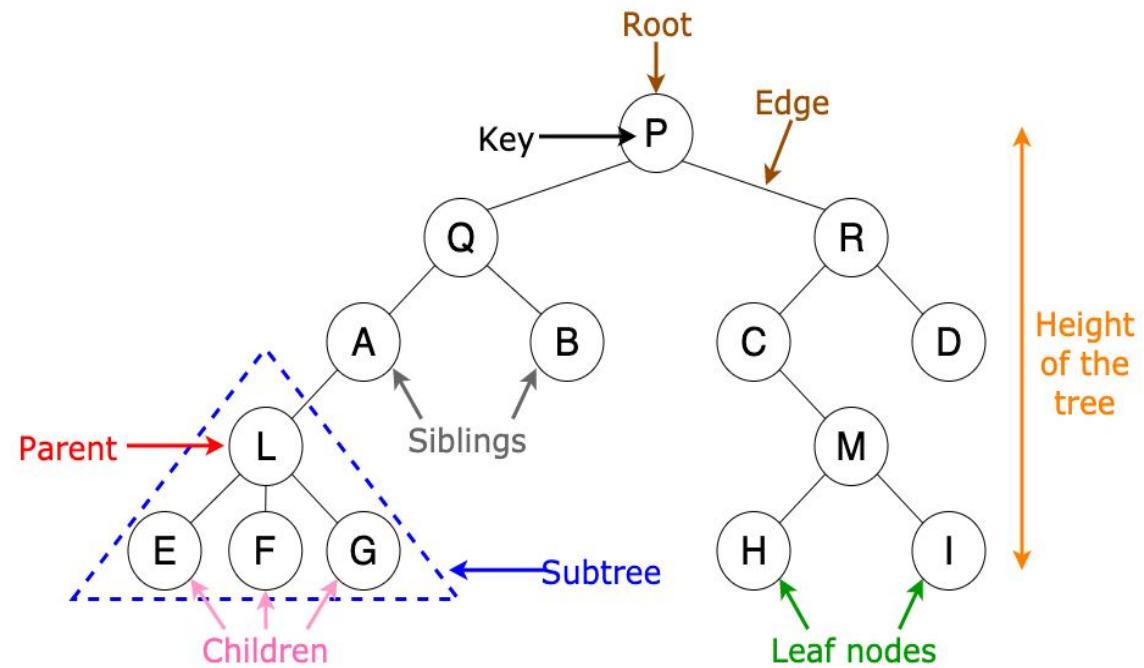
# Why Trees Are Essential in Data Organization:

- Trees provide a **natural hierarchy** for organizing data, making them fundamental to systems like file directories, databases, websites, and GUIs.
- With trees, algorithms can be implemented more quickly and efficiently, allowing for faster performance in data-intensive applications.



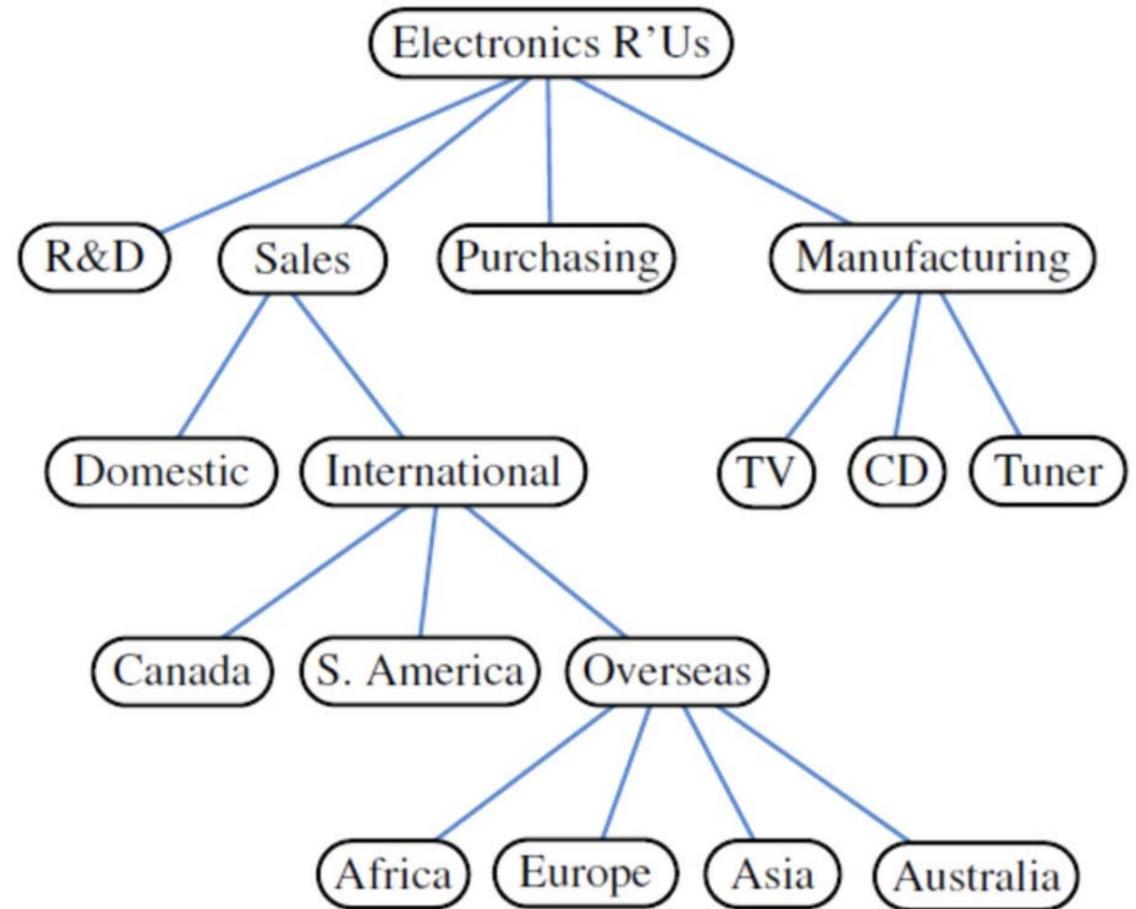
# Understanding Nonlinearity in Trees:

- Trees are "nonlinear" because their structure is hierarchical rather than sequential.
  - In a tree, relationships aren't simply "before" or "after"; instead, they form a hierarchy where some nodes are **"above"** others (parents) and some are **"below"** (children).
  - This hierarchy closely resembles family trees, giving rise to terms like **parent**, **child**, **ancestor**, and **descendant** to define relationships.



# Formal Definition of Tree

- Tree: an ADT that stores elements hierarchically
- Each element has
  - A parent element
  - Zero or more child elements
  - Root: top element



# Formal Definition of Tree

Definition: A tree  $T$  is a set of nodes storing elements and a parent-child relation

If  $T$  is non-empty, it has a special node, called root of  $T$ , that has no parent

Every non-root node has a unique parent  $w$ ; every node with parent  $w$  is a child of  $w$

# Basic Terms

Sibling: two nodes that are children of the same parent

External node (leaves): a node  $v$  is external if it has no children

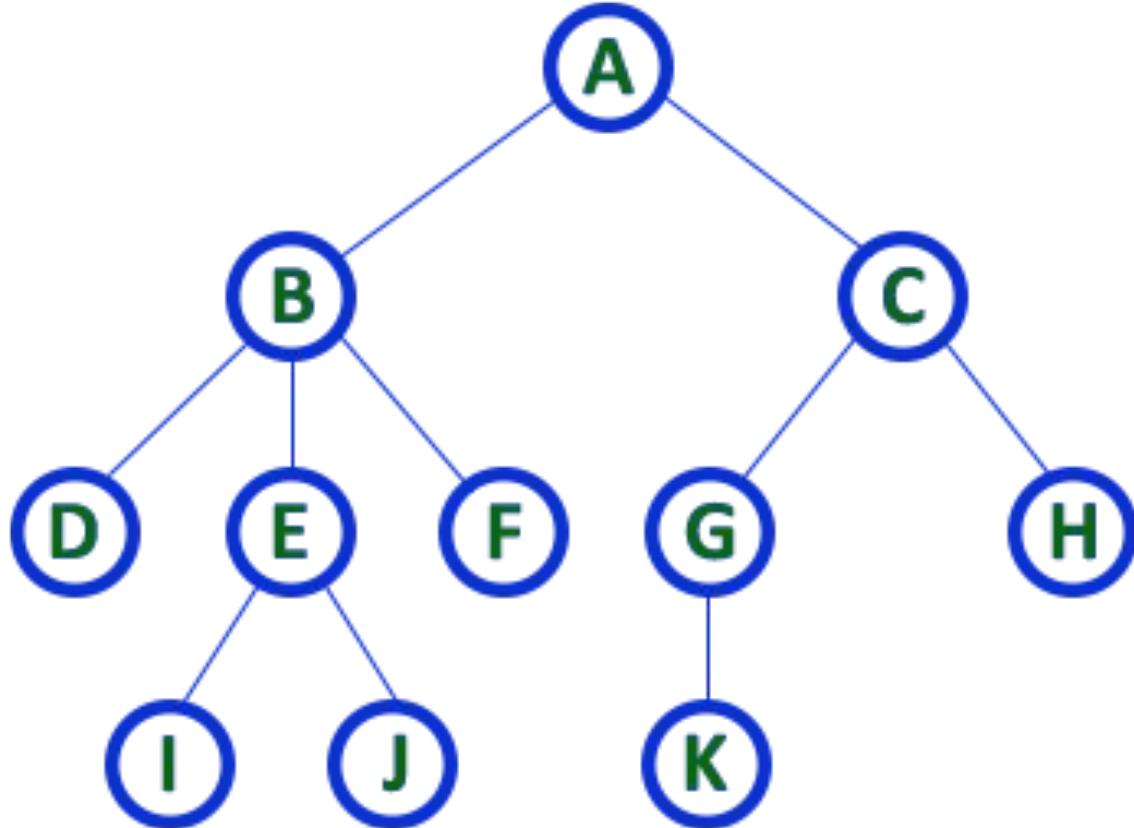
Internal node: a node  $v$  is internal if it has one or more children

Ancestor:  $u$  is an ancestor of  $v$  if  $u = v$  or  $u$  is an ancestor of the parent of  $v$

Descendant:  $v$  is a descendant of  $u$  if  $u$  is an ancestor of  $v$

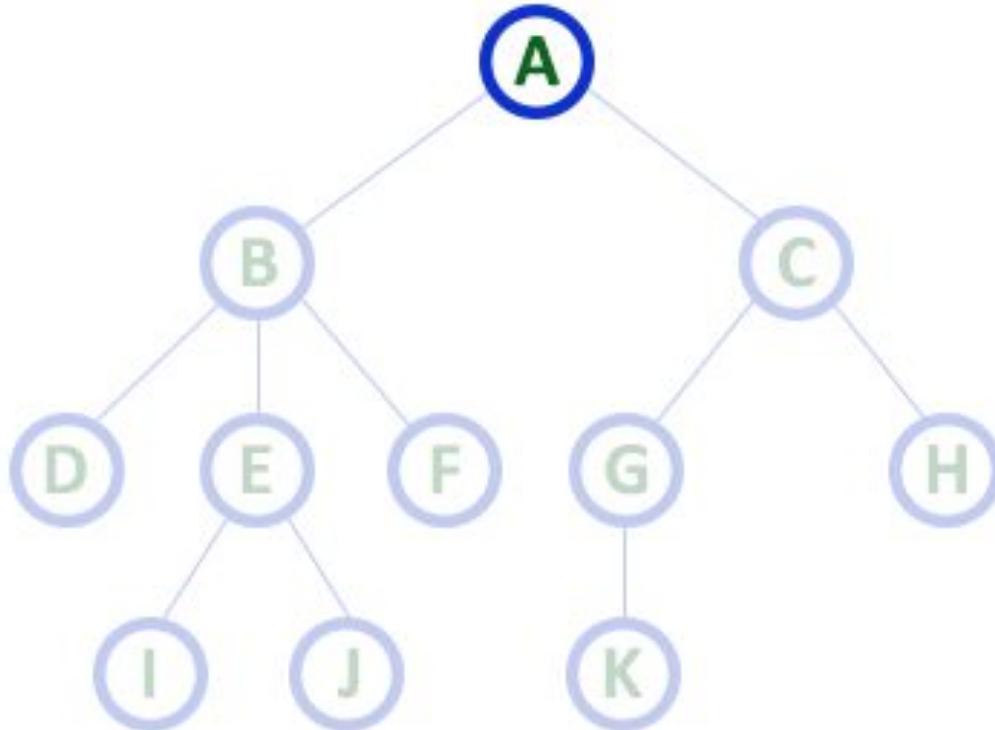
# Basic Terms

- Subtree  
of  $T$  rooted at  $v$ : the tree consisting of all the descendants of  $v$  in  $T$  (including  $v$  itself)
- Edge: an edge of tree  $T$  is a pair of nodes  $(u, v)$  such that  $u$  is the parent of  $v$ , or vice versa
- Path: a path of  $T$  is a sequence of nodes such that any two consecutive sequence form an edge



