



Today's agenda

- ↳ Dynamic programming Intro
- ↳ when to use DP
- ↳ steps for DP
- ↳ # of stairs
- ↳ Sol



AlgoPrep

→ DRY → Don't Repeat yourself



Q) Nth fibonacci number

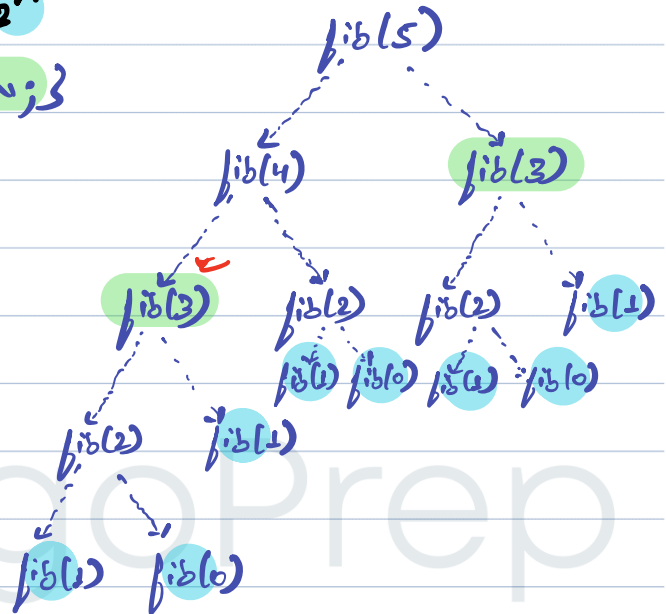
↳ 0 1 1 2 3 5 8 13 21 34

```
int fib(int n) {    T.C:  $2^n$   
    if (n == 0 || n == 1) return n;  
    }
```

```
    int a = fib(n-1);  
    int b = fib(n-2);
```

```
    return a+b;
```

```
}
```



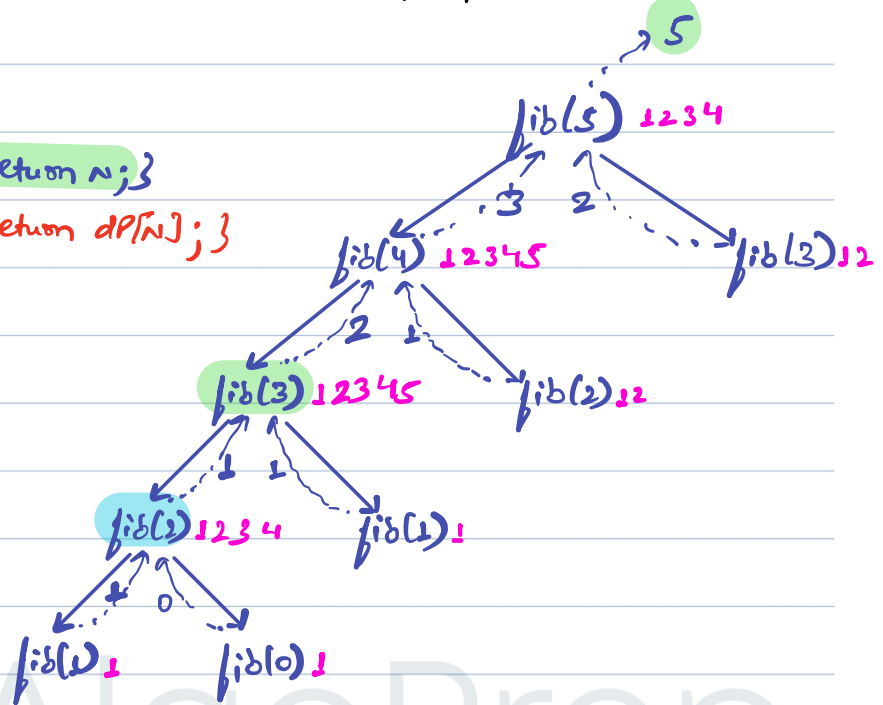


```
int dp[N+1] = {-1};
```

dp

0	1	2	3	4	5
-1	-1	-1	2	3	5

```
int fib(int n) {  
    1 if (n == 0 || n == 1) return n;  
    2 if (dp[n] != -1) return dp[n];  
  
    3 int a = fib(n-1);  
    4 int b = fib(n-2);  
  
    5 dp[n] = a + b;  
    6 return a + b;  
}
```



