

## 1 Key Concepts

- Continuum Hypothesis
- Viscosity
- Hydrostatic Pressure
- Surface Tension
- First Law of Thermodynamics
- Adiabatic and Reversible Processes
- Entropy
- Ideal Gases
- Conservation of Mass and Momentum
- Bernoulli's Equation
- Lagrangian vs. Eulerian View
- Stream Lines, Path Lines, Streak lines
- Material Derivative
- Compressible vs. Incompressible flow
- Rotational vs. Irrotational flow
- Reynold's Transport Theorem
- Conservation of Mass, Energy, and Momentum
- Line Integrals
- Integrals in polar, cylindrical, and spherical coordinates

## 2 Important Equations

### 2.1 Basic Definitions

$$\tau = \mu \frac{du}{dx} \quad (\text{Newton's Law of Viscosity})$$

$$P = \frac{F}{A} \quad (\text{Definition of Pressure})$$

$$h = u + \rho v \quad (\text{Definition of Enthalpy})$$

$$Tds = dq \quad (\text{Definition of Entropy})$$

$$C_v = \left( \frac{du}{dT} \right) \Big|_V \quad \text{and} \quad C_p = \left( \frac{dh}{dT} \right) \Big|_P \quad (\text{Heat Capacities})$$

## 2.2 Calculus Rules

$$\frac{Df}{Dt} = \frac{df}{dt} + \vec{u} \cdot \nabla f \quad (\text{Material Derivative})$$

$$\int_V \vec{\nabla} \cdot \vec{F} dV = \oint_S \vec{F} \cdot \vec{n} dS \quad (\text{Gauss's Theorem})$$

$$\oint_C \vec{u} \cdot \vec{t} ds = \int_A (\vec{\nabla} \times \vec{u}) \cdot \vec{n} dA \quad (\text{Stoke's Theorem})$$

$$\frac{d}{dt} \int_{V(t)} F(\vec{u}, t) dV = \int_{V(t)} \frac{dF}{dt} dV + \int_{S(t)} F(\vec{u}, t) \vec{b} \cdot \vec{n} dS \quad (\text{Reynolds Transport Theorem})$$

$$dA = r dr d\theta \quad (\text{Differential Surface Area in Cylindrical Coordinates})$$

$$dV = r dr d\theta dz \quad (\text{Differential Volume in Cylindrical Coordinates})$$

$$dA = \rho^2 \sin(\phi) d\phi d\theta \quad (\text{Differential Surface Area in Spherical Coordinates})$$

$$dV = \rho^2 \sin(\phi) d\phi d\theta d\rho \quad (\text{Differential Volume in Spherical Coordinates})$$

## 2.3 Conservation Equations

$$\frac{dM}{dt} = \int_S \rho \vec{u} \cdot \vec{n} dS \quad (\text{Conservation of Mass})$$

$$\frac{dM}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad (\text{Conservation of Mass (simple)})$$

$$de = dq + dw \quad \text{or} \quad \frac{de}{dt} = \dot{w} + \dot{q} \quad (\text{First Law of Thermodynamics})$$

$$\int_V \frac{d}{dt} \rho \vec{u} dV + \int_A \rho \vec{u} (\vec{u} \cdot \vec{n}) dA = \int_V \rho \vec{g} dV + \int_A \vec{f} dA \quad (\text{Integral Conservation of Momentum})$$

$$\rho \left( \frac{d\vec{u}}{dt} + \vec{u} \cdot \nabla \vec{u} \right) = \rho \vec{g} - \nabla P \quad (\text{Differential Conservation of Momentum})$$

