Answers to Selected Exercises

CHAPTER 3

3.1. (a) $\bar{x} = 16.029$. (b) $s = 0.0202$.

3.3. (a) $\bar{x} = 952.9$. (b) $s = 3.7$.

3.5. (a) $\bar{x} = 121.25$. (b) $s = 22.63$.

3.13. Both the normal and lognormal distributions appear to be reasonable models for the data.

3.15. The lognormal distribution appears to be a reasonable model for the concentration data.

3.19. (a) $\bar{x} = 89.476$. (b) $s = 4.158$.

3.23. *sample space*: {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

$$p(x) = \begin{cases} 1/36; x = 2 & 2/36; x = 3 \\ 3/36; x = 4 & 4/36; x = 5 \\ 5/36; x = 6 & 6/36; x = 7 \\ 5/36; x = 8 & 4/36; x = 9 \\ 3/36; x = 10 & 2/36; x = 11 \\ 1/36; x = 12 & 0;\ \text{otherwise} \end{cases}$$

3.25. (a) 0.0196. (b) 0.0198.
(c) Cutting occurrence rate reduces probability from 0.0198 to 0.0100.

3.27. (a) $k = 0.05$. (b) $\mu = 1.867$, $\sigma^2 = 0.615$.
(c) $F(x) = \{0.383; x = 1 \quad 0.750; x = 2$
$1.000; x = 3$

3.29. (a) Approximately 11.8%. (b) Decrease profit by $5.90/calculator.

3.31. Decision rule means 22% of samples will have one or more nonconforming units.

3.33. 0.921

3.35. (a) 0.633. (b) 0.659. Approximation is not satisfactory.
(c) $n/N = 0.033$. Approximation is satisfactory. (d) $n = 11$

3.37. $\Pr\{x = 0\} = 0.364$, $\Pr\{x \geq 2\} = 0.264$

3.39. $\Pr\{x \geq 1\} = 0.00001$

3.41. $\mu = 1/p$

3.43. $\Pr\{x \leq 35\} = 0.159$. Number failing minimum spec is 7950.
$\Pr\{x > 48\} = 0.055$. Number failing maximum spec is 2750.

3.45. Process is centered at target, so shifting process mean in either direction increases nonconformities. Process variance must be reduced to 0.015^2 to have at least 999 of 1000 conform to specification.

3.47. $\Pr\{x > 1000\} = 0.0021$

3.49. If $c_2 > c_1 + 0.0620$, then choose process 1.

CHAPTER 4

4.1. (a) $Z_0 = 6.78$. Reject H_0.
(b) $P = 0$. (c) $8.2525 \leq \mu \leq 8.2545$.

4.3. (a) $t_0 = 1.952$. Reject H_0.
(b) $25.06 \leq \mu \leq 26.94$.

4.5. (a) $t_0 = -3.089$. Reject H_0.
(b) $13.39216 \leq \mu \leq 13.40020$.

4.7. $n = 246$

4.9. (a) $t_0 = -6.971$. Reject H_0.
(b) $9.727 \leq \mu \leq 10.792$
(c) $\chi_0^2 = 14.970$. Do not reject H_0.
(d) $0.738 \leq \sigma \leq 1.546$
(e) $\sigma \leq 1.436$

4.11. (a) $t_0 = 0.11$. Do not reject H_0.
(c) $-0.127 \leq (\mu_1 - \mu_2) \leq 0.141$
(d) $F_0 = 0.8464$. Do not reject H_0.
(e) $0.165 \leq \sigma_1^2/\sigma_2^2 \leq 4.821$
(f) $0.007 \leq \sigma^2 \leq 0.065$

4.13. (a) $t_0 = -0.77$. Do not reject H_0.
(b) $-6.7 \leq (\mu_1 - \mu_2) \leq 3.1$.
(c) $0.21 \leq \sigma_1^2/\sigma_2^2 \leq 3.34$

4.15. (a) $Z_0 = 4.0387$. Reject H_0. (b) $P = 0.00006$.
(c) $p \leq 0.155$.