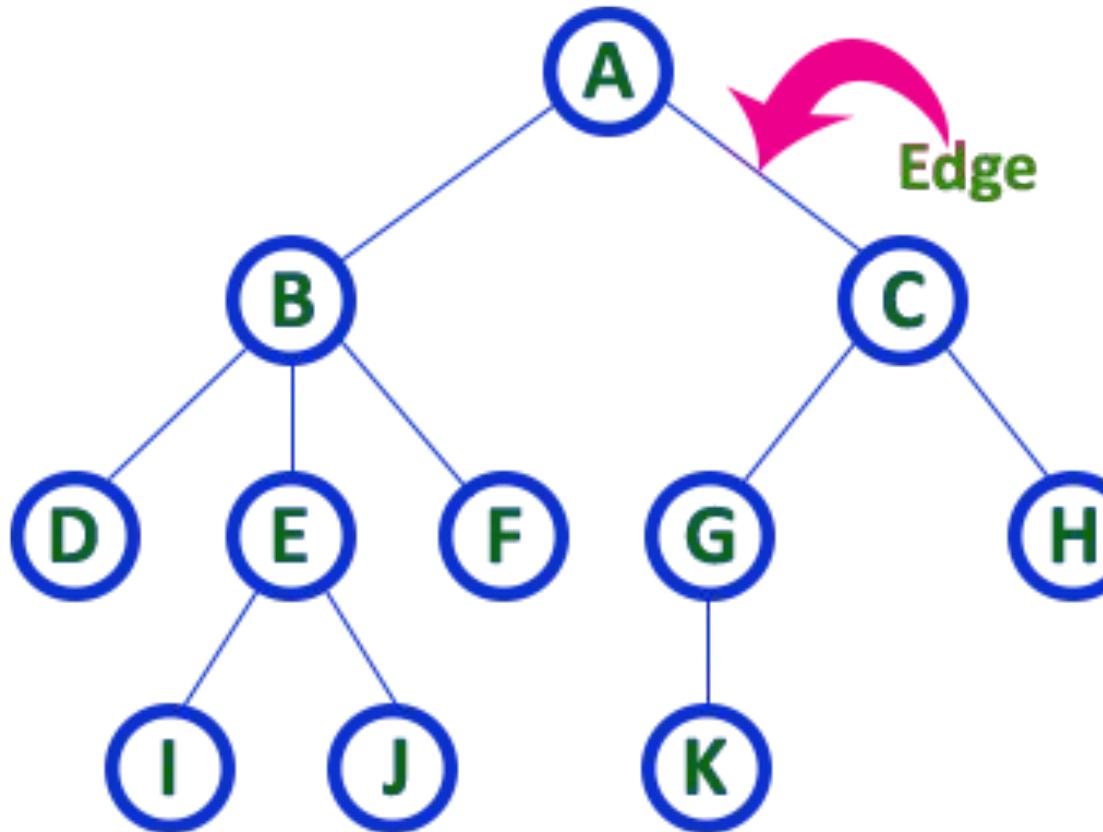
## TREE with 11 nodes and 10 edges

- In any tree with ' $N$ ' nodes there will be maximum of ' $N-1$ ' edges
- In a tree every individual element is called as 'NODE'

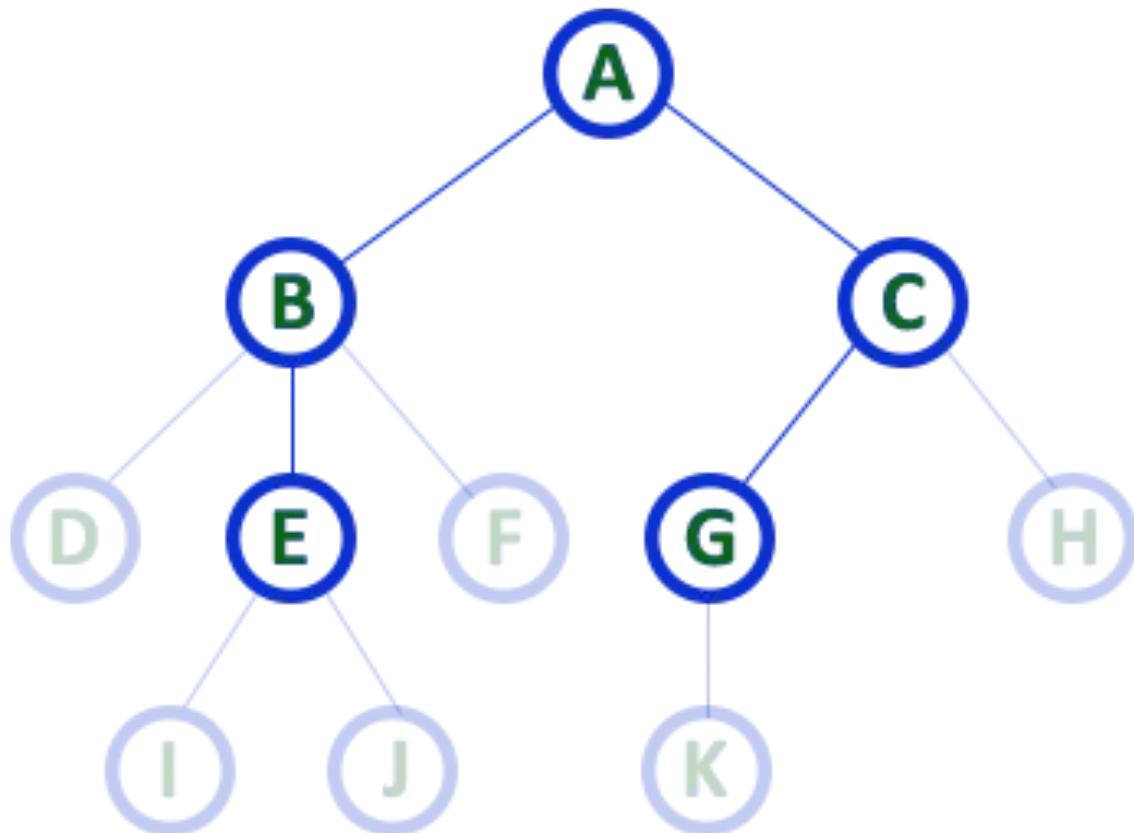


**Here 'A' is the 'root' node**

- In any tree the first node is  
called as **ROOT** node

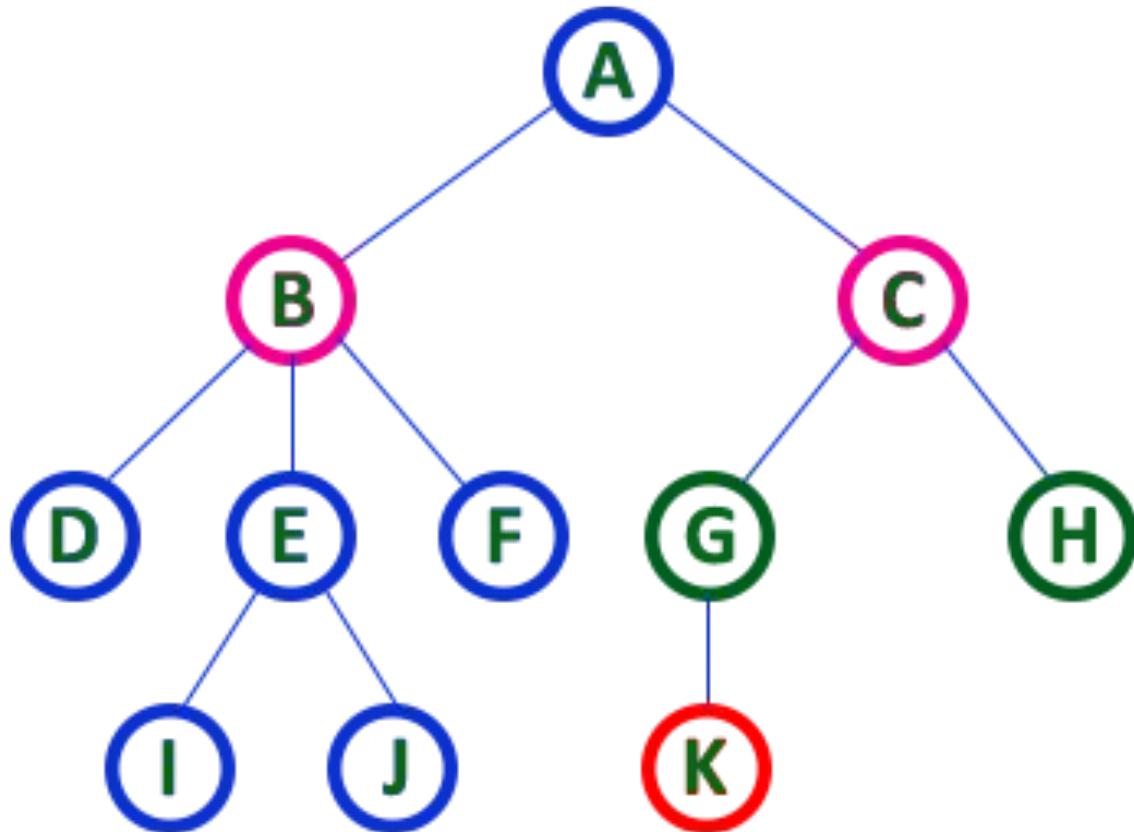


- In any tree, 'Edge' is a connecting link between two nodes.



Here A, B, C, E & G are Parent nodes

- In any tree the node which has child / children is called 'Parent'
- A node which is predecessor of any other node is called 'Parent'

