HOMEWORK 2

Page 78, Question 1
If Socrates is human, then Socrates is mortal is the same as p $\rightarrow$ q
The argument form is mudus ponens, which is a valid argument form.
Because the hypothesis is true, the conclusion must be true also.

Page 78, Question 2
If George does not have eight legs, then he is not a spider follows the form of p $\rightarrow$ q
The argument form is modus tollens, which is a valid argument form.
Because the hypothesis is true, the conclusion must be true also.

Page 78, Question 5
Let:
P : "Randy works hard"
Q : "Randy is a dull boy"
J : "Randy gets the job"

If Randy works hard, then he is a dull boy becomes P$\rightarrow$ Q If Randy is a dull boy, then he will not get the job becomes Q$\rightarrow \neg$J
So if Randy works hard, then he is a dull boy. If he is a dull boy, he will not get the job.
Therefore, if Randy works hard, he will not get the job.

Proof:

1.)P	Given
2.)P$\rightarrow$ Q	Given
3.)Q	Modus ponens(2 and 3)
4.)Q$\rightarrow \neg$J	Given
5.)$\neg$ J	Modus ponens(3 and 4)

Page 78, Question 6
Let:
R : "It rains"
F : "It is foggy"
P : "The sailing race will be held"
Q : "The life saving demonstration will go on"
T : "The trophy will be awarded"

If it does not rain or if it is not foggy, then the sailing race will be held and the life-saving demonstration will go on, becomes $(\neg R \vee \neg F) \rightarrow (P \wedge Q)$
If the sailing race is held, then the trophy will be awarded, becomes P$\rightarrow$T
The trophy was not awarded, becomes $\neg$T

Proof:

1.)$\neg$T	Hypothesis
2.) P$\rightarrow$T	Hypothesis

3.)$\neg$P	Modus tollens(1 and 2)
4.)($\neg$R$\vee$$\neg$F)$\rightarrow$(P$\wedge$Q)	Hypothesis
5.)($\neg$(P$\wedge$Q))$\rightarrow$ $\neg$($\neg$R$\vee$$\neg$F)	Contrapositive of 4
6.)($\neg$P$\vee$$\neg$Q)$\rightarrow$(R$\wedge$F)	De Morgan's law
7.)$\neg$P$\vee$$\neg$Q	Addition with 3
8.)R$\wedge$F	Modus ponens (6 and 7)
9.)R	Simplify 8

Page 91, Question 1
Assume a and b are two odd integers.
a = 2x + 1 and b = 2y + 1
When added:
2x+1 + 2y + 1 = 2x + 2y + 2
2(x + y + 2) must be an even number since it is multiplied by 2.
Therefore the sum of any two odd integers will be even.

Page 91, Question 2
Let a and b be two even integers.
a = 2x and b = 2y
When added:
2x + 2y = 2(x + y) which must be even since it is multiplied by 2.
Therefore the sum of any two even integers will be even. **Page 91, Question 3**

Let x be an even number.
If x = 2y
Then $x^2 = (2y)^2 = 4y^2$
Which further simplifies to $2(2y^2)$ which must be an even number. Therefore the square of any even number will be an even number.

Page 91, Question 6
Let a and b be two odd integers.
a = 2x + 1 and n = 2y + 1
When muliplied:
(2x+1)·(2y+1) = 4xy + 2x + 2y + 1
2(2xy + x + y) + 1 which must be odd since it is adding one to an even number.

Page 91, Question 9
Let a be some rational number, z be some irrational number, and r be another rational number.
Let r = a + z
Adding -a to both sides is equivalent to subtracting so,
r + (-a) = z
This expression states that the sum of two rational numbers yields an irrational number, which is contradictory to the previously made assumption that the sum of two rational numbers is rational. Therefore, the sum of an irrational number and a rational number is irrational.

Page 91, Question 11
Let a = $\sqrt{2}$, an irrational number.
When squared we get: $\sqrt{2} \cdot \sqrt{2} = 2$, a rational number. Therefore, the product of two irrational numbers is not always irrational.

Page 91, Question 17
(a)
Let n be some odd number.
Then, n = 2x + 1, for some number x
Substituting n we get, $(2x+1)^3$ + 5 = $8x^3$ + $12x^2$ + 6x + 6
Simplified further: 2($4x^3$ + $6x^2$ + 3x + 3).
Therefore, n^3 + 5 must be even since it is some number mulitplied by 2.

(b)
Let n be some odd number, and assume is n^3 + 5 is also odd.
Since n is odd, and we know that the product of odd numbers is also odd, therefore, n^3 is also odd.
Also, it is known that the difference of two odd numbers is even, however, in this case, it is not true.
(n^3 + 5) - n^3 is an odd number subtracted from an odd number, however the result is 5, an odd number.
Therefore, if n^3 + 5 is odd, then n must be even.

Page 91, Question 18
(a)
Assume that n is some odd number.
Then, n = 2x + 1, for some integer x.
Substituting n in 3n + 2:
3(2x + 1) + 2 = 6x + 5 = 6x + 4 + 1
2(3x + 2) + 1 is adding one to some even number, therefore odd.
Therefore, if 3n+2 is even, then n must also be even.

(b)
Assume that n is some odd number and that 3n + 2 is even.
Since the product of two odd numbers will be odd, then 3n must odd.
It follows that 3n + 2 will also be odd since it is adding 2 to an odd number.
Following this logic, assuming that 3n + 2 is even was incorrect. By contradiction, if 3n + 2 is even, then n must also be even.