দ্বিপদী বিস্তৃতি / উপপাদ্য

সূত্র, $^{(a+x)^n}T_{r+1} = {}^nC_r\, a^{n-r} x^r$ [r তম পদ বের] লাফ (পাওয়ার সূত্র)

যদি 19 তম পদ বের হয়

$$^{(a+x)^n}T_{18+1} = {}^nC_{18}\, a^{\cancel{18}\, n-18}\, x^{18}$$

$$^{20}T_{19+1} = {}^{20}C_{19}$$

* পাওয়ার জোড় (যোগ) ①: মধ্যপদ ১ টি

মধ্যপদ $= \frac{n}{2} + 1$

* পাওয়ার বিজোড় (বং ②): মধ্যপদ ২ টি

$\frac{n+1}{2} + 1$, $\frac{n-1}{2} + 1$

* মাথার উপর যত থাকে তার থেকে 1 বেশি পদসংখ্যা বা মোট পদ।

$\Rightarrow (a+x)^3 = a^3 + 3a^2x + 3ax^2 + x^3$

এর ৩য় পদ নির্ণয় - সূত্র দিয়ে,

$$^{(a+x)^3}T_{2+1} = {}^3C_2\, a^{3-2} . x^2$$
$$= 3ax^2$$

* লাফ দেওয়ার পর ২টি কাজ মাইনাস সামনে নিয়ে আসা ও ভেরিয়েবলগুলো এক জায়গায় করা।

উদা

$\left(\frac{a}{2} - \frac{2}{x}\right)^{33}$ এর 16 তম পদ নির্ণয়

$$^{\left\{\frac{a}{2}+\left(-\frac{2}{x}\right)\right\}^{33}}T_{\cancel{32+1}\; 15+1} = {}^{33}C_{15} \left(\frac{a}{2}\right)^{33-15} . \left(-\frac{2}{x}\right)^{15}$$

* x বর্জিত / x মুক্ত / constant / ধ্রুব পদ হলে $x^0 = 1$ হবে।

x এর অবিক্রম শক্তি অনুসারে যদি বলে তাহলে x^1 = লিখব।

$\Rightarrow$ এই অধ্যায়গুলো অপ্রকৃত $\left(\frac{28}{27}\right)$ ভগ্নাংশ বেশি আছে।

* বিন্যাসের প্রমাণ, কর,

$$(2n)! = 2^n n! \{1.3.5 \ldots\ldots (2n-1)\}$$

L.H.S =

$$(2n)!$$

$$= \{1.2.3.4.5.6 \ldots\ldots (2n-1)\, 2n\}$$

$$= \{1.3.5.7 \ldots\ldots (2n-1)\} \{(2.1), (2.2), (2.3) \ldots \underbrace{2n}_{\text{শেষ পদ } 2n}\}$$

$$= \{\underbrace{1.3.5.7 \ldots\ldots}_{\text{থাকবে}} (2n-1)\} \{2^n \{\underbrace{1.2.3. \ldots\ldots n}_{n! \text{ হবে}}\}\}$$

$$= 2^n . n! \{1.3.5.7 \ldots (2n-1)\}$$

প্রমাণ কর,

$$^{2n}P_n = 2^n \{1.3.5. \ldots (2n-1)\}$$

$$\text{L.H.S} = {}^{2n}P_n = \frac{2n!}{(2n-n)!} = \frac{2n!}{n!}$$

$$= \frac{2^n . \cancel{n!} \{1.3.5. \ldots (2n-1)\}}{\cancel{n!}}$$

$$= 2^n \{1.3.5. \ldots (2n-1)\}$$

$(a+x)^n$ এর মূল নিয়মে বিস্তৃতি -

১ম পদ $T_{0+1} = {}^nC_0\, a^{n-0} x^0 = a^n$ —— (i)

২য় পদ, $T_{1+1} = {}^nC_1\, a^{n-1} x^1$ —— (ii)

৩য় পদ, $T_{2+1} = {}^nC_2\, a^{n-2} x^2$ —— (iii)

৪র্থ পদ, $T_{3+1} = {}^nC_3\, a^{n-3} x^3$ —— (iv)

$(n+1)$ তম পদ $T_{n+1} = {}^nC_n\, a^{n-n} x^n = x^n$ —— (v)

$$(a+x)^n = a^n + {}^nC_1 a^{n-1} x^1 + {}^nC_2 a^{n-2} x^2 + {}^nC_3 x^{n-3} x^3 \ldots + x^n$$