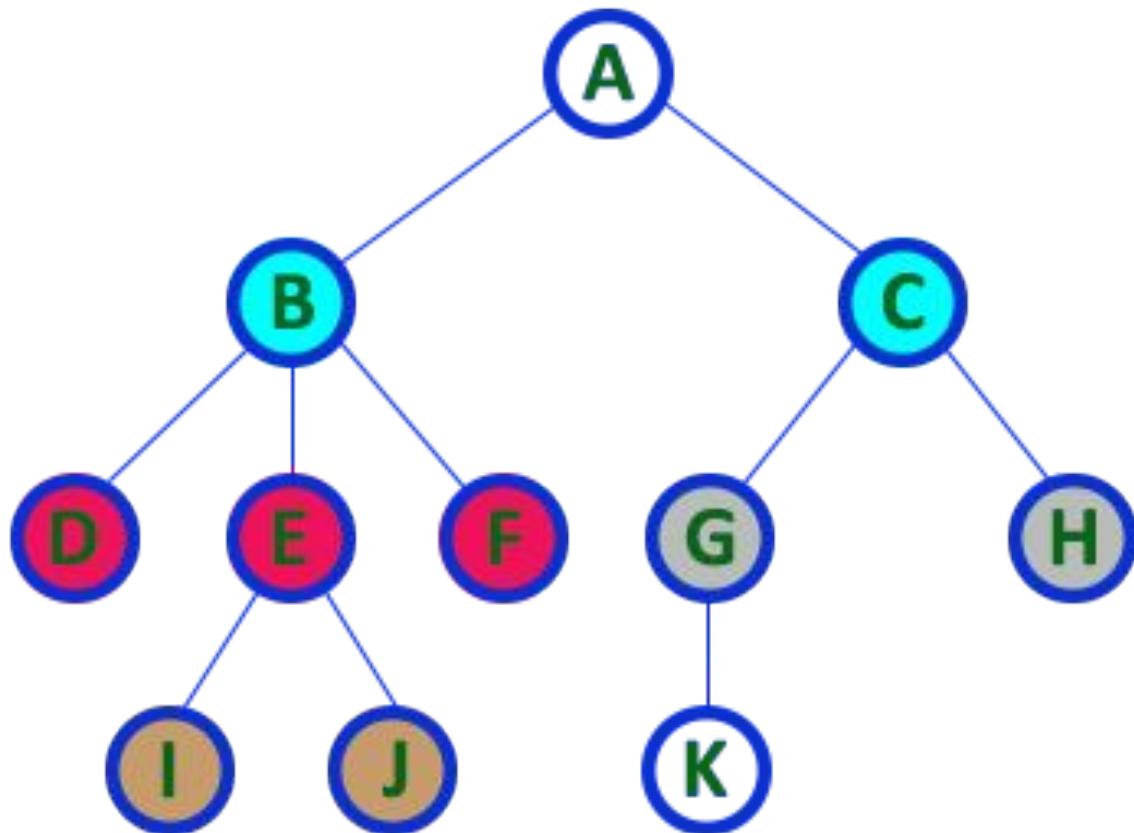


Here B & C are Children of A

Here G & H are Children of C

Here K is Child of G

- descendant of any node is called as CHILD Node



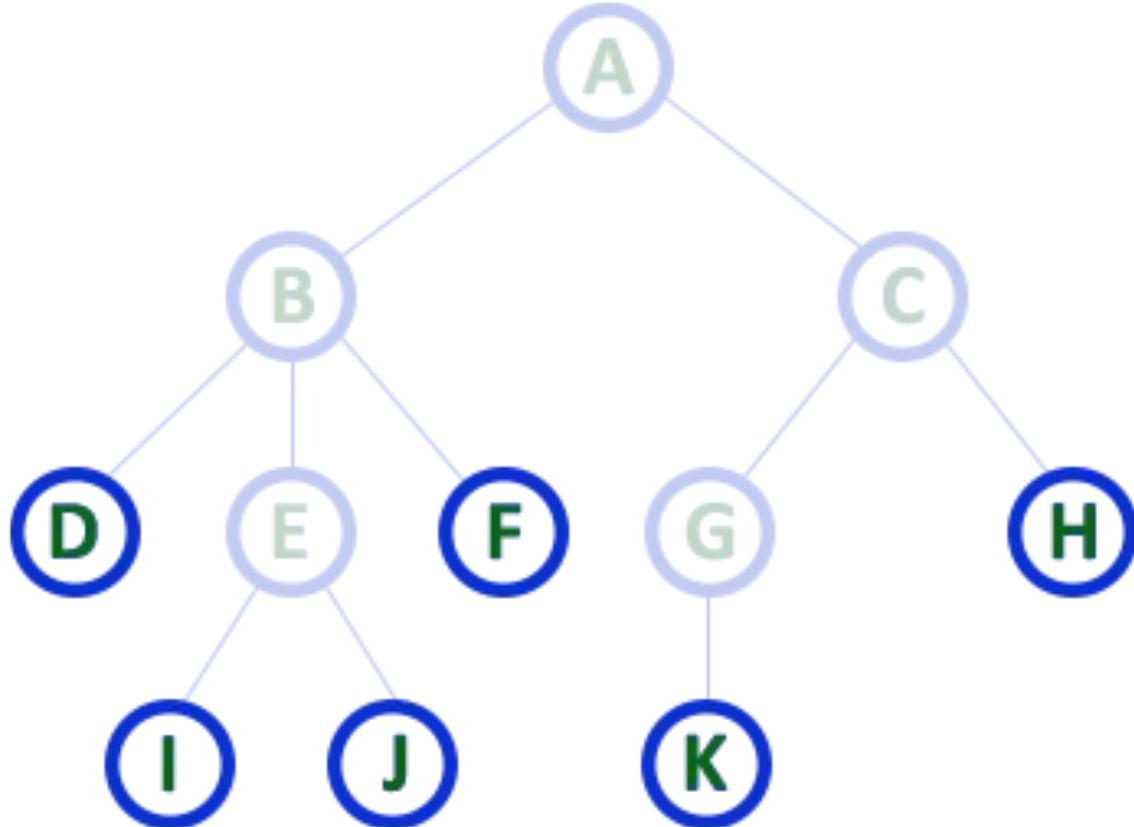
Here B & C are Siblings

Here D E & F are Siblings

Here G & H are Siblings

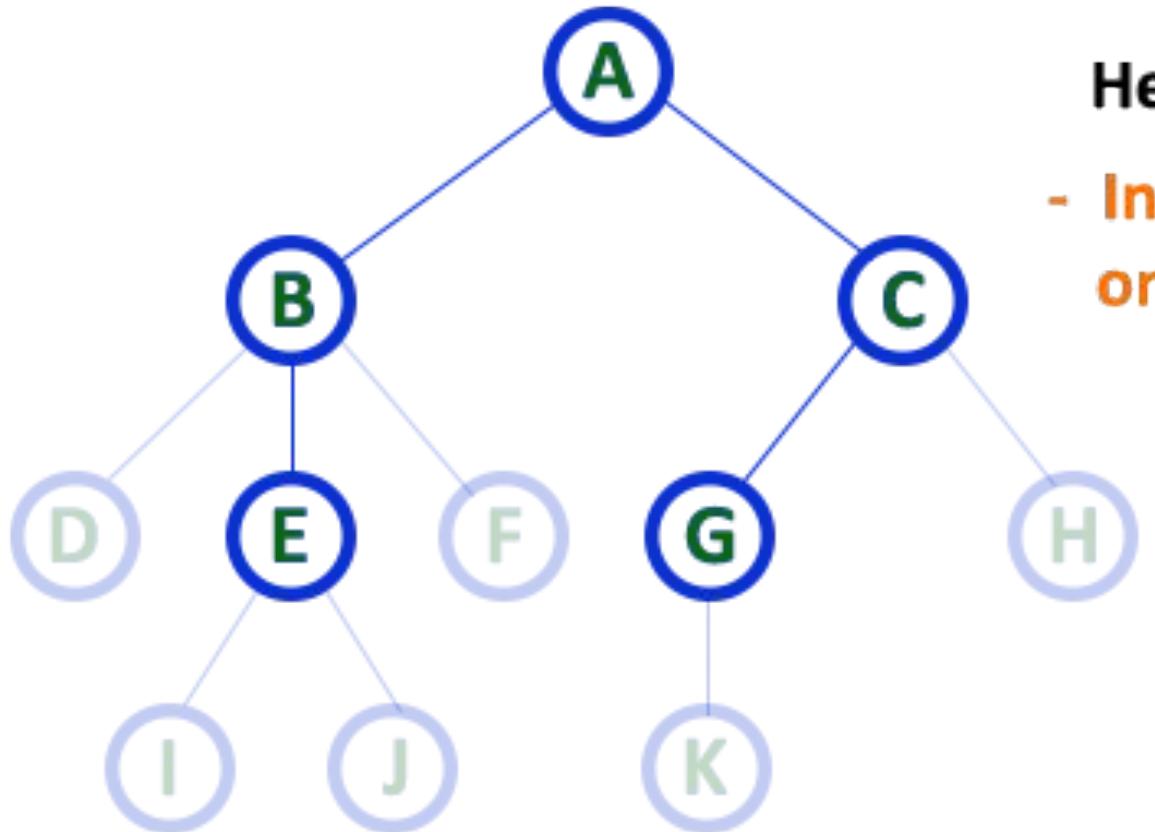
Here I & J are Siblings

- In any tree the nodes which has same Parent are called 'Siblings'
- The children of a Parent are called 'Siblings'



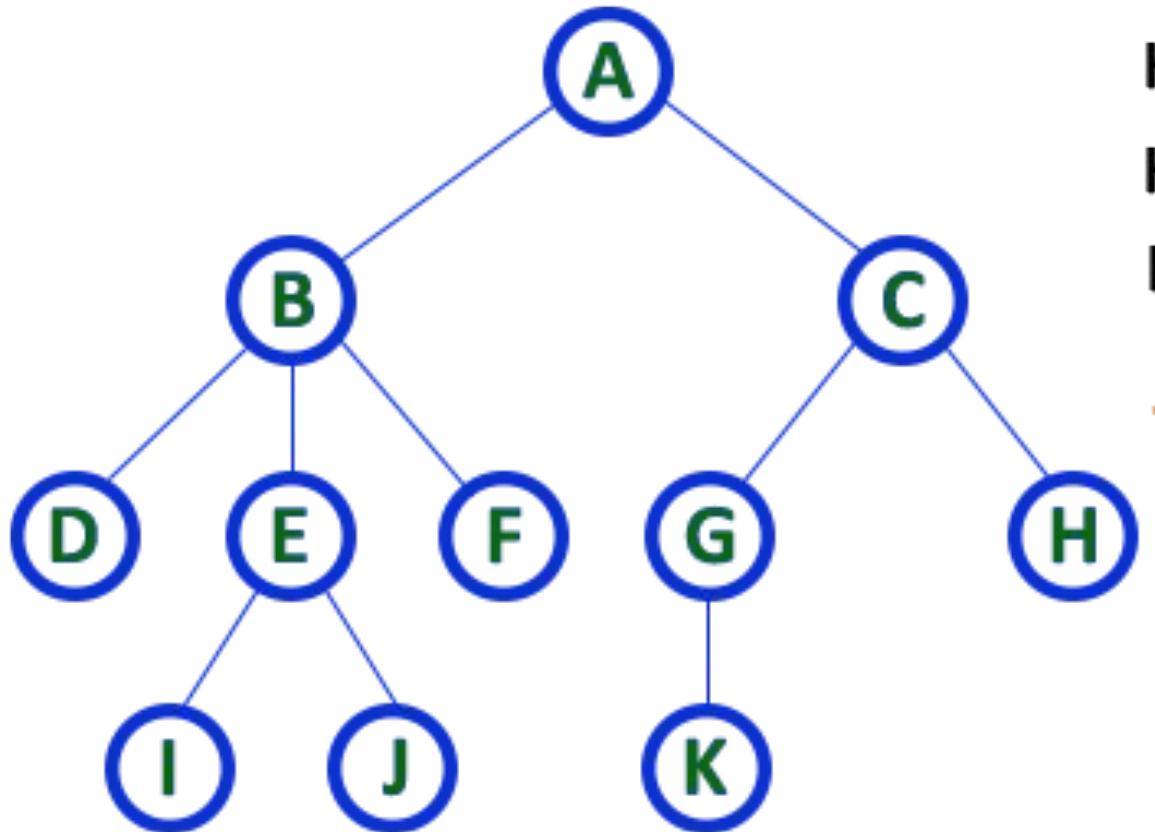
Here D, I, J, F, K & H are Leaf nodes

- In any tree the node which does not have children is called 'Leaf'
- A node without successors is called a 'leaf' node



Here A, B, C, E & G are **Internal nodes**

- In any tree the node which has atleast one child is called '**Internal**' node
  - Every non-leaf node is called as '**Internal**' node

