1 Key Concepts

- Navier-Stokes Equations
- Low-Reynolds Number Flows

2 Important Equations

$$\frac{du_r}{dt} + \left(\overrightarrow{u} \cdot \overrightarrow{\nabla}\right) u_r - \frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{dP}{dr} + \nu \left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{du_\theta}{d\theta}\right) \tag{1}$$

$$\frac{du_\theta}{dt} + \left(\overrightarrow{u} \cdot \overrightarrow{\nabla}\right) u_\theta + \frac{u_\theta u_r}{r} = -\frac{1}{\rho r}\frac{dP}{d\theta} + \nu \left(\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{du_r}{d\theta}\right) \tag{2}$$

$$\frac{du_z}{dt} + \left(\overrightarrow{u} \cdot \overrightarrow{\nabla}\right) u_z = -\frac{1}{\rho}\frac{dP}{dz} + \nu \nabla^2 u_z \tag{Navier-Sokes Equations - Cylindrical}$$

$$\overrightarrow{u} \cdot \overrightarrow{\nabla} = u_r \frac{d}{dr} + \frac{u_\theta}{r}\frac{d}{d\theta} + u_z \frac{d}{dz} \tag{3}$$

$$\nabla = \left\langle \frac{d}{dr}, \frac{1}{r}\frac{d}{d\theta}, \frac{d}{dz} \right\rangle \tag{4}$$

$$\nabla^2 = \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) + \frac{1}{r^2}\frac{d^2}{d\theta^2} + \frac{d^2}{dz^2} \tag{5}$$

$$\frac{D\overrightarrow{u}}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\overrightarrow{u} \tag{Conservation of Momentum - low Re}$$

$$Re = \frac{\rho u D}{\mu} \tag{Reynolds Number}$$

3 Practice Problems

1. (Section 9.2 of *Fluid Mechanics, Sixth Edition*) Circular Couette Flow is the flow of a fluid between two cylinders of radii R_1 and R_2, where $R_2 > R_1$, each rotating with angular velocities Ω_1 and Ω_2, respectively. Determine the velocity profile of the fluid between the cylinders. You may assume that the fluid is in a laminar flow, and $\vec{u} = \langle 0, u_\theta(r), 0 \rangle$.

   (a) What occurs as $R_2 \to \infty$?

   (b) What occurs as $R_1 \to 0$?

2. The Darcy Weisbach equation allows for the approximation of viscous flow pressure drops in any flow domain within a pipe of diameter D:

$$\Delta P_{loss} = f_D(Re, \varepsilon)\frac{\rho}{2}\frac{u^2 L}{D} \tag{6}$$

   Where L is the pipe length, and f_D is the dimensionless Darcy Friction Factor, which is an empirically-determined function of the Reynolds number and the pipe roughness ε. Two correlations for f_D within different flow regions are:

$$f_D = \frac{64}{Re} \text{ , Re} < 2000 \tag{7}$$

$$f_D = \frac{0.316}{Re^{1/4}} \text{ , } 4000 < Re < 10^5 \tag{8}$$

   Given a smooth pipe 2 meters long, with a diameter of 0.076 m, what is the pressure loss is $u = 0.01$ m/s If $u = 0.5$ m/s?