দ্বিপদী (sheet)

1. x বর্জিত পদটির মান নির্ণয় –

(i) $(2x+\frac{1}{6x})^{10}$

T_{r+1}

${}^{10}C_r (2x)^{10-r}\cdot(\frac{1}{6x})^r$

$\Rightarrow {}^{10}C_r\, 2^{10-r}\cdot x^{10-r}\,\frac{1}{6^r}\cdot\frac{1}{x^r}$

$\Rightarrow {}^{10}C_r\, 2^{10-r}\cdot\frac{1}{6^r}\,x^{10-2r}$

শর্তমতে,

$x^{10-2r}=x^0$

বা, $10-2r=1$

$r=5$.

∴ নির্ণেয় x বর্জিত পদটি,

${}^{10}C_5\, 2^{10-5}\cdot\frac{1}{6^5}=\frac{28}{27}$ (Ans)

(ii) $(x^2-2+\frac{1}{x^2})^6$

$=(x^2-2x\frac{1}{x}+\frac{1}{x^2})^6=(x-\frac{1}{x})^{12}$

$T_{r+1}={}^{12}C_r\, x^{12-r}(-\frac{1}{x})^r$

$={}^{12}C_r\, x^{12-2r}(-1)^r$

শর্তমতে, $x^{12-2r}=x^0$

∴ $r=6$

∴ নির্ণেয় x বর্জিত পদটি $=(-1)^6\cdot{}^{12}C_6$

$=924$ (Ans)

(iii) $(1+x)^p(1+\frac{1}{x})^q$

$=(1+x)^p(\frac{1+x}{x})^q$

$=(1+x)^{p+q}\,\frac{1}{x^q}$

$T_{r+1}={}^{p+q}C_r\, x^{r}\cdot\frac{1}{x^q}$

$={}^{p+q}C_r\, x^{r-q}$

শর্তমতে, $x^{r-q}=x^0$

∴ $r=q$

∴ নির্ণেয় x বর্জিত পদটি =

${}^{p+q}C_q=\frac{|\underline{p+q}}{|\underline{q}\;|\underline{p+q-q}}$

$=\frac{|\underline{p+q}}{|\underline{q}\;|\underline{p}}$ (Answer)

(iv) $(2x^2-\frac{1}{x^3})^{10}$

$T_{r+1}={}^{10}C_r\cdot(2x^2)^{10-r}\cdot(-\frac{1}{x^3})^r$

$={}^{10}C_r\, 2^{10-r}\cdot x^{20-2r}\cdot\frac{1}{x^{3r}}\cdot(-1)^r$

$=(-1)^r\,{}^{10}C_r\, 2^{10-r}\cdot x^{20-5r}$

শর্তমতে, $x^{20-5r}=x^0$

$r=4$

∴ নির্ণেয় x বর্জিত পদটি =

$(-1)^4\cdot{}^{10}C_4\cdot 2^6=13440$ (Ans)

(b) $\left(\frac{x^4}{y^3}+\frac{y^2}{2x}\right)^{10}$

$T_{r+1} = {}^{10}C_r\left(\frac{x^4}{y^3}\right)^{10-r}\cdot\left(\frac{y^2}{2x}\right)^r$

$= {}^{10}C_r\ x^{40-4r}\cdot y^{-30+3r}\cdot y^{2r}\cdot\frac{1}{2^r}\cdot x^{-r}$

$\Rightarrow {}^{10}C_r\ x^{40-5r}\cdot y^{-30+5r}\cdot\frac{1}{2^r}$

শর্তমতে,

$y^{-30+5r} = y^0$

$r = 6$

∴ নির্ণেয় y বর্জিত পদটি,

${}^{10}C_6\ x^{40-30}\cdot\frac{1}{2^6} = \frac{105}{32}x^{10}$ (Ans)

(2)

$(1+x)^{44}$

$T_{20+1} = {}^{44}C_{20}\ x^{20}$

$T_{21+1} = {}^{44}C_{21}\cdot x^{21}$

শর্তমতে, ${}^{44}C_{21}x^{21} = {}^{44}C_{20}x^{20}$

$\Rightarrow x = \frac{7}{8}$ (Ans)

