



Today's agenda

↳ Pow(a,n)

↳ TC & SC of recursion



AlgoPrep



Q) Given a and n , calculate a^n .

Ex:

a n

2 3 \rightarrow 8

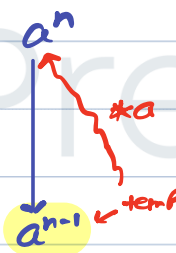
3 5 \rightarrow 243

```
int Pow (int a, int n){  
    if (n==1) { return a; }
```

Faith: Given a and n , calculate and return a^n .

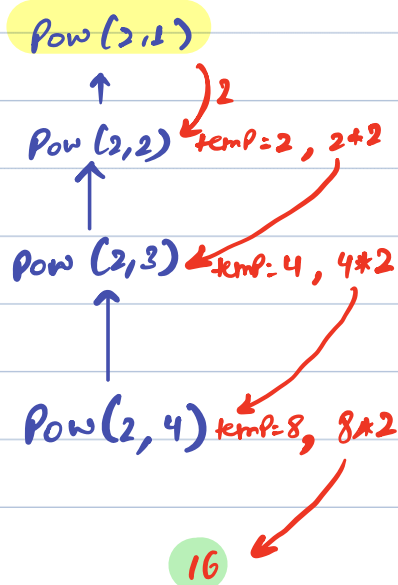
```
    int temp = Pow(a, n-1);  
    return temp * a;  
}
```

main logic?



base case:

$n=0 \rightarrow 1$
or
 $n=1 \rightarrow a$



No. of function: $4 \rightarrow n$

T.C of one function: $O(1)$

overall T.C: $O(1) * N : O(N)$



// Give me something better than $O(N)$.

① $a^n = a^{n-1} * a$

$$\begin{array}{c} 2^5 \\ \downarrow \\ 2^4 \\ \downarrow \\ 2^3 \\ \downarrow \\ 2^2 \\ \downarrow \\ 2^1 \end{array}$$

$a^n \rightarrow a^{n/2}$ ^{temp}

↳ if $n == \text{even} \rightarrow a^n = a^{n/2} * a^{n/2}$ ^{temp temp}

↳ if $n == \text{odd} \rightarrow a^n = a^{n/2} * a^{n/2} * a$

$\text{temp} * \text{temp}$
 $a^{n/2} * a^{n/2} = a^n$

$2^{16} \xrightarrow{\text{temp: } 256} 256 * 256 = 65536$

$2^8 \xrightarrow{\text{temp: } 16} 16 * 16 = 256$

$2^4 \xrightarrow{\text{temp: } 4} 4 * 4 = 16$

$2^2 \xrightarrow{\text{temp: } 2} 2 * 2 = 4$

2^1

$2^{17} \rightarrow 256 * 256 * 2$

2^8



$$a^n \rightarrow a^{n/2} \quad \text{temp}$$

$$\hookrightarrow \text{if } n == \text{even} \rightarrow a^n = a^{n/2} * a^{n/2} \quad \text{temp} \quad \text{temp}$$

$$\hookrightarrow \text{if } n == \text{odd} \rightarrow a^n = a^{n/2} * a^{n/2} * a$$

```
int Pow (int a, int n) {
```

```
    if (n == 1) { return a; }
```

Faith: Given a and n , calculate and return a^n .

T.C: $O(\log N)$

```
    int temp = Pow(a, n/2);
```

main logic: temp

```
    if (n % 2 == 0) return temp * temp; }  $a^n \rightarrow a^{n/2}$ 
```

$$\hookrightarrow \text{if } n == \text{even} \rightarrow a^n = a^{n/2} * a^{n/2} \quad \text{temp} \quad \text{temp}$$

```
    else { return temp * temp * a; }
```

$$\hookrightarrow \text{if } n == \text{odd} \rightarrow a^n = a^{n/2} * a^{n/2} * a$$

```
}
```

base case:

$$a = 2 \quad n = -3$$

$$\hookrightarrow \frac{1}{2^3}$$



```
int Pow (int a, int n){
```

```
    if (n == 1) { return a; }
```

```
    int temp = Pow(a, n/2);
```

```
    if (n % 2 == 0) { return temp * temp; }
```

```
    else { return temp * temp * a; }
```

```
}
```

a = 2

N = 37



N = 18



N = 9



N = 4



N = 2



N = 1

Overall T.C: (T.C of 1 function) *
(Total no. of function)
↳ $\log N$
↳ $O(\log N)$

$$\rightarrow a^n = a^{n/2} * a^{n/2}$$

⇓

$$a^n = a^{n/3} * a^{n/3} * a^{n/3}$$



TC of recursive code

Overall T.C: $(T.C \text{ of } 1 \text{ function}) * \overset{O(1)}{\text{(Total no. of function)}}$

```
int sum (int n) {  
    if (n == 1) return 1; }  
}
```

T.C of 1 function: $O(1)$
no. of function: N

```
int temp = sum (N-1);
```

```
return temp + N;
```

3

S.C: $O(N)$

Total T.C: $O(N)$

sum {	N = 1	→ $O(1)$
sum {	temp N = 2	→ $O(1)$
sum {	temp N = 3	→ $O(1)$
sum {	temp N = 4	→ $O(1)$
sum {	temp N = 5	→ $O(1)$

↳ if you delete a space that is already considered in your space complexity and create new space in place of previous space, you are not increasing your space complexity.

→ T.C of fibonacci



```
int fib (int N) {
```

```
    if (N==0 || N==1) {return N;}
```

T.C of 1 function: $O(1)$

No. of function: $2^N \Rightarrow O(2^N)$

```
    int temp1 = fib(N-1);
```

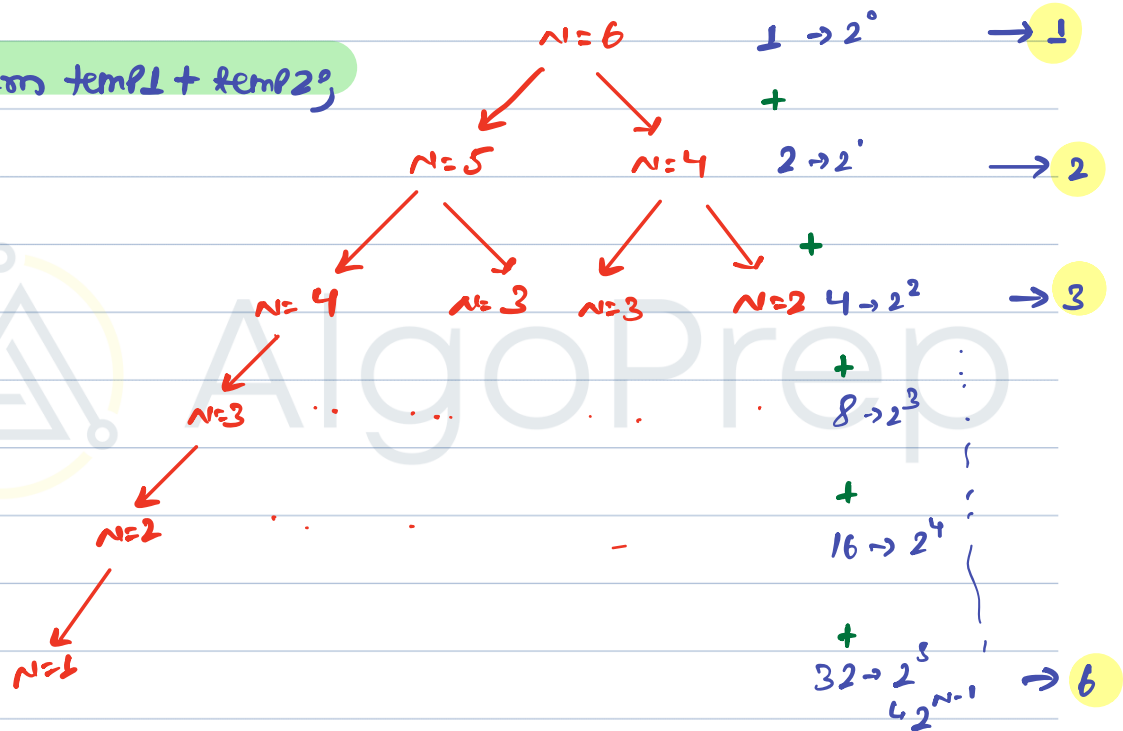
```
    int temp2 = fib(N-2);
```

Total T.C:

```
    return temp1 + temp2;
```

```
}
```

S.C:



$$2^0 + 2^1 + 2^2 + \dots + 2^{N-1} = \text{Sum of A.P.} = a * \frac{(2^N - 1)}{2 - 1}$$



→ S.C: $O(N)$

$$= 1 * \frac{2^N - 1}{2 - 1}$$

$$= 2^N - 1$$

Total S.C: (Space complexity of one function) * max no. of levels.



No. of calls = 2^n \Rightarrow No. of functions = 2^n
Count of levels = n

Break till 9:42 PM

Space Complexity:

↳ How much space are you using.

iterative: variables, Array, String
 ↓ ↓ ↓
 $O(1)$ $n \times \text{size}$ $n \times \text{size}$
 ↓ ↓
 $O(n)$ $O(n)$

recursive: variables, Array, String, Call Stack

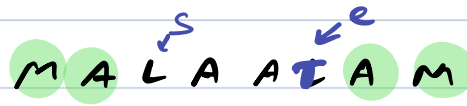
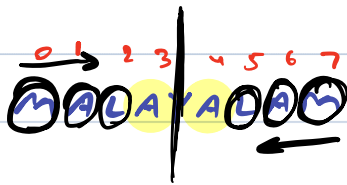
note: Input or output won't be considered in space complexity (Big O notation).



Q) Given an ^{character} array, check if it is Palindrome or not?
↳ recursion

Ex: MALAYALAM → true

Ex: a a b b a → false



```
boolean isPalindrome (char ch[], int s, int e) {  
    if (s == e) { return true; }
```

Faith: check and return whether

ch array is Palindrome between s to e.

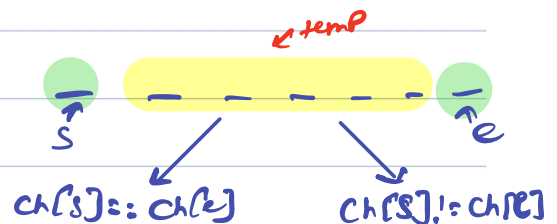
```
    if (ch[s] == ch[e]) {
```

```
        boolean temp = isPalindrome (ch, s+1, e-1);
```

```
        return temp;
```

```
    } else { return false; }
```

Main logic:



temp == false

temp == true

return false

return false

return true

base case:



```
boolean isPalindrom (char ch[], int s, int e) {
    1 if (s == e || s > e) { return true; }
```

0 1 2 3 4 5 6 7
M A L A A L A M

```
2 if (ch[s] == ch[e]) {
    a boolean temp = isPalindrom (ch, s+1, e-1);
    b return temp;
3 } else { return false; }
```

isPalin { s=4 e=3 } 1 2a 2b
isPalin { s=3 e=4 } temp = true 1 2a 2b
isPalin { s=2 e=5 } temp = true 1 2a 2b
isPalin { s=1 e=6 } temp = true 1 2a
isPalin { s=0 e=7 } temp = true 1 2a
true

0 1 2 3 4
a b c d a

isPalin { s=1 e=3 } 1 3
isPalin { s=0 e=4 } temp = false 1 2a 2b
false

```
boolean isPalindrom (char ch[], int s, int e) {
    1 if (s == e || s > e) { return true; }
    2 if (ch[s] == ch[e]) {
        a boolean temp = isPalindrom (ch, s+1, e-1);
        b return temp;
    3 } else { return false; }
```

T.C: $O(1) * \frac{n}{2} \approx O(n/2) \approx O(n)$

S.C: $O(1) * \frac{n}{2} \approx O(n/2) \approx O(n)$