4.17. (a) $F_0 = 1.0987$. Do not reject H_0.
(b) $t_0 = 1.461$. Do not reject H_0.

4.19. $t_0 = -1.10$. There is no difference between mean measurements.

4.21. (a) $\chi_0^2 = 42.75$. Do not reject H_0.
(b) $1.14 \le \sigma \le 2.19$.

4.23. $n = [(Z_{\alpha/2} + Z_\beta)\sigma/\delta]^2$

4.27. $Z_0 = 0.3162$. Do not reject H_0.

4.29. (a) $F_0 = 3.59$, $P = 0.053$.

4.31. (a) $F_0 = 1.87$, $P = 0.214$.

4.33. (a) $F_0 = 1.45$, $P = 0.258$.

4.35. (a) $F_0 = 30.85$, $P = 0.000$.

CHAPTER 5

5.17. Pattern is random.

5.19. There is a nonrandom, cyclic pattern.

5.21. Points 17, 18, 19, and 20 are outside lower 1-sigma area.

5.23. Points 16, 17, and 18 are 2 of 3 beyond 2 sigma of centerline. Points 5, 6, 7, 8, and 9 are of 5 at 1 sigma or beyond of centerline.

CHAPTER 6

6.1. (a) Samples 12 and 15 exceed $\bar{x}$ UCL.
(b) $\bar{p} = 0.00050$.

6.3. (a) $\bar{x}$ chart: CL = 10.9, UCL = 50.35, LCL = –28.55
R chart: CL = 54.1, UCL = 123.5, LCL = 0
Process is in statistical control.
(b) $\bar{\sigma}_x = 23.3$. (c) $\hat{C}_P = 1.43$.

6.5. (a) $\bar{x}$ chart: CL = –0.003, UCL = 1.037, LCL = –1.043
s chart: CL = 1.066, UCL = 1.830, LCL = 0.302
(b) R chart: CL = 3.2, UCL = 5.686, LCL = 0.714
(c) s^2 chart: CL = 1.2057, UCL = 3.6297, LCL = 0.1663

6.7. $\bar{x}$ chart: CL = 10.33, UCL = 14.73, LCL = 5.92
s chart: CL = 2.703, UCL = 6.125, LCL = 0

6.9. (a) chart: CL = 74.00118, UCL = 74.01458, LCL = 73.98777
R chart: CL = 0.02324, UCL = 0.04914, LCL = 0 (b) No. (c) $\hat{C}_P = 1.668$.

6.11. $\bar{x}$ chart: CL = 80, UCL = 89.49, LCL = 70.51
s chart: CL = 9.727, UCL = 16.69, LCL = 2.76

6.13. (a) $\bar{x}$ chart: CL = 20, UCL = 22.34, LCL = 17.66
s chart: CL = 1.44, UCL = 3.26, LCL = 0
(b) LNTL = 15.3, UNTL = 24.7
(c) $\hat{C}_P = 0.85$
(d) $\hat{p}_{\text{rework}} = 0.0275$, $\hat{p}_{\text{scrap}} = 0.00069$, Total = 2.949%
(e) $\hat{p}_{\text{rework}} = 0.00523$, $\hat{p}_{\text{scrap}} = 0.00523$, Total = 1.046%

6.15. (a) $\bar{x}$ chart: CL = 79.53, UCL = 84.58, LCL = 74.49
R chart: CL = 8.75, UCL = 18.49, LCL = 0
Process is in statistical control.
(b) Several subgroups exceed UCL on R chart.