(3) $\left(x-\frac{1}{x}\right)^{2n}$ এর মধ্যপদ,

$\frac{2n}{2}+1 = (n+1)$

$T_{n+1} = {}^{2n}C_n\ x^{2n-n}\cdot\left(-\frac{1}{x}\right)^n$

$= {}^{2n}C_n\cdot \cancel{x^n}\ \frac{1}{\cancel{x^n}}\cdot(-1)^n$

$= \frac{2n!}{n!\,(2n-n)!}\cdot(-1)^n$

$= \frac{2^n\cdot \cancel{n!}\,\{1,3,5\ \ldots\ (2n-1)\}\cdot(-1)^n}{\cancel{n!}\ n!}$

$= (-2)^n\,\frac{\{1,3.5.\ \ldots\ (2n-1)\}}{n!}$

(proved)

(4)

$\left(3+\frac{x}{2}\right)^n$

$T_{r+1} = {}^nC_r\ 3^{n-r}\cdot\left(\frac{x}{2}\right)^r$

$= {}^nC_r\ 3^{n-r}\ x^r\ \frac{1}{2^r}$

1st condition	2nd condition
$x^r = x^7$	$x^r = x^8$
$r = 7$	$r = 8$

শর্তমতে,

শর্তমতে,

$$^nC_8\, 3^{n-8} \cdot \frac{1}{2^8} = {}^nC_7\, 3^{n-7} \cdot \frac{1}{2^7}$$

$$\Rightarrow {}^nC_8 \frac{3^n}{3^8} \frac{1}{2^8} = {}^nC_7 \frac{3^n}{3^7} \cdot \frac{1}{2^7}$$

$$\Rightarrow \frac{n-7}{8} = 6$$

$$\Rightarrow n = 48 + 7 = 55$$ (Answer)

(5)

$$\left(2x^2 + \frac{p}{x^3}\right)^{10} \quad T_{r+1}$$

$$^{10}C_r\, (2x^2)^{10-r} \cdot \left(\frac{p}{x^3}\right)^r$$

$$\Rightarrow {}^{10}C_r\, 2^{10-r} \cdot x^{20-2r} \cdot p^r \frac{1}{x^{3r}}$$

$$\Rightarrow {}^{10}C_r\, 2^{10-r}\, x^{20-5r} \cdot p^r$$

1^{st} condition	2^{nd} condition
$x^{20-5r} = x^5$	$x^{20-5r} = x^{15}$
$r = 3$	$r = 1$

শর্তমতে, $^{10}C_3\, 2^{10-3} \cdot p^3 = {}^{10}C_1\, 2^{10-1} \cdot p^1$

$$\Rightarrow {}^{10}C_3\, 2^7\, p^2 = {}^{10}C_1\, 2^9$$

$$\Rightarrow p = \frac{1}{\sqrt{3}}$$ (Answer)

(6 → 5, 6 same)

(7) $(1+x)^n \quad T_{r+1} = {}^nC_r\, x^r$

$$T_{(r+2)+1} = {}^nC_{r+2}\, x^{r+2}$$

শর্তমতে, $^nC_{r+2} = {}^nC_r$ বড় বানাটি উপরে

$$\Rightarrow \frac{n-r}{r+2} = 1$$ → 2 দিতে ভুল হয়!

$$\Rightarrow n - r - r - 2 = 0$$

$$\Rightarrow n - 2 = 2r$$

$$\therefore 2r = (n-2)$$ (showed)

(8)

$$\left(2x^2 + \frac{3}{x}\right)^{19} \quad T_{r+1}$$

$$= {}^{19}C_r\, (2x^2)^{19-r} \cdot \left(\frac{3}{x}\right)^r$$

$$= {}^{19}C_r\, 2^{19-r} \cdot x^{38-2r} \cdot 3^r \cdot \frac{1}{x^r}$$

$$\Rightarrow {}^{19}C_r\, 2^{19-r} \cdot 3^r \cdot x^{38-3r}$$

Again, $T_{(r+1)+1} = {}^nC_{r+1}\, (2x^2)^{19-r-1} \left(\frac{3}{x}\right)^{r+1}$

$$\Rightarrow {}^nC_{r+1}\, 2^{18-r} \cdot x^{36-2r} \cdot 3^{r+1} \cdot x^{r+1}$$

$$\Rightarrow {}^nC_{r+1}\, 2^{18-r}\, x^{35-3r} \cdot 3^{r+1}$$

শর্তমতে,

$$^{19}C_n 2^{19-n} \cdot 3^n = {}^{19}C_{n+1} \cdot 2^{18-n} \cdot 3^{n+1}$$

$$\Rightarrow {}^{19}C_n \frac{2^{19}}{2^n} \cdot 3^n = {}^{19}C_{n+1} \frac{2^{18}}{2^n} \cdot 3^n \cdot 3$$

$$\Rightarrow \frac{2}{3} = \frac{19-n}{n+1}$$

$$\Rightarrow \cancel{19-n-2n-2=0}$$

$$\Rightarrow 57-3n-2n-2=0$$

$$\Rightarrow 5n = 55$$

$$\therefore n = 11$$ (Answer)

