## Background

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## Background

• Linear algebra

Probabilities

Tensors

Introduction to pytorch

## Linear algebra and gradients

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#### Overview

• Notation: vector, matrix

Definition: vector and matrix operations

Gradients and chain rule

#### What is a vector?

An array of numbers

• Notation: v

bold lower case

• Size:  $size(\mathbf{v}) = n$ 

• Order:  $\dim(\mathbf{v}) = 1$ 

• Indexing: V

 $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ 

4.31.29.92.3

1.2 4.4

#### What is a matrix?

An 2D array of numbers

• Notation:

bold upper case

• Size:  $size(\mathbf{M}) = n \times m$ 

• Order:  $dim(\mathbf{M}) = 2$ 

• Indexing:  $\mathbf{M}_{ii}$  1.2 1.1 4.4 3.2

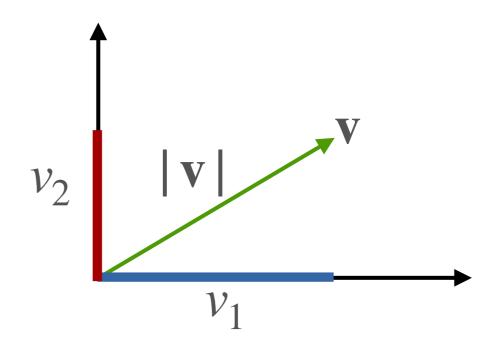
 1
 4

 2
 5

 3
 6

#### Vector norm

$$|\mathbf{v}| = \sqrt{\sum_{i}^{n} v_i^2}$$



### Element wise operations

$$\mathbf{v} + \mathbf{w} = \begin{bmatrix} v_0 + w_0 \\ v_1 + w_1 \\ \dots \\ v_n + w_n \end{bmatrix} \quad \mathbf{v} * \mathbf{w} = \begin{bmatrix} v_0 * w_0 \\ v_1 * w_1 \\ \dots \\ v_n * w_n \end{bmatrix} \quad \cdots$$

$$\mathbf{v} - \mathbf{w} = \begin{bmatrix} v_0 - w_0 \\ v_1 - w_1 \\ \dots \\ v_n - w_n \end{bmatrix} \quad \mathbf{v} / \mathbf{w} = \begin{bmatrix} v_0 / w_0 \\ v_1 / w_1 \\ \dots \\ v_n / w_n \end{bmatrix} \quad \cdots$$

### Inner product

$$\mathbf{v} \cdot \mathbf{w} = v_0 \cdot w_0 + v_1 \cdot w_1 + \dots + v_n \cdot w_n$$

## Outer product

$$\mathbf{v} \otimes \mathbf{w} = \begin{bmatrix} v_0 w_0 & v_0 w_1 & \dots & v_0 w_n \\ v_1 w_0 & v_1 w_1 & \dots & v_1 w_n \\ & & & \dots & & \\ v_n w_0 & v_n w_1 & \dots & v_n w_n \end{bmatrix}$$

#### Frobenius norm

$$|\mathbf{M}| = \sqrt{\sum_{i}^{n} \sum_{j}^{m} M_{ij}^{2}}$$

#### Transpose

 $\mathbf{M}^{ op}$ 

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \qquad \begin{bmatrix} a_{00} & a_{10} \\ a_{01} & a_{11} \\ a_{02} & a_{12} \end{bmatrix}$$

### Matrix multiplication

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \end{bmatrix} \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \\ b_{30} & b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} c_{00} & c_{01} & c_{02} \\ c_{10} & c_{11} & c_{12} \end{bmatrix}$$

2x4

4x3

2x3

## Matrix-vector multiplication

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \end{bmatrix} \begin{bmatrix} b_{00} \\ b_{10} \\ b_{20} \\ b_{30} \end{bmatrix} = \begin{bmatrix} c_{00} \\ c_{10} \end{bmatrix}$$

