জটিল

1. বর্গমূল নির্ণয়

① $7-30\sqrt{-2}$

$= 5^2 - 2.5.3\sqrt{-2} + (3\sqrt{-2})^2$

$= (5-3\sqrt{-2})^2$

$= \pm(5-3\sqrt{-2})$

$= \pm(5-3i\sqrt{2})$ (Ans)

(ii) $\frac{1}{2}(1-3\sqrt{-7})$

$= \frac{1}{4}(2-6\sqrt{-7})$

$= \frac{1}{4}\{3^2 - 2.3\sqrt{-7} + (\sqrt{-7})^2\}$

$= \frac{1}{4}(3-\sqrt{-7})^2$

$= \{\frac{1}{2}(3-i\sqrt{7})\}^2$

$= \pm\frac{1}{2}(3-i\sqrt{7})$

$\therefore$ নির্ণেয় বর্গমূল $= \pm\frac{1}{2}(3-i\sqrt{7})$ (Ans)

(iii) $x + i\sqrt{1-x^2}$

$= x + i\sqrt{(1+x)}\sqrt{(1-x)}$

$\Rightarrow \frac{1}{2}\{2x + 2i\sqrt{(1+x)}.\sqrt{(1-x)}$

$\Rightarrow \frac{1}{2}\{(\sqrt{1+x})^2 + 2\sqrt{(1+x)}\,i\sqrt{(1-x)} + (i\sqrt{(1-x)})^2$

$\Rightarrow \frac{1}{2}(\sqrt{1+x} + i\sqrt{1-x})^2$

$\Rightarrow \{\frac{1}{\sqrt{2}}(\sqrt{1+x} + i\sqrt{1-x})\}^2$

$\Rightarrow \pm\frac{1}{\sqrt{2}}(\sqrt{1+x} + i\sqrt{1-x})$

$\therefore$ নির্ণেয় বর্গমূল $\pm\frac{1}{\sqrt{2}}(\sqrt{1+x} + i\sqrt{1-x})$ Ans

(iv) $-8-6\sqrt{-1}$

যদি আমাদের আর্টি স্কোয়ারে ২য় তাহলে ছোট মানটি আমাদের হবে। স্কয়ারে ২টি বড় টি। জটিল মানটি যখনও ৮ নং আগে থাকবে।

$\Rightarrow 1^2 - 2.3\sqrt{-1} + (3\sqrt{-1})^2$

$1 + 9.-1$

$= -8$

$\Rightarrow (1-3\sqrt{-1})^2$

$= (1-3i)^2$

$= \pm(1-3i)$

$\therefore$ নির্ণেয় বর্গমূল $= \pm(1-3i)$ (Ans)

②

$i^{45454} = -1$

$i^{4457} = i$

$i^{\cancel{14587}} =$

$i^{14587} = -i$

$i^{44444} = 1$

③

$w^{21548} = w^2$

$w^{255} = w^0 = 1$

$w^{56879} = w^2$

$w^{1222} = w$

(4)

$$\sqrt[4]{-144}$$
$$\Rightarrow \sqrt[4]{(12i)^2}$$
$$\Rightarrow \sqrt{\pm 12i}$$
$$\Rightarrow \sqrt{6}\sqrt{\pm 2i}$$
$$\Rightarrow \sqrt{6}\sqrt{1 \pm 2i + i^2}$$
$$\Rightarrow \sqrt{6}\sqrt{(1 \pm i)^2}$$
$$\Rightarrow \sqrt{6} \pm (1 \pm i)$$
$$\Rightarrow \pm\sqrt{6}(1 \pm i)$$
(Ans)

(6) $\sqrt[4]{-81}$

$$\Rightarrow \sqrt[4]{(9i)^2}$$
$$\Rightarrow \sqrt{\pm 9i}$$
$$\Rightarrow 3\sqrt{\pm i}$$
$$\Rightarrow \frac{3}{\sqrt{2}}\sqrt{\pm 2i}$$
$$\Rightarrow \frac{3}{\sqrt{2}}\sqrt{1 \pm 2i + i^2}$$
$$= \frac{3}{\sqrt{2}}\sqrt{(1 \pm i)^2}$$
$$\Rightarrow \pm\frac{3}{\sqrt{2}}(1 \pm i)$$
(Ans)

(5)

$$x = \sqrt[6]{-64}$$
$$(x^2)^3 = -64$$
$$\Rightarrow (x^2)^3 + 4^3 = 0$$
$$\Rightarrow (x^2+4)(x^4 - 4x + 16) = 0$$