6.17. (a) $\bar{x}$ chart: CL = 34.00, UCL = 37.50, LCL = 30.50
R chart: CL = 3.42, UCL = 8.81, LCL = 0
(b) Detect shift more quickly.
(c) $\bar{x}$ chart: CL = 34.00, UCL = 36.14, LCL = 31.86
R chart: CL = 5.75, UCL = 10.72, LCL = 0.78

6.19. (a) $\bar{x}$ chart: CL = 223, UCL = 237.37, LCL = 208.63
R chart: CL = 34.29, UCL = 65.97, LCL = 2.61
(b) $\hat{\mu} = 223$; $\hat{\sigma}_x = 12.68$
(c) $\hat{C}_P = 0.92$. (d) $\hat{p} = 0.00578$

6.21. (a) $\hat{\sigma}_x = 1.60$
(b) $\bar{x}$ chart: UCL = 22.14, LCL = 17.86
s chart: UCL = 3.13, LCL = 0
(c) Pr{in control} = 0.57926

6.23. $\hat{C}_P = 0.8338$

6.25. (a) $\bar{x}$ chart: UCL = 22.63, LCL = 17.37
R chart: UCL = 9.64, LCL = 0
(b) $\hat{\sigma}_x = 1.96$. (c) $\hat{C}_P = 0.85$.
(d) Pr{not detect} = 0.05938

6.27. The process continues to be in a state of statistical control.

6.29. (a) $\bar{x}$ chart: CL = 449.68, UCL = 462.22, LCL = 437.15
s chart: CL = 17.44, UCL = 7.70, LCL = 0

6.31. Pr{detecting shift on 1st sample} = 0.37

6.33. (a) $\bar{\bar{x}} = 20.26$, $\text{UCL}_{\bar{x}} = 23.03$, $\text{LCL}_{\bar{x}} = 17.49$; $\bar{R} = 4.8$, $\text{UCL}_R = 10.152$, $\text{LCL}_R = 0$
(b) $\hat{p} = 0.0195$

6.35. (a) $\bar{x}$ Recalculating limits without samples 1, 12, and 13:
$\bar{x}$ chart: CL = 1.45, UCL = 5.46, LCL = (2.57
R chart: CL = 6.95, UCL = 14.71, LCL = 0
(b) Samples 1, 12, 13, 16, 17, 18, and 20 are out-of-control, for a total 7 of the 25 samples, with runs of points both above and below the centerline. This suggests that the process is inherently unstable, and that the sources of variation need to be identified and removed.

6.37. (a) $\overline{R} = 45.0$, $\text{UCL}_R = 90.18$, $\text{LCL}_R = 0$
(b) $\hat{\mu} = 429.0$, $\hat{\sigma}_x = 17.758$
(c) $\hat{C}_p = 0.751$; $\hat{p} = 0.0533$
(d) To minimize fraction nonconforming the mean should be located at the nominal dimension (440) for a constant variance.

6.39. (a) $\text{UCL}_{\bar{x}} = 108, \text{LCL}_{\bar{x}} = 92$.
(b) $\text{UCL}_{\bar{x}} = 111.228, \text{LCL}_{\bar{x}} = 88.772$.

6.41. $\text{ARL}_1 = 2.986$

6.43. (a) $\text{CL}_s = 9.213, \text{UCL}_s = 20.88, \text{LCL}_s = 0$.
(b) $\text{UCL}_{\bar{x}} = 209.8, \text{LCL}_{\bar{x}} = 190.2$.

6.45. (a) $\bar{\bar{x}} = 90, \text{UCL}_{\bar{x}} = 91.676, \text{LCL}_{\bar{x}} = 88.324$;
$\overline{R} = 4, \text{UCL}_R = 7.696, \text{LCL}_R = 0.304$
(b) $\hat{\sigma}_x = 1.479$.
(c) $\bar{s} = 1.419$, $\text{UCL}_s = 2.671$, $\text{LCL}_s = 0.167$.

6.47. Pr{detect shift on 1st sample} = 0.1587

6.49. (a) $\alpha = 0.0026$. (b) $\hat{C}_P = 0.667$.
(c) Pr{not detect on 1st sample} = 0.5000.
(d) $\text{UCL}_{\bar{x}} = 362.576, \text{LCL}_{\bar{x}} = 357.424$.