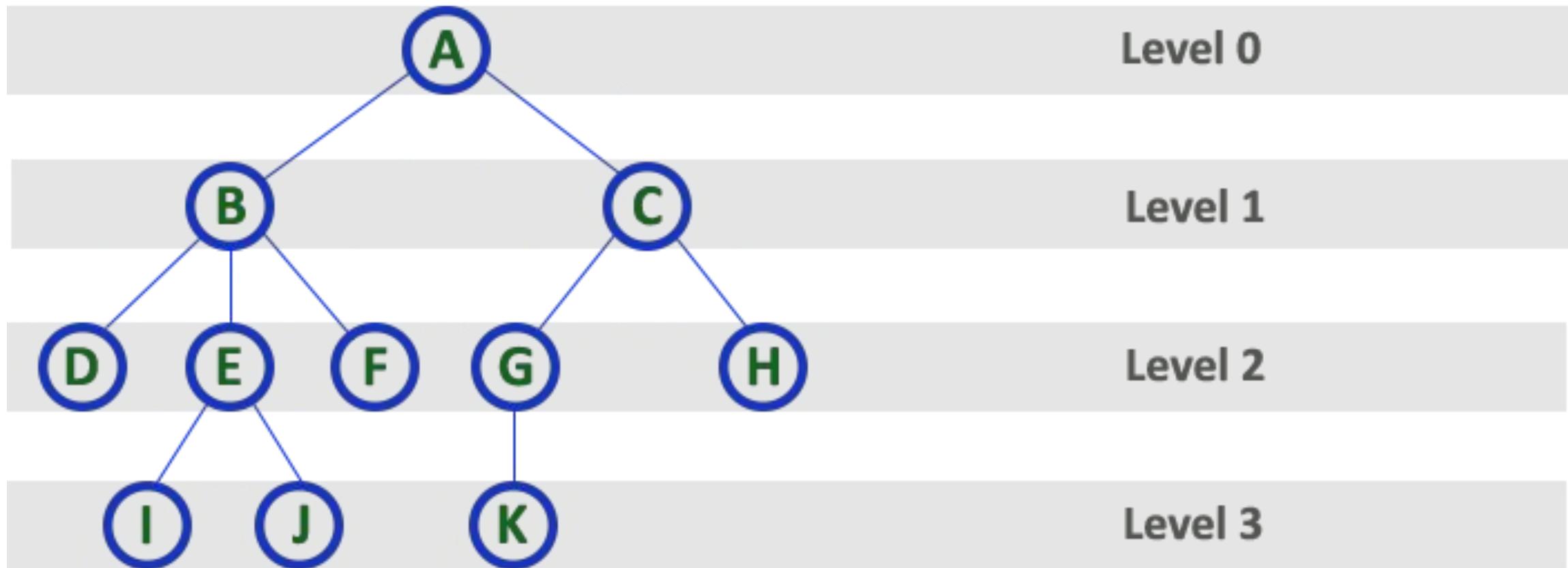


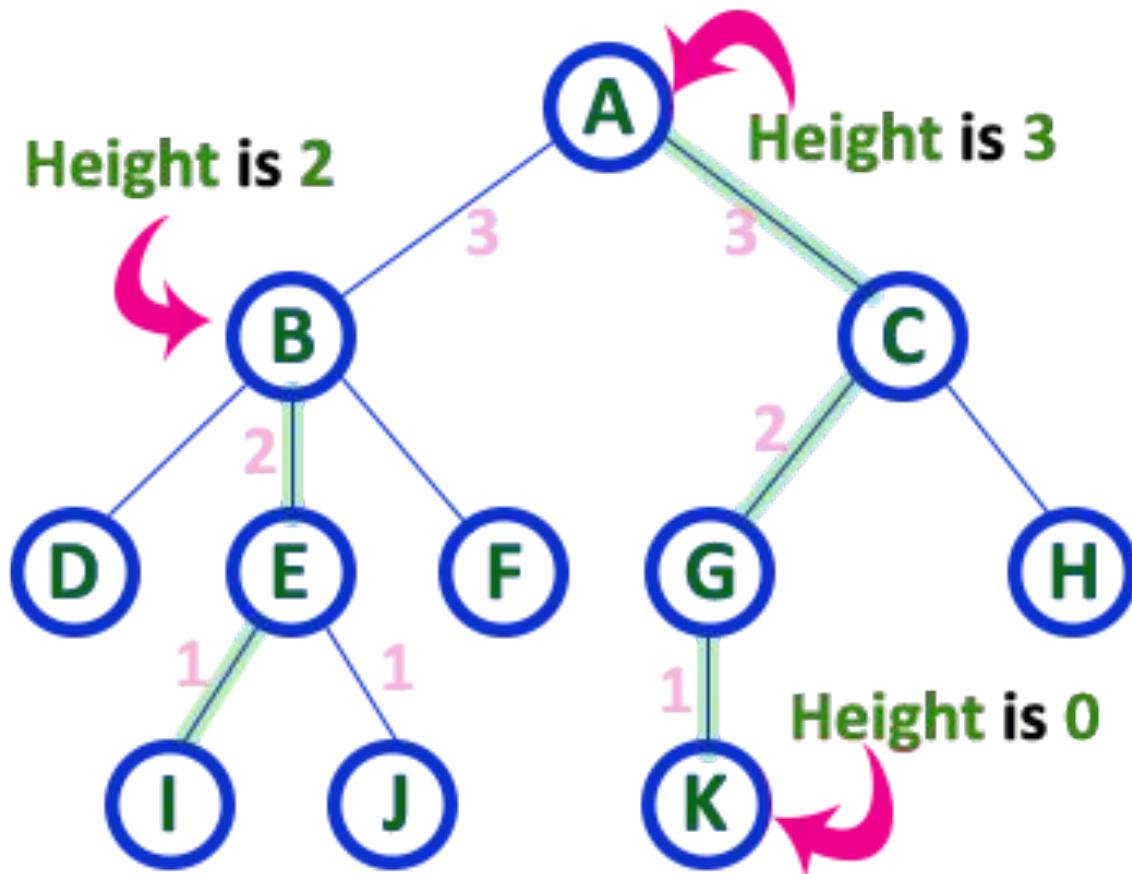
Here Degree of B is 3

Here Degree of A is 2

Here Degree of F is 0

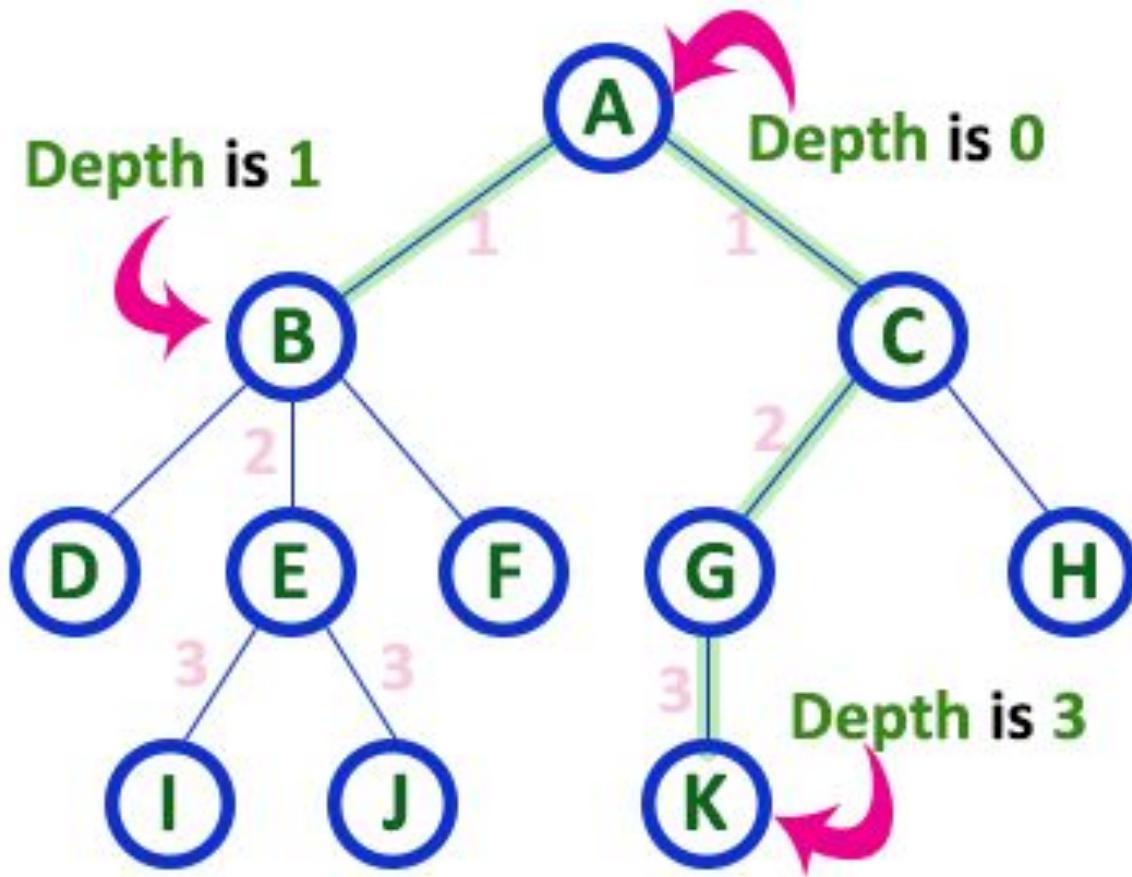
- In any tree, 'Degree' of a node is total number of children it has.





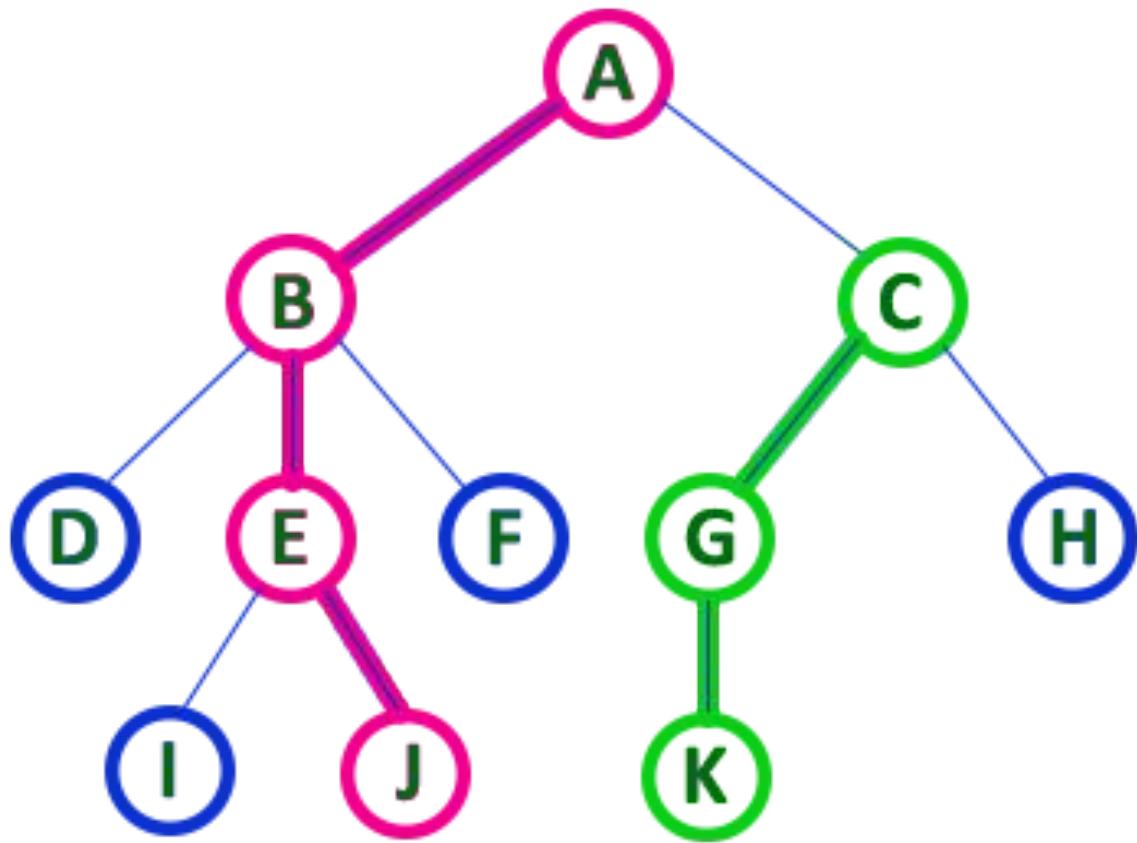
Here Height of tree is 3

- In any tree, 'Height of Node' is total number of Edges from leaf to that node in longest path.
- In any tree, 'Height of Tree' is the height of the root node.



Here Depth of tree is 3

- In any tree, 'Depth of Node' is total number of Edges from root to that node.
- In any tree, 'Depth of Tree' is total number of edges from root to leaf in the longest path.



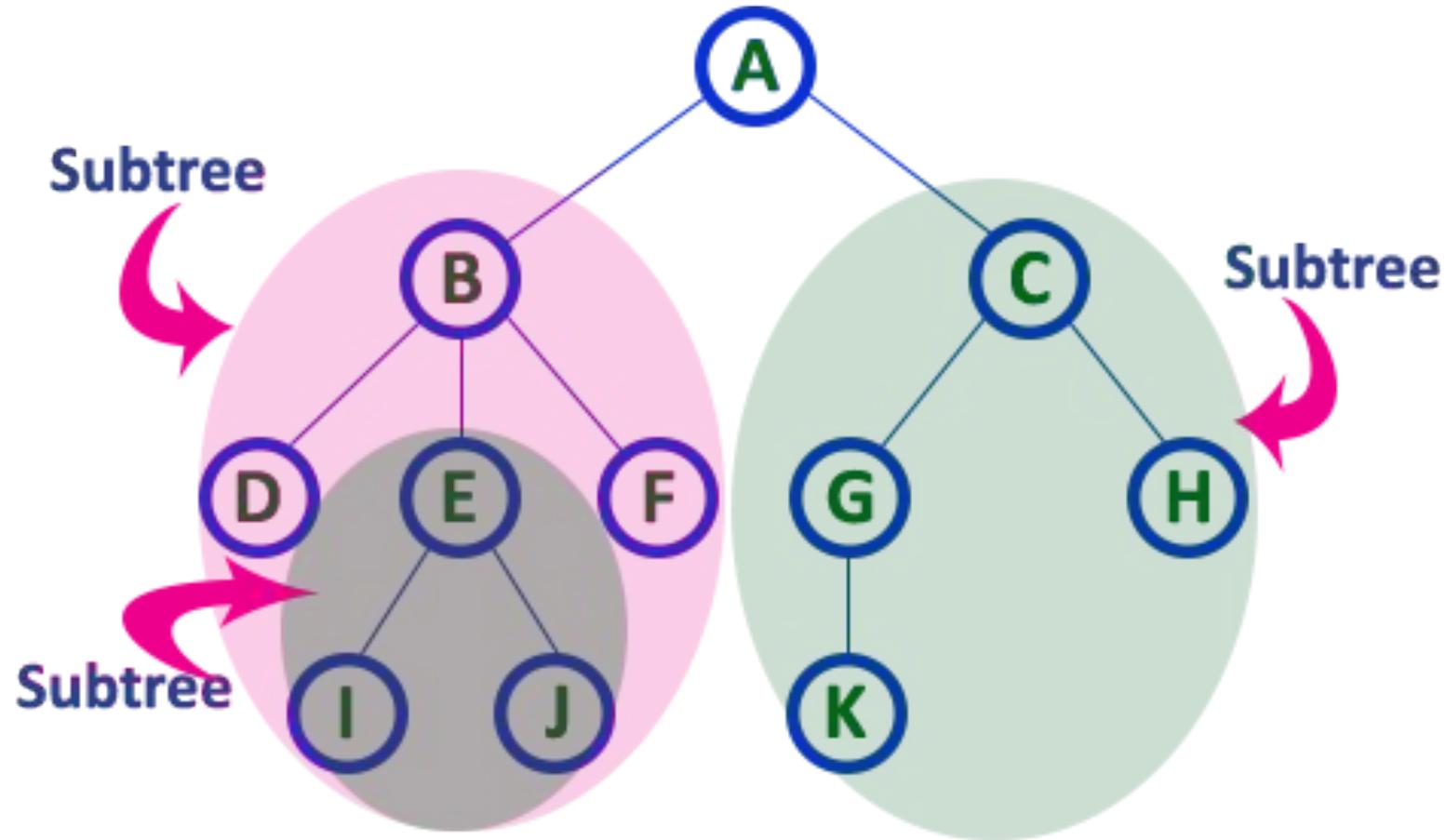
- In any tree, '**Path**' is a sequence of nodes and edges between two nodes.

Here, '**Path**' between A & J is

A - B - E - J

Here, '**Path**' between C & K is

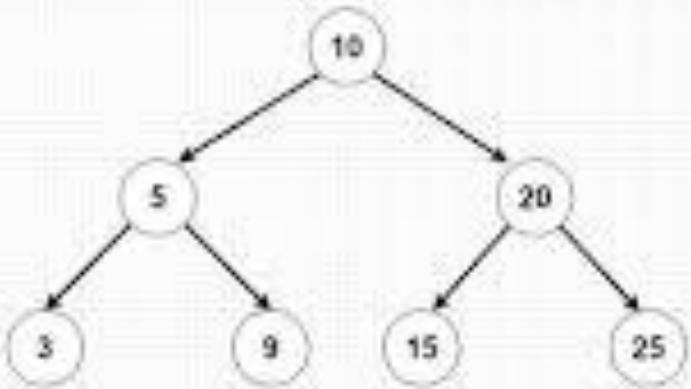
C - G - K



# Tree Traversal



## Binary Tree Traversals



LEVEL-ORDER `[ [10], [5, 20], [3, 9, 15, 25] ]`

PRE-ORDER `[ 10, 5, 3, 9, 20, 15, 25 ]`

IN-ORDER `[ 3, 5, 9, 10, 15, 20, 25 ]`

POST-ORDER `[ 3, 9, 5, 15, 25, 20, 10 ]`

# Tree Traversal

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- Tree traversal refers to the process of visiting each node in a tree data structure in a specific order.

