দ্বিঘাত

① $4x^2-6x+1=0$

$\alpha+\beta=\frac{\cancel{6}^{3}}{\cancel{4}_{2}} \quad \frac{3}{2}$

$\alpha\beta=\frac{1}{4}$

$(\alpha+\frac{1}{\beta})$ ও $(\beta+\frac{1}{\alpha})$ এর যোগফল,

$\alpha+\frac{1}{\beta}+\beta+\frac{1}{\alpha}$

$=\frac{\alpha^2\beta+\alpha+\beta+\alpha\beta^2}{\alpha\beta}$

$=\frac{\alpha\beta(\alpha+\beta)+\alpha+\beta}{\alpha\beta}$

$=\frac{\frac{1}{4}\times\frac{3}{2}+\frac{3}{2}}{\frac{1}{4}}$

$=\frac{3+12}{\cancel{8}_{2}}\times\cancel{4}=\frac{15}{2}$

গুণফল $=(\alpha+\frac{1}{\beta})(\beta+\frac{1}{\alpha})$

$\Rightarrow \alpha\beta+1+1+\frac{1}{\alpha\beta}$

$\Rightarrow \frac{1}{4}+2+4$

$=\frac{1+24}{4}=\frac{25}{4}$

∴ নির্ণেয় সমীকরণটি হবে,

$x^2-\frac{15}{2}x+\frac{25}{4}=0$

$=\frac{4x^2-30x+25}{4}=0$

$=4x^2-30x+25=0$

② $ax^2+bx-a=0$

$\alpha+\beta=\frac{-b}{a}$, $\alpha\beta=\frac{-a}{a}=-1$

$(a\alpha+b)(a\beta+b)$ এর যোগফল =

$a\alpha+b+a\beta+b$

$=a(\alpha+B)+2b$

$=\cancel{a}.\frac{-b}{\cancel{a}}+2b=b$

গুণফল $(a\alpha+b)(a\beta+b)$

$=a^2\alpha\beta+ab\alpha+ab\beta+b^2$

$=a^2(-1)+ab(\alpha+B)+b^2$

$=-a^2+\cancel{a}b\frac{-b}{\cancel{a}}+b^2$

$=-a^2-\cancel{b^2}+\cancel{b^2}=-a^2$

∴ নির্ণেয় সমীকরণটি, $x^2-bx-a^2=0$

③

$x^2-2bx+b^2-a^2=0$

let মূলদ্বয় α,β

$\alpha+\beta=2b$, $\alpha\beta=b^2-a^2$

$(\alpha-\beta)^2=(2b)^2-4(b^2-a^2)$

$=\cancel{4b^2}-\cancel{4b^2}+4a^2$

$\alpha-\beta=2a$

∴ নির্ণেয় সমীকরণটি হবে,

$x^2-(2b+2a)x+4ab=0$

$\Rightarrow x^2-2(a+b)x+4ab=0$ (Ans)

④ $ax^2+bx+c=0$

Let, $\alpha+\beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

$\frac{1}{\alpha^2}$ ও $\frac{1}{\beta^2}$ এর যোগফল =

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}$$

$$= \frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}$$

$$= \frac{\frac{b^2}{a^2}-2\frac{c}{a}}{\frac{c^2}{a^2}}$$

$$= \frac{b^2-2ac}{a^2}\times\frac{a^2}{c^2}$$

$$= \frac{b^2-2ac}{c^2}$$

গুণফল, $= \frac{1}{\alpha^2\beta^2} = \frac{a^2}{c^2}$

∴ নির্ণেয় সমীকরণটি =

$$x^2-\frac{b^2-2ac}{c^2}x+\frac{a^2}{c^2}=0$$

$$\Rightarrow \frac{x^2c^2-(b^2-2ac)x+a^2}{c^2}=0$$

$$\Rightarrow c^2x^2-(b^2-2ac)x+a^2=0$$

(Ans)