2×4

4

2

## Vector operations

- Length / norm: |v|
- Element-wise operations: v + w, v w, ...
- Inner (dot) product:  $\mathbf{v}^{\mathsf{T}}\mathbf{w} = \mathbf{v} \cdot \mathbf{w}$
- Outer product:  $\mathbf{v}\mathbf{w}^{\top} = \mathbf{v} \otimes \mathbf{w}$

## Matrix Operations

- Frobenius norm |M|
- ullet Transpose  $\mathbf{M}^{\top}$
- Matrix multiplication AB Av

#### Functions of vectors

• Definition:  $f: \mathbf{x} \to \mathbf{y}$ 

## Derivatives of vector valued functions

Gradient of scaler function

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial f}{\partial \mathbf{x}_0} & \frac{\partial f}{\partial \mathbf{x}_1} & \dots & \frac{\partial f}{\partial \mathbf{x}_n} \end{bmatrix}$$

• Jacobian: Derivative of vector g(x) by vector x:

$$\frac{\partial \mathbf{g}(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial \mathbf{g}}{\partial \mathbf{x}_0} & \frac{\partial \mathbf{g}}{\partial \mathbf{x}_1} & \dots & \frac{\partial \mathbf{g}}{\partial \mathbf{x}_n} \end{bmatrix}$$
 vector

## Chain rule for scalar functions

- Nested functions: f(g(x))
  - where  $f: \mathbb{R} \to \mathbb{R}$  and  $g: \mathbb{R} \to \mathbb{R}$
  - and y = g(x)
- Derivative:  $\frac{\partial f(g(x))}{\partial x} = \frac{\partial f(y)}{\partial y} \frac{\partial g(x)}{\partial x}$

## Chain rule for vector-valued functions

- Nested functions:  $f(g(\mathbf{x}))$ 
  - where  $f: \mathbb{R}^m \to \mathbb{R}^n$  and  $g: \mathbb{R}^p \to \mathbb{R}^m$
  - and y = g(x)
- Derivative:  $\frac{\partial f(g(\mathbf{x}))}{\partial \mathbf{x}} = \frac{\partial f(\mathbf{y})}{\partial \mathbf{y}} \frac{\partial g(\mathbf{x})}{\partial \mathbf{x}}$

nxp nxm mxp

#### Summary

- Vector v
- Matrix M
- Gradients  $\frac{\partial f(x)}{\partial x}$
- Chain rule  $\frac{\partial f(g(x))}{\partial x} = \frac{\partial f(y)}{\partial y} \frac{\partial g(x)}{\partial x}$
- More reading
   [The Matrix Cookbook, Petersen and Pedersen 2012]

# Probability, likelihood, sampling and expectation

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#### Overview

Probabilities and likelihood

Sampling

Linearity of expectation

## What is a (discrete) distribution?

- Informal: bunch of positive numbers that some to one
- Distribution:  $P: a \rightarrow [0,1]$
- Event:  $a \in [0,...,n-1]$
- Probability: P(a)
  - Chance of a occurring

# What is a conditional probability?

• Conditional probability:

 $P(a \mid \theta)$ 

- Chance of a occurring given  $\theta$
- Example  $\theta$ 's:
  - Other event
  - Model parameters

#### What is the likelihood?

- Informal: same as probability
- Formal: a function of parameter  $\theta$  that describes the probability of observing data  $x^n$  given  $\theta$ .
- Definition:  $L(\theta) \equiv L(\theta; x^n) = P(x^n | \theta)$
- Usually refers to past events

## Sampling

- Definition:  $a \sim P$ 
  - Produce events a following P(a)
- Sampling bias
  - Empirical probability of samples != P(a)

## Expectation

- Definition:
  - $\mathbb{E}_{a \sim P}[f(a)]$
- ullet For any function f
  - $\sum_{a} P(a)f(a)$
  - $\frac{1}{N} \sum_{a \sim P} f(a)$

## Linearity of expectation

•  $\mathbb{E}_{a \sim P}[\alpha f(a)] = \alpha \mathbb{E}_{a \sim P}[f(a)]$ 

•  $\mathbb{E}_{a \sim P}[f(a) + g(a)] = \mathbb{E}_{a \sim P}[f(a)] + \mathbb{E}_{a \sim P}[g(a)]$ 