# Tree Traversal Technique

S



## Tree Traversal Techniques

Depth First Traversal  
(DFS)

Breadth First Traversal  
(Level Order Traversal or BFS)

Preorder  
Traversal

Inorder  
Traversal

Postorder  
Traversal

# Tree Traversal Techniques

## Breadth-First Search (BFS)

- BFS explores all nodes at the current depth level before moving on to the next level.
- Use Queue DS

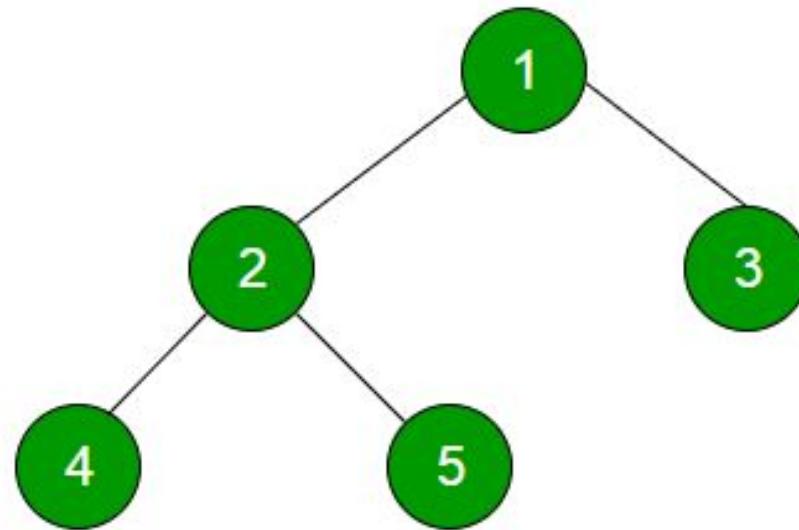
## Depth-First Search (DFS)

- DFS explores as far as possible along each branch before backtracking.
- Use Stack Ds

# Level Order Traversal (Breadth First Search or BFS)

- **Level Order Traversal** technique is defined as a method to traverse a Tree such that all nodes present in the same level are traversed completely before traversing the next level.

Output: 1 2 3 4 5



# Depth First Search

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In-Order Traversal  
(Left, Root, Right)

---

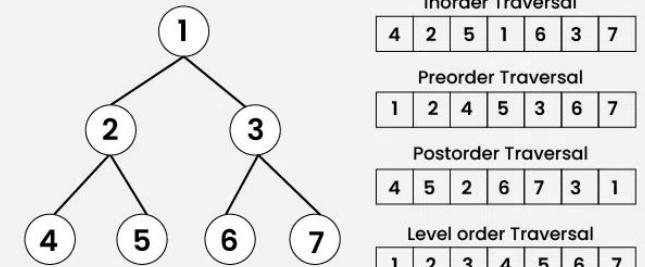
Pre-Order Traversal  
(Root, Left, Right)

---

Post-Order Traversal  
(Left, Right, Root)

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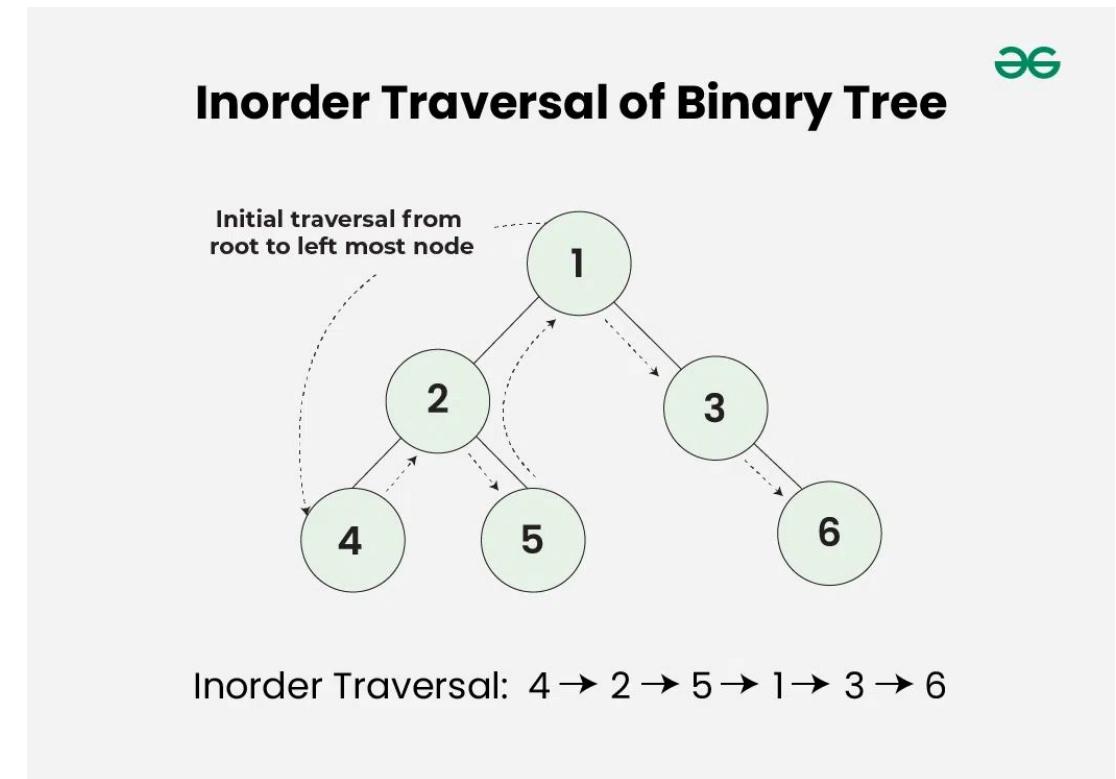
Tree  
Traversal  
Techniques



Inorder Traversal	4   2   5   1   6   3   7
Preorder Traversal	1   2   4   5   3   6   7
Postorder Traversal	4   5   2   6   7   3   1
Level order Traversal	1   2   3   4   5   6   7

# In-Order Traversal (Left, Root, Right)

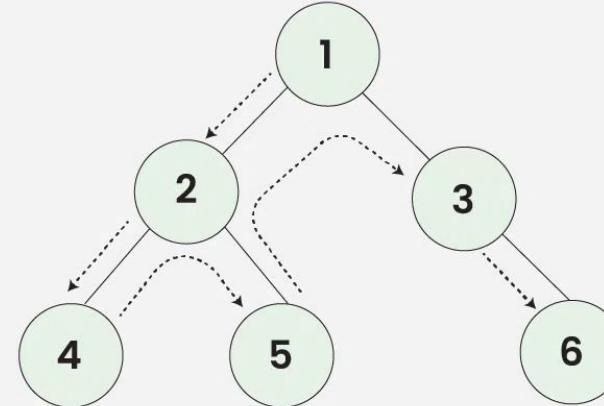
- Inorder traversal visits the node in the order: **Left -> Root -> Right**



# Pre-Order Traversal (Root, Left, Right)

- Preorder traversal visits the node in the order: **Root -> Left -> Right**

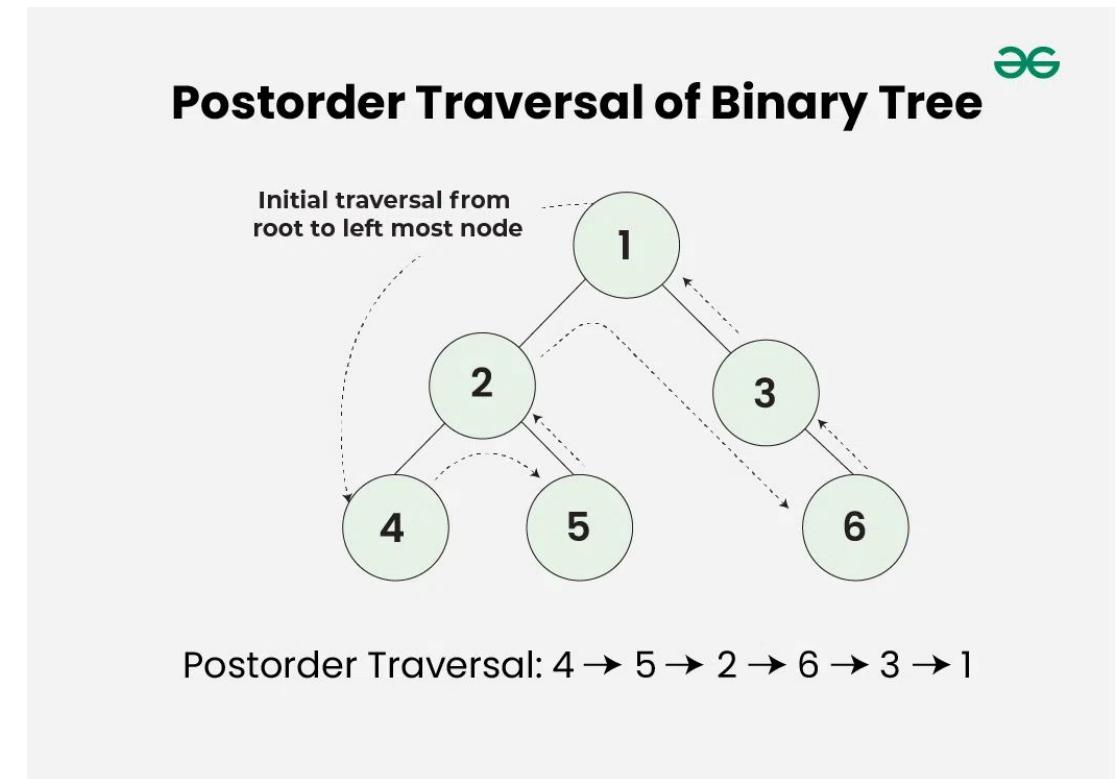
## Preorder Traversal of Binary Tree



Preorder Traversal: 1 → 2 → 4 → 5 → 3 → 6

# Post-Order Traversal (Left, Right, Root)

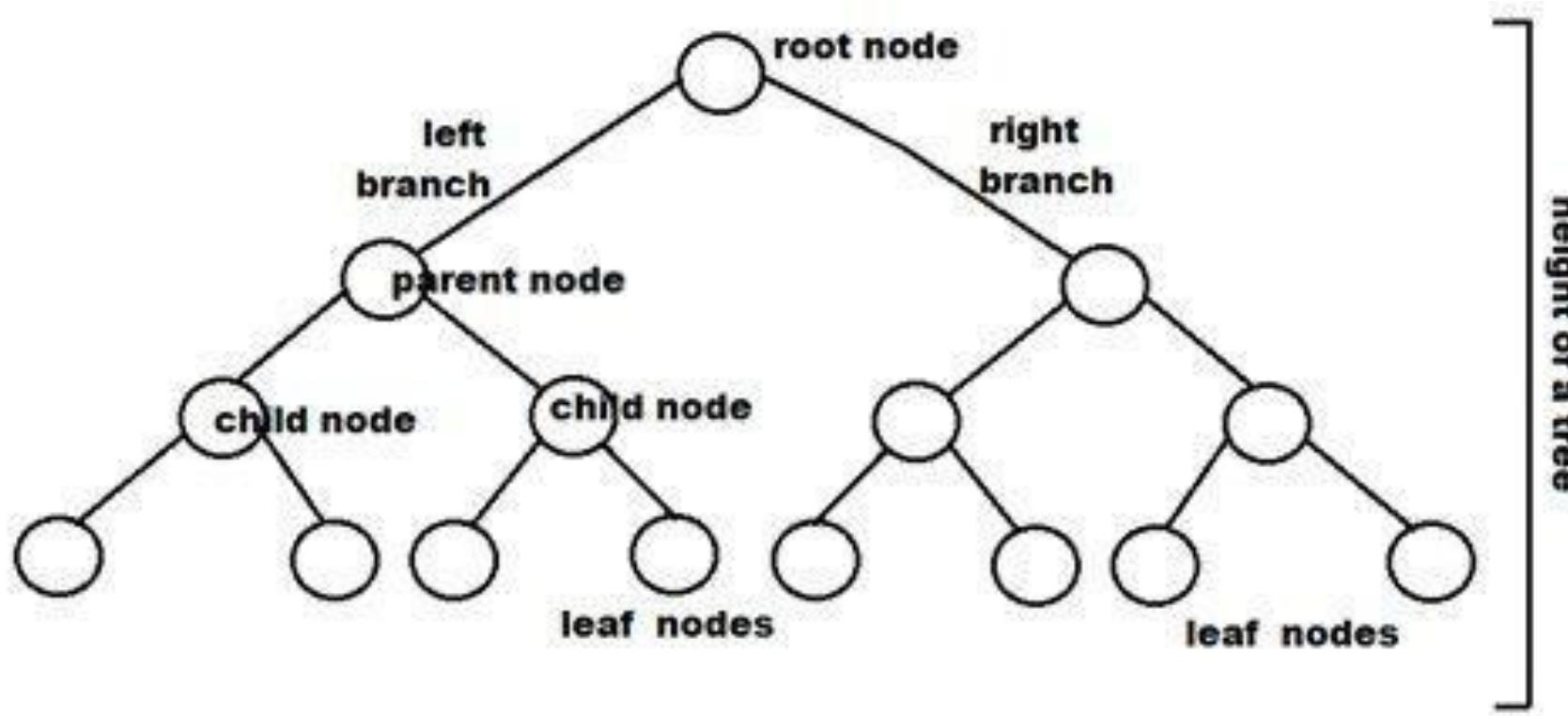
- Post-order traversal visits the node in the order: **Left -> Right -> Root**



# Binary Tree

# Binary Tree

- A **binary tree** is a tree-type non-linear data structure with a maximum of two children for each parent.