⑤ দেয়া আছে,

$a+b+c=0$

$a+b=-c$

$b+c=-a$

$a+c=-b$

L.H.S =

$$(b+c-a)x^2+(c+a-b)x+(a+b-c)=0$$

$$\Rightarrow (-2a)x^2+(-2b)x+(-2c)=0$$

$$\Rightarrow -2(ax^2+bx+c)=0$$

$$\Rightarrow ax^2+bx+c=0$$

মূলদ্বয় মূলদ হলে D পূর্ণবর্গ হবে।

$$D = b^2-4ac$$

$$= (a+c)^2-4ac$$

$$= (a-c)^2$$

যেহেতু ইহা পূর্ণবর্গ তাই মূলদ্বয় মূলদ।

⑥

L.H.S =

$$(a^2-b^2)x^2+2(a^2+b^2)x+(a^2-b^2)$$

মূলদ্বয় মূলদ হলে D পূর্ণবর্গ হবে।

$$D = \{2(a^2+b^2)\}^2-4(a^2-b^2)(a^2-b^2)$$

$$\Rightarrow \{4(a^4+2a^2b^2+b^4)\}-4(a^4-2a^2b^2+b^4)$$

$$\Rightarrow 4a^4+8a^2b^2+4b^4-4a^4+8a^2b^2-4b^4$$

$= 16a^2b^2$ যা একটি পূর্ণবর্গ সংখ্যা।

⑦ দেয়া আছে $b = ka + \frac{c}{k}$

L.H.S = $ax^2 + bx + c = 0$

$D = b^2 - 4ac$

$= (ka + \frac{c}{k})^2 - 4ac$

$= k^2a^2 + 2ka\frac{c}{k} + \frac{c^2}{k^2} - 4ac$

$= (k^2a^2 - 2ac + \frac{c^2}{k^2})$

$= (ka - \frac{c}{k})^2$ ইহা পূর্ণবর্গ।

(Ans)

⑧

$2x^2 - 2(a+b)x + a^2 + b^2 = 0$

মূলগুলি বাস্তব হবে D ধনাত্মক হবে।

$D = \{-2(a+b)\}^2 - 4.2(a^2+b^2)$

$= 4a^2 + 8ab + 4b^2 - 8a^2 - 8b^2$

$= -4a^2 + 8ab - 4b^2$

$= -4(a^2 - 2ab + b^2)$

$= \{-4(a-b)^2\}$

অর্থাৎ পূর্ণবর্গ রাশির সাথে (−) মানের যেগুলোর গুনফল ঋণাত্মকই। অতএব মূলদ্বয় অবাস্তব।

$a = b$ হলে,

$\{-4(a-b)^2\}$

$= -4(b-b)$

$= 0$

যা বাস্তব।

∴ $a \neq b$ না হলে মূলদ্বয় বাস্তব হতে পারেনা।

⑨ $(k-1)x^2 - (k+2)x + 4 = 0$

$D = 0$ হলে মূলদ্বয় বাস্তব ও সমান হবে −

$D = \{-(k+2)\}^2 - 4(k-1).4 = 0$

$= k^2 + 4k + 4 - 16k + 16 = 0$

$= k^2 - 12k + 20 = 0$

$\therefore k = 2, 10$ (Ans)

⑩

$x^2 - 2(p-2)x + 2p - 10$

D ধনাত্মক হলে p বাস্তব হবে −

$D = \{-2(p-2)\}^2 - 4(2p-10)$

$= 4p^2 - 16p + 16 - 8p + 40$

$= 4p^2 - 24p + 56$

$= (2p-6)^2 + 20$

যেহেতু ইহা যোগবোধক তাই ~~p ধনাত্মক~~ মূলদ্বয় বাস্তব।

এখন, let, মূলদ্বয় α, β

$\alpha + \beta = 2p - 4$

$\alpha\beta = 2p - 10$

$(\alpha - \beta)^2 = (2p-4)^2 - 4(2p-10) = 6^2$

$= 4p^2 - 16p + 16 - 8p + 40 - 36 = 0$

$= 4p^2 - 24p + 20 = 0$

$\therefore p = 1, 5$ (Ans)

