

EXAM 1 - PRACTICE QUESTIONS

Rules:

- Show all your work and reasoning how you arrive at your answer. Just writing an answer is not sufficient for most problems.
- When in doubt, *draw the picture!!!*

Problems:

1. Let X_1, \dots, X_n be a random sample from a uniform distribution $U[-1, \theta]$, where $\theta > -1$.
 - a) Obtain the MLE for EX_1 .
 - b) Obtain the MLE for $\text{var}(X_1)$.
 - c) Obtain two Method of Moments estimators for $\text{var}(X_1)$.
 - d) Now suppose we know that $\theta \geq 0$. Obtain the MLE for θ .
2. You participate in a study that aims to develop a new vaccine to protect against Covid-19 infection. The new vaccine has been applied in two independent trials. The first trial has led to a sample of size n from a Bernoulli(θ_1) model and the second trial to a sample of size m from a Bernoulli(θ_2) model, where θ_1, θ_2 are the success probabilities that the vaccine is effective.
 - a) Obtain the MLEs $\hat{\theta}_1, \hat{\theta}_2$ for θ_1, θ_2 .
 - b) Provide arguments why θ_1, θ_2 might be the same or might be different. What about $\hat{\theta}_1, \hat{\theta}_2$?
 - c) It is now assumed that $\theta_1 = \theta_2$. Calculate $\text{var}(\hat{\theta}_1)$, $\text{var}(\hat{\theta}_2)$.
 - d) Under this assumption, a scientist suggests to combine the two MLEs in a suitable way to obtain an improved estimator that takes all of the available information into account. For that, one considers combinations $\hat{\theta}_c = c\hat{\theta}_1 + (1 - c)\hat{\theta}_2$ for $0 \leq c \leq 1$. Find the best c such that $\text{var}(\hat{\theta}_c)$ is minimized.
3. Consider a sample of size n from an exponential distribution with parameter $\beta > 0$. Assume the r.v. X has this distribution.
 - a) Find a method of moments estimator for β .
 - b) Find a method of moments estimator for $P(0 \leq X \leq 2)$.
 - c) Find a method of moments estimator for the median of the distribution.