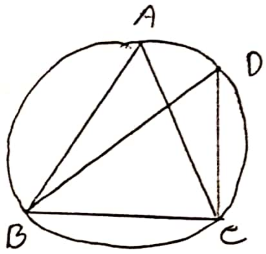
সূত্রাবলি

(1) <u>সাইনসূত্র</u> $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$



প্রমাণঃ A যখন সূক্ষ্মকোণ

ABDC এ কোণ $\angle C$ = সমকোণ

$\therefore \sin \angle BDC = \dfrac{BC}{BD}$

$\sin \angle BDC = \dfrac{a}{BD}$

$\sin \angle BAC = \dfrac{a}{2R}$

$\therefore \sin A = \dfrac{a}{2R}$

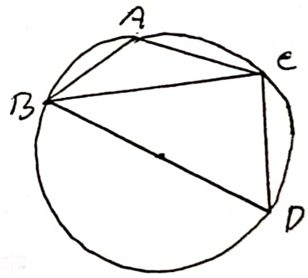
$\boxed{\dfrac{a}{\sin A} = 2R}$

অনুরূপভাবে

$\dfrac{b}{\sin B} = 2R$

এবং

$\dfrac{c}{\sin C} = 2R$



A যখন স্থূলকোণ

ABDC এ $\angle C$ = সমকোণ

$\sin \angle BDC = \dfrac{BC}{BD}$

$\Rightarrow \sin \angle BDC = \dfrac{a}{2R}$

$\Rightarrow \sin(180 - A) = \dfrac{a}{2R}$

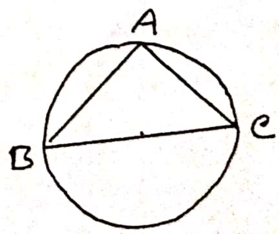
$\Rightarrow \sin A = \dfrac{a}{2R}$

$\therefore \boxed{\dfrac{a}{\sin A} = 2R}$

অনুরূপভাবে

$\dfrac{b}{\sin B} = 2R$

$\dfrac{c}{\sin C} = 2R$



A যখন সমকোণ

$\sin A = \sin 90^\circ$

$= 1$

$= \dfrac{BC}{BC}$

$= \dfrac{a}{2R}$

$\therefore \boxed{\dfrac{a}{\ldots}}$

অনুরূপভাবে

$\dfrac{b}{\sin B} = 2R$ এবং

$c = 2R$

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② প্রমাণ কর: $\frac{b-c}{b+c} = \frac{\sin B - \sin C}{\sin B + \sin C}$

$$L.H.S = \frac{b-c}{b+c}$$

$$= \frac{2R\sin B - 2R\sin C}{2R\sin B + 2R\sin C}$$

$$= \frac{\sin B - \sin C}{\sin B + \sin C}$$

$$= R.H.S$$

③ প্রমাণ কর: $\frac{a-b}{a+b} = \tan\frac{A-B}{2}\tan\frac{C}{2}$

$$L.H.S = \frac{a-b}{a+b}$$

$$= \frac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin B}$$

$$= \frac{\sin A - \sin B}{\sin A + \sin B}$$

$$= \frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}$$

$$= \cot\frac{A+B}{2}\tan\frac{A-B}{2}$$

$$= \cot\frac{180-C}{2}\tan\frac{A-B}{2}$$

$$= \tan\frac{A-B}{2}\tan\frac{C}{2}$$

$$= R.H.S$$

④ $\cos\frac{B-C}{2} = \frac{b+c}{a}\sin\frac{A}{2}$ or $\sin\frac{B-C}{2} = \frac{b-c}{a}\cos\frac{A}{2}$

$$R.H.S = \frac{b+c}{a}\sin\frac{A}{2}$$

$$= \frac{2R\sin B + 2R\sin C}{2R\sin A}\cdot\sin\frac{A}{2}$$

$$= \frac{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}}\cdot\sin\frac{A}{2}$$

$$= \frac{\sin\frac{180-A}{2}\cos\frac{B-C}{2}}{\cos\frac{A}{2}}$$

$$= \frac{\cos\frac{A}{2}\cos\frac{B-C}{2}}{\cos\frac{A}{2}}$$

$$= \cos\frac{B-C}{2} \quad \text{(proved)}$$

⑤ $a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) = 0$

$L.H.S = 2R\sin A(\sin B - \sin C) + 2R\sin B(\sin C - \sin A) + 2R\sin C(\sin A - \sin B)$

$$= 2R\cdot 0$$

$$= 0$$

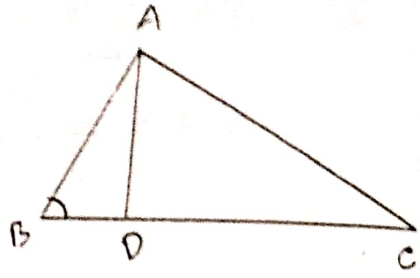
⑰

*লম্ব অভিক্ষেপের সূত্র

$$\left\{\begin{array}{l} a = b\cos C + c\cos B \\ b = a\cos C + c\cos A \\ c = a\cos B + b\cos A \end{array}\right\}$$

$$\underline{a = c\cos B + b\cos C}$$

প্রমাণ: $BC = BD + CD$ — (i)