6.51. (a) $\hat{\mu} = 706.00$; $\hat{\sigma}_x = 1.827$.
(b) $\text{UNTL} = 711.48, \text{LNTL} = 700.52$.
(c) $\hat{p} = 0.1006$.
(d) Pr{detect on 1st sample} = 0.9920
(e) Pr{detect by 3rd sample}

6.53. $\bar{\bar{x}} = 16.1052$; $\hat{\sigma}_x = 0.021055$;
$\overline{\text{MR2}} = 0.02375$. Assumption of normally distributed coffee can weights is valid. %underfilled = 0.0003%.

6.55. (a) Viscosity measurements appear to follow a normal distribution.
(b) The process appears to be in statistical control, with no out-of-control points, runs, trends, or other patterns.
(c) $\hat{\mu} = 2928.9$; $\hat{\sigma}_x = 131.346$;
$\overline{\text{MR2}} = 148.158$

6.57. (a) The process is in statistical control. The normality assumption is reasonable.
(b) It is clear that the process is out of control during this period of operation.
(c) The process has been returned to a state of statistical control.

6.59. The measurements are approximately normally distributed. The out-of-control signal on the moving range chart indicates a significantly large difference between successive measurements (7 and 8). Consider the process to be in a state of statistical process control.

6.61. (a) The data are not normally distributed. The distribution of the natural-log transformed uniformity measurements is approximately normally distributed.
(b) $\bar{x}$ chart: CL = 2.653, UCL = 3.586, LCL = 1.720
R chart: CL = 0.351, UCL = 1.146, LCL = 0

6.63. x chart: $\bar{x} = 16.11$, $\text{UCL}_x = 16.17$, $\text{LCL}_x = 16.04$
MR chart: $\overline{\text{MR2}} = 0.02365$, $\text{UCL}_{\text{MR2}} = 0.07726$, $\text{LCL}_{\text{MR2}} = 0$

6.65. x chart: $\bar{x} = 2929$, $\text{UCL}_x = 3338$, $\text{LCL}_x = 2520$
MR chart: $\overline{\text{MR2}} = 153.7$, $\text{UCL}_{\text{MR2}} = 502.2$, $\text{LCL}_{\text{MR2}} = 0$

6.67. (a) $\hat{\sigma}_x = 1.157$. (b) $\hat{\sigma}_x = 1.682$. (c) $\hat{\sigma}_x = 1.137$
(d) $\hat{\sigma}_{x,\text{ span }3} = 1.210$, $\hat{\sigma}_{x,\text{ span }4} = 1.262$, ...,
$\hat{\sigma}_{x,\text{ span }19} = 1.406$, $\hat{\sigma}_{x,\text{ span }20} = 1.435$

6.69. (a) $\bar{x}$ chart: CL = 11.76, UCL = 11.79, LCL = 11.72
R chart (within): CL = 0.06109, UCL = 0.1292, LCL = 0
(c) I chart: CL = 11.76, UCL = 11.87, LCL = 11.65
MR2 chart (between): CL = 0.04161, UCL = 0.1360, LCL = 0

6.71. (b) R chart (within): CL = 0.06725, UCL = 0.1422, LCL = 0
(c) I chart: CL = 2.074, UCL = 2.159, LCL = 1.989
MR2 chart (between): CL = 0.03210, UCL = 0.1049, LCL = 0
(d) Need lot average, moving range between lot averages, and range within a lot.
I chart: CL = 2.074, UCL = 2.133, LCL = 2.015

MR2 chart (between): CL = 0.02205, UCL = 0.07205, LCL = 0
R chart (within): CL = 0.1335, UCL = 0.2372, LCL = 0.02978

CHAPTER 7

7.1. $\bar{p} = 0.0585$, UCL = 0.1289, LCL = 0. Sample 12 exceeds UCL. Without sample 12: $\bar{p} = 0.0537$, UCL = 0.1213, LCL = 0.

