

## 1 Key Concepts

- Conservation of Mass, Energy, and Momentum
- Line Integrals
- Integrals in polar, cylindrical, and spherical coordinates

## 2 Important Equations

$$M = \int_V \rho dV \quad (\text{Formulation of Mass})$$

$$\frac{dM}{dt} = \int_S \rho \vec{u} \cdot \vec{n} dS \quad (\text{Conservation of Mass})$$

$$\frac{dM}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad (\text{Conservation of Mass (simple)})$$

$$H = \frac{v^2}{2} + \vec{g} \cdot \vec{r} + \frac{p}{\rho} \quad (\text{Bernoulli's Equation})$$

$$dV = \rho^2 \sin(\phi) d\phi d\theta d\rho \quad (\text{Differential Volume in Spherical Coordinates})$$

### 3 Practice Problems

1. Derive the integral conservation of mass equation.
2. Derive the integral conservation of momentum equation.
3. Evaluate the line integral  $\int_C (1 + xy^2) ds$  from  $(0,0)$  to  $(1,2)$  along the curve  $C$  given by parametric equations  $x = t$  and  $y = 2t$ .
4. An ideal fluid travels through a pipe of diameter  $D_1$  at position  $z_1$  with average velocity  $v_1$ . What is the change in pressure experienced by the fluid as it travels up the pipe to an elevation  $z_2$  with a new diameter  $D_2$ ? What is the change in velocity? What is the mass flow rate?
5. Suppose there is a gas giant planet whose density changes with a constant  $k$  of its depth. If the planet has an outer radius  $R$ , and an internal density  $\rho_0$ , then what is its total mass?