(9)

প্রথম পদ, $a^n = b$ —— (i)

২য় পদ,

$$^nC_1 a^{n-1} (3x)^1 = \frac{21}{2} bx$$

$$\Rightarrow {}^nC_1 a^n \frac{1}{a} 3x = \frac{21}{2} a^n x$$

$$\Rightarrow \frac{n}{a} = \frac{7}{2}$$

$$\Rightarrow a = \frac{2n}{7}$$ —— (ii)

৩য় পদ, $$^nC_2 a^{n-2} (3x)^2 = \frac{189}{4} bx^2$$

$$\frac{n(n-1)}{1.2} a^n \cdot \frac{49}{4n^2} 9x^2 = \frac{189}{4} a^n x^2$$

$$\Rightarrow \frac{n-1}{n} = \frac{3}{7} \times 2$$

$$\Rightarrow 7n-7-6n=0$$

$$n=7$$

n এর মান (ii) এ বসাই,

$$a = \frac{14}{7} = 2$$

n ও a এর মান (i) এ বসাই,

$$2^7 = b$$

$$\therefore b = 128$$

$\therefore$ নির্ণেয় মান, $a=2, b=128$ ও $n=7$ (Ans)

(10)

১ম সংখ্যা,

$$x^n = 729$$ —— (i)

২য় সংখ্যা,

$$^nC_1 x^{n-1} a^1 = 7290$$

$$\Rightarrow n \cdot 729 \cdot \frac{1}{x} a = 7290$$

$$\Rightarrow an = 10x$$

$$\Rightarrow x = \frac{an}{10}$$

৩য় পদ,

$$^nC_2 x^{n-2} \cdot a^2 = 30375$$

$$\Rightarrow \frac{n(n-1)}{1.2} \cdot 729 \cdot \frac{100}{a^2 n^2} \cdot a^2 = 30375$$

$$\Rightarrow \frac{n-1}{n} = \frac{30375}{72900} \times 2$$

$$\Rightarrow \frac{n-1}{n} = \frac{5}{6}$$

$$= 6n-6-5n=0$$

$$n=6$$

n এর মান ① এ বসাই,

$x^6 = 729$

$\Rightarrow x^6 = 3^6$

$\Rightarrow x = 3$

x ও n এর মান ⑪ এ বসাই,

$a = \frac{10\times 3}{6} = 5$

$\therefore a = 5$ (Answer)

(13)

প্রশ্নমতে,

$\left(x^p + \frac{1}{x^p}\right)^{2\times 5}$

T_{r+1}

$= {}^{10}C_r \cdot \cancel{x^{10p}} (x^p)^{10-r} \cdot \left(\frac{1}{x^p}\right)^r$

$= {}^{10}C_r \cdot x^{10p - pr - pr}$

$= {}^{10}C_r \cdot x^{10p-2pr}$

শর্তমতে,

$x^{10p-2pr} = x^0$

$10p = 2pr$

$r = 5$

$\therefore$ পদটি,

${}^{10}C_5 \cdot x^{10p-10p}$

$= 252$ (Answer)

(11)

$(1+x)(a-bx)^{12}$

$= (1+x)\{a^{12} + {}^{12}C_1 a^{12-1}(-bx)^1 + {}^{12}C_2 a^{12-2}(-bx)^2 + \ldots\ldots {}^{12}C_7 a^{12-7}(-bx)^7 + {}^{12}C_8 a^{12-8}(-bx)^8 + \ldots\}$

প্রশ্ন শর্তমতে,

x^8 এর সহগ, ${}^{12}C_8 a^4 b^8 + {}^{12}C_7 a^5 (-b)^7 = 0$

বা, ${}^{12}C_8 a^4 b^8 = {}^{12}C_7 a^5 b^7$

বা, $\frac{{}^{12}C_8}{{}^{12}C_7} = \frac{a}{b}$

বা $\frac{a}{b} = \frac{5}{8}$ (Answer)