(11) $a^2x^2+6abx+ac+8b^2=0$

$D=0$ হলে মূলদ্বয় সমান হবে।

$(6ab)^2-4a^2(ac+8b^2)=0$

$=36a^2b^2-4a^3c-32a^2b^2=0$

$\Rightarrow 4a^2b^2-4a^3c=0$

$\Rightarrow 4a^2(b^2-ac)=0$

$\therefore b^2=ac$

এখন, $\cancel{L.H.S}=$ $ac(x+1)^2=4b^2x$

$\Rightarrow b^2(x^2+2x+1)=4b^2x$

$=b^2x^2+2b^2x+b^2-4b^2x=0$

$=b^2x^2-2b^2x+b^2=0$

$D=(-2b^2)^2-4b^4$

$=0$

$\therefore$ ১ম সমীকরণের মূলদ্বয় সমান হলে ২য় সমীকরণের মূলদ্বয় সমান হবে।

(12) $x^2-6x-1+k(2x+1)=0$

$\Rightarrow x^2-6x-1+2kx+k=0$

$\Rightarrow x^2+x(2k-6)+(k-1)=0$

$D=0$ হলে মূলদ্বয় সমান হবে।

$D=(2k\pm6)^2-4(k-1)$

$=4k^2\pm24k+36-4k+4=0$

$=4k^2\mp28k+40=0$

$\Rightarrow k=5,2$ (Ans)

(13) দেয়া আছে,

$ax+by=1$

বা, $y=\dfrac{1-ax}{b}$

এখন, $ax^2+by^2=1$

বা, $ax^2+b\dfrac{(1-ax)^2}{b^2}=1$

এখানে ভুল হয়? $\Rightarrow \dfrac{abx^2+1-2ax+a^2x^2}{b}=1$

$\Rightarrow x^2(ab+a^2)-2ax+1-b=0$

$D=0$ হলে x এর মান সমান হবে।

$\Rightarrow (-2a)^2-4(ab+a^2)(1-b)=0$

$=\cancel{4a^2}-4ab+4ab^2-\cancel{4a^2}+4a^2b=0$

$\Rightarrow 4ab(-1+b+a)=0$

$\Rightarrow a+b=1$ (proved)

(14) $(k+1)x^2+2(k+3)x+(2k+3)$

$D=\{4(k+3)^2\}-4(k+1)(2k+3)=0$

$\Rightarrow 4k^2+24k+36-8k^2-12k-8k-12=0$

$=-4k^2+4k+24=0$

$\therefore k=-2,3$ (Ans)

(15) let সাধারণ মূল α

$$a\alpha^2 + b\alpha + c = 0$$
$$c\alpha^2 + b\alpha + a = 0$$

$$\frac{\alpha^2}{ab-bc} = \frac{\alpha}{c^2-a^2} = \frac{1}{ab-bc}$$

(i) (iii) হতে,

$$\alpha^2 = 1$$
$$\alpha = \pm 1$$

ক্ষেত্রে $\pm$ থাকলে (i) (ii) Term নিয়ে হবে দেখবে মিলে যায়।

(ii) (iii) হতে,

$$\frac{\pm 1}{(c+a)(c-a)} = \frac{1}{-b(c-a)}$$

$\Rightarrow c+a = \pm b$ (proved)

(16) let সাধারণ মূল α

$$p\alpha^2 + q\alpha + 1 = 0$$
$$q\alpha^2 + p\alpha + 1 = 0$$

$$\frac{\alpha^2}{q-p} = \frac{\alpha}{q-p} = \frac{1}{p^2-q^2}$$

(i) (iv) হতে,

$$\alpha = 1$$

(ii) (iii) হতে,

$$\frac{1}{(q-p)} = \frac{1}{-(q-p)(q+p)}$$

$\Rightarrow (q+p) = -1$

$\Rightarrow p+q+1 = 0$ (proved)

(17) let সাধারণ মূল α

$$\alpha^2 + k\alpha - 6k = 0$$
$$\alpha^2 - 2\alpha - k = 0$$

$$\frac{\alpha^2}{-k^2+12k} = \frac{\alpha}{-6k+k} = \frac{1}{-2-k}$$

(i) (ii) হতে,

$$\alpha = \frac{-(k^2+12k)}{-5k}$$

(iv) (iii) হতে,

$$\alpha = \frac{-5k}{-(2+k)}$$

এখন, $\frac{k^2+12k}{5k} = \frac{5k}{2+k}$

$\Rightarrow 2k^2 + k^3 + 24k + 12k^2 - 25k^2 = 0$

$\Rightarrow k^3 - 11k^2 + 24k = 0$

$\Rightarrow k(k^2 - 11k + 24k) = 0$

$\therefore k = 0, 3, 8$ (Ans)