$x^2 = -4$

$\cancel{x = \pm 2}$

$x = \sqrt{-4}$

$= \pm 2i$

$x^4 - 4x + 16$

$$x^2 = \frac{4 \pm \sqrt{16-64}}{2}$$
$$x^2 = \frac{4 \pm \sqrt{-48}}{2}$$
$$x^2 = \frac{4 \pm 4i\sqrt{3}}{2}$$
$$x^2 = \frac{\cancel{2}(2 \pm 2i\sqrt{3})}{\cancel{2}}$$
$$x^2 = \{(\sqrt{3})^2 \pm 2i\sqrt{3} + i^2\}^2$$
$$x^2 = (\sqrt{3} \pm i)^2$$
$$x = \pm(\sqrt{3} \pm i)$$

∴ নির্ণেয় মূল $= \pm 2i, \pm(\sqrt{3} \pm i)$ (Ans)

(7)

$$\sqrt[3]{a+ib} = x+iy$$
$$a+ib = (x+iy)^3$$
$$= x^3 + 3x^2 iy + 3x(iy)^2 + (iy)^3$$
$$= x^2 + 3x^2 iy - 3xy - iy^3$$
$$= x^2 - 3xy + i(3x^2 y - y^3)$$

$$\therefore a = x^2 - 3xy$$
$$b = 3x^2 y - y^3$$

আবার, $\sqrt[3]{a-ib} = x - iy$

$$a - ib = (x-iy)^3$$
$$a-ib = x^2 - 3xy - i(3x^2y - y^3)$$
$$= x^2 - 3xy - 3x^2 iy + iy^3$$
$$= x^2 - 3x^2 iy + 3xi^2y^2 - (iy)^3$$
$$= (x-iy)^3$$
(Showed)

(৪)

$$\sqrt[3]{x+iy} = p+iq$$

$$x+iy = (p+iq)^3$$

$$= p^3 + 3p^2iq + 3p(iq)^2 + (iq)^3$$

$$= p^3 + 3p^2iq - 3pq^2 - iq^3$$

$$= p^3 - 3pq^2 + i(3p^2q - q^3)$$

$$x = p^3 - 3pq^2$$

$$y = 3p^2q - q^3$$

$$R.H.S = \frac{x}{p} + \frac{y}{q}$$

$$= \frac{p^3 - 3pq^2}{p} + \frac{3p^2q - q^3}{q}$$

$$= \frac{\cancel{p}(p^2 - 3q^2)}{\cancel{p}} + \frac{\cancel{q}(3p^2 - q^2)}{\cancel{q}}$$

$$= p^2 - 3q^2 + 3p^2 - q^2$$

$$= 4p^2 - 4q^2$$

$$= 4(p^2 - q^2) \text{ (showed)}$$

(৩)

$$\left\{\frac{1}{2}(1+\sqrt{-3})\right\}^6$$

$$\Rightarrow \frac{1}{64}\left\{(1+\sqrt{-3})^3\right\}^2$$

$$\Rightarrow \cancel{\frac{1}{64}\left\{1+2.\sqrt{-3}+(\sqrt{-3})^2\right\}^3}$$

$$\Rightarrow \frac{1}{64}\left\{1+3\sqrt{-3}+3(\sqrt{-3})^2+(\sqrt{-3})^3\right\}^2$$

$$= \frac{1}{64}(1+3\sqrt{-3}-9-3\sqrt{-3})^2$$

$$= \frac{1}{64}(-8)^2$$

$$= \frac{1}{64} \times 64 = 1$$

(১০) দেয়া আছে, $\alpha^2+\alpha+1=0$

$$\alpha^2+\alpha = -1$$

প্রমান ভুল থাকলো

$$L.H.S = (x-1)(x-\alpha)(x-\alpha^2)$$

$$= (x-1)(x^2 - \alpha^2 x - \alpha x + \alpha^3)$$

$$= (x-1)(x^2 - x(\alpha^2+\alpha) + 1) \quad [\omega^3 = \alpha^3 = 1]$$

$$= (x-1)(x^2+x+1)$$

$$= x^3 - 1 \text{ (Ans)}$$

(১১)