(12)

$(1-x)^8 (1+x)^7$

$\Rightarrow (1+x)^7 (1-x)^7 \cdot (1-x)$

$\Rightarrow (1-x)(1-x^2)^7$

$\Rightarrow (1-x)\{1 + {}^7C_1(-x^2)^1 + {}^7C_2(-x^2)^2 + {}^7C_3(-x^2)^3 + {}^7C_4(-x^2)^4 + \ldots\}$

শর্তমতে, x^7 এর সহগ $= {}^7C_3 = 35$ (Answer)

(14) $\left(x^4 - \frac{1}{x^3}\right)^{11}$ এর মধ্যপদ দুটি,

২য় পদ্ধতি

$\frac{n+1}{2}+1$ ও $\frac{n-1}{2}+1$

$= \frac{11+1}{2}+1$, $\frac{11-1}{2}+1$

$= 7$ ও 6

$T_{6+1} = {}^{11}C_6 (x^4)^{11-6} \cdot \left(-\frac{1}{x^3}\right)^6$

$= {}^{11}C_6 \, x^2 = 462x^2$ (Ans)

$T_{5+1} = {}^{11}C_5 (x^4)^{11-5} \cdot \left(-\frac{1}{x^3}\right)^5$

$= (-1)^5 \, {}^{11}C_5 \, x^9$

$= -462x^9$ (Answer)

(15)

ধরি ক্রমিক পদ দুটি, T_{r+1} ও T_{r+2}

$T_{r+1} = {}^{24}C_r \cdot x^r$

$T_{(r+1)+1} = {}^{24}C_{r+1} \, x^{r+1}$

শর্তমতে,

$\frac{{}^{24}C_{r+1}}{{}^{24}C_r} = \frac{4}{1}$

$\Rightarrow \frac{24-r}{r+1} = \frac{4}{1}$

$\Rightarrow 24-r-4r-4=0$

$r = \cancel{5}\,4$

∴ পদ দুটি, ~~(5+1) = 6 ও (5+2) = 7~~

$(4+1) = 5$ $(4+2) = 6$

অথবা,

$\frac{{}^{24}C_{r+1}}{{}^{24}C_r} = \frac{1}{4}$

$\Rightarrow \frac{24-r}{r+1} = \frac{1}{4}$

$\Rightarrow 96-4r-r-1=0$

$\Rightarrow r = 19$

অথবা পদ দুটি, $(19+1) = 20$ ও

$(19+2) = 21$ Ans

(16)

$(x+1)^{20}$

$T_{(r-1)+1} = {}^{20}C_{r-1} \cdot x^{20-r+1}$

$= {}^{20}C_{r-1} \, x^{21-r}$

$T_{(r+3)+1} = {}^{20}C_{r+3} \cdot x^{20-r-3}$

$= {}^{20}C_{r+3} \, x^{17-r}$

শর্তমতে,

${}^{20}C_{r+3} = {}^{20}C_{r-1}$

$\frac{20-r+1}{r+3} = 1$

~~20-r-r-3=0~~

$\Rightarrow 20-r+1-r-3=0$

$\Rightarrow r = \frac{18}{2} = 9$

$\therefore r = 9$ (Answer)

(17) $\left(x^3 - \frac{5}{x^2}\right)^{12}$

T_{r+1}

$= {}^{12}C_r (x^3)^{12-r} \cdot \left(-\frac{5}{x^2}\right)^r$

$= {}^{12}C_r \, x^{36-3r} \cdot 5^r \cdot \frac{1}{x^{2r}} (-1)^r$

$= (-1)^r \, {}^{12}C_r \, x^{36-5r} \, 5^r$

শর্তমতে,

$x^{36-5r} = x^{11}$

$36-5r = 11$

বা, $r = 5$

$\therefore$ x^{11} এর সহগ,

$- {}^{12}C_5 \, 5^5$ (Answer)

(18) $\left(\frac{x}{y} + \frac{y}{x}\right)^{10}$ এর মধ্যপদ,

$\frac{10}{2} + 1 = 6^{th}$

$T_{5+1} = {}^{10}C_5 \left(\frac{x}{y}\right)^{10-5} \cdot \left(\frac{y}{x}\right)^5$

$= {}^{10}C_5 \, x^5 \cdot y^{-5} \cdot y^5 \cdot x^{-5}$

$= {}^{10}C_5 = 252$ (Answer)