(18)(19) let সাধারণ মূল α,

$$\alpha^2 + b\alpha + c = 0$$
$$\alpha^2 + m\alpha + n = 0$$

$$\frac{\alpha^2}{bn-mc} = \frac{\alpha}{c-n} = \frac{1}{m-b}$$

(i) (iii) হতে,

$$\alpha = \sqrt{\frac{bn-mc}{m-b}}$$ (showed)

(18) $x^2+px+q=0$

let মূলদ্বয়, α, β_1

$\alpha\beta_1 = q$

$x^2+qx+p=0$

let $\alpha\beta_2$

$\cancel{\alpha+\beta_2} = \alpha\beta_2 = p$

$\alpha^2+p\alpha+q=0$

$\alpha^2+q\alpha+p=0$

$\frac{\alpha^2}{p^2-q^2} = \frac{\alpha}{q-p} = \frac{1}{q-p}$

(ii) (iii) হতে,

$\alpha = 1$

(i) (ii)

$\frac{1}{(p+q)\cancel{(p-q)}} = \frac{1}{-\cancel{(p-q)}}$

$= p+q = -1$

$\therefore$ সমীকরণটি, $x^2-(\overset{\beta_1+\beta_2}{p+q})x+pq=0$

$\Rightarrow x^2+x+pq=0$

(Showed) (Ans)

(20) let সাধারণ মূল α

$p\alpha^2+2\alpha+1=0$

$\alpha^2+2\alpha+p=0$

$\frac{\alpha^2}{2p-2} = \frac{\alpha}{1-p^2} = \frac{1}{2p-2}$

(i) (iii) নিয়ে,

$\alpha = \pm 1$

(ii) (iii) নিয়ে,

$\frac{\cancel{\pm 1}}{-2\cancel{(1-p)}} = \frac{\pm 1}{(1+p)(1-p)} = \frac{1}{-2(1-p)}$

$\Rightarrow 1+p = \pm 2$

$\therefore p = 1, -3$ (Ans)

(21)

$ax^2+bx+c=0$

let মূলদ্বয়, $3\alpha, 4\alpha$

যোগফল, $7\alpha = \frac{-b}{a}$

$\alpha = \frac{-b}{7a}$

গুণফল, $12\alpha^2 = \frac{c}{a}$

$\Rightarrow 12\frac{b^2}{49a^{\cancel{2}}} = \frac{c}{\cancel{a}}$

$\Rightarrow 12b^2 = 49ac$

(Proved)

(22) $ax^2+bx+b=0$

let মূলদ্বয় $m\alpha, n\alpha$

যোগফল, $m\alpha+n\alpha = \frac{-b}{a}$

$\alpha(m+n) = \frac{-b}{a}$

$(m+n) = \frac{-b}{a\alpha}$

গুণফল, $mn\alpha^2 = \frac{b}{a}$

$mn = \frac{b}{\alpha^2 a}$

L.H.S $= \sqrt{\frac{m}{n}}+\sqrt{\frac{n}{m}}+\sqrt{\frac{b}{a}}$

$= \frac{m+n}{\sqrt{mn}}+\sqrt{\frac{b}{a}}$

$= \frac{\frac{-b}{a\alpha}}{\sqrt{\frac{b}{\alpha^2 a}}}+\sqrt{\frac{b}{a}}$

$= \frac{-b}{a\alpha}\times\frac{\alpha\sqrt{a}}{\sqrt{b}}+\sqrt{\frac{b}{a}}$

$= -\sqrt{\frac{b}{a}}+\sqrt{\frac{b}{a}} = 0$ (proved)

