Angel and Shreiner: Interactive Computer Graphics, Sixth Edition

Chapter 11 Solutions

11.1 Let's do the problem in two dimensions. The solution in three dimensions is essentially the same. Assume that the vertices are used in a consistent clockwise or counterclockwise manner. Starting at some vertex, that vertex and the next determine a line of the form

$$ax + by + c = 0.$$

If we evaluate $ax + by + c$ for a given point, the result will be positive or negative depending on which side of the line the point lies. If we are following the vertices in a clockwise manner, the point is inside the polygon if and only if it is to the right of each of these lines.

11.2 For each face of the polygon, we take the first three vertices and form the equation of a plane in the form

$$ax + by + cz + d = 0.$$

Again, as in Exercise 13.1, if we use a consistent order and evaluate $ax + by + cz + d$ for polygon for the given point, we will be inside if and only if we get all positive or all negative values, the sign depending on which order we follow the vertices.

11.3 Consider two identical circles of radius r centered at $(a, 0)$ and $(-a, 0)$. We can describe them through the single implicit equation

$$((x - a)^2 + y^2 - r^2)((x + a)^2 + y^2 - r^2),$$

by simply multiplying together their individual implicit equations. We can form the torus by rotation these circles about the y axis which is equivalent to replacing x^2 by $x^2 + z^2$.

11.4 If we make the usual substitutions

$$x = x_0 + td_x,$$

$$y = y_0 + td_y,$$

$$z = z_0 + td_z,$$

in the equation for the torus, we obtain an equation whose highest power in t is 4. The roots of this equation can be obtained analytically.

11.5 The line from the center of the circle to the closest point on the ray must be perpendicular to the ray. Thus, if the ray is written as $\mathbf{p} = \mathbf{p_0} + \mathbf{td}$ and the circle has radius r and center $\mathbf{p}_c$, we can solve

$$\mathbf{d} \cdot (\mathbf{p_0} + \mathbf{td} - \mathbf{p_c}) = \mathbf{0},$$

for t. We can then check the distance between this point and the center. If it is greater than r, the ray misses the sphere.

11.6 Although each pixel has five rays through it, we do only twice as much work because all the arrays through the corners are on a grid with the same resolution which is shifted one half pixel from the original grid.

11.7 Generally, the depth information has to be retained so that the raster processors can determine which entities are in front.

11.8 The greatest difficulty with blending is that order in which objects are rendered affect the final image. Thus, simply sending the depth and frame buffers is not sufficient to get a consistent rendering. One strategy might be to send the information on all pixels that are in front of the closest opaque object to the raster processors.

11.9 As was discussed in the text, pipeline strategies can be adapted to non–shared-memory architectures. Ray tracing is more difficult to adapt because if there are multiple reflections or translucent objects, all object must be available when a ray is traced. For large data sets, a distributed memory architecture may not have sufficient memory to allow storage of the entire object database on each processor. In this case, a shared-memory machine has a huge advantage.

11.10 Although ugly, the solution is no different in principle from the simple case in the text. Perhaps the easiest approach is to use rotations and translations to convert the arbitrary positioning to the easy case.

11.12 $i + j + k$

11.13 The highest power is $3i$ so there can be $3i + 1$ terms in the polynomial.

11.17 Eventually each processor is responsible for only a small area of the screen. Thus, most primitives will overlap boundaries between these regions and will have to be sent to multiple processors which will negate the advantage of sort first.

11.18 There are 256 (2^8) ways to color the vertices of a cube. If we take out symmetries (rotations, swapping colors), there are 14 distinct cases. Of these 4 cases gave a face with two whites on one diagonal and two blacks on the other and thus have an ambiguous interpretation.