Read in the following dictionary:

```
3.0
               +7.00x_1 - 6.00x_2 - 7.00x_3 + 9.00x_4 - 1.00x_5 - 10.00x_6 - 8.00x_7 + 1.00x_8 + 2.00x_9
x_{10}
       68.0
               -8.00x_1 - 5.00x_2 - 9.00x_3 - 8.00x_4 - 10.00x_5 + 3.00x_6 - 7.00x_7 - 2.00x_8 - 9.00x_9
x_{11}
       41.0
x_{12}
               -3.00x_1 -5.00x_2 -7.00x_3 +6.00x_4 -9.00x_5
                                                                            -10.00x_7 - 4.00x_8 + 1.00x_9
               -6.00x_1 + 4.00x_2 - 4.00x_3 - 10.00x_4 - 5.00x_5 + 6.00x_6 - 10.00x_7 - 5.00x_8 - 5.00x_9
       57.0
x_{13}
       -4.0
                        +2.00x_2 +8.00x_3 +4.00x_4 +7.00x_5 -4.00x_6
                                                                                       +5.00x_8 -6.00x_9
x_{14}
               -3.00x_1 + 6.00x_2 - 4.00x_3 + 10.00x_4 + 5.00x_5 - 6.00x_6 + 6.00x_7 + 5.00x_8 - 1.00x_9
      -11.0
x_{15}
       0.0
               -1.00x_1 - 4.00x_2 + 3.00x_3 + 3.00x_4 - 3.00x_5 - 3.00x_6 - 2.00x_7 - 3.00x_8 + 1.00x_9
 z
```

0.1 Initialization Phase: Dual Problem Solving

New Objective in primal was changed to:

$$\max \sum_{j=1}^{9} -x_j$$

Primal variable x_j corresponds to dual variable y_j for j = 1, ..., 15 Dual Dictionary (with objective changed is):

```
-7.00y_{10} +8.00y_{11} +3.00y_{12} +6.00y_{13}
y_1
            +6.00y_{10} +5.00y_{11} +5.00y_{12} -4.00y_{13} -2.00y_{14} -6.00y_{15}
     1.0
y_2
            +7.00y_{10} +9.00y_{11} +7.00y_{12} +4.00y_{13} -8.00y_{14} +4.00y_{15}
     1.0
y_3
            -9.00y_{10} +8.00y_{11} -6.00y_{12} +10.00y_{13} -4.00y_{14} -10.00y_{15}
y_4
     1.0
     1.0
            +1.00y_{10} +10.00y_{11} +9.00y_{12} +5.00y_{13} -7.00y_{14} -5.00y_{15}
y_5
     1.0
           +10.00y_{10} -3.00y_{11}
                                                  -6.00y_{13} +4.00y_{14} +6.00y_{15}
y_6
            +8.00y_{10} +7.00y_{11} +10.00y_{12} +10.00y_{13}
     1.0
y_7
            -1.00y_{10} +2.00y_{11} +4.00y_{12} +5.00y_{13} -5.00y_{14} -5.00y_{15}
y_8
     1.0
            -2.00y_{10} +9.00y_{11} -1.00y_{12} +5.00y_{13} +6.00y_{14} +1.00y_{15}
            -3.00y_{10} -68.00y_{11} -41.00y_{12} -57.00y_{13} +4.00y_{14} +11.00y_{15}
```

