1 Key Concepts

- Kelvin's and Helmholtz's Theorems
- Biot-Savart Law

2 Important Equations

$$\frac{D\overrightarrow{\omega}}{Dt} = (\overrightarrow{\omega} \cdot \nabla)\overrightarrow{u} + \nu \nabla^2 \overrightarrow{u} \qquad \text{(Conservative Body Vorticity Equation)}$$

$$\overrightarrow{u}(\overrightarrow{x}, t) = \frac{1}{4\pi} \int_V' \frac{\overrightarrow{\omega}(\overrightarrow{x'}, t) \times \left(\overrightarrow{x} - \overrightarrow{x'}\right)}{|\overrightarrow{x} - \overrightarrow{x'}|^3} d^3x' \qquad \text{(Biot-Savart Induced Flow)}$$

$$\overrightarrow{u}(\overrightarrow{x}, t) = \frac{\Gamma}{4\pi} \int_{vortex} \overrightarrow{e}_\omega \times \frac{\left(\overrightarrow{x} - \overrightarrow{x'}\right)}{|\overrightarrow{x} - \overrightarrow{x'}|^3} dl \qquad \text{(Biot-Savart Induced Flow - Thin Vortex)}$$

$$\frac{D\Gamma}{Dt} = \int_C \frac{1}{\rho} \frac{d\tau_{ij}}{dx_j} dx_i \qquad \text{(Material Derivative of Circulation)}$$

3 Practice Problems

1. (Example 5.3 of *Fluid Mechanics, Sixth Edition*) In cylindrical coordinates, pure-strain extensional (stretching) flow along the z-axis is given by: $u_r = -(\gamma/2)r$ and $u_z = \gamma z$ where γ is the strain rate. A vortex aligned with the z-axis has vorticity $\omega_z = \omega_0(r)$ at $t = 0$. What is $\omega_z(r,t)$ in this flow when the fluid is inviscid? When γ is positive, does the vorticity at $r = 0$ strengthen or weaken as t increases? When $\gamma \neq 0$, does the vortex's circulation change?

2. (Example 5.4 of *Fluid Mechanics, Sixth Edition*) Consider a thin ideal vortex segment of uniform strength Γ that lies along the z-axis between z_1 and z_2, and has a sense of rotation that points along the z-axis. Use the Biot-Savart law to show that the induced velocity $\overrightarrow{u}$ at the location (r, θ, z) will be $\overrightarrow{u}(\overrightarrow{x}, t) = (\Gamma/4\pi r)(cos\theta_1 - cos\theta_2)\overrightarrow{e}_\theta$, were θ_1 and θ_2 are the polar angles given in the provided illustration.

3. (Example 5.2 of *Fluid Mechanics, Sixth Edition*) For planar shear flow $\overrightarrow{u} = (u_1(x_2), 0)$, what is $D\Gamma/Dt$ when Γ is computed from a small rectangular contour, ABCD - centered on (x_1, x_2) with sides Δx_1 and Δx_2 - when ρ and μ are constants?