•  $\mathbb{E}_{a \sim P}[f(a)g(a)] \neq \mathbb{E}_{a \sim P}[f(a)]\mathbb{E}_{a \sim P}[g(a)]$ 

#### Summary

- Event:  $a \in [0,...,n-1]$
- Distribution:  $P: a \rightarrow [0,1]$
- Probability: P(a)
- Sampling:  $a \sim P$
- Expectation:  $\mathbb{E}_{a \sim P}[f(a)]$

[Introduction to Probability, Bertsekas and Tsitsiklis 2002] [All of Statistics, Wasserman 2004]

#### Tensors

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#### Overview

• Tensors: order-d matrices

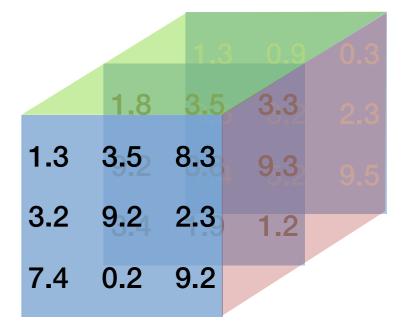
#### Tensors

- Notation: T
  - Bold upper case
- Size:  $size(T) = s_1 \times s_2 \times ... \times s_d$
- Order:  $\dim(\mathbf{T}) = d$
- Indexing:  $T_{ij...k}$

1.2 3.2 5.8

1.3 3.5 8.33.2 9.2 2.3

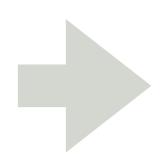
7.4 0.2 9.2



#### What are Tensors used for?

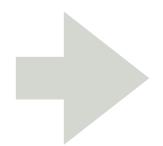
- In deep learning: everything
- Data
- Parameters
- Intermediate representations
- Input and output of almost any deep network operation

H



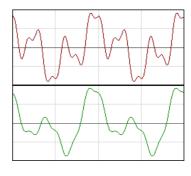
HxWx3 tensor

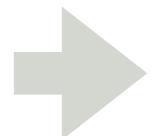
To understand the world, we humans constantly need to relate the present to the past, and put events in context. In this paper, we enable existing video models to do the same. We propose a long-term feature bank—supportive information extracted over the entire span of a video—to augment state-of-the-art video models that otherwise would only view short clips of 2-5 seconds. Our experiments demonstrate that augmenting 3D convolutional networks with a long-term feature bank yields state-of-the-art results on three challenging video datasets: AVA, EPIC-Kitchens, and Charades.



N tensor

N-char. doc





TxM tensor

T sec. of audio

#### Summary

• Tensors: order-d matrices

Basic building block of deep networks

#### January 23, 2024

Prerequisites: \* python 3.6 or newer (e.g. from here http://conda.io) \* pytorch and torchvision: https://pytorch.org/ \* Pillow: https://pillow.readthedocs.io/en/stable/ \* ipython and notebook \* numpy, scipy and matplotlib

```
[1]: import torch
    from PIL import Image
    import numpy as np
[2]: torch.zeros(10)
[2]: tensor([0., 0., 0., 0., 0., 0., 0., 0., 0.])
    torch.ones(10)
[3]:
[3]: tensor([1., 1., 1., 1., 1., 1., 1., 1., 1.])
    torch.ones([4,5])
[4]: tensor([[1., 1., 1., 1., 1.],
             [1., 1., 1., 1., 1.]
             [1., 1., 1., 1., 1.],
             [1., 1., 1., 1., 1.]])
[5]: v = torch.ones(5)
    print( v.dtype, v.shape )
    torch.float32 torch.Size([5])
[6]: v = torch.arange(100)
    v
[6]: tensor([0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,
            18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35,
            36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53,
            54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71,
            72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89,
            90, 91, 92, 93, 94, 95, 96, 97, 98, 99])
```

```
[7]: v.shape
 [7]: torch.Size([100])
 [8]: m = v.view((10,10))
[8]: tensor([[ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
              [10, 11, 12, 13, 14, 15, 16, 17, 18, 19],
              [20, 21, 22, 23, 24, 25, 26, 27, 28, 29],
              [30, 31, 32, 33, 34, 35, 36, 37, 38, 39],
              [40, 41, 42, 43, 44, 45, 46, 47, 48, 49],
              [50, 51, 52, 53, 54, 55, 56, 57, 58, 59],
              [60, 61, 62, 63, 64, 65, 66, 67, 68, 69],
              [70, 71, 72, 73, 74, 75, 76, 77, 78, 79],
              [80, 81, 82, 83, 84, 85, 86, 87, 88, 89],
              [90, 91, 92, 93, 94, 95, 96, 97, 98, 99]])
 [9]: m.shape
 [9]: torch.Size([10, 10])
[10]: I = Image.open('cat.jpg')
[10]:
```



