HOMEWORK 2

Page 78, 1
If Socrates is human, then Socrates is mortal = P$\rightarrow$ Q
Socrates is human. : Socrates is mortal = P:Q
The argument form is the valid form of modus ponens.

Page 78, 2
If George does not have eight legs, then he is not a spider = P$\rightarrow$ Q
George is a spider. : George has eight legs. = Q : P
The argument form is the valid form of modus tollens.

Page 78, 5
W : "Randy works hard"
D : "Randy is a dull boy"
J : "Randy gets the job"

If Randy works hard, then he is a dull boy= W$\rightarrow$ D
If Randy is a dull boy, then he will not get the job = D$\rightarrow \neg$J

Proof:

(1) W	Hypothesis
(2) W$\rightarrow$D	Hypothesis
(3) D	Modus ponens using (2) and (3)
(4) D$\rightarrow \neg$J	Given
(5) $\neg$ J	Modus ponens using (3) and (4)

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Let:
R : "It rains"
F : "It is foggy"
S : "The sailing race will be held"
L: : "The life saving demonstration will go on"
T : "The trophy will be awarded"

If it does not rain or if it is not foggy, then the sailing race will be held and the life-saving demonstration will go on = ($\neg$R$\vee\neg$F)$\rightarrow$(S$\wedge$L)
If the sailing race is held, then the trophy will be awarded = S$\rightarrow$T
The trophy was not awarded, becomes $\neg$T

Proof:

(1) $\neg$T	Hypothesis
(2) P$\rightarrow$T	Hypothesis
(3) $\neg$P	Modus tollens using (1) and (2)
(4) ($\neg$R$\vee\neg$F)$\rightarrow$(P$\wedge$Q)	Hypothesis
(5) ($\neg$(P$\wedge$Q))$\rightarrow \neg$($\neg$R$\vee\neg$F)	Contrapositive of (4)
(6) ($\neg$P$\vee\neg$Q)$\rightarrow$(R$\wedge$F)	De Morgan's
(7) $\neg$P$\vee\neg$Q	Addition with (3)

(8) R∧F Modus ponens using (6) and (7)
(9) R Simplification of (8)

Page 91, 1
Let a and b be two odd integers.
a = 2x + 1 and b = 2y + 1, where x and y are integers.
a + b = 2x+1 + 2y + 1 = 2x + 2y + 2
2(x + y + 2) must be an even number.
Therefore, the sum of any two odd integers must yield an even result.

Page 91, 2
Let a and b be two even integers.
a = 2x and b = 2y, where x and y are integers,
a+ b = 2x + 2y = 2(x + y) which must be even.
Therefore, the sum of any two even integers must yield an even result.

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Let x be an even integer.
x = 2y, where y is an integer,
$x^2 = (2y)^2 = 4y^2$
Simplify:
$2(2y^2)$
This operation yielded an even result. Therefore, the square of any even number must yield an even result.

Page 91, 6
Let a and b be two odd integers.
a = 2x + 1 and n = 2y + 1, where x and y are integers,
a ·b = (2x+1)·(2y+1) = 4xy + 2x + 2y + 1
Simplify:
2(2xy + x + y) + 1
This operation yielded an odd result.
Therefore, the product of two odd integers must yield an odd result.

Page 91, 9
Let a and b be rational numbers.
Let x be an irrational number.
Let a = b + x
Subtract a from both sides of the equation:
a + (-b) = b + x + (-b)
a + (-b) = x
The result concluded that the sum of two rational numbers yielded an irrational number.
Therefore, by contradiction, the sum of a rational number and an irrational number must yield an irrational result.

Page 91, 11
Let a = $\sqrt{5}$.

$a^2 = \sqrt{5}^2 = 5$.
This operation squared an irrational number and yielded a rational number.
Therefore, the product of two irrational numbers must not always yield an irrational result.

Page 91, 17
(a)
Let n be an odd number.
n = 2x + 1, where x is an integer,
Substitution: $(2x + 1)^3$ + 5 = $8x^3$ + $12x^2$ + 6x + 6
Simplify:
2($4x^3$ + $6x^2$ + 3x + 3).
Therefore, n^3 + 5 must be even since 2 is a factor.

(b)
Let n be an odd number.
Let n^3 + 5 also be an odd number.
Since n is odd, and the product of odd numbers is must yield an odd result, n^3 must also be an odd number.
(n^3 + 5) - n^3 is an odd number subtracted from an odd number which yields an odd result.
However, it known that the difference between two odd numbers should yield an even result.
Therefore, if n^3 + 5 yields an odd result, n must be an even number.

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(a)
Let n be an odd number.
Then, n = 2x + 1, where x is an integer.
Substitution: 3(2x + 1) + 2 = 6x + 5 = 6x + 4 + 1
Simplify:
2(3x + 2) + 1
This operation yields an odd number when an odd number is substituted in.
Therefore, if 3n+2 yields an even result, n must also be even.

(b)
Let n be an odd number.
Let 3n + 2 be an even number..
Since the product of two odd numbers must yield an odd result, then 3n must be an odd number.
Therefore, 3n + 2 must also be an odd number.
Therefore, by contradiction, if 3n + 2 yields an even result, then n must also be even.