1 Key Concepts

- Velocity Gradient Tensor and Components
- Compressible vs. Incompressible flow
- Rotational vs. Irrotational flow
- Reynold's Transport Theorem

2 Important Equations

$$\frac{du_i}{dx_j} = S_{ij} + \frac{1}{2}R_{ij} \qquad \text{(Velocity Gradient Tensor)}$$

$$S_{ij} = \frac{1}{2}\left(\frac{du_i}{dx_j} + \frac{du_j}{dx_i}\right) \qquad \text{(Strain Rate Tensor)}$$

$$R_{ij} = \frac{du_i}{dx_j} - \frac{du_j}{dx_i} \qquad \text{(Rotation Tensor)}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{u} = 0 \qquad \text{(Criteria for Incompressible Flow)}$$

$$\int_V \overrightarrow{\nabla} \cdot \overrightarrow{F} dV = \oint_S \overrightarrow{F} \cdot \overrightarrow{n} dS \qquad \text{(Gauss's Theorem)}$$

$$\overrightarrow{\nabla} \times \overrightarrow{u} = \overrightarrow{0} \qquad \text{(Criteria for Irrotational Flow)}$$

$$\frac{d}{dt}\int_{V(t)} F\left(\overrightarrow{u}, t\right) dV = \int_{V(t)} \frac{dF}{dt} dV + \int_{S(t)} F\left(\overrightarrow{u}, t\right) \overrightarrow{b} \cdot \overrightarrow{n} dS \qquad \text{(Reynolds Transport Theorem)}$$

$$dA = r dr d\theta \qquad \text{(Differential Surface Area in Cylindical Coordinates)}$$

$$dV = r dr d\theta dz \qquad \text{(Differential Volume in Cylindical Coordinates)}$$

$$dA = \rho^2 sin(\phi) d\phi d\theta \qquad \text{(Differential Surface Area in Spherical Coordinates)}$$

$$dV = \rho^2 sin(\phi) d\phi d\theta d\rho \qquad \text{(Differential Volume in Spherical Coordinates)}$$

3 Practice Problems

1. Determine if the following velocity fields are incompressible, and calculate their vorticity fields.
   (a) $u(x, y, z) = \langle x, y, 0\rangle$
   (b) $u(x, y, z) = \langle x, -y, 0\rangle$
   (c) $u(x, y, z) = \langle a, b, c\rangle$ where a, b, and c are constants.
   (d) $u(x, y, z) = \left\langle x^2, x - z, z^2 - y\right\rangle$

2. (Example 3.4 of *Fluid Mechanics, Sixth Edition*) A steady two-dimension flow field incorporating fluid element rotation and strain is given in the 2D Cartesian coordinates by $\overrightarrow{u}(u, v) = (qy/x^2, qy^2/x^3)$ where q is a positive constant. Determine the vorticity and strain rate tensor in this flow away from $x = 0$.

3. (Example 3.5 of *Fluid Mechanics, Sixth Edition*) A steady two-dimension flow field incorporating fluid element rotation and strain is given in the 2D Cartesian coordinates by $\overrightarrow{u}(u, v) = (Ax, -Ay)$ where A is a positive constant. Determine the streamlines, vorticity and strain rate tensor in this flow away from $x = 0$.

4. (Example 3.6 of *Fluid Mechanics, Sixth Edition*) The base radius r of a fixed-height right circular cone is increasing at the rate $\dot{r}$. Use the Reynolds transport theorem to determine the rate at which the cone's volume is increasing when the cone's base radius is r_0 if its height is h.

5. For an incomprehensible fluid entering a pipe of area A_1 with uniform velocity v_1, and exiting through a cross section A_2, use Reynold's Transport Theory to derive the exit velocity v_2.