$$\frac{1-ix}{1+ix} = a-ib$$

$$\Rightarrow 1-ix = (a-ib)(1+ix)$$

$$= a + aix - ib \,\cancel{+}\, i^2bx$$

$$= a + aix - ib + bx$$

$$= (a+bx) - i(\cancel{ax}\; b - ax)$$

উভয় পক্ষ থেকে বাস্তব ও অবাস্তব অংশ সমীকৃত করে,

$$a + bx = 1$$

$$\text{বা, } x = \frac{1-a}{b} \quad \text{------ (i)}$$

$$x = b - ax$$

$$\Rightarrow x(1+a) = b$$

$$x = \frac{b}{1+a} \quad \text{------ (ii)}$$

(i) (ii) হতে,

$$\frac{1-a}{b} = \frac{b}{1+a}$$

$$\text{বা, } b^2 = 1-a^2$$

$$\text{বা, } a^2+b^2 = 1 \text{ (showed)}$$

(12) দেয়া আছে,

$\alpha = \frac{1+\sqrt{-1}}{\sqrt{2}}$

$\alpha^2 = \left(\frac{1+\sqrt{-1}}{2}\right)^2$

$\alpha^2 = \frac{1+2\sqrt{-1}+(\sqrt{-1})^2}{2}$

$\alpha^2 = \frac{\cancel{1}+\cancel{2}\sqrt{-1}-\cancel{1}}{\cancel{2}}$

$\alpha^2 = \sqrt{-1} = i$

L.H.S =

$\alpha^6+\alpha^4+\alpha^2+1$

i^3+i^2+i+1

$-\cancel{i}-1+\cancel{i}+1 = 0$ (proved)

(13) দেয়া আছে,

$x = 2+i$ (i বর্গ করে)

$\Rightarrow (x-2)^2 = i^2$

$\Rightarrow x^2-4x+4+1=0$

$\Rightarrow x^2-4x+5=0$

L.H.S =

$x^4-4x^3+6x^2-4x+5$

$\Rightarrow x^4-4x^3+5x^2+x^2-4x+5$

$\Rightarrow x^2(x^2-4x+5)+1(x^2-4x+5)$

$\Rightarrow x^2\times 0+1\times 0 = 0$

RHS (showed)

(14) দেয়া আছে

$x = -1+i\sqrt{2}$

$\Rightarrow (x+1)^2 = (i\sqrt{2})^2$

$\Rightarrow x^2+2x+1 = -2$

$\Rightarrow x^2+2x+3=0$

L.H.S = $x^4+4x^3+6x^2+4x+9$

$\Rightarrow x^4+2x^3+3x^2+2x^3+4x^2+6x-x^2-2x-3+12$

$\Rightarrow x^2(x^2+2x+3)+2x(x^2+2x+3)-1(x^2+2x+3)+12$

$\Rightarrow x^2(0)+2x(0)-1(0)+12$

$= 12$ (R.H.S)

(Showed)

(15) 1 এর ঘনমূল

let $x = \sqrt[3]{1}$

$x^3 = 1$

$x^3-1=0$

$(x-1)(x^2+x+1)$

$x = 1$ অথবা, $\frac{-1\pm\sqrt{1-4.1}}{2}$

$= \frac{-1\pm\sqrt{-3}}{2}$

এখানে, $\boxed{\frac{-1\pm\sqrt{-3}}{2},\ \frac{-1\pm\sqrt{-3}}{2},\ 1}$

হলো 1 এর ঘনমূল।

* -1

let, $x = \sqrt[3]{-1}$

$x^3 = -1$

$x^3 + 1^3 = 0$

$(x+1)(x^2 - x + 1) = 0$

$x + 1 = 0$ অথবা,

$x = -1$

$x^2 - x + 1 = 0$

$x = \frac{+1 \pm \sqrt{1-4}}{2}$

$= \frac{1 \pm \sqrt{-3}}{2}$

$\therefore$ নির্ণেয় ঘনমূল $\boxed{-1, \frac{1+\sqrt{-3}}{2}, \frac{1-\sqrt{-3}}{2}}$ (Ans)

** $-i$

let, ~~$x = \sqrt[3]{-i}$~~ $x = \sqrt[3]{-i}$

$x^3 = -i$

~~$x^3 + i = 0$~~

$x^3 - i^3 = 0$

$(x - i)(x^2 + ix + i^2) = 0$

$x = i$ অথবা,

$x = \frac{-i \pm \sqrt{i^2 - 4(-1)}}{2}$

$= \frac{-i \pm \sqrt{+3}}{2}$

$\frac{-i + \sqrt{3}}{2}, \frac{-i - \sqrt{+3}}{2}$

$\therefore -i$ এর ঘনমূল, $\boxed{i, \frac{-i + \sqrt{3}}{2}, \frac{-i - \sqrt{+3}}{2}}$ (Ans)

* i

let, $x = \sqrt[3]{i}$

$\Rightarrow x^3 = i$ → Technique ...

