1 Key Concepts

- Thin-Film Lubrication
- Free Vorticies

2 Important Equations

$$\frac{\rho u_\theta^2}{r} = \frac{dP}{dr} \tag{1}$$

$$\rho g = -\frac{dP}{dz} \qquad \text{(Solid-Body Rotational Field)}$$

$$u_\theta(r) = \begin{cases} \frac{\Gamma r}{2\pi R^2} & r \leq R \\ \frac{\Gamma}{2\pi r} & r > R \end{cases} \qquad \text{(Rankine Vortex)}$$

$$u(x,y,t) \cong -\frac{h^2(x,t)}{2\mu}\frac{dp(x,t)}{dx}\frac{y}{h(x,t)}\left(1 - \frac{y}{h(x,t)}\right) + (U_h(t) - U_0(t))\frac{y}{h(x,t)} + U_0(t) \qquad \text{(Thin-Film Velocity Approximation)}$$

3 Practice Problems

1. (Example 5.1 of *Fluid Mechanics, Sixth Edition*) The surface of a quiescent pool of water is deflected symmetrically downward near $r = 0$ because of vortical flow within the pool. In cylindrical coordinates, the water surface profile is $z = h(r)$. Assume the velocity field in the water has only an angular component, $\overrightarrow{u} = \langle 0, u_\theta(r), 0\rangle$ and determine $u_\theta(r)$ when the pressure on the surface of the water is p_0

2. (Exercise 9.28 of *Fluid Mechanics, Sixth Edition*) A circular lubricated journal bearing of radius a holds a stationary round shaft. The bearing hub rotates at angular rate Ω. A load per unit depth of the shaft, W causes the center of the shaft to be displaced from the center of the rotating hub by a distance ϵh_0, where h_0 is the average gap thickness and $h_0 \ll a$. The gap is filled with a incompressible oil of viscosity μ. You may neglect the shear stress contribution of W.

    (a) Determine W by assuming a lubrication flow profile in the gap and $h(\theta) = h_0(1+\epsilon cos(\theta))$ with $\epsilon \ll 1$.