(23) $ax^2+bx+c=0$

let মূলদ্বয় $\alpha, \alpha r$

যোগফল, $\alpha(1+r) = \frac{-b}{a}$

$\alpha = \frac{-b}{a(1+r)}$

গুণফল $\alpha^2 r = \frac{c}{a}$

$\Rightarrow \frac{b^2 r}{a^2(1+r)^2} = \frac{c}{a}$

$\Rightarrow \frac{r}{(1+r)^2} = \frac{ac}{b^2}$

$\Rightarrow \frac{(1+r)^2}{r} = \frac{b^2}{ac}$ (proved)

(24) $27x^2+6x-(p+2)=0$

let মূলদ্বয়, α, α^2

$\alpha+\alpha^2 = \frac{-6}{27} = -\frac{2}{9}$

বা, $9\alpha^2+9\alpha+2=0$

$\alpha = -\frac{2}{3}, -\frac{1}{3}$

গুণফল, $\alpha^3 = \frac{-p-2}{27}$

$(-\frac{2}{3})$ নিলে, $-\frac{8}{27} = \frac{-p-2}{27}$

$\Rightarrow -p = -8+2$

$p = 6$

$(-\frac{1}{3})$ নিলে, $-\frac{1}{27} = \frac{-p-2}{27}$

$\Rightarrow -p = 2-1 = 1$

$\therefore p = -1$

$\therefore p = -1, 6$ (Ans)

একটি অক্ষর

(25) $ax^2+bx+c=0$

let মূলদ্বয়, α, α^2

যোগফল, $\alpha+\alpha^2=\frac{-b}{a}$ —— (1)

গুনফল, $\alpha^3=\frac{c}{a}$

(1) নং কে ঘন করে,

$(\alpha+\alpha^2)^3=\alpha^3+\alpha^6+3\alpha^3(\alpha+\alpha^2)$

$\Rightarrow \frac{c}{a}+\frac{c^2}{a^2}+3\frac{c}{a}\cdot\frac{-b}{a}=\frac{-b^3}{a^3}$

$=\frac{ac+c^2-3bc}{a^2}=\frac{-b^3}{a^3}$

$\Rightarrow a^2c+ac^2+b^3=3abc$ (showed)

Again,

$c(a-b)^3=a(c-b)^3$

$\Rightarrow \left\{\frac{a-b}{c-b}\right\}^3=\frac{a}{c}$

L.H.S $=$

$\left(\frac{a-b}{c-b}\right)^3=\left\{\frac{a\left(1-\frac{b}{a}\right)}{a\left(\frac{c}{a}-\frac{b}{a}\right)}\right\}^3$

$=\left\{\frac{1+\alpha+\alpha^2}{\alpha^3+\alpha+\alpha^2}\right\}^3$

$=\left\{\frac{(1+\alpha+\alpha^2)}{\alpha(1+\alpha+\alpha^2)}\right\}^3$

$=\frac{1}{\alpha^3}=\frac{a}{c}$ (showed)

(26) $x^2+px+q=0$

let মূলদ্বয়, α, α^2

$\alpha+\alpha^2=-p$ —— (1)

$\alpha^3=q$

(1) নং ঘন করে,

$\alpha^3+\alpha^6+3\alpha^3(\alpha+\alpha^2)=-p^3$

$\Rightarrow q+q^2+3q(-p)+p^3=0$

$\Rightarrow p^3-q(3p-1)+q^2=0$ (showed)

(27) $x^2+px+q=0$

let মূলদ্বয় α, β

$\alpha+\beta=-p \qquad \alpha\beta=q$

$\alpha-\beta=1$

L.H.S $=p^2+4q^2$

$\Rightarrow(\alpha+\beta)^2+4q^2$

$\Rightarrow(\alpha-\beta)^2+4\alpha\beta+4q^2$

$=1+4q+4q^2$

$=(1+2q)^2$ (showed)

(28) $x^2 - bx + c = 0$

Let α, β $\alpha+\beta = b$, $\alpha\beta = c$

$\alpha - \beta = K$

$\Rightarrow b^2 - 4c = K^2$

Again,

$x^2 - cx + b = 0$

$\alpha_1 + \beta_1 = c$ $\alpha_1\beta_1 = b$

$(\alpha_1 - \beta_1)^2 = c^2 - 4b = K^2$

প্রশ্ন, $b^2 - 4c = c^2 - 4b$

$\Rightarrow b^2 - c^2 = 4c - 4b$

$\Rightarrow (b+c)(b-c) = -4(b-c)$

$\Rightarrow b + c + 4 = 0$ (proved)

