1 Key Concepts

- Differential Equation Basics
- Potential flow

2 Important Equations

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x},\ u = \frac{\partial \phi}{\partial x},\ v = \frac{\partial \phi}{\partial y} \quad \text{(Potential Flow Definitions)}$$

$$\nabla^2 \Phi = q(r) \quad \text{(Poisson Equation)}$$

$$\nabla^2 \Phi = 0 \quad \text{(Laplace Equation)}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\Phi}{dr}\right) + \frac{1}{r^2}\frac{d^2\Phi}{d\theta^2} = 0 \quad \text{(Laplace Equation in Polar Coordinates)}$$

$$\phi_{point} = \frac{q_s}{2\pi} ln|\overrightarrow{x} - \overrightarrow{x}'| \quad \text{(Potential of a Point-Source Element)}$$

$$\phi_{Uniform} = U\overrightarrow{x} \quad \text{(Potential of a Uniform Flow Field)}$$

$$H = p + \frac{1}{2}\rho|\nabla\phi|^2 = p + \frac{1}{2}\rho|\nabla\psi|^2 \quad \text{(Bernoulli's Equation with Potential Flows)}$$

3 Practice Problems

1. Given a point source $q_s(x_0, y_0)$, determine the solution of the Poisson Equation $\nabla^2\Phi = q(x_0, y_0)$
2. Suppose the differential equation $py''(x) + qy'(x) + ry(x) = 0$ has two solutions, $y_1(x)$ and $y_2(x)$. Show that the superpostion $y_1(x) + y_2(x)$ is also a solution.
3. A *doublet* is a flow system consisting of a point-source and point-sink of flow with strength q_s located a distance ϵ away from each other on the y-axis. Determine the potential flow field.
4. A *half-body* is a flow system consisting of a point-source at the origin superimposed on a uniform flow field. Determine the velocity fields and pressure distribution.