Initialization succeeded in finding final dual dictionary with 6 pivots

```
-1.20y_{14} + 10.40y_{11} + 1.20y_{12} + 9.00y_{13} - 0.30y_4 - 9.70y_{10}
y_1
            +0.40y_{14} +0.20y_{11} +8.60y_{12} -10.00y_{13} +0.60y_4 +11.40y_{10}
y_2
             -5.00y_{14} +6.00y_{11} +12.00y_{12} +0.00y_{13} +0.50y_4 +5.50y_{10}
y_5
             -9.60y_{14} + 12.20y_{11} + 4.60y_{12} + 8.00y_{13} - 0.40y_4 + 3.40y_{10}
      1.4
y_3
             -0.40y_{14} \ +0.80y_{11} \ -0.60y_{12} \ +1.00y_{13} \ -0.10y_{4} \ -0.90y_{10}
      0.1
y_{15}
            +1.60y_{14} +1.80y_{11} -3.60y_{12} +0.00y_{13} -0.60y_4 +4.60y_{10}
y_6
            +2.40y_{14} +2.20y_{11} +13.60y_{12} +4.00y_{13} +0.60y_4 +13.40y_{10}
y_7
      0.5
             -3.00y_{14} -2.00y_{11} +7.00y_{12} +0.00y_{13} +0.50y_4 +3.50y_{10}
y_8
             +5.60y_{14} +9.80y_{11} -1.60y_{12} +6.00y_{13} -0.10y_4 -2.90y_{10}
y_9
            -0.40y_{14} - 59.20y_{11} - 47.60y_{12} - 46.00y_{13} - 1.10y_4 - 12.90y_{10}
```

Primal Dictionary is:

```
0.4
             +1.20x_1 -0.40x_2 +5.00x_5 +9.60x_3 +0.40x_{15} -1.60x_6 -2.40x_7 +3.00x_8 -5.60x_9
      59.2
             -10.40x_1 -0.20x_2 -6.00x_5 -12.20x_3 -0.80x_{15} -1.80x_6 -2.20x_7 +2.00x_8 -9.80x_9
x_{11}
x_{12}
      47.6
             -1.20x_1 -8.60x_2 -12.00x_5 -4.60x_3 +0.60x_{15} +3.60x_6 -13.60x_7 -7.00x_8 +1.60x_9
             -9.00x_1 + 10.00x_2 -0.00x_5 -8.00x_3 -1.00x_{15} -0.00x_6 -4.00x_7 -0.00x_8 -6.00x_9
      46.0
x_{13}
             +0.30x_1 -0.60x_2 -0.50x_5 +0.40x_3 +0.10x_{15} +0.60x_6 -0.60x_7 -0.50x_8 +0.10x_9
x_4
      1.1
      12.9
             +9.70x_1 -11.40x_2 -5.50x_5 -3.40x_3 +0.90x_{15} -4.60x_6 -13.40x_7 -3.50x_8 +2.90x_9
x_{10}
      -1.1
              -1.30x_1 -0.40x_2 -0.50x_5 -1.40x_3 -0.10x_{15} -1.60x_6 -0.40x_7 -0.50x_8 -1.10x_9
 z
```

Primal Dictionary with original objective is:

```
+1.20x_1 -0.40x_2 +5.00x_5 +9.60x_3 +0.40x_{15} -1.60x_6 -2.40x_7 +3.00x_8 -5.60x_9
x_{14}
      0.4
x_{11}
      59.2
            -10.40x_1 -0.20x_2 -6.00x_5 -12.20x_3 -0.80x_{15} -1.80x_6 -2.20x_7 +2.00x_8 -9.80x_9
      47.6
             -1.20x_1 -8.60x_2 -12.00x_5 -4.60x_3 +0.60x_{15} +3.60x_6 -13.60x_7 -7.00x_8 +1.60x_9
x_{12}
      46.0
             -9.00x_1 + 10.00x_2 -0.00x_5 -8.00x_3 -1.00x_{15} -0.00x_6 -4.00x_7 -0.00x_8 -6.00x_9
x_{13}
             +0.30x_1 -0.60x_2 -0.50x_5 +0.40x_3 +0.10x_{15} +0.60x_6 -0.60x_7 -0.50x_8 +0.10x_9
      1.1
x_4
      12.9
             +9.70x_1 -11.40x_2 -5.50x_5 -3.40x_3 +0.90x_{15} -4.60x_6 -13.40x_7 -3.50x_8 +2.90x_9
x_{10}
             -0.10x_1 -5.80x_2 -4.50x_5 +4.20x_3 +0.30x_{15} -1.20x_6 -3.80x_7 -4.50x_8 +1.30x_9
 z
      3.3
```