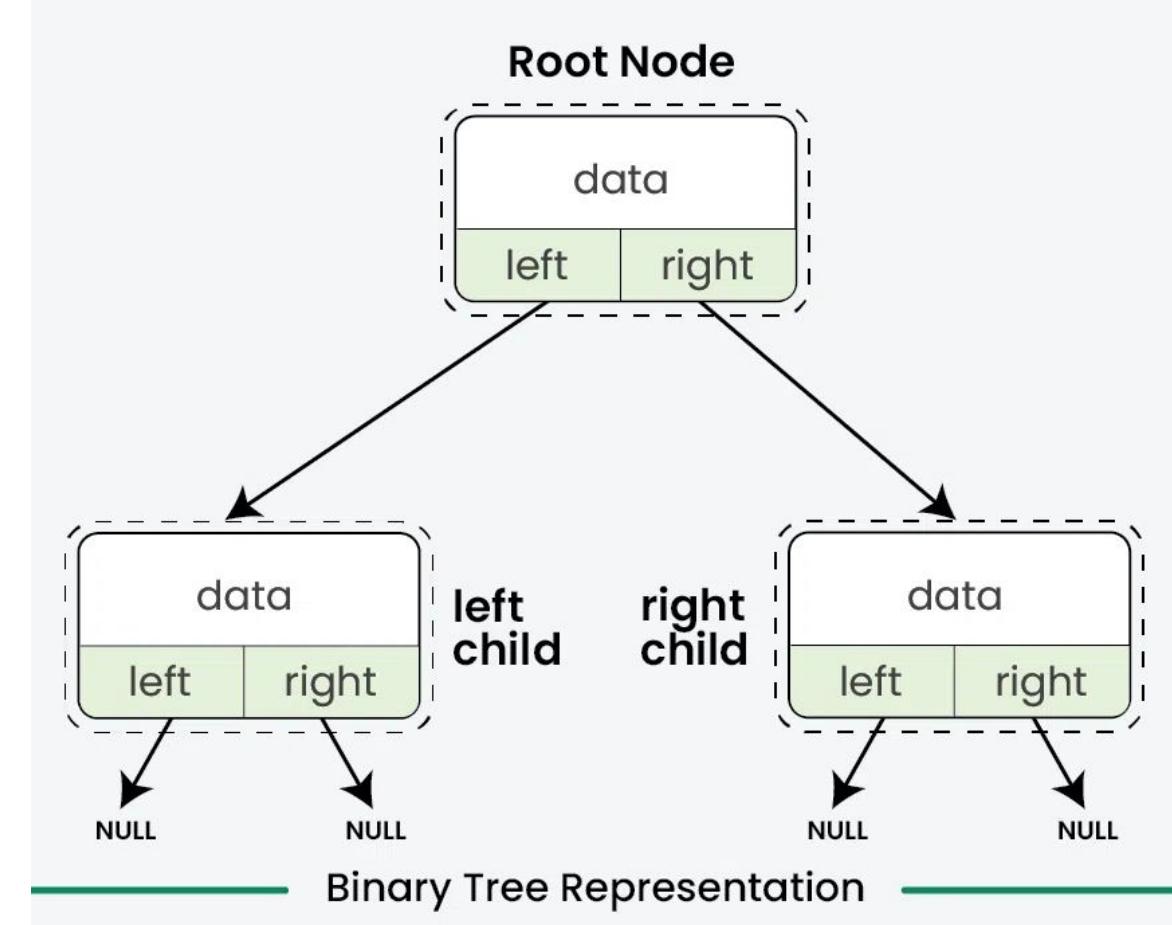


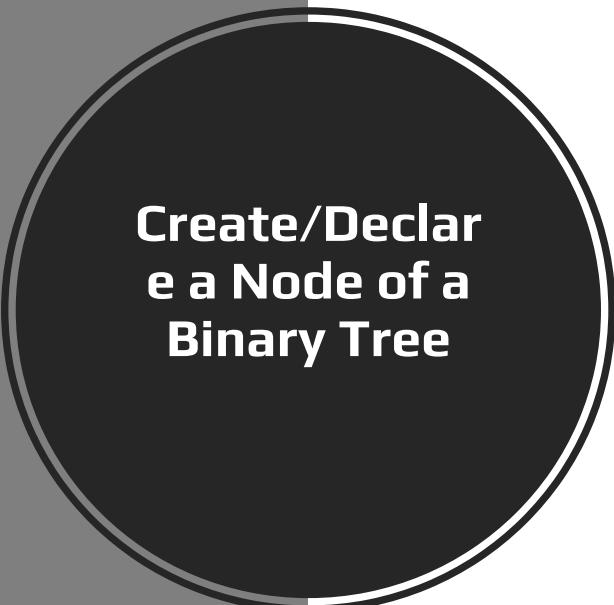
# Representation of Binary Tree

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Each node in a Binary Tree has three parts:

- Data
- Pointer to the left child
- Pointer to the right child

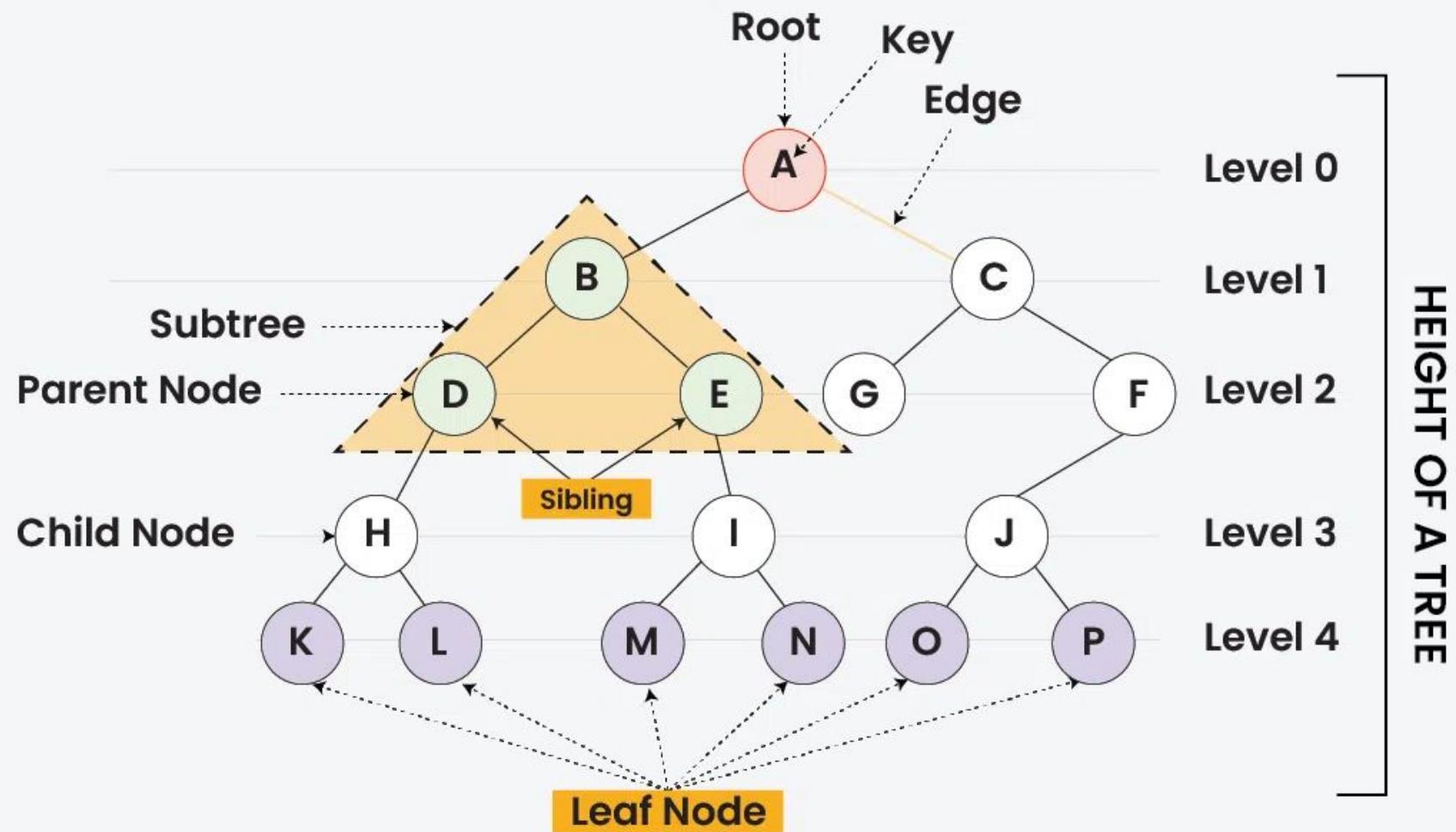




Create/Declare  
a Node of a  
Binary Tree

```
class Node {  
    int key;  
    Node left, right;  
  
    public Node(int item) {  
        key = item;  
        left = right = null;  
    }  
}
```

## Terminologies in Binary Tree



Terminologies in Binary Tree in Data Structure

# Why use binary trees

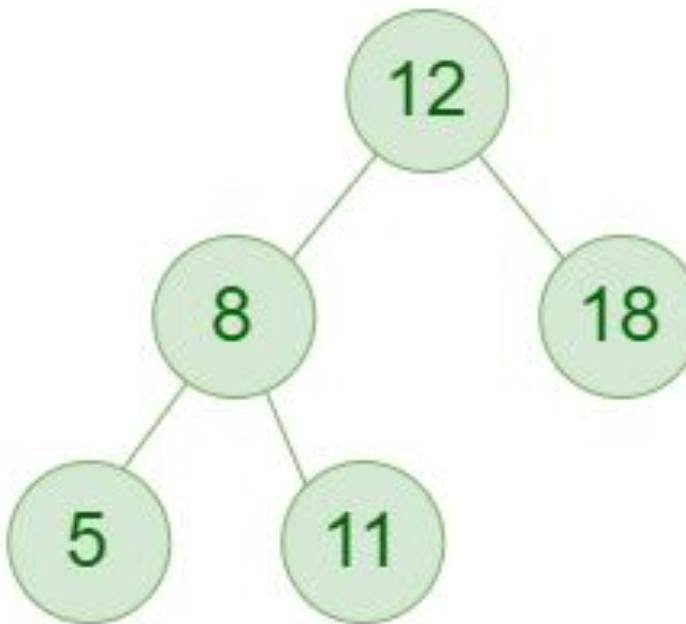
- Efficiency in Search Operations
  - Binary Search Tree (BST)
- Simplified Traversals
  - Binary trees have a **clear structure** with a left and right child for each node.
  - allows for efficient tree traversals (in-order, pre-order, post-order, and level-order)
- Memory Efficiency
  - In binary trees, each node typically contains two pointers to its children whereas general trees may require a more complex structure
- Balanced Tree Structures for Sorting
  - AVL and Red Black Tree for efficient **sorting algorithms**

# Types of Binary Tree

## Full Binary Tree

- A *full binary tree* is a binary tree with either zero or two child nodes for each node.

### Full Binary Tree

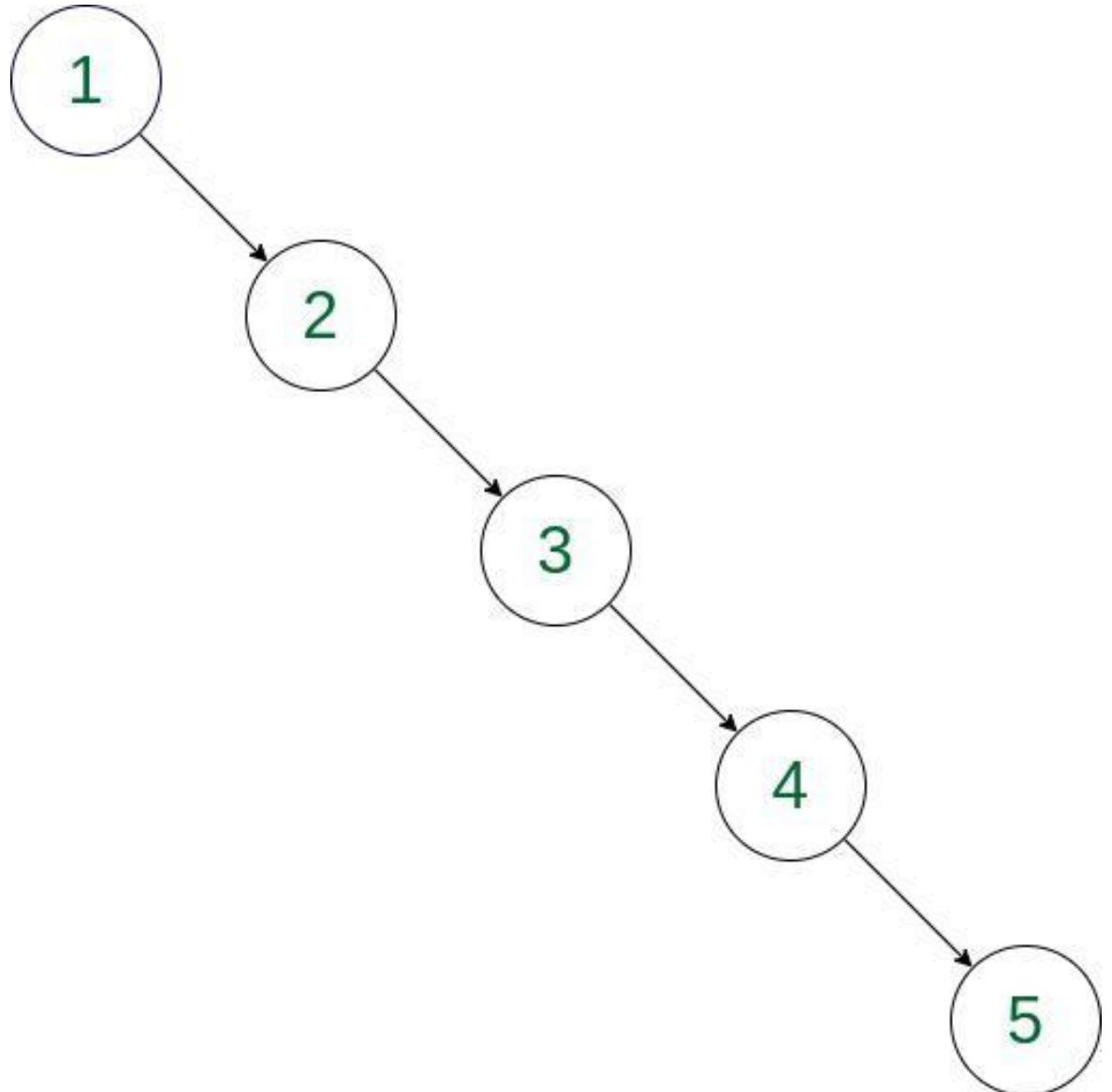


## Types of Binary Tree

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### Degenerate Binary Tree

Every **non-leaf node** has just **one** child in a binary tree known as a **Degenerate Binary tree**. The tree effectively transforms into a linked list as a result, with each node linking to its **single** child.



