1 Key Concepts

- Lagrangian vs. Eulerian View
- Stream Lines
- Material Derivative
- Compressible vs. Incompressible flow
- Rotational vs. Irrotational flow
- Reynold's Transport Theorem

2 Important Equations

$$\frac{Df}{Dt} = \frac{df}{dt} + \overrightarrow{u} \cdot \nabla f \qquad \text{(Material Derivative)}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{u} = 0 \qquad \text{(Criteria for Incompressible Flow)}$$

$$\int_V \overrightarrow{\nabla} \cdot \overrightarrow{F} dV = \oint_S \overrightarrow{F} \cdot \overrightarrow{n} dS \qquad \text{(Gauss's Theorem)}$$

$$\overrightarrow{\nabla} \times \overrightarrow{u} = \overrightarrow{0} \qquad \text{(Criteria for Irrotational Flow)}$$

$$\frac{d}{dt} \int_{V(t)} F(\overrightarrow{u}, t)\, dV = \int_{V(t)} \frac{dF}{dt} dV + \int_{S(t)} F(\overrightarrow{u}, t)\, \overrightarrow{b} \cdot \overrightarrow{n} dS \qquad \text{(Reynold's Transport Theorem)}$$

3 Practice Problems

1. Determine if the following velocity fields are incompressible, and calculate their vorticity fields.

   (a) $u(x, y, z) = \langle x, y, 0\rangle$

   (b) $u(x, y, z) = \langle x, -y, 0\rangle$

   (c) $u(x, y, z) = \langle a, b, c\rangle$ where a, b, and c are constants.

   (d) $u(x, y, z) = \left\langle x^2, x - z, z^2 - y\right\rangle$

2. (Example 3.2 of *Fluid Mechanics, Sixth Edition*) A fluid particle in a steady flow moves along the x-axis. Its distance from the origin is r_0 at time t_0 and its trajectory is $r(t) = \left[K(t - t_0) + r_0^3\right]^{1/3}$, where K is a constant. Determine the flow's Eulerian velocity and acceleration, $u(x)$ and $a(x)$, and show that $a(x)$ may also be obtained from $Du(x)/Dt$.

3. (Example 3.3 of *Fluid Mechanics, Sixth Edition*) In 2D Cartesian coordinates, determine the steamline, path line, and streak line that pass through the origin of coordinates at $t = t'$ in the unsteady flow two-dimensional near-surface flow field typical of long-wavelength water waves with amplitude ξ_0: $u = \omega\xi_0 cos(\omega t)$ and $v = \omega\xi_0 sin(\omega t)$

4. For an incomprehensible fluid entering a pipe of area A_1 with uniform velocity v_1, and exiting through a cross section A_2, use Reynold's Transport Theory to derive the exit velocity v_2.

5. Suppose we have some extrinsic property F that can be related to its intrinsic form f by $F = \rho f$. What is the material derivative of the extrinsic property in terms of the material derivative of the intrinsic property?