(29)

$x^2 - px + q = 0$

Let মূলদ্বয়, α, $\alpha+1$

যোগফল, $\cancel{\alpha+1}\cdot \alpha + \alpha + 1 = p$

$2\alpha + 1 = p$

গুণফল, $\alpha^2 + \alpha = q$

L.H.S $= p^2 - 4q - 1 \cancel{= 0}$

$(2\alpha+1)^2 - 4(\alpha^2+\alpha) - 1 \cancel{= 0}$

$\Rightarrow 4\alpha^2 + \cancel{8}4\alpha + 1 - 4\alpha^2 - 4\alpha - 1$

$= 0$ R.H.S (showed)

(30) $\frac{1}{x} + \frac{1}{p-x} = \frac{1}{q}$

$= \frac{p - x + x}{x(p-x)} = \frac{1}{q}$

$= xp - x^2 - pq = 0$

$= x^2 - xp + pq = 0$

Let মূলদ্বয়, α, β

$\alpha + \beta = p$, $\alpha\beta = pq$

$(\alpha - \beta)^2 = p^2 - 4pq = r^2$

$= p^2 - 4pq - r^2 = 0$

$\therefore p = \frac{4q \pm \sqrt{16p^2 + 4r^2}}{2}$ (Ans)

(31) Let মূলদ্বয়, $\alpha, \frac{1}{\alpha}$।

$(k^2-3)x^2 + 3kx(3k+1) = 0$

$\alpha \times \frac{1}{\alpha} = \frac{3k+1}{k^2-3}$

$\Rightarrow k^2 - 3 - 3k - 1 = 0$

$\Rightarrow k^2 - 3k - 4 = 0$

$\therefore k = 4, -1$ (Ans)

(32) $\frac{1}{x} + \frac{1}{x+a} = \frac{1}{m} + \frac{1}{m+a}$

$\Rightarrow \frac{x+a+x}{x(x+a)} = \frac{m+a+m}{m(m+a)}$

$\Rightarrow \frac{2x+a}{x^2+ax} = \frac{2m+a}{m^2+am}$

$\Rightarrow 2mx^2 + x^2a + \cancel{2max} + a^2x =$

$2m^2x + \cancel{2amx} + m^2a + a^2m = 0$

$\Rightarrow x^2(2m+a) + x(a^2-2m^2) - (am^2+a^2m) = 0$

মূলদ্বয় সমান ও বিপরীত চিহ্ন হলে $b = 0$ হবে।

শর্তমতে,

$a^2 - 2m^2 = 0$

বা, $a^2 = 2m^2$ (showed)

(33)

$ax^2+bx+c=0$

$\alpha+\beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

এখন, $cx^2 - 2bx + 4a = 0$

বা, $\frac{c}{a}x^2 - \frac{2b}{a}x + 4 = 0$

বা, $\alpha\beta x + 2(\alpha+\beta)x + 4 = 0$

বা, $\alpha\beta x^2 + 2\alpha x + 2\beta x + 4 = 0$

বা, $\alpha x(\beta x+2) + 2(\beta x+2) = 0$

বা, $x = \frac{-2}{\alpha}, x = \frac{-2}{\beta}$

$\boxed{\text{Ans}}$

(34) $x^2+px+q=0$

Let $\alpha+\beta = -p$, $\alpha\beta = q$

এখন, $q^2x^2 - (p^2-2q)x + 1 = 0$

$\Rightarrow \cancel{(\alpha+\beta)^2x^2}$

$\Rightarrow \alpha^2\beta^2x^2 - \{(\alpha+\beta)^2 - 2\alpha\beta\}x + 1 = 0$

$\Rightarrow \alpha^2\beta^2x^2 - (\alpha^2+\beta^2)x + 1 = 0$

$\Rightarrow \alpha^2\beta^2x^2 - \alpha^2x - \beta^2x + 1 = 0$

$\Rightarrow \alpha^2x(\beta^2x-1) - 1(\beta^2x-1) = 0$

$\Rightarrow x = \frac{1}{\alpha^2}, x = \frac{1}{\beta^2}$

$\boxed{\text{Ans}}$

(35)