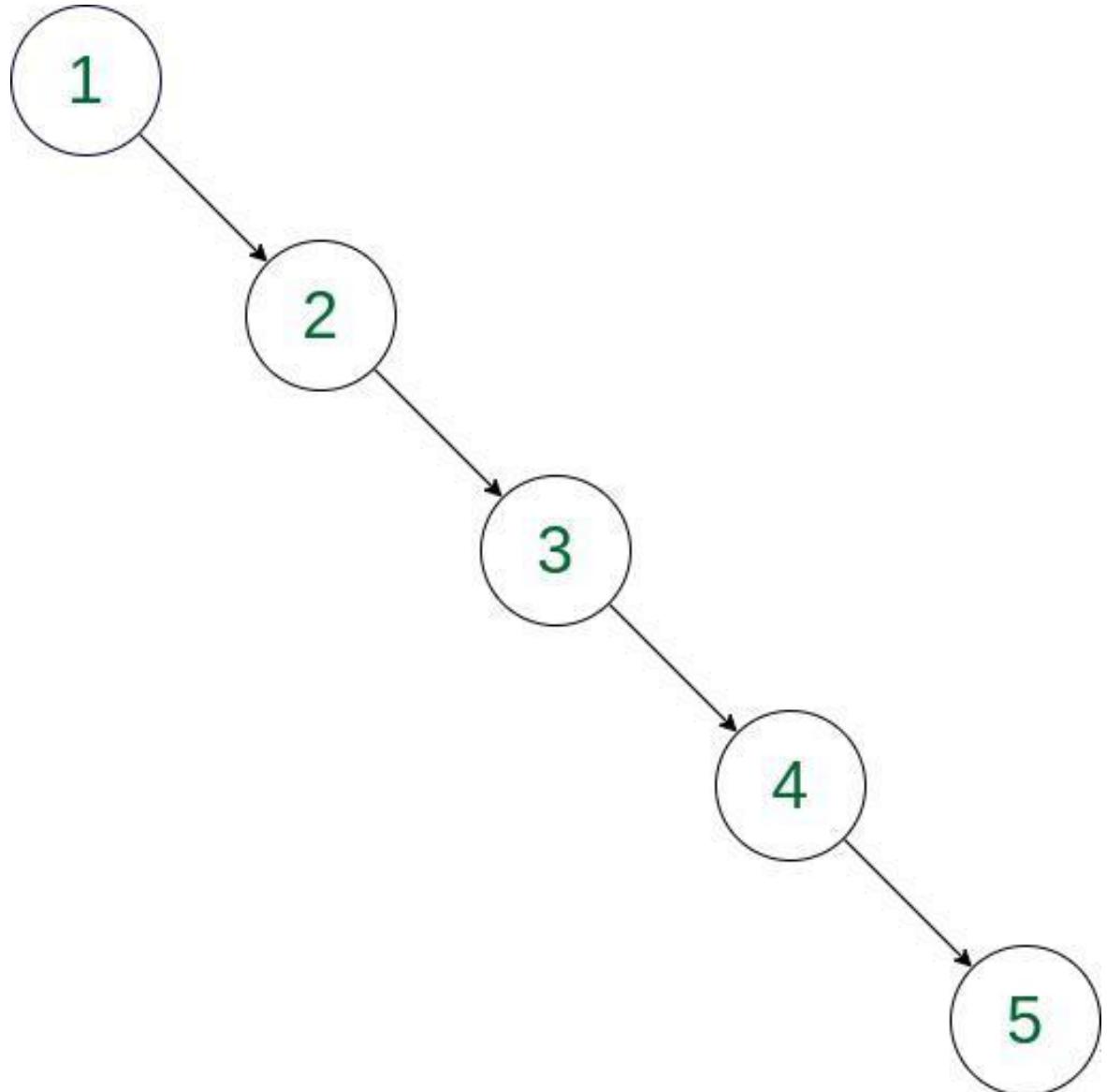
# Types of Binary Tree

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## Degenerate Binary Tree

Degenerate Binary tree is of two types:

- **Left-skewed Tree:** If all the nodes in the degenerate tree have only a left child.
- **Right-skewed Tree:** If all the nodes in the degenerate tree have only a right child.



# Types of Binary Tree

## Skewed Binary Tree

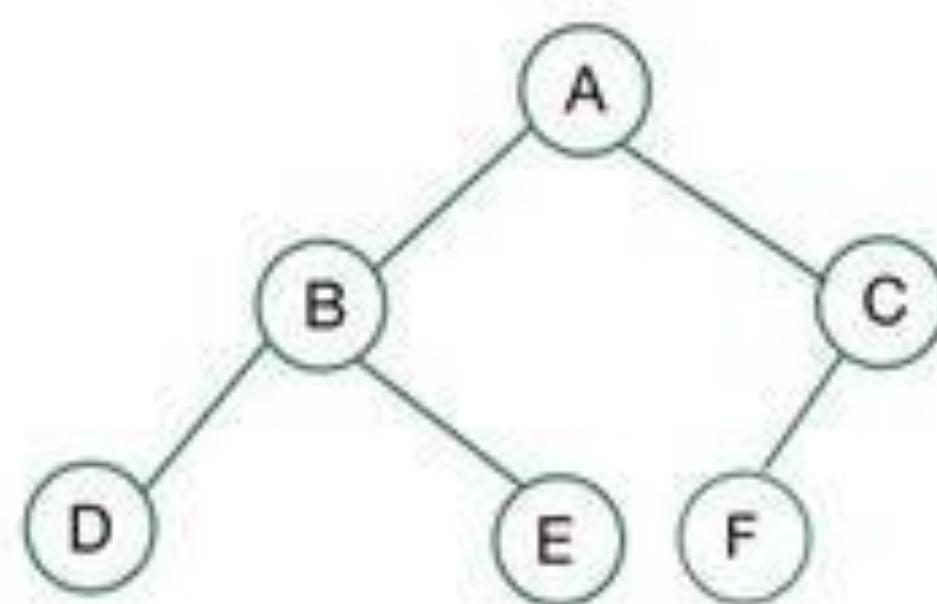
A skewed binary tree is a type of binary tree in which all the nodes have only either one child or no child.



# Types of Binary Tree

## Complete Binary Tree

A complete binary tree is a special type of binary tree where all the levels of the tree are filled completely except the lowest level nodes which are filled from as left as possible.

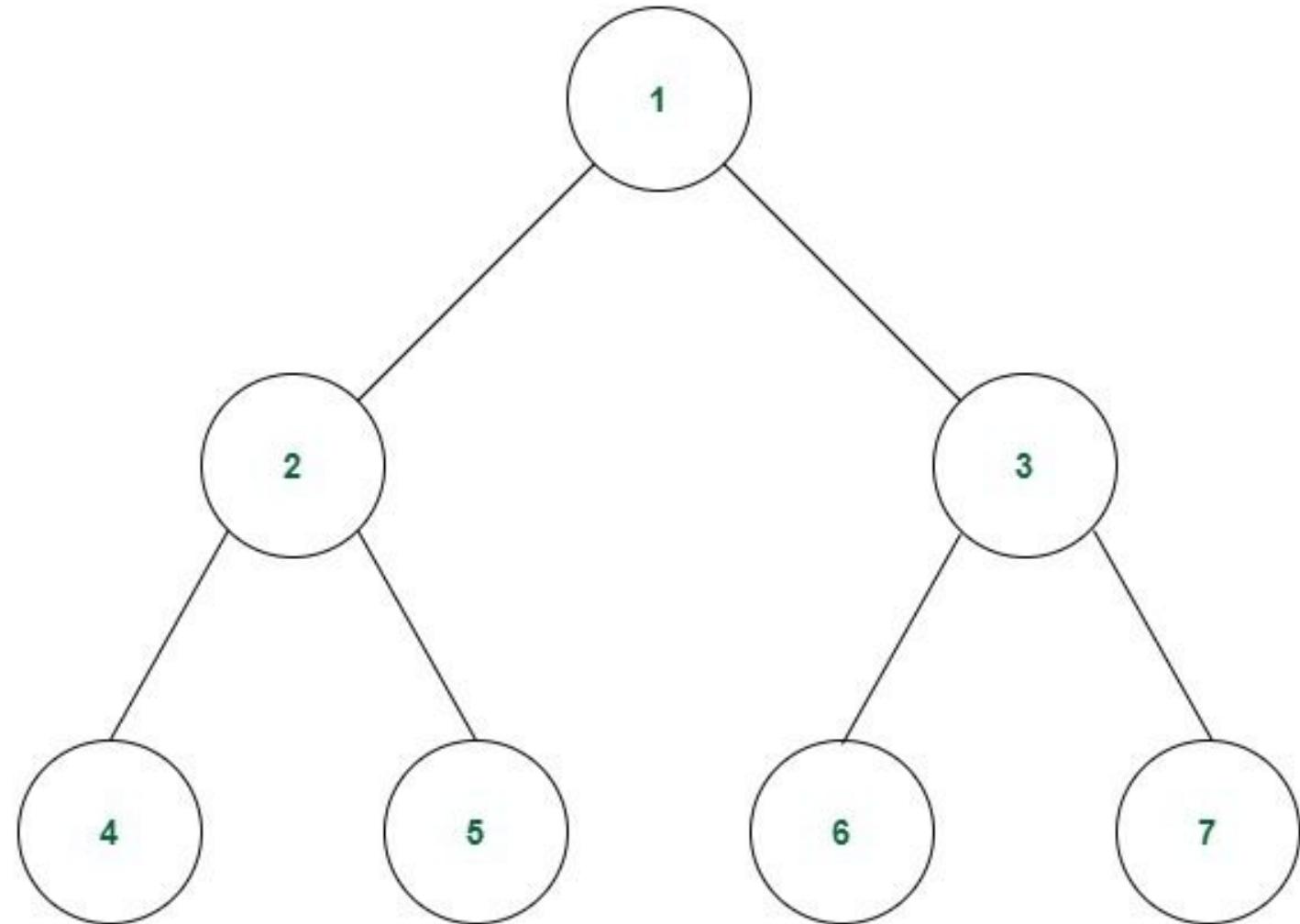


# Types of Binary Tree

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## Perfect Binary Tree

A **perfect binary tree** is a special type of binary tree in which all the leaf nodes are at the same depth, and all non-leaf nodes have two children.

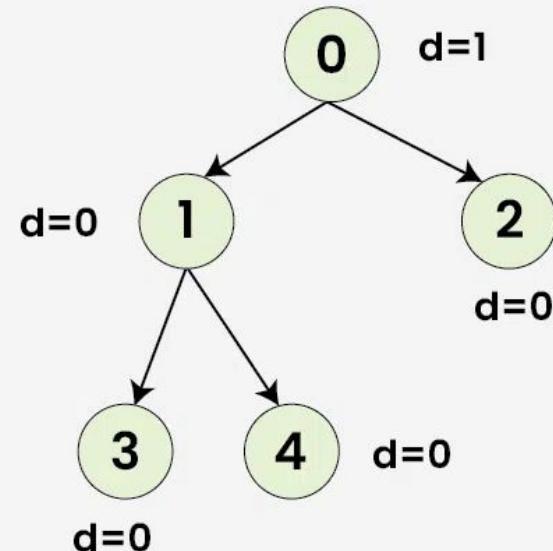


# Types of Binary Tree

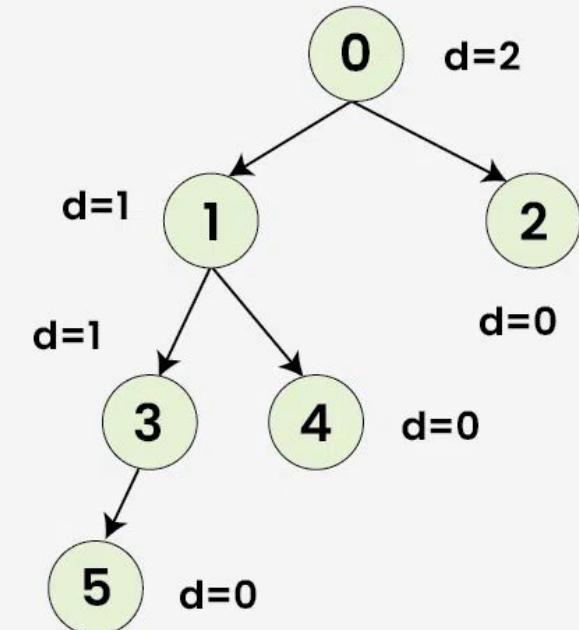
## Balanced Binary Tree

- It is a type of binary tree in which the difference between the height of the left and the right subtree for each node is either 0 or 1.

