Math 351: Homework 1 (Due September 14)

Name: Jack Ellert-Beck

Section 0.2

Problem 1

Prove the following equivalences:

a) $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$

We can write a truth table that lists the truth values of $\neg(P \vee Q)$ and $(\neg P \wedge \neg Q)$ for all truth values of P and Q:

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$(\neg P \wedge \neg Q)$
T	T	F	F	T	**F**	**F**
T	F	F	T	T	**F**	**F**
F	T	T	F	T	**F**	**F**
F	F	T	T	F	**T**	**T**

Since the two statements have the same truth value in all cases, they are equivalent.

b) $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$

We can write a truth value to test every case:

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$
T	T	F	F	T	**F**	**F**
T	F	F	T	F	**T**	**T**
F	T	T	F	F	**T**	**T**
F	F	T	T	F	**T**	**T**

Since both statements have the same truth value for any value of P or Q, they are equivalent.

Problem 2

Prove that $P \implies Q \equiv (\neg P) \vee Q$. Deduce that the negation of $P \implies Q$ is $P \wedge (\neg Q)$.

We write a truth table to show that $P \implies Q$ has the same truth value as $(\neg P) \vee Q$ in all cases:

P	Q	$\neg P$	$(\neg P) \vee Q$	$P \implies Q$
T	T	F	**T**	**T**
T	F	F	**F**	**F**
F	T	T	**T**	**T**
F	F	T	**T**	**T**

Since $P \implies Q$ is equivalent to $(\neg P) \vee Q$, their negations will be equivalent. So we negate $(\neg P) \vee Q$ and manipulate using equivalences from Problem 1 to find the negation:

$$\begin{aligned}\neg(P \implies Q) &= \neg(\neg P \vee Q)\\ &= \neg(\neg P \vee \neg(\neg Q))\\ &= \neg(\neg(P \wedge (\neg Q)))\\ &= P \wedge (\neg Q).\end{aligned}$$

We also used the fact that $\neg(\neg P) \equiv P$, which we now show with a truth table:

P	$\neg P$	$\neg(\neg P)$
T	F	**T**
F	T	**F**

Problem 3

Find the negation of the following statements.

a) $\neg(P \wedge \neg Q) \vee R$

$$\begin{aligned}\neg(\neg(P \wedge \neg Q) \vee R) &= (P \wedge \neg Q) \wedge \neg R\\ &= P \wedge \neg Q \wedge \neg R.\end{aligned}$$

b) $P \implies (Q \vee R)$

$$\begin{aligned} \neg(P \implies (Q \vee R)) &= P \wedge \neg(Q \vee R) \\ &= P \wedge \neg Q \wedge \neg R. \end{aligned}$$

c) $\neg(P \vee Q) \implies (R \vee S)$

$$\begin{aligned} \neg(\neg(P \vee Q) \implies (R \vee S)) &= \neg((P \vee Q) \vee (R \vee S)) \\ &= \neg(P \vee Q) \wedge \neg(R \vee S) \\ &= \neg P \wedge \neg Q \wedge \neg R \wedge \neg S. \end{aligned}$$

Problem 5

Prove that an implication is equivalent to its contrapositive.

The contrapositive of $P \implies Q$ is the statement $\neg Q \implies \neg P$. We can manipulate the statement $\neg P \vee Q$, noting that it is equivalent to $P \implies Q$:

$$\begin{aligned} \neg P \vee Q &= Q \vee \neg P \\ &= \neg(\neg Q) \vee \neg P \\ &= \neg Q \implies \neg P. \end{aligned}$$

Problem 6

An example of an implication whose converse is not true:

If polygon $ABCD$ is a square, then its interior angles are all 90 degrees.

An example of an implcation whose converse is true:

$$(x = 4) \implies (x + 2 = 6)$$

Problem 7

Section 0.3

Problem 1

Problem 2

Problem 3

Problem 4

Problem 5

Problem 17