1 Optimization Phase Simplex

Starting Dictionary is:

```
0.4
             +1.20x_1 -0.40x_2 +5.00x_5 +9.60x_3 +0.40x_{15} -1.60x_6 -2.40x_7 +3.00x_8 -5.60x_9
x_{14}
             -10.40x_1 -0.20x_2 -6.00x_5 -12.20x_3 -0.80x_{15} -1.80x_6 -2.20x_7 +2.00x_8 -9.80x_9
      59.2
x_{11}
             -1.20x_1 -8.60x_2 -12.00x_5 -4.60x_3 +0.60x_{15} +3.60x_6 -13.60x_7 -7.00x_8 +1.60x_9
x_{12}
      47.6
      46.0
             -9.00x_1 + 10.00x_2 - 0.00x_5 - 8.00x_3 - 1.00x_{15} - 0.00x_6 - 4.00x_7 - 0.00x_8 - 6.00x_9
x_{13}
      1.1
             +0.30x_1 -0.60x_2 -0.50x_5 +0.40x_3 +0.10x_{15} +0.60x_6 -0.60x_7 -0.50x_8 +0.10x_9
x_4
      12.9
             +9.70x_1 -11.40x_2 -5.50x_5 -3.40x_3 +0.90x_{15} -4.60x_6 -13.40x_7 -3.50x_8 +2.90x_9
x_{10}
      3.3
             -0.10x_1 -5.80x_2 -4.50x_5 +4.20x_3 +0.30x_{15} -1.20x_6 -3.80x_7 -4.50x_8 +1.30x_9
```

 x_3 enters and x_{10} leaves

```
36.8235294118
                                                                                                                                                                                                                 +28.59x_1 - 32.59x_2 - 10.53x_5 - 2.82x_{10} + 2.94x_{15} - 14.59x_6 - 40.24x_7 - 6.88x_8 + 2.50x_{10} + 2.
x_{14}
x_{11}
                                                     12.9117647059
                                                                                                                                                                                                                   -45.21x_1 + 40.71x_2 + 13.74x_5 + 3.59x_{10} - 4.03x_{15} + 14.71x_6 + 45.88x_7 + 14.56x_8 - 20.
                                                     30.1470588235
                                                                                                                                                                                                                 -14.32x_1 + 6.82x_2 - 4.56x_5 + 1.35x_{10} - 0.62x_{15} + 9.82x_6 + 4.53x_7 - 2.26x_8 - 2.36x_{10} - 2.26x_{10} + 2.26x_{10} - 2.26x_
x_{12}
                                                     15.6470588235
                                                                                                                                                                                                                   -31.82x_1 + 36.82x_2 + 12.94x_5 + 2.35x_{10} - 3.12x_{15} + 10.82x_6 + 27.53x_7 + 8.24x_8 - 12.
x_{13}
                                                     2.61764705882
                                                                                                                                                                                                                      +1.44x_1 -1.94x_2 -1.15x_5 -0.12x_{10} +0.21x_{15} +0.06x_6 -2.18x_7 -0.91x_8 +0.46x_1
    x_4
                                                     3.79411764706
                                                                                                                                                                                                                        +2.85x_1 -3.35x_2 -1.62x_5 -0.29x_{10} + 0.26x_{15} -1.35x_6 -3.94x_7 -1.03x_8 +0.8
      x_3
                                                                                                                                                                                                                 +11.88x_1 - 19.88x_2 - 11.29x_5 - 1.24x_{10} + 1.41x_{15} - 6.88x_6 - 20.35x_7 - 8.82x_8 + 4.86x_{10} + 1.88x_{10} + 1.8
                                                     19.2352941176
         z
```

