Math 351: Homework 1 (Due September 14)

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From the **0.1** exercises on pages 9, work exercises 1,2,3,5,6,7.

Problem 1

Prove the following equivalences:

a) $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$(\neg P \wedge \neg Q)$
T	T	F	F	T	**F**	**F**
T	F	F	T	T	**F**	**F**
F	T	T	F	T	**F**	**F**
F	F	T	T	F	**T**	**T**

b) $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$(\neg P \vee \neg Q)$
T	T	F	F	T	**F**	**F**
T	F	F	T	F	**T**	**T**
F	T	T	F	F	**T**	**T**
F	F	T	T	F	**T**	**T**

Problem 2

Prove that $P \implies Q \equiv (\neg P) \vee Q$. Deduce that the negation of $P \implies Q$ is $P \wedge (\neg Q)$.

P	Q	$\neg P$	$(\neg P) \vee Q$	$P \implies Q$
T	T	F	**T**	**T**
T	F	F	**F**	**F**
F	T	T	**T**	**T**
F	F	T	**T**	**T**

Since $P \implies Q$ is equivalent to $(\neg P) \vee Q$, their negations will be equivalent. So we negate $(\neg P) \vee Q$ and manipulate using equivalences from Problem 1 to find the negation:

$$\begin{aligned} \neg(P \implies Q) &= \neg(\neg P \vee Q) \\ &= \neg(\neg P \vee \neg(\neg Q)) \\ &= \neg(\neg(P \wedge (\neg Q))) \\ &= P \wedge (\neg Q). \end{aligned}$$

We also used the fact that $\neg(\neg P) \equiv P$, which we now show with a truth table:

P	$\neg P$	$\neg(\neg P)$
T	F	**T**
F	T	**F**

Problem 3

Find the negation of the following statements.

a) $\neg(P \wedge \neg Q) \vee R$

$$\begin{aligned} \neg(\neg(P \wedge \neg Q) \vee R) &= (P \wedge \neg Q) \wedge \neg R \\ &= P \wedge \neg Q \wedge \neg R. \end{aligned}$$

b) $P \implies (Q \vee R)$

$$\begin{aligned} \neg(P \implies (Q \vee R)) &= P \wedge \neg(Q \vee R) \\ &= P \wedge \neg Q \wedge \neg R. \end{aligned}$$

c) $\neg(P \vee Q) \implies (R \vee S)$

$$\begin{aligned} \neg(\neg(P \vee Q) \implies (R \vee S)) &= \neg(\neg(P \vee Q) \wedge \neg(R \vee S)) \\ &= (P \vee Q) \vee (R \vee S) \\ &= P \vee Q \vee R \vee S. \end{aligned}$$