### [11]: np.array(I) [11]: array([[[194, 206, 220], [194, 206, 220], [194, 206, 220], [159, 181, 205], [156, 180, 204], [155, 179, 203]], [[195, 207, 221], [195, 207, 221], [195, 207, 221], [159, 181, 205], [156, 180, 204], [155, 179, 203]], [[196, 208, 222], [196, 208, 222], [195, 207, 221], ..., [159, 181, 205], [156, 180, 204], [155, 179, 203]], [[236, 239, 244], [236, 239, 244], [236, 239, 244], [226, 234, 237], [225, 233, 236], [224, 232, 235]], [[236, 239, 244], [236, 239, 244], [236, 239, 244], [224, 232, 235], [225, 233, 236], [224, 232, 235]], [[236, 239, 244],

[236, 239, 244],

```
[236, 239, 244],
              [219, 227, 230],
              [220, 228, 231],
              [219, 227, 230]]], dtype=uint8)
[12]: np.array(I).shape
[12]: (853, 1280, 3)
[13]: from torchvision import transforms
      image_to_tensor = transforms.ToTensor()
      image_tensor = image_to_tensor(I)
      image_tensor
[13]: tensor([[[0.7608, 0.7608, 0.7608, ..., 0.6235, 0.6118, 0.6078],
               [0.7647, 0.7647, 0.7647, ..., 0.6235, 0.6118, 0.6078],
               [0.7686, 0.7686, 0.7647, ..., 0.6235, 0.6118, 0.6078],
               [0.9255, 0.9255, 0.9255, ..., 0.8863, 0.8824, 0.8784],
               [0.9255, 0.9255, 0.9255, ..., 0.8784, 0.8824, 0.8784],
               [0.9255, 0.9255, 0.9255, ..., 0.8588, 0.8627, 0.8588]],
              [[0.8078, 0.8078, 0.8078, ..., 0.7098, 0.7059, 0.7020],
               [0.8118, 0.8118, 0.8118, ..., 0.7098, 0.7059, 0.7020],
               [0.8157, 0.8157, 0.8118, ..., 0.7098, 0.7059, 0.7020],
               [0.9373, 0.9373, 0.9373, ..., 0.9176, 0.9137, 0.9098],
               [0.9373, 0.9373, 0.9373, ..., 0.9098, 0.9137, 0.9098],
               [0.9373, 0.9373, 0.9373, ..., 0.8902, 0.8941, 0.8902]],
              [[0.8627, 0.8627, 0.8627, ..., 0.8039, 0.8000, 0.7961],
               [0.8667, 0.8667, 0.8667, ..., 0.8039, 0.8000, 0.7961],
               [0.8706, 0.8706, 0.8667, ..., 0.8039, 0.8000, 0.7961],
               [0.9569, 0.9569, 0.9569, ..., 0.9294, 0.9255, 0.9216],
               [0.9569, 0.9569, 0.9569, ..., 0.9216, 0.9255, 0.9216],
               [0.9569, 0.9569, 0.9569, ..., 0.9020, 0.9059, 0.9020]]])
[14]: image_tensor.shape
[14]: torch.Size([3, 853, 1280])
[15]: tensor to image = transforms.ToPILImage()
      tensor_to_image(image_tensor)
[15]:
```