 x_1 enters and x_{11} leaves

```
44.9889394925
                                                                                                                                                                -0.63x_{11} -6.85x_2 -1.84x_5 -0.55x_{10} +0.39x_{15} -5.29x_6 -11.22x_7 +2.32x_8 -10.19x
x_{14}
 x_1
                                      0.285621340273
                                                                                                                                                                -0.02x_{11} + 0.90x_2 + 0.30x_5 + 0.08x_{10} - 0.09x_{15} + 0.33x_6 + 1.01x_7 + 0.32x_8 - 0.45x_9
x_{12}
                                          26.0559531555
                                                                                                                                                               +0.32x_{11} - 6.07x_2 - 8.91x_5 + 0.22x_{10} + 0.66x_{15} + 5.16x_6 - 10.01x_7 - 6.88x_8 + 4.08x_9 + 4.08x_{10} + 4.08x_
                                          6.55757970072
                                                                                                                                                                +0.70x_{11} + 8.17x_2 + 3.27x_5 - 0.17x_{10} - 0.28x_{15} + 0.47x_6 - 4.77x_7 - 2.01x_8 + 1.40x_5
x_{13}
                                                                                                                                                                -0.03x_{11} - 0.64x_2 - 0.71x_5 - 0.00x_{10} + 0.08x_{15} + 0.53x_6 - 0.71x_7 - 0.45x_8 - 0.20x_9
   x_4
                                          3.02927781392
                                             4.6089785296
                                                                                                                                                                -0.06x_{11} - 0.78x_2 - 0.75x_5 - 0.07x_{10} + 0.01x_{15} - 0.42x_6 - 1.05x_7 - 0.11x_8 - 0.42x_6
   x_3
                                                                                                                                                                -0.26x_{11} - 9.18x_2 - 7.68x_5 - 0.29x_{10} + 0.35x_{15} - 3.02x_6 - 8.29x_7 - 5.00x_8 - 0.43x_9 + 0.00x_{10} + 0.00x_{
                                          22.6291476903
```

 x_{15} enters and x_1 leaves

```
x_{14}
    46.2481751825
                 -0.73x_{11} -2.88x_2 -0.50x_5 -0.20x_{10} -4.41x_1 -3.85x_6 -6.74x_7 +3.74x_8 -12.16x_8
    3.20437956204
                 -0.25x_{11} + 10.10x_2 + 3.41x_5 + 0.89x_{10} - 11.22x_1 + 3.65x_6 + 11.39x_7 + 3.61x_8 - 5.01x_9
x_{15}
    28.1678832117
                 +0.15x_{11} +0.58x_2 -6.66x_5 +0.80x_{10} -7.39x_1 +7.57x_6 -2.50x_7 -4.50x_8 +0.77x_9
x_{12}
    5.65693430657
                 x_{13}
    3.27737226277
                  -0.05x_{11} + 0.14x_2 - 0.45x_5 + 0.07x_{10} - 0.87x_1 + 0.81x_6 + 0.17x_7 - 0.17x_8 - 0.59x_9
                 x_3
    4.64233576642
    23.7591240876
```

 x_{10} enters and x_{13} leaves

```
-1.10x_{11} -5.45x_2 -1.62x_5 + 0.48x_{13} - 5.93x_1 - 3.59x_6 -2.90x_7 +5.21x_8 -13.52x_1
x_{14}
                                      15.1034482759
                                                                                                                                                    +1.38x_{11} + 21.31x_2 + 8.28x_5 - 2.10x_{13} - 4.59x_1 + 2.48x_6 - 5.38x_7 - 2.76x_8 + 0.90x_9
x_{15}
                                                                                                                                                    +1.62x_{11}+10.69x_2-2.28x_5-1.90x_{13}-1.41x_1+6.52x_6-17.62x_7-10.24x_8+6.10x_9+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.00x_1+6.
                                      38.8965517241
x_{10}
                                      13.3620689655
                                                                                                                                                    +1.83x_{11} + 12.59x_2 + 5.47x_5 - 2.36x_{13} + 7.45x_1 - 1.31x_6 - 18.83x_7 - 7.16x_8 + 6.64x_5 + 6.64x
   x_4
                                       4.15517241379
                                                                                                                                                    +0.07x_{11} +0.97x_2 -0.09x_5 -0.16x_{13} -0.38x_1 +0.72x_6 -1.07x_7 -0.64x_8 -0.16x_9
                                      3.86206896552
                                                                                                                                                     -0.31x_{11} -5.34x_2 -6.36x_5 -0.05x_{13} -3.79x_1 -1.76x_6 -4.69x_7 -3.88x_8 -2.05x_9
                                    24.0517241379
```

Final Dictionary Solution: 24.0517241379 Num Pivots: 5