Math notes

Memory growth factor of 2

Memory used with growth factor of 2

$$\sum_{i=0}^{n} 2^i = 2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^{n-1} + 2^n$$
$$= 1 + 2 + 4 + 8 + \ldots + 2^n$$

Previous memory step:

$$\sum_{i=0}^{n-1} 2^i = 1 + 2 + 4 + 8 + \ldots + 2^{n-1}$$

Let S be total history of memory freed

$$S = 1 + 2 + 4 + 8 + \ldots + 2^{n-1}$$
$$2S = 2 + 4 + 8 + 16 + \ldots + 2^n$$

To get S again:

$$\begin{aligned} S &= 2S - S \\ &= \left(2 + 4 + 8 + \ldots + 2^{n-1} + 2^n\right) - \left(1 + 2 + 4 + 8 + \ldots + 2^{n-1}\right) \\ &= 2^n - 1 \end{aligned}$$

This will always be smaller than the new memory needed:

$$2^n + 1 < 2^n$$

Optimal growth factor

Let

- x be generalized growth factor
- n be realloc iteration

$$1, x, x^2, x^3, \ldots, x^n$$

Summing series

Using formula for a geometric series $S_k = \frac{r^k - 1}{r - 1}$:

$$\frac{x^{n-1} - 1}{x - 1} \geq x^n$$

Determining equation for limit calc

Let R_n be ratio between reusable blocks and new block:

$$R_n = \frac{\sum_{i=0}^{n-2} x^i}{x^n}$$

If $R_n \geq 1$ we should be able to reuse previously freed memory.

$$\lim_{n\to\infty} R_n \geq 1$$

$$\lim_{n\to\infty} \frac{\sum_{i=0}^{n-2} x^i}{x^n} \geq 1$$

$$\lim_{n\to\infty} \left(\frac{\frac{x^{n-1}-1}{x-1}}{x^n}\right) \geq 1$$

$$\lim_{n\to\infty} \left[\frac{x^{n-1}}{x^n(x-1)} - \frac{1}{x^n(x-1)}\right] \geq 1$$

Quotient rule lets us subtract exponents of the same base:

$$\begin{aligned} &\frac{x^{n-1}}{x^n} \\ &= x^{n-1-n} \\ &= x^{-1} \\ &= \frac{1}{x} \end{aligned}$$

Thus

$$\lim_{n\to\infty} \left[\frac{x^{n-1}}{x^n(x-1)} - \frac{1}{\frac{x^n}{x-1}}\right] \geq 1$$

$$\lim_{n\to\infty} \left[\frac{1}{x(x-1)} - \frac{1}{x^n(x-1)}\right] \geq 1$$

Evaluating limit

$$\begin{aligned} &\lim_{n\to\infty} \left[\frac{1}{x(x-1)} - \frac{1}{x^n(x-1)}\right] \\ &= \lim_{n\to\infty} \left[\frac{1}{x(x-1)} - 0\right] \\ &= \frac{1}{x(x-1)} \end{aligned}$$

Evaluating inequality

$$\begin{aligned} \frac{1}{x(x-1)} &\geq 1 \\ 1 &\geq x(x-1) \\ 1 &\geq x^2 - x \\ 0 &\geq x^2 - x - 1 \end{aligned}$$

Solvng this quadratic equation yields:

$$x_1 = \frac{1+\sqrt{5}}{2} \approx 1.618$$

$$x_2 = \frac{1-\sqrt{5}}{2} \approx -0.618$$

the first of which is the golden ratio.

Compromise operation

$$n \times 1.5 = n + \left(\frac{n}{2}\right)$$

which is

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n + (n >> 1)
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which should be pretty fast for a computer to do