[]:

### January 23, 2024

```
[1]: import torch
[2]: a = torch.arange(10)
     b = torch.arange(10)*10
     print( a, b )
    tensor([0, 1, 2, 3, 4, 5, 6, 7, 8, 9]) tensor([0, 10, 20, 30, 40, 50, 60, 70,
    80, 90])
[3]: a+b
[3]: tensor([ 0, 11, 22, 33, 44, 55, 66, 77, 88, 99])
[4]: a.shape
[4]: torch.Size([10])
[5]: a[None].shape
[5]: torch.Size([1, 10])
[6]: a[:,None].shape
[6]: torch.Size([10, 1])
[7]: c = a[:,None]
     c.shape
[7]: torch.Size([10, 1])
[8]: c[:,0]
[8]: tensor([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
[9]: d = a[None]
     d.shape
[9]: torch.Size([1, 10])
```

```
[10]: d[0]
[10]: tensor([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
[11]: d[0,:]
[11]: tensor([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
[12]: torch.ones([3,2]) + 10
[12]: tensor([[11., 11.],
              [11., 11.],
              [11., 11.]])
[13]: torch.ones([3,2]) + torch.ones([2,3])
      RuntimeError
                                                 Traceback (most recent call last)
      Cell In[13], line 1
       ----> 1 torch.ones([3,2]) + torch.ones([2,3])
      RuntimeError: The size of tensor a (2) must match the size of tensor b (3) at 11
        onon-singleton dimension 1
[14]: torch.ones([3,1]) + torch.ones([1,3])
[14]: tensor([[2., 2., 2.],
              [2., 2., 2.],
              [2., 2., 2.]])
[15]: a[None,:] + b[:,None]
[15]: tensor([[ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
              [10, 11, 12, 13, 14, 15, 16, 17, 18, 19],
              [20, 21, 22, 23, 24, 25, 26, 27, 28, 29],
              [30, 31, 32, 33, 34, 35, 36, 37, 38, 39],
              [40, 41, 42, 43, 44, 45, 46, 47, 48, 49],
              [50, 51, 52, 53, 54, 55, 56, 57, 58, 59],
              [60, 61, 62, 63, 64, 65, 66, 67, 68, 69],
              [70, 71, 72, 73, 74, 75, 76, 77, 78, 79],
              [80, 81, 82, 83, 84, 85, 86, 87, 88, 89],
              [90, 91, 92, 93, 94, 95, 96, 97, 98, 99]])
[16]: a[None,:].repeat(10,1)
```

```
[16]: tensor([[0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
              [0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
              [0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
              [0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
              [0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
              [0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
              [0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
              [0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
              [0, 1, 2, 3, 4, 5, 6, 7, 8, 9],
              [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]])
[17]: b[:,None].repeat(1,10)
[17]: tensor([[ 0, 0, 0, 0, 0, 0, 0, 0, 0],
              [10, 10, 10, 10, 10, 10, 10, 10, 10, 10],
              [20, 20, 20, 20, 20, 20, 20, 20, 20, 20],
              [30, 30, 30, 30, 30, 30, 30, 30, 30],
              [40, 40, 40, 40, 40, 40, 40, 40, 40, 40],
              [50, 50, 50, 50, 50, 50, 50, 50, 50, 50],
              [60, 60, 60, 60, 60, 60, 60, 60, 60],
              [70, 70, 70, 70, 70, 70, 70, 70, 70, 70]
              [80, 80, 80, 80, 80, 80, 80, 80, 80, 80],
              [90, 90, 90, 90, 90, 90, 90, 90, 90, 90]])
[18]: a[None,:].repeat(10,1) + b[:,None].repeat(1,10)
[18]: tensor([[ 0, 1, 2, 3, 4, 5, 6, 7,
                                                8, 9],
              [10, 11, 12, 13, 14, 15, 16, 17, 18, 19],
              [20, 21, 22, 23, 24, 25, 26, 27, 28, 29],
              [30, 31, 32, 33, 34, 35, 36, 37, 38, 39],
              [40, 41, 42, 43, 44, 45, 46, 47, 48, 49],
              [50, 51, 52, 53, 54, 55, 56, 57, 58, 59],
              [60, 61, 62, 63, 64, 65, 66, 67, 68, 69],
              [70, 71, 72, 73, 74, 75, 76, 77, 78, 79],
              [80, 81, 82, 83, 84, 85, 86, 87, 88, 89],
              [90, 91, 92, 93, 94, 95, 96, 97, 98, 99]])
[19]: a[None,:] * b[:,None]
[19]: tensor([[
                0,
                      0,
                           0,
                                0,
                                     0,
                                          0,
                                               0,
                                                    0,
                                                         0,
                                                               0],
              0,
                     10,
                          20,
                               30,
                                    40,
                                         50,
                                              60,
                                                   70,
                                                        80,
              60,
                                    80, 100, 120, 140, 160, 180],
                 0,
                     20,
                          40,
              0,
                     30,
                          60,
                               90, 120, 150, 180, 210, 240, 270],
              80, 120, 160, 200, 240, 280, 320, 360],
                 0,
                     40,
              0,
                     50, 100, 150, 200, 250, 300, 350, 400, 450],
              60, 120, 180, 240, 300, 360, 420, 480, 540],
                0,
              [ 0,
                     70, 140, 210, 280, 350, 420, 490, 560, 630],
```

```
[ 0, 80, 160, 240, 320, 400, 480, 560, 640, 720],
[ 0, 90, 180, 270, 360, 450, 540, 630, 720, 810]])
```