$ax^2+bx+c = 0$ —— (i)

$cx^2+bx+a = 0$ —— (ii)

Let

(i) নং এর মূল 2α

(ii) নং " " α

$4\alpha^2a + 2\alpha b + c = 0$

$c\alpha^2 + b\alpha + a = 0$

$\frac{\alpha^2}{2ab-bc} = \frac{\alpha}{c^2-4a^2} = \frac{1}{4ab-2bc}$

(i) (iii) নিয়ে,

$\alpha = \frac{2ab-bc}{c^2-4a^2}$

(ii) (iii) নিয়ে,

$\alpha = \frac{c^2-4a^2}{4ab-2bc}$

Now.

$$\frac{2ab-bc}{c^2-4a^2} = \frac{c^2-4a^2}{4ab-2bc}$$

$\Rightarrow (c^2-4a^2)^2 = b(2a-c).2b(2a-c)$

$\Rightarrow \{(c+2a)(c-2a)\}^2 = 2b^2(2a-c)^2$

$\Rightarrow \{-(2a-c).(2a+c)\}^2 = 2b^2(2a-c)^2$

$\Rightarrow (2a+c)^2 = 2b^2$ $\boxed{\text{Ans}}$

অথবা,

$(2a+c)^2(2a-c)^2 - 2b^2(2a-c)^2 = 0$

$\Rightarrow (2a-c)^2\{(2a-c)^2-2b^2\} = 0$

$\Rightarrow 2a-c = 0$

$\Rightarrow 2a = c$ (proved)

(36)

$x^2+px+q=0$ ——— (i)

$x^2+p_1x+q_1=0$ ——— (ii)

(i) নং এর মূলদ্বয় α, β

$\alpha+\beta=-p$, $\alpha\beta=q$

(ii) নং এর মূলদ্বয় α_1, β_1

$\alpha_1+\beta_1=-p_1$ $\alpha_1\beta_1=q_1$

শর্তমতে, $\frac{\alpha_1}{\beta_1} = \frac{\alpha}{\beta}$

$= \frac{(\alpha_1+\beta_1)^2}{\alpha_1\beta_1} = \frac{(\alpha+\beta)^2}{\alpha\beta}$

$= \frac{p_1^2}{q_1} = \frac{p^2}{q} \Rightarrow p^2q_1 = p_1^2q$

[Showed]

(37) $ax^2-bx+c=0$

let মূলদ্বয় α, β

$\alpha+\beta=\frac{b}{a}$, $\alpha\beta=\frac{c}{a}$

$(\alpha-\beta)^2 = \frac{b^2}{a^2} - 4\frac{c}{a}$

$= \frac{b^2-4ac}{a^2}$

$bx^2-cx+a=0$

$\alpha_1+\beta_1=\frac{c}{b}$, $\alpha_1\beta_1=\frac{a}{b}$

$(\alpha_1-\beta_1)^2 = \frac{c^2}{b^2} - 4\frac{a}{b}$

$= \frac{c^2-4ab}{b^2}$

শর্তমতে,

$$\frac{b^2-4ac}{a^2} = \frac{c^2-4ab}{b^2}$$

$\Rightarrow b^4-4ab^2c = a^2c^2-4a^3b$

$\Rightarrow b^4-a^2c^2 = \cancel{4ab(b^2-}$
$4ab(bc-a^2)$

$\therefore b^4-a^2c^2 = 4ab(bc-a^2)$

[Proved]

(38)

$x^2-(p+q)x+pq=0$

let মূলদ্বয় α, β

$\alpha+\beta=p+q$, $\alpha\beta=pq$

$x^2+bx+c=0$

যোগফল, $2(\alpha+\beta)=-b$

বা, $b=-2(p+q)$

গুণফল, $4\alpha\beta=c$

বা, $c=4pq$

$\boxed{\text{Ans}}$

(39)