## 2.4 Derived Equations

$$H = \frac{v^2}{2} + \vec{g} \cdot \vec{r} + \frac{p}{\rho} \quad (\text{Bernoulli's Equation})$$

$$P = \rho RT \quad \text{or} \quad PV = nR_0 T \quad (\text{Ideal Gas Equations})$$

$$F_1 A_1 = F_2 A_2, \text{ where } z_1 = z_2 \quad (\text{Pascal's Law})$$

$$\Delta P = \rho g z \quad (\text{Change in Hydrostatic Pressure})$$

$$F_b = \rho g V \quad (\text{Buoyancy Force})$$

$$\vec{\nabla} \cdot \vec{u} = 0 \quad (\text{Criteria for Incompressible Flow})$$

$$\vec{\nabla} \times \vec{u} = \vec{0} \quad (\text{Criteria for Irrotational Flow})$$

$$\frac{du_i}{dx_j} = S_{ij} + \frac{1}{2} R_{ij} \quad (\text{Velocity Gradient Tensor})$$

$$S_{ij} = \frac{1}{2} \left( \frac{du_i}{dx_j} + \frac{du_j}{dx_i} \right) \quad (\text{Strain Rate Tensor})$$

$$R_{ij} = \frac{du_i}{dx_j} - \frac{du_j}{dx_i} \quad (\text{Rotation Tensor})$$

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \quad (\text{Streamline Criteria})$$

$$\begin{aligned} \frac{d\rho}{dt} + \nabla \cdot (\rho \vec{u}) &= 0 \quad (\text{Euler Equations}) \\ \rho \left( \frac{d\vec{u}}{dt} + \vec{u} \cdot \nabla \vec{u} \right) &= \rho \vec{g} - \nabla P \end{aligned}$$

### 3 Modeling Approach

1. Write out in standard form what is given (e.g.  $P$ ,  $\vec{u}$ ,  $F$ , etc.)
2. Write out in standard form what property, if any, is being requested.
3. Sketch the problem. If forces are involved, sketch a free-body diagram including all applied forces and body forces (weight).
4. Based on the given and requested properties, write out the equations that relate the given and requested properties.
  - (a) Conservation Equations **always** apply.
  - (b) If forces, momentum, or pressure is involved with a flowing fluid  $\rightarrow$  Conservation of Momentum Equation, using the free-body diagram to determine the net forces applied
  - (c) If forces, momentum, or pressure is involved with a flowing fluid **and** steady, inviscid, and incompressible flow can be assumed between two points  $\rightarrow$  Bernoulli's Equation
  - (d) If temperature, heat, or work of a fluid is involved  $\rightarrow$  First Law of thermodynamics
  - (e) Hydrostatic Pressure of still fluid  $\rightarrow$  Pascal's Law, Buoyancy Force, Change in Hydrostatic Pressure
  - (f) Behavior of Individual particles  $\rightarrow$  Lagrangian forms of Newton's Laws, Velocity, Path lines
  - (g) Behavior of Flow at a location  $\rightarrow$  Eulerian forms of Velocity, Material derivative, and conservation of momentum.
  - (h) A time derivative of an extrinsic property  $\rightarrow$  Reynolds Transport Theorem of the intrinsic property
5. Eliminate unnecessary components using appropriate approximations
  - (a) Steady flow  $\rightarrow \frac{du}{dt} = 0$
  - (b) Incompressible flow  $\rightarrow \frac{d\rho}{dt} = 0$ ,  $\vec{\nabla} \cdot \vec{u} = 0$
  - (c) Inviscid/Irrotational flow  $\rightarrow \mu = 0$ ,  $\vec{\nabla} \times \vec{u} = \vec{0}$
6. Use algebra and calculus to solve for the requested property with respect to the given properties
  - (a) Gauss's Theorem and Stoke's Theorem to convert bounds of integrated differentials
  - (b) Conversion of Lagrangian to Eulerian coordinates
  - (c) Converting line integrals over curves to Cartesian coordinates

## 4 Practice Problems

1. A Venturi device is commonly used to determine the mass flow rate of a fluid passing through a pipe. Assuming the entrance cross section of the Venturi has an area  $A_1$ , which narrows down to a cross sectional area  $A_2$ . If a differential pressure gauge reads a differential pressure of  $P_D$ , then what is the mass flow rate? You may assume an incompressible, ideal fluid of density  $\rho$ .
2. (Exercise 3.10 of *Fluid Mechanics, Sixth Edition*) Compute and compare the streamline, path line, and streak line that pass through  $(1, 1, 0)$  for the Cartesian velocity field  $\vec{u} = (x, -yt, 0)$ .