Calculus BC Important Info Sheet

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Part I

Information

1 Functions

1.1 Even and Odd Functions

A function $y = f(x)$ is **even** if $f(-x) = f(x)$ for every x in the function's domain. Every even function is summertric about the y-axis.

$$\text{Example:} \qquad (-x)^2 = x^2$$

A function $y = f(x)$ is **odd** if $f(x) = -f(x)$ for every x in the function's domain. Every odd function is symmertric about the origin.

$$\text{Example:} \qquad (-x)^3 = -(x)^3$$

1.2 Periodicity

A function $f(x)$ is **periodic** with period p $(p > 0)$ if $f(x + p) = f(x)$ for every value of x.

Sinusoids are examples of periodic functions. Specifically, the period of the function $y = A\sin(Bx + C)$ or $y = A\cos(Bx + C)$ is $\frac{2\pi}{|B|}$. The amplitude is $|A|$. The period of $y = \tan(x)$ is π.

1.3 Inverse Functions

If f and g are two functions such that $f(g(x)) = x$ for every x in the domain of g, and $g(f(x)) = x$ for every x in the domain of f, then f and g are inverse functions of one another.

A function f has an inverse if and only if no horizontal line intersects its graph more than once.

If f is either increasing or decreasing in an interval, then f has an inverse.

For information on the derivatives of inverse functions, see THE THING THAT SHOULD BE HERE.

2 Limits

2.1 Definition of a Limit

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. Then $\lim_{x\to c} f(x) = L$ means that for each $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

2.1.1 Symbolic Definitions of Limits

- Definition of a finite limit at a specific point:

$$\lim_{x\to c} = L \Leftrightarrow (\forall\varepsilon > 0, \exists\delta > 0, 0 < |x-c| < \delta \Rightarrow |f(x)-L| < \varepsilon)$$

- Definition of an undefined limit at a specific point:

$$\lim_{x\to c} = \infty \Leftrightarrow (\forall M > 0, \exists\delta > 0, 0 < |x-c| < \delta \Rightarrow f(x) > M)$$

$$\lim_{x\to c} = -\infty \Leftrightarrow (\forall M < 0, \exists\delta > 0, 0 < |x-c| < \delta \Rightarrow f(x) < M)$$

- Definition of a finite limit at infinity:

$$\lim_{x\to\infty} = L \Leftrightarrow (\forall\varepsilon > 0, \exists M, x > M \Rightarrow |f(x)-L| < \varepsilon)$$

$$\lim_{x\to-\infty} = L \Leftrightarrow (\forall\varepsilon > 0, \exists M, x < M \Rightarrow |f(x)-L| < \varepsilon)$$

2.2 Continuity

A function $y = f(x)$ is **continuous** at $x = a$ if $f(a)$ exists, $\lim\limits_{x\to a} f(x)$ exists, and $\lim\limits_{x\to a} f(x) = f(a)$. $y = f(x)$ is continuous on (a, b) if $f(x)$ is continuous for every $x \in (a, b)$.

2.3 Horizontal and Vertical Asymptotes

A line $y = b$ is a **horizontal asymptote** of the graph of $y = f(x)$ if either $\lim\limits_{x\to\infty} f(x) = b$ or $\lim\limits_{x\to-\infty} f(x) = b$.

A line $x = a$ is a **vertical asymptote** of the graph of $y = f(x)$ if either $\lim\limits_{x\to a^+} f(x) = \pm\infty$ or $\lim\limits_{x\to a^-} f(x) = \pm\infty$.

2.4 Evaluating Limits

2.4.1 Limits of Rational Functions as $x \to \pm\infty$

- $\lim\limits_{x\to\pm\infty} \dfrac{f(x)}{g(x)} = 0$ if the degree of $f(x)$ is less than the degree of $g(x)$.

$$\text{Example:} \qquad \lim_{x\to\infty} \frac{x^2-2x}{x^3+3} = 0$$

- $\lim\limits_{x\to\pm\infty} \dfrac{f(x)}{g(x)}$ is infinite if the degree of $f(x)$ is greater than the degree of $g(x)$.

$$\text{Example:} \qquad \lim_{x\to\infty} \frac{x^3+2x}{x^2-8} = \infty$$

- $\lim\limits_{x\to\pm\infty} \dfrac{f(x)}{g(x)}$ is finite if the degree of $f(x)$ is equal to the degree of $g(x)$. The limit will be equal to the ratio of the leading coefficients of $f(x)$ to $g(x)$.

$$\text{Example:} \qquad \lim_{x\to\infty} \frac{2x^2-3x+2}{10x-5x^2} = -\frac{2}{5}$$

Lorem Ipsum