T.C: $O(N)$

S.C: $O(N)$

→ Optimal Substructure

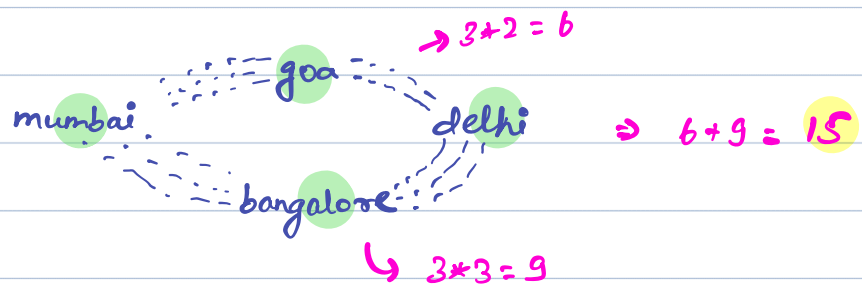
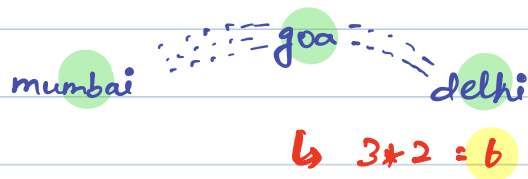
↳ you can divide the problem into subproblem.

→ overlapping subproblems

↳ same problem repeating



// quick questions



AlgoPrep

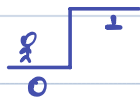


// N Stairs

↳ Given N , how many ways we can go from 0 - N th stairs.

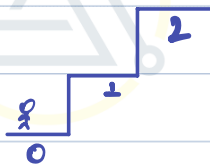
Note: you can take steps of length 1 or 2.

$N=1$



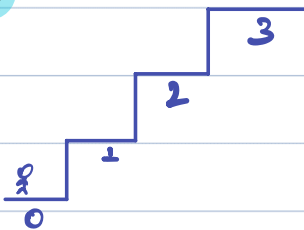
ways = 1

$N=2$



ways = 2
1 1
2

$N=3$



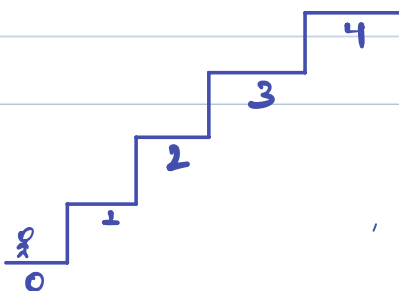
ways = 3

1 1 1

1 2

2 1

$N=4$



ways = 5

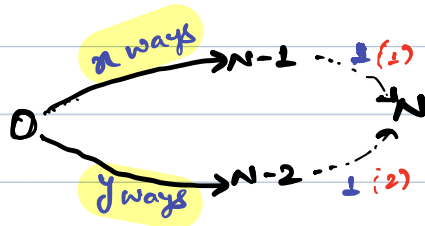
1 1 1 1

1 1 2

1 2 1

2 1 1

2 2



$$x * 1 + y * 1 \Rightarrow x + y$$

No. of ways (N) = No. of ways (N-1) + No. of ways (N-2)

$$N=1 \rightarrow 1$$

$$N=2 \rightarrow 2$$

```
int dp[N+1] = {-1};
```

```
int Stairs(int n) {
```

```
1 if (n==1 || n==2) { return n; }
```

```
2 if (dp[n] != -1) { return dp[n]; }
```

```
3 int a = Stairs(n-1);
```

```
4 int b = Stairs(n-2);
```

```
5 dp[n] = a+b;
```

```
6 return a+b;
```

```
}
```

Break till 9:25 PM



11 Steps for DP → ① optimal substructure
→ ② overlapping subproblem

① dp state: what are we solving → fib(i)

② recurrence relation

↳ relation betⁿ problem and subproblem.