$x^2+ax+\frac{1}{4}(a^2-b^2)=0$

$\alpha+\beta=-a$

$\alpha\beta=\frac{1}{4}(a^2-b^2)$

$(\alpha-\beta)^2=(-a)^2-4\cdot\frac{1}{4}(a^2-b^2)$

$=a^2-a^2+b^2$

$\alpha\pm\beta=\pm b$

$\therefore$ সমীকরণটি $=x^2-(-a\pm b)x\pm ab=0$

$\Rightarrow x^2+(a\pm b)x\pm ab=0$

$\boxed{\text{Ans}}$

(40)

$ax^2+bx+c=0$

let মূলদ্বয় α, β

$\alpha+\beta=-\frac{b}{a}$, $\alpha\beta=\frac{c}{a}$

$\Rightarrow a\alpha+b=-a\beta$

$a\beta+b=-a\alpha$

$L.H.S=(a\alpha+b)^{-2}+(a\beta+b)^{-2}$

$=\frac{1}{(a\alpha+b)^2}+\frac{1}{(a\beta+b)^2}$

$=\frac{1}{a^2\beta^2}+\frac{1}{a^2\alpha^2}$

$=\frac{\alpha^2+\beta^2}{a^2\alpha^2\beta^2}=$

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{a^2(\alpha\beta)^2}$

$=\frac{\frac{b^2}{a^2}-2\frac{c}{a}}{a^2(\alpha\beta)^2}=\frac{\frac{b^2-2ac}{\cancel{a^2}}}{\cancel{a^2}(\alpha\beta)^2}$

$=\frac{b^2-2ac}{a^2}\times\frac{a^2}{c^2}\times\cancel{a^2}$

$=\frac{b^2-2ac}{a^2c^2}$

(proved)

(41)

$x^2-px+q=0$

$\alpha+\beta=p$ ——(i)

$\alpha\beta=q$

~~(i) নং থেকে পাই,~~

$\cancel{(\alpha+\beta)^3=}\ p=$

ত্রিঘাতের যোগফল,

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$\Rightarrow p^3-3pq$

[proved]

(42) $4x^2+2x-1=0$

let, অপর মূল β

$\alpha+\beta = \frac{-2}{4} = \frac{-1}{2}$, $\alpha\beta = \frac{-1}{4}$

এখন,

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\alpha^2+\beta^2 = \frac{1}{4} + 2\cdot\frac{1}{4}$

$\beta^2 = \frac{3}{4} - \alpha^2$

$\beta = \frac{3-4\alpha^2}{4\beta}$ → পরেশান হবে

$= \frac{3-4\alpha^2}{4\times\frac{-1}{4\alpha}}$ → শুধু বিজাং মান α এ সাথে তাদের কেটে যাবে।

$= 3-4\alpha^2(-\alpha)$

$= 4\alpha^3 - 3\alpha$ (proved)

(43) একটি মূল $\sqrt{-5}-1 = -1+\sqrt{-5}$

জটিল মূলগুলি একটি অপরটির অনুবন্ধী

∴ অপর মূলটি $-1-\sqrt{-5}$

যোগফল $= -1+\cancel{\sqrt{-5}}-1-\cancel{\sqrt{-5}}$

$= -2$

গুনফল $= (-1+\sqrt{-5})(-1-\sqrt{-5})$

$= (-1)^2 - (\sqrt{-5})^2$

$= 6$

∴ নির্ণেয় সমীকরনটি $x^2+2x+6=0$

$\boxed{\text{Ans}}$

(44) $p+q+r=0$

$p=-q-r$

$q=-p-r$

$r=-p-q$

$x^2+px+qr=0$ —— (i)

$\Rightarrow x^2-qx-rx+qr=0$ [$p=-q-r$ বসিয়ে]

$\Rightarrow x(x-q)-r(x-q)=0$

$\Rightarrow x=r,\ x=q$

$x^2+qx+rq=0$ —— (ii)

$q=-p-r$ বসিয়ে (i) এর অনুরূপ

$x=p \quad x=r$

$x^2+rx+pq=0$ —— (iii)

$r=-p-q$ বসিয়ে, (i) নং এর অনুরূপ

$x=p \quad x=q$

(i) (ii) এর সাধারন মূল r

(ii) (iii) " " " p

(iii) (i) " " " q

(proved)