[]:

#### January 23, 2024

[1]: %pylab inline

```
import torch
     # Load tensorboard to monitor traning (pip install -U tb-nightly)
     %load_ext tensorboard
    %pylab is deprecated, use %matplotlib inline and import the required libraries.
    Populating the interactive namespace from numpy and matplotlib
[2]: import torch.utils.tensorboard as tb
     import tempfile
     log_dir = tempfile.mkdtemp()
     %tensorboard --logdir {log_dir} --reload_interval 1
    <IPython.core.display.HTML object>
[3]: logger = tb.SummaryWriter(log_dir+'/test3', flush_secs=1)
     logger.add_scalar('first/some_number', 0, global_step=1)
     logger.add_scalar('first/some_number', 1, global_step=2)
[4]: logger.add_histogram('plots/hist1', np.array([10,2,5,4,2]), global_step=1)
     logger.add_histogram('plots/hist2', np.random.rand(20), global_step=1)
[5]: logger.add_image('img/1', (np.random.rand(3,100,100)*255).astype(np.uint8),
      ⇔global_step=1)
[6]: logger.add_histogram('plots/hist2', np.random.rand(20), global_step=2)
[7]: logger.add_image('img/1', (np.random.rand(3,100,100)*255).astype(np.uint8),
      ⇔global_step=2)
[8]: for i, x in enumerate(np.random.rand(100)):
         logger.add_scalar('first/noise', x+0.1*i, global_step=i)
[]:
```

## Summary

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# Linear algebra

Vectors & matrices

operators

1.2 3.2 5.8

1.3 3.5 8.33.2 9.2 2.37.4 0.2 9.2

gradients & Jacobians

[The Matrix Cookbook, Petersen and Pedersen 2012]

## Probabilities

- Event:  $a \in [0,...,n-1]$
- Probability: P(a)
- Sampling:  $a \sim P$
- Expectation:  $\mathbb{E}_{a \sim P}[f(a)]$

[Introduction to Probability, Bertsekas and Tsitsiklis 2002] [All of Statistics, Wasserman 2004]

### Tensors

Tensors: order-d
 matrices

 Basic building block of deep networks