③ dp table

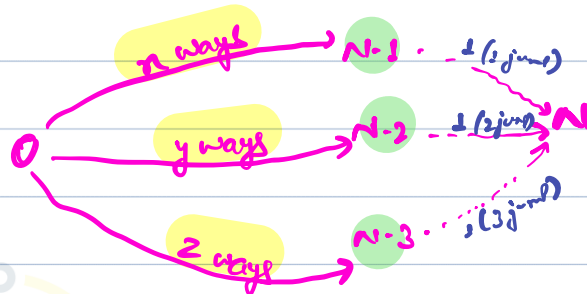
↳ where we store the answer.



// N Stairs

↳ Given N , how many ways we can go from 0 - N th stairs.

Note: you can take steps of length 1 or 2 or 3



Golden rule of recursion:

↳ No. of Calls = No. of Choices.



Q) Find minimum number of Perfect Squares required to Sum = N.

N=6

$$1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 = 6$$

$$2^2 + 1^2 + 1^2 = 3$$

N=10

$$1^2 + 1^2 + \dots + 1^2 = 10$$

$$2^2 + 2^2 + 1^2 + 1^2 = 4$$

$$3^2 + 1^2 = 2$$

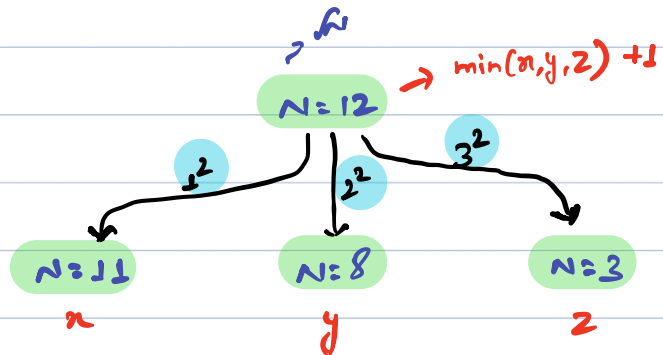
N=9

$$3^2 = 1$$

N=12

$$3^2 + 1^2 + 1^2 + 1^2 = 4$$

$$2^2 + 2^2 + 2^2 = 3$$



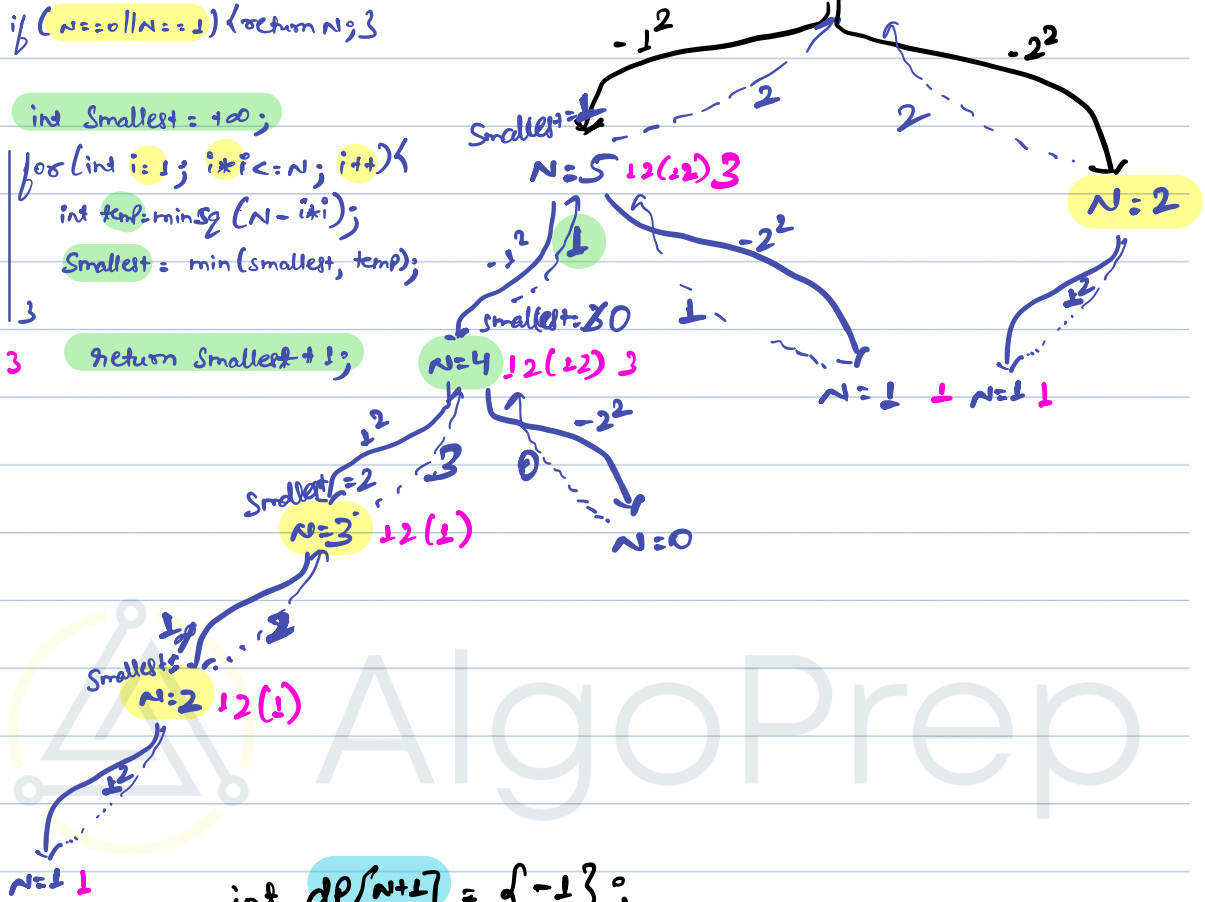
```
int minSq (int N) {  
    if (N==0 || N==1) {return N;}  
    int Smallest = 100;  
    for (int i=1; i*i<=N; i++) {  
        int temp = minSq (N - i*i);  
        Smallest = min (Smallest, temp);  
    }  
    return Smallest + 1;  
}
```

3



```
int minSq (int N) {  
