Let $y = \ln\left(\dfrac{6x^{1/5}}{20+4x^{1/5}}\right)$. Find the derivative of $y$ with respect to $x$.
First, we will rewrite $y$ in a more pleasant way:

$$\begin{aligned}
\ln\left(\frac{6x^{1/5}}{20+4x^{1/5}}\right) &= \ln(6x^{1/5}) - \ln(20+4x^{1/5}) \\
&= \ln(6) + \frac{1}{5}\ln(x) - \ln(4(5+x^{1/5})) \\
&= \ln(6) + \frac{1}{5}\ln(x) - \ln(4) - \ln(5+x^{1/5})
\end{aligned}$$

Now, differentiate

$$\begin{aligned}
\frac{d}{dx}\left[\ln\left(\frac{6x^{1/5}}{20+4x^{1/5}}\right)\right] &= \frac{d}{dx}\left[\ln(6) + \frac{1}{5}\ln(x) - \ln(4) - \ln(5+x^{1/5})\right] \\
&= 0 + \frac{1}{5x} - 0 - \frac{\frac{1}{5}x^{-4/5}}{5+x^{1/5}} \\
&= \frac{1}{5x} - \frac{1}{5x^{4/5}\left(5+x^{1/5}\right)}
\end{aligned}$$

where ever $y$ is defined.

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Let $y = \sqrt{\dfrac{1}{x(x+1)}}$. Then

$$\begin{aligned}
y &= \left(\frac{1}{x(x+1)}\right)^{1/2} \\
&= \frac{1^{1/2}}{(x(x+1))^{1/2}} \\
&= \frac{1}{x^{1/2}(x+1)^{1/2}}
\end{aligned}$$

That is, $y = \sqrt{\dfrac{1}{x(x+1)}} = \dfrac{1}{x^{1/2}(x+1)^{1/2}}$. Natural Log both sides:

$$\begin{aligned}
\ln(y) &= \ln\left(\frac{1}{x^{1/2}(x+1)^{1/2}}\right) \\
&= \ln(1) - \ln\left(x^{1/2}(x+1)^{1/2}\right) \\
&= 0 - \ln(x^{1/2}) - \ln((x+1)^{1/2}) \\
&= -\frac{1}{2}\ln(x) - \frac{1}{2}\ln(x+1) \\
&= -\frac{1}{2}\left(\ln(x) + \ln(x+1)\right)
\end{aligned}$$

Multiply both sides by $-2$ and then implicate differentiate:

$$\begin{aligned}
\frac{d}{dx}\left[-2\ln(y)\right] &= \frac{d}{dx}\left[\ln(x) + \ln(x+1)\right] \\
-2\frac{1}{y}\frac{dy}{dx} &= \frac{1}{x} + \frac{1}{x+1} \\
\frac{dy}{dx} &= -\frac{y}{2}\left(\frac{1}{x} + \frac{1}{x+1}\right)
\end{aligned}$$

Where $y = \sqrt{\dfrac{1}{x(x+1)}}$

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Let $y = \ln\left(\frac{(x^2+8)^5}{\sqrt{4-x}}\right)$. Then

$$y = \ln\left(\frac{(x^2+8)^5}{\sqrt{4-x}}\right) = ... = 5\ln(x^2+8) - \frac{1}{2}\ln(4-x)$$

If we set $f(x) = \ln(x^2+8)$ and $g(x) = \ln(4-x)$. Then

$$y = \ln\left(\frac{(x^2+8)^5}{\sqrt{4-x}}\right) = 5f(x) - \frac{1}{2}g(x)$$

and

$$y' = \frac{d}{dx}\left[\ln\left(\frac{(x^2+8)^5}{\sqrt{4-x}}\right)\right] = 5f'(x) - \frac{1}{2}g'(x)$$

Now, find $f'(x)$ and $g'(x)$.

$$f'(x) = \left(\ln(x^2+8)\right)' = \frac{\left(x^2+8\right)'}{x^2+8} = \frac{2x}{x^2+8}$$

and

$$g'(x) = (\ln(4-x))' = \frac{(4-x)'}{4-x} = \frac{-1}{4-x}$$

Therefore, putting everything together we have:

$$y' = \frac{d}{dx}\left[\ln\left(\frac{(x^2+8)^5}{\sqrt{4-x}}\right)\right] = 5f'(x) - \frac{1}{2}g'(x) = 5\left(\frac{2x}{x^2+8}\right) - \frac{1}{2}\left(\frac{-1}{4-x}\right)$$

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Let $y = \cot^{-1}\left(\sqrt{4x^2-1}\right) + \sec^{-1}(2x)$, $f(x) = \sqrt{4x^2-1}$, and $g(x) = 2x$. Then

$$f'(x) = \frac{d}{dx}\left[(4x^2-1)^{1/2}\right] = \frac{(4x^2-1)'}{2\sqrt{4x^2-1}} = \frac{8x}{2\sqrt{4x^2-1}} \tag{1}$$

and

$$g'(x) = \frac{d}{dx}[2x] = 2 \tag{2}$$

Since $\frac{d}{dx}\left[\cot^{-1}(x)\right] = -\frac{1}{x^2+1}$ and chain rule we have

$$\frac{d}{dx}\left[\cot^{-1}(f(x))\right] = -\frac{1}{\left(f(x)\right)^2+1} \cdot f'(x) \tag{3}$$

Since $\frac{d}{dx}\left[\sec^{-1}(x)\right] = \frac{1}{|x|\sqrt{x^2-1}}$ and chain rule we have

$$\frac{d}{dx}\left[\sec^{-1}(g(x))\right] = \frac{1}{|g(x)|\sqrt{\left(g(x)\right)^2-1}} \cdot g'(x) \tag{4}$$

By Equations (1) and (3) we have

$$\frac{d}{dx}\left[\cot^{-1}\left(\sqrt{4x^2-1}\right)\right] = -\frac{1}{\left(\sqrt{4x^2-1}\right)^2+1} \cdot \frac{8x}{2\sqrt{4x^2-1}} = ... = -\frac{1}{x\sqrt{x^2-1}} \tag{5}$$

By Equations (2) and (4) we have

$$\frac{d}{dx}\left[\sec^{-1}(2x)\right] = \frac{1}{|2x|\sqrt{(2x)^2 - 1}} \cdot (2) \tag{6}$$

Therefore, by Equations (5) and (6) we have

$$\frac{d}{dx}\left[\cot^{-1}\left(\sqrt{4x^2 - 1}\right) + \sec^{-1}(2x)\right] = -\frac{1}{\left(\sqrt{4x^2 - 1}\right)^2 + 1} \cdot \frac{8x}{2\sqrt{4x^2 - 1}} + \frac{2}{|2x|\sqrt{(2x)^2 - 1}}$$

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