ΔABD-এ

$$\cos B = \frac{BD}{AB}$$

$$\Rightarrow BD = AB\cos B$$

$$= c\cos B \quad \text{— (ii)}$$

ΔACD-এ

$$\cos C = \frac{CD}{AC}$$

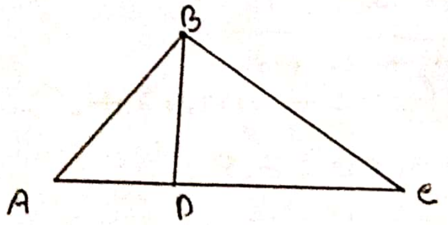
$$\Rightarrow CD = AC\cos C$$

$$= b\cos C \quad \text{— (iii)}$$

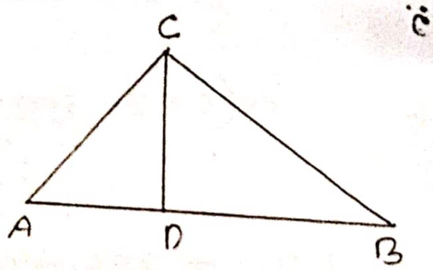
(ii) ও (iii) এর মান (i) এ বসিয়ে

$$\boxed{a = c\cos B + b\cos C}$$

অনুরূপভাবে,



$b = a\cos C + c\cos A$



$c = a\cos B + b\cos A$

⑥ প্রমাণ কর: $a(\cos B + \cos C) = 2(b+c)\sin^2\frac{A}{2}$

$$L.H.S = a(\cos B + \cos C)$$

$$= a\cos B + a\cos C$$

$$= c - b\cos A + b - c\cos A$$

$$= (b+c) - \cos A\,(b+c)$$

$$= (b+c)(1-\cos A)$$

$$= (b+c)\,2\sin^2\frac{A}{2}$$

$$= 2(b+c)\sin^2\frac{A}{2}$$

$$= R.H.S$$

11-45 ⑦ প্রমাণ কর যে, $a(\cos C - \cos B) = 2(b-c)\cos^2\frac{A}{2}$

⑧ $\sin A + \sin B + \sin C = \frac{S}{R}$

$L.H.S = \sin A + \sin B + \sin C$

$= \frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R}$

$= \frac{a+b+c}{2R}$

$= \frac{2S}{2R}$

$= \frac{S}{R} = R.H.S$

⑨ $\frac{1}{a}\sin A + \frac{1}{b}\sin B + \frac{1}{c}\sin C = \frac{6\Delta}{abc}$

$L.H.S = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{2R}$

$= \frac{3}{2R}$

$= \frac{6}{4R} \times \frac{abc}{abc}$

$= \frac{6\Delta}{abc} = R.H.S$

⑩ $\cos A = \sin B - \cos C$ হলে দেখাও যে ত্রিভুজটি সমকোণী

Given, $\cos A = \sin B - \cos C$

$\Rightarrow \cos A + \cos C = \sin B$

$\Rightarrow 2\cos\frac{A+C}{2}\cos\frac{A-C}{2} = \sin B$

$\Rightarrow 2\cos\frac{\pi-B}{2}\cos\frac{A-C}{2} = \sin B$

$\Rightarrow 2\sin\frac{B}{2}\cos\frac{A-C}{2} = 2\sin\frac{B}{2}\cos\frac{B}{2}$

$\Rightarrow \cos\frac{A-C}{2} = \cos\frac{B}{2}$

$\Rightarrow \frac{A-C}{2} = \frac{B}{2}$ $\therefore A = B + C$ (সমকোণী)

