

Calculus BC Important Info Sheet

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Part I

Information

1 Functions

1.1 Even and Odd Functions

A function $y = f(x)$ is **even** if $f(-x) = f(x)$ for every x in the function's domain. Every even function is symmetric about the y -axis.

$$\text{Example: } (-x)^2 = x^2$$

A function $y = f(x)$ is **odd** if $f(x) = -f(-x)$ for every x in the function's domain. Every odd function is symmetric about the origin.

$$\text{Example: } (-x)^3 = -(x)^3$$

1.2 Periodicity

A function $f(x)$ is **periodic** with period p ($p > 0$) if $f(x + p) = f(x)$ for every value of x .

Sinusoids are examples of periodic functions. Specifically, the period of the function $y = A \sin(Bx + C)$ or $y = A \cos(Bx + C)$ is $\frac{2\pi}{|B|}$. The amplitude is $|A|$. The period of $y = \tan(x)$ is π .

1.3 Inverse Functions

The inverse of a function $f(x)$ is often written as $f^{-1}(x)$, or given a new name such as $g(x)$.

If f and g are two functions such that $f(g(x)) = x$ for every x in the domain of g , and $g(f(x)) = x$ for every x in the domain of f , then f and g are inverse functions of one another.

A function f has an inverse if and only if no horizontal line intersects its graph more than once.

If f is either increasing or decreasing in an interval, then f has an inverse.

For information on the derivatives of inverse functions, see Section 3.4.

2 Limits

2.1 Definition of a Limit

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. Then $\lim_{x \rightarrow c} f(x) = L$ means that for each $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

2.1.1 Symbolic Definitions of Limits

- Definition of a finite limit at a specific point:

$$\lim_{x \rightarrow c} = L \Leftrightarrow (\forall \varepsilon > 0, \exists \delta > 0, 0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon)$$

- Definition of an undefined limit at a specific point:

$$\lim_{x \rightarrow c} = \infty \Leftrightarrow (\forall M > 0, \exists \delta > 0, 0 < |x - c| < \delta \Rightarrow f(x) > M)$$

$$\lim_{x \rightarrow c} = -\infty \Leftrightarrow (\forall M < 0, \exists \delta > 0, 0 < |x - c| < \delta \Rightarrow f(x) < M)$$

- Definition of a finite limit at infinity:

$$\lim_{x \rightarrow \infty} = L \Leftrightarrow (\forall \varepsilon > 0, \exists M, x > M \Rightarrow |f(x) - L| < \varepsilon)$$

$$\lim_{x \rightarrow -\infty} = L \Leftrightarrow (\forall \varepsilon > 0, \exists M, x < M \Rightarrow |f(x) - L| < \varepsilon)$$

2.2 Continuity

A function $y = f(x)$ is **continuous** at $x = a$ if $f(a)$ exists, $\lim_{x \rightarrow a} f(x)$ exists, and $\lim_{x \rightarrow a} f(x) = f(a)$. $y = f(x)$ is continuous on (a, b) if $f(x)$ is continuous for every $x \in (a, b)$.

2.3 Horizontal and Vertical Asymptotes

A line $y = b$ is a **horizontal asymptote** of the graph of $y = f(x)$ if either $\lim_{x \rightarrow \infty} f(x) = b$ or $\lim_{x \rightarrow -\infty} f(x) = b$.

A line $x = a$ is a **vertical asymptote** of the graph of $y = f(x)$ if either $\lim_{x \rightarrow a^+} f(x) = \pm\infty$ or $\lim_{x \rightarrow a^-} f(x) = \pm\infty$.

2.4 Evaluating Limits

2.4.1 Limits of Rational Functions as $x \rightarrow \pm\infty$

- $\lim_{x \rightarrow \pm\infty} \frac{f(x)}{g(x)} = 0$ if the degree of $f(x)$ is less than the degree of $g(x)$.

$$\text{Example: } \lim_{x \rightarrow \infty} \frac{x^2 - 2x}{x^3 + 3} = 0$$

- $\lim_{x \rightarrow \pm\infty} \frac{f(x)}{g(x)}$ is infinite if the degree of $f(x)$ is greater than the degree of $g(x)$.

$$\text{Example: } \lim_{x \rightarrow \infty} \frac{x^3 + 2x}{x^2 - 8} = \infty$$

- $\lim_{x \rightarrow \pm\infty} \frac{f(x)}{g(x)}$ is finite if the degree of $f(x)$ is equal to the degree of $g(x)$. The limit will be equal to the ratio of the leading coefficients of $f(x)$ to $g(x)$.

$$\text{Example: } \lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 2}{10x - 5x^2} = -\frac{2}{5}$$

2.4.2 Remarkable Limits

See THE THING THAT HAPPENS LATER.

2.5 Intermediate Value Theorem

A function $y = f(x)$ that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$. As a more specific result, if f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ differ in sign, then the equation $f(x) = 0$ has at least one solution in the open interval $[a, b]$.

3 Derivatives

3.1 Notation of Derivatives

The derivative of the function $y = f(x)$ is commonly written as any of the following:

$$\begin{array}{ccc} f'(x) & [f(x)]' & y' \\ \frac{dy}{dx} & \frac{d}{dx}f(x) & \end{array}$$

Higher order derivatives can be written in a number of ways. Below are the respective notations for second, fourth, and nth order derivatives in three different styles.

$$\begin{array}{ccc} \frac{d^2y}{dx^2} & \frac{d^4y}{dx^4} & \frac{d^ny}{dx^n} \\ f''(x) & f^{IV}(x) & f^N(x) \\ f^{(2)}(x) & f^{(4)}(x) & f^{(n)} \end{array}$$

3.2 Rate of Change

If (x_0, y_0) and (x_1, y_1) are points on the graph of $y = f(x)$, then the **average rate of change** of y with respect to x over the interval $[x_0, x_1]$ is

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y}{\Delta x}$$

If (x_0, y_0) is a point on the graph of $y = f(x)$, then the **instantaneous rate of change** of y with respect to x at x_0 is $f'(x_0)$.

3.3 Definition of a Derivative

The derivative of a function $f(x)$ at the point $x = a$ can be defined in either of two ways:

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\ f'(a) &= \lim_{b \rightarrow a} \frac{f(b) - f(a)}{b - a} \end{aligned}$$

If this limit exists at a , then $f(x)$ is said to be **differentiable** at a .

If a function is differentiable at a point $x = a$, it is continuous at that point. The converse is false, i.e. continuity does not imply differentiability.

3.4 Derivatives of Inverse Functions

If f is differentiable at every point on an interval I , and $f'(x) \neq 0$ on I , then $g = f^{-1}(x)$ is differentiable at every point of the interior of the interval $f(I)$ and $g'(f(x)) = \frac{1}{f'(x)}$.

3.5 Finding Maxima and Minima

3.6 Monotonicity

3.7 Concavity

3.8 Rolle's Theorem

3.9 Mean Value Theorem

3.10 Extreme Value Theorem

3.11 Linear Approximation

3.12 Newton's Method

3.13 L'Hôpital's Rule

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