$\Rightarrow x^3 + i^3 = 0$

$\Rightarrow (x+i)(x^2 - xi + i^2) = (x+1)(x^2 - xi - 1)$

$x = -i$ অথবা, ~~$x^2 - xi + i^2$~~ $x^2 - xi - 1$

~~$\Rightarrow \frac{+i \pm \sqrt{(-i)^2 - 4(-1)}}{2}$~~

~~$= \frac{i \pm 1 -}{}$~~

** $x = \sqrt[3]{i}$

$x^3 + i^3$

$\Rightarrow (x+i)(x^2 - xi + i^2)$

$x = -i \mid x^2 - xi - 1$

$x = \frac{+i \pm \sqrt{(-i)^2 - 4\cdot(-1)}}{2}$

$= \frac{i \pm \sqrt{-1+4}}{2}$

$\Rightarrow \frac{i \pm \sqrt{3}}{2}$

$\therefore i$ এর ঘনমূল,

$-i, \frac{i \pm \sqrt{3}}{2}$, $\frac{i - \sqrt{3}}{2}$ (Ans)

(16) L.H.S $= \sqrt{[-1-\sqrt{\{-1-\sqrt{-1- \cdots\cdots \infty)\}]}}}$

Let,

$$x = \sqrt{[-1-\sqrt{\{-1-\sqrt{-1 \cdots \infty)\}]}}}$$

$$\Rightarrow x^2 = -1-\sqrt{[-1-\sqrt{\{-1-\sqrt{-1- \cdots \infty)\}]}}}$$

$$\Rightarrow x^2 = -1-x$$

$$\Rightarrow x^2+x+1=0$$

$$\therefore x = \frac{-1\pm\sqrt{1-4}}{2}$$

$$= \frac{-1\pm\sqrt{-3}}{2}$$

$= \omega$ বা ω^2 (showed)

(17)

Let, ~~$x = \sqrt{[-1-\sqrt{\{-1-\sqrt{-1 \cdots \infty)\}]}}}$~~

$$x = \sqrt{[-2+2\sqrt{\{-2+2\sqrt{(-2+ \cdots \infty)\}]}}}$$

$$\Rightarrow x^2 = -2+2\sqrt{[-2+2\sqrt{\{-2+2\sqrt{(-2+ \cdots \infty)\}]}}}$$

$$\Rightarrow x^2 = -2+2x$$

$$\Rightarrow x^2-2x+2=0$$

$$\therefore x = \frac{2\pm\sqrt{(-2)^2-4.2}}{2}$$

$$= \frac{2\pm\sqrt{4-8}}{2}$$

$$= \frac{2\pm 2i}{2} = \frac{\cancel{2}(1\pm i)}{\cancel{2}}$$

$= (1\pm i)$ (Ans)

(18) $x:y = a+ib : c+id$

$$\frac{x}{y} = \frac{a+ib}{c+id}$$

$$\Rightarrow cx + idx = ay + yib$$

$$\Rightarrow cx - ay = +i(by - dx)$$

$$\Rightarrow (cx-ay)^2 = -1(by-dx)^2$$

$\Rightarrow$ ~~$c^2x^2-2cxay+a^2y^2$~~

$$\Rightarrow (cx-ay)^2 + (by-dx)^2 = 0$$

$$\Rightarrow c^2x^2-2cxay+a^2y^2+b^2y^2-2bydx+d^2x^2$$

$$\Rightarrow x^2(c^2+d^2)-2(ca+bd)xy+y^2(a^2+b^2) =$$

(showed)

(19) (a)

$$(1-\omega^2)(1-\omega^4)(1-\omega^8)(1-\omega^{10})$$

$$\Rightarrow (1-\omega^2)(1-\omega^2)(1-\omega)(1-\omega)$$

$$\Rightarrow (1-2\omega^2+\omega)(1-2\omega+\omega^2)$$

$$\Rightarrow (1+\omega+\omega^2-3\omega^2)(1+\omega+\omega^2-3\omega)$$

$$= (-3\omega^2)(-3\omega)$$

$$= 9\omega^3 = 9$$

(b) ω এর মেথোড না খাটালে 1 হবে

(b) $(x+y)^2+(x\omega+y\omega^2)^2+(x\omega^2+y\omega)^2$

$\Rightarrow x^2+2xy+y^2+x^2\omega^2+2xy+y^2\omega+x^2\omega$
$+2xy+y^2\omega^2$

$\Rightarrow x^2(1+\omega+\omega^2)+y^2(1+\omega+\omega^2)+6xy$

$= x^2(0)+y^2(0)+6xy$

$= 6xy$ (showed)