(11) <u>কোসাইন কোন</u>

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

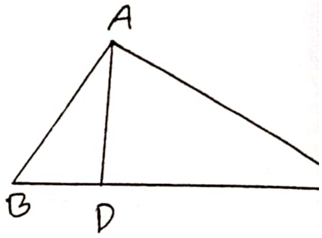
প্রমান: ΔABC এ স্থূল অভিক্ষেপের সূত্রানুযায়ী

$$AB^2 = BC^2 + AC^2 - 2BC \cdot CD$$

$$\Rightarrow c^2 = a^2+b^2 - 2a \cdot b\cos C$$

$$\Rightarrow 2ab\cos C = a^2+b^2-c^2$$

$$\therefore \cos C = \frac{a^2+b^2-c^2}{2ab}.$$



(proved)

অনুরূপভাবে,

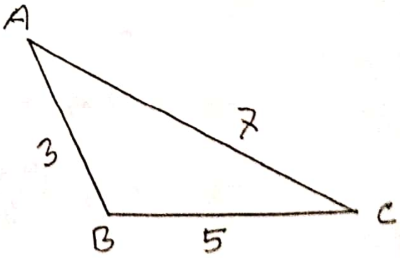
$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

and

$$\cos B = \frac{a^2+c^2-b^2}{2bc}$$

(12) কোন ত্রিভুজের বাহুর দৈর্ঘ্য 3, 5, 7 স্থূলকোনটির মান বের কর।

We know,



$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{5^2+3^2-7^2}{2 \cdot 5 \cdot 3}$$

$$= -1/2$$

$$\Rightarrow \cos B = \cos 120$$

$$\therefore B = 120^\circ$$

(13) $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ হলে দেখাও যে $c = 6$

Given,

$$\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$$

$$\Rightarrow \frac{a+b+2c}{(a+c)(b+c)} = \frac{3}{a+b+c}$$

$$\Rightarrow (a+b+c)(a+b+2c) = 3(a+c)(b+c)$$

$$\Rightarrow (a+b+c)^2 + c(a+b+c) = 3(ab+ac+bc+$$

$$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca+ab+bc+c$$

$$= 3ab+3bc+3ac+3c^2$$

$$\Rightarrow a^2+b^2-c^2 = ab$$

$$\Rightarrow a^2+b^2-c^2 = 1/2\,(2ab)$$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = 1/2$$

$$\Rightarrow \cos C = 1/2$$

$$\Rightarrow \cos C = \cos 60^\circ$$

$$C = 60^\circ$$

(14)

$(a+b+c)(b+c-a) = 3bc$ হলে $A = ?$

Given, $(a+b+c)(b+c-a) = 3bc$

$$\Rightarrow (b+c)^2 - a^2 = 3bc$$

$$\Rightarrow b^2+2bc+c^2-a^2 = 3bc$$

$$\Rightarrow b^2+c^2-a^2 = 1/2\,(2bc)$$

$$\Rightarrow \frac{b^2+c^2-a^2}{2bc} = 1/2$$

$$\Rightarrow \cos A = 1/2$$

$$\therefore A = 60^\circ$$

(15) $a^4+b^4+c^4 = 2c^2(a^2+b^2)$ হলে দেখাও যে, $C = 45^\circ$ অথবা 135°

$$a^4+b^4+c^4 = 2a^2c^2+2b^2c^2$$

$$\Rightarrow (a^2)^2+(b^2)^2+(c^2)^2+2a^2b^2-2b^2c^2-2c^2a^2 = 2a^2b^2$$

$$\Rightarrow (a^2+b^2-c^2)^2 = 2a^2b^2$$

$$\Rightarrow a^2+b^2-c^2 = \pm\sqrt{2}\,ab$$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = \pm\frac{\sqrt{2}}{2}$$

$$\Rightarrow \cos C = \pm 1/\sqrt{2}$$

$\cos C = 1/\sqrt{2}$	$\cos C = \frac{-1}{\sqrt{2}}$
$\Rightarrow \cos C = \cos 45^\circ$	$\cos C = \cos 135^\circ$
$\therefore C = 45^\circ$	$\therefore C = 135^\circ$

(16) + ত্রিভুজ ABC তে প্রমাণ কর, $a\sin(A/2+B) = (b+c)\sin A/2$

$$\frac{b+c}{a}\sin A/2 = \sin(A/2+B)$$

R.H.S

$$\frac{2R\sin B+2R\sin C}{2R\sin A}\sin A/2$$

$$\frac{\sin B+\sin(\pi-(A+B))}{\sin A}\sin A/2$$

$$= \frac{[\sin B+\sin(A+B)]\sin A/2}{\sin A}$$

$$= \frac{\cancel{2}\sin\frac{B+A+B}{2}\cos\left(\frac{A+B-B}{2}\right)\cancel{\sin A/2}}{\cancel{2}\sin A/2\cancel{\cos A/2}}$$

$$= \sin(A/2+B)$$