### Balanced Binary Tree



### Unbalanced Binary Tree



**Depth of a node (d) = [height of left child - height of right child]**

# Types of Binary Tree

## Binary Search Tree

- A **Binary Search Tree** (or **BST**) is a data structure used in computer science for organizing and storing data in a sorted manner.
- Each node in a **Binary Search Tree** has at most two children, a **left** child and a **right** child
- **left** child containing values less than the parent node and the **right** child containing values greater than the parent node.

# Types of Binary Tree

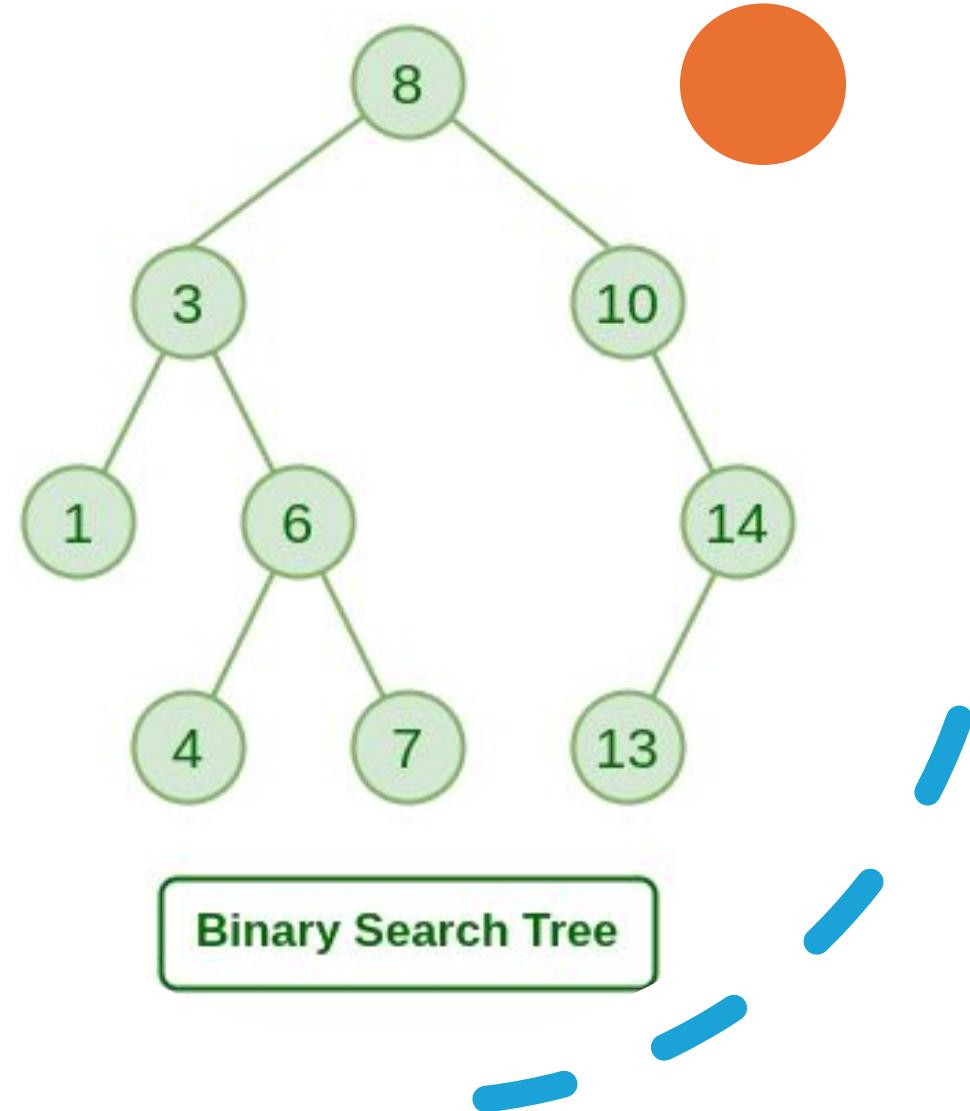
## Binary Search Tree

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# Types of Binary Tree

## Binary Search Tree

- This hierarchical structure allows for **efficient searching, insertion, and deletion** operations on the data stored in the tree.



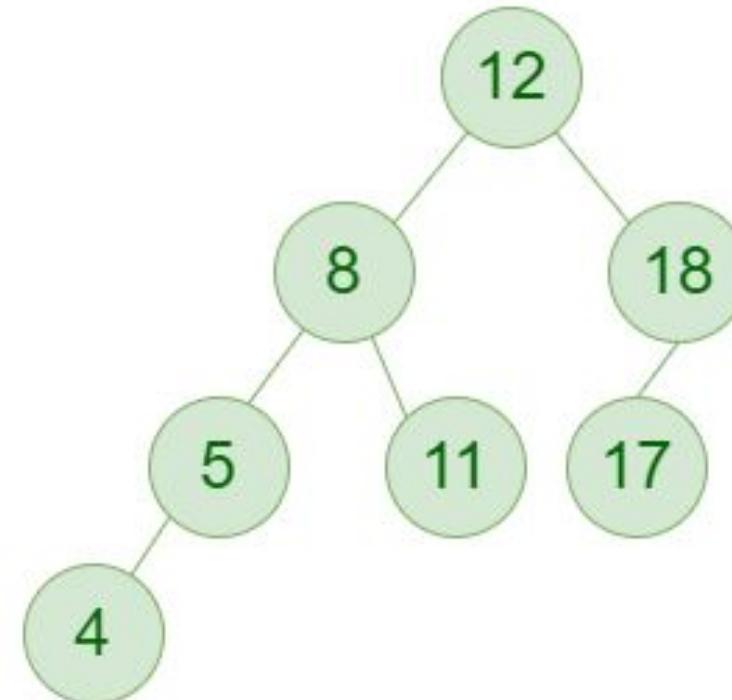
# Types of Binary Tree

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- AVL Tree

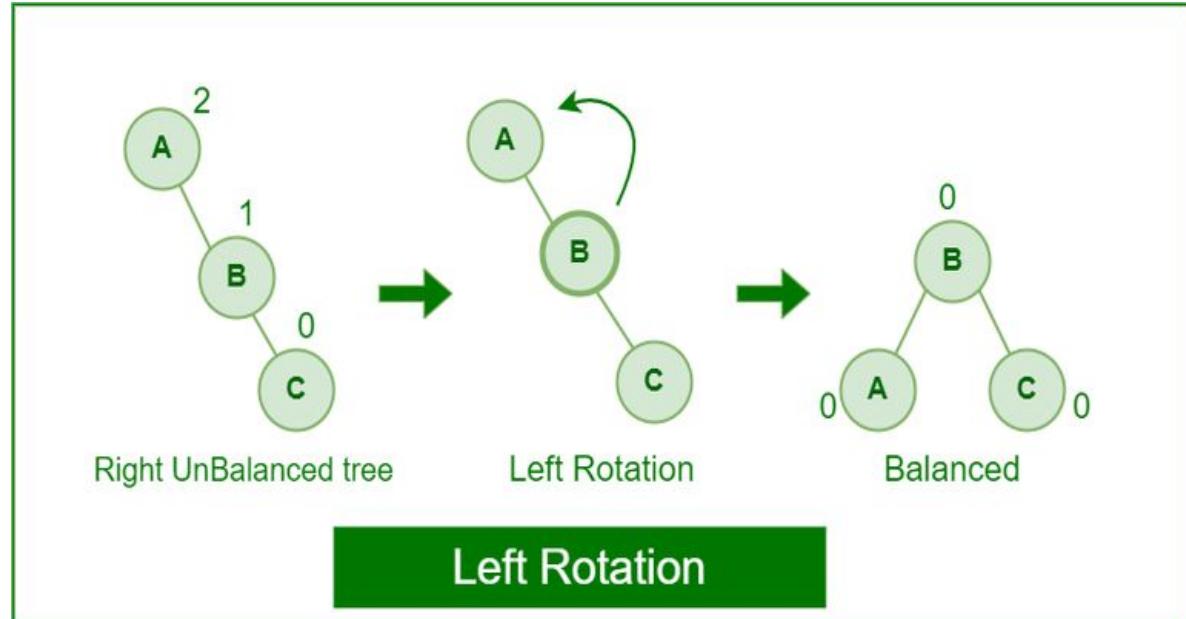
An **AVL tree** defined as a self-balancing [Binary Search Tree \(BST\)](#) where the difference between heights of left and right subtrees for any node cannot be more than one.

AVL Tree



# Types of Binary Tree

- Rotating the subtrees in an AVL Tree:
- An AVL tree may rotate in one of the following four ways to keep itself balanced:
  - Left Rotation
  - Right Rotation
  - Left-Right Rotation
  - Right-Left rotation



# Types of Binary Tree

## Red-Black Tree

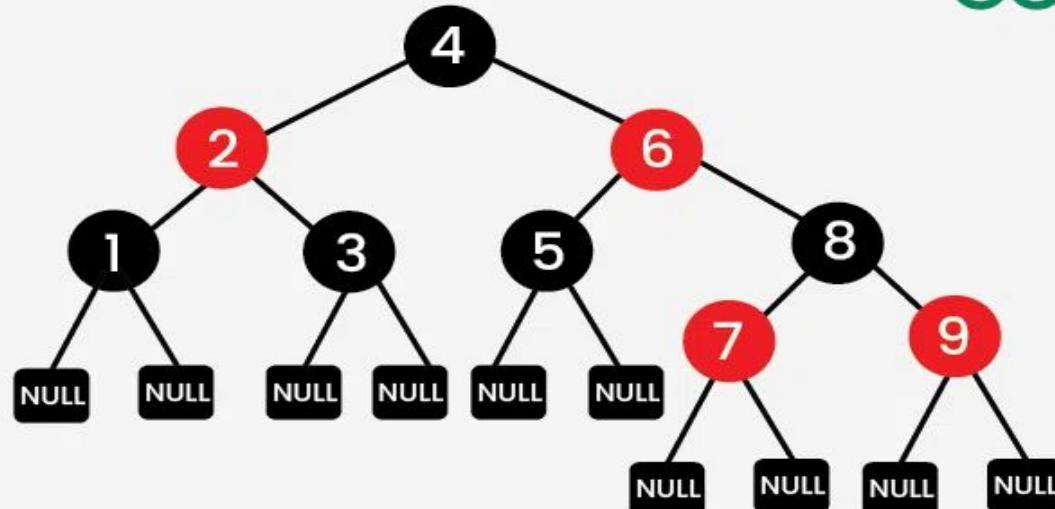
- A **Red-Black Tree** is a self-balancing binary search tree where each node has an additional attribute: a color, which can be either **red** or **black**.
- The primary objective of these trees is to maintain balance during insertions and deletions, ensuring efficient data retrieval and manipulation.

# Types of Binary Tree

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## Red-Black Tree

**Red-Black  
Tree**



# Types of Binary Tree

## Red-Black Tree

**Properties of Red-Black Trees:** A Red-Black Tree have the following properties:

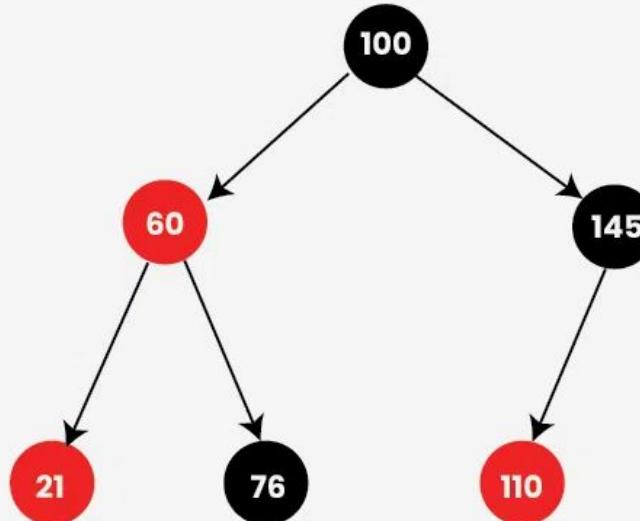
- **Node Color:** Each node is either red or **black**.
- **Root Property:** The root of the tree is always **black**.
- **Red Property:** Red nodes cannot have red children (no two consecutive red nodes on any path).
- **Black Property:** Every path from a node to its descendant null nodes (leaves) has the same number of **black** nodes.
- **Leaf Property:** All leaves (NIL nodes) are **black**.

# Types of Binary Tree

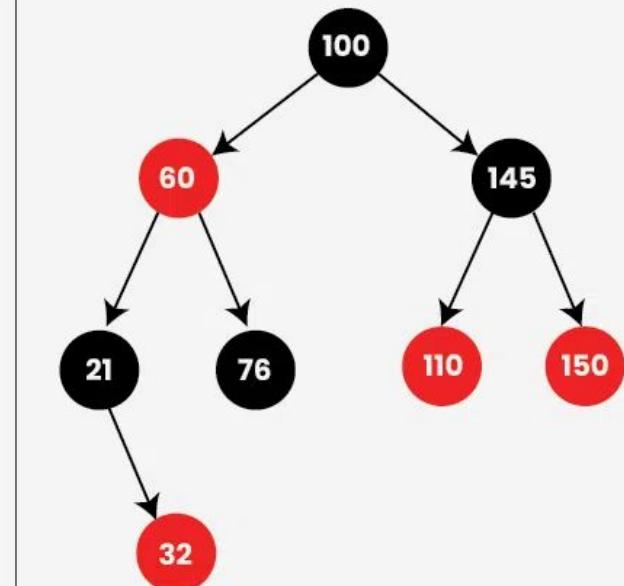
Red Black Tree



## Example of Red-black Tree



A incorrect Red-black Tree



A correct Red-black Tree

# Types of Binary Tree

We will discuss the following Binary tree types in detail:

- Binary Search Tree
- AVL Tree
- Red Black Tree