20।

$z = x+iy$

ⓐ $[z-5] = 3$

$|x+iy-5| = 3$ এখানে রুট দিয়ে ২টা মান [illegible])

$= \sqrt{(x-5)^2+y^2} = 3$

$= (x-5)^2+y^2 = 9$

(Ans)

(21)

b) $x+iy = (a+ib)(c+id)$

$= x+iy = ac+aid+bci-bd$

$\therefore x = ac-bd$

$y = ad+bc$

আবার, $(a-ib)(c-id)$

$\Rightarrow ac-aid-bci+i^2bd$

$\Rightarrow ac \cancel{+} bd - i(ad+bc)$

$\Rightarrow x-iy$ (showed)

(22)

$r = \sqrt{(-1)^2+(\sqrt{3})^2}$

$= \sqrt{1+3} = 2$

আবার, $\theta = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$= \tan^{-1}\tan 60^\circ$

$= 60^\circ$ (Ans)

(23)

$z = x+iy$

$3|z-1| = 2|z-2|$

$\Rightarrow 3|x+iy-1| = 2|x+iy-2|$

$\Rightarrow |3x-3+3iy| = |2x-4+2iy|$

$\Rightarrow \sqrt{(3x-3)^2+(3y)^2} = \sqrt{(2x-4)^2+(2y)^2}$

$\Rightarrow 9x^2-18x+9+9y^2 = 4x^2-16x+16+4y^2$

$\Rightarrow 5x^2+5y^2 = 2x+7$

$5(x^2+y^2) = 2x+7$ (proved)

(24)

$z = x+iy$

$|2z-1| = |z-2|$

$\Rightarrow |2x-1+2iy| = |x-2+iy|$

$\Rightarrow \sqrt{(2x-1)^2+(2y)^2} = \sqrt{(x-2)^2+y^2}$

$\Rightarrow 4x^2-\cancel{4x}+1+4y^2 = x^2-4x+4+y^2$

$\Rightarrow 3x^2-3y^2 = 3$

$\Rightarrow 3(x^2+y^2) = 3$

$x^2+y^2 = 1$ (proved)

(26) $(-1+\sqrt{-3})^4 + (-1-\sqrt{-3})^4$

$\Rightarrow \left(\frac{-1+\sqrt{-3}}{2}\right)^4 \times 16 + \left(\frac{-1-\sqrt{-3}}{2}\right)^4 \times 16$

$= \omega \times 16 + \omega^2 \times 16$

$\Rightarrow 16(\omega + \omega^2)$

$= -16$ (Ans)

(27)

L.H.S $= \sqrt{i} + \sqrt{-i}$

let, $x^2 = (\sqrt{i} + \sqrt{-i})^2$

$= (\sqrt{i})^2 + 2\sqrt{i}\sqrt{-i} + (\sqrt{-i})^2$

$\Rightarrow i + 2\sqrt{1} - i$

$= 2$

$\therefore x = \pm\sqrt{2}$ (Ans)

(29)

$x^2 + y^2 + z^2$

$\rightarrow (p+q)^2 + (p\omega + q\omega^2)^2 + (p\omega^2 + q\omega)^2$

$\Rightarrow p^2 + 2pq + q^2 + p^2\omega^2 + 2pq + q^2\omega + p^2\omega + 2pq + q^2\omega^2$

$\Rightarrow p^2(1+\omega+\omega^2) + q^2(1+\omega+\omega^2) + 6pq$

$\Rightarrow 6pq$ (Ans)

(30) let $n = 3m$ বা 3 দ্বারা বিভাজ্য

L.H.S =

$\left(\frac{-1+\sqrt{-3}}{2}\right)^n + \left(\frac{-1-\sqrt{-3}}{2}\right)^n$

$\Rightarrow (\omega)^{3m} + (\omega^2)^{3m}$

$\Rightarrow \omega^{3m} + (\omega^3)^{2m}$

$\Rightarrow 1 + 1 = 2$ (showed)

Again

$\left(\frac{-1+\sqrt{-3}}{2}\right)^{3m+1} + \left(\frac{-1-\sqrt{-3}}{2}\right)^{3m+1}$

$\Rightarrow \omega^{3m+1} + (\omega^2)^{3m+1}$

$= \omega^{3m}\cdot\omega + (\omega^{3m})^{2m}\cdot\omega^2$

$\Rightarrow \omega + \omega^2 = -1$ (showed)