1 if (N==0 || N==1) {return N;}
```

```
2   int Smallest = 100;  
   for (int i=1; i*i<=N; i++) {  
       int temp = minSq (N - i*i);  
       Smallest = min (Smallest, temp);  
3   }  
3   return Smallest + 1;
```



```
int dp[N+1] = {-1};
```

```
int minSq (int N, int[] dp) {  
if (N==0 || N==1) {return N;}  
if (dp[N] != -1) {return dp[N];}
```

T.C:

S.C:

```
int Smallest = 100;  
for (int i=1; i*i<=N; i++) {  
    int temp = minSq (N - i*i, dp);  
    Smallest = min (Smallest, temp);  
3 }
```

```
dp[N] = Smallest + 1;
```

```
return Smallest + 1;
```

3



```
int Smallest = +∞;  
for (int i = 1; i*i ≤ N; i++) {  
    int temp = minSq(N - i*i);  
    Smallest = min(Smallest, temp);  
}
```



AlgoPrep

```
int Smallest = +∞;  
int temp1 = minSq(N - 12);  
Smallest = Math.min(Smallest, temp1);  
int temp2 = minSq(N - 22);  
Smallest = Math.min(Smallest, temp2);  
int temp3 = minSq(N - 32);  
Smallest = Math.min(Smallest, temp3);  
return Smallest + 1;
```