7.3. For $n = 80$, $\text{UCL}_i = 0.1397$, $\text{LCL}_i = 0$. Process is in statistical control.

7.5. (a) $\bar{p} = 0.1228$, UCL = 0.1425, LCL = 0.1031
(b) Data should not be used since many subgroups are out of control.

7.7. Pr{detect shift on 1st sample} = 0.278, Pr{detect shift by 3rd sample} = 0.624

7.9. $\bar{p} = 0.10$, UCL = 0.2125, LCL = 0
$p = 0.212$ to make $\beta = 0.50$. $n \geq 82$ to give positive LCL.

7.11. $n = 81$

7.13. (a) $\bar{p} = 0.07$, UCL = 0.108, LCL = 0.032
(b) Pr{detect shift on 1st sample} = 0.297
(c) Pr{detect shift on 1st or 2nd sample} = 0.506

7.15. (a) Less sample 3: $n\bar{p} = 14.78$, UCL = 27.421, LCL = 4.13
(b) Pr{detect shift on 1st sample} = 0.813

7.17. (a) $n\bar{p} = 40$, UCL = 58, LCL = 22.
(b) Pr{detect shift on 1st sample} = 0.583.

7.19. ARL1 = 1.715 $\approx - 2$

7.21. (a) CL = = 0.0221
for $n = 100$: UCL = 0.0622, LCL = 0
for $n = 150$: UCL = 0.0581, LCL = 0
for $n = 200$: UCL = 0.0533, LCL = 0
for $n = 250$: UCL = 0.0500, LCL = 0
(b) $Z_i = (\hat{p}_i - 0.0221)\Big/\sqrt{0.0221(1-0.0221)/n_i}$

7.23. $Z_i = (\hat{p}_i - 0.0221)\Big/\sqrt{0.0216/n_i}$

7.25. $n \geq 892$

7.27. (a) $L = 2.83$. (b) $n\bar{p} = 20$, UCL = 32.36, LCL = 7.64.
(c) Pr{detect shift on 1st sample} = 0.140.

7.29. (a) $n \geq 397$. (b) $n = 44$.

7.31. (a) $\bar{p} = 0.02$, UCL = 0.062, LCL = 0.
(b) Process has shifted to $\bar{p} = 0.038$.

7.33. $n\bar{p} = 2.505$, UCL = 7.213, LCL = 0

7.35. $Z_i = (\hat{p}_i - 0.06)\Big/\sqrt{0.0564/n_i}$

7.37. Variable u: CL = 0.7007; $\text{UCL}_i = 0.7007 + 3\sqrt{0.7007/n_i}$; $\text{LCL}_i = 0.7007 - 3\sqrt{0.7007/n_i}$
Averaged u: CL = 0.701, UCL = 1.249, LCL = 0.1527

7.39. $Z_i = (u_i - 0.7007)\Big/\sqrt{0.7007/n_i}$

7.41. $\bar{c} = 8.59$, UCL = 17.384, LCL = 0. Process is not in statistical control.

7.43. (a) c chart: CL = 15.43, UCL = 27.21, LCL = 3.65
(b) u chart: CL = 15.42, UCL = 27.20, LCL = 3.64

7.45. (a) c chart: CL = 4, UCL = 10, LCL = 0
(b) u chart: CL = 1, UCL = 2.5, LCL = 0

7.47. (a) c chart: CL = 9, UCL = 18, LCL = 0
(b) u chart: CL = 4, UCL = 7, LCL = 1

7.49. $\bar{c} = 7.6$, UCL = 13.00, LCL = 2.20

7.51. $\bar{u} = 7$; $\text{UCL}_i = 7 + 3\sqrt{7/n_i}$; $\text{LCL}_i = 7 - 3\sqrt{7/n_i}$

7.53. $\bar{c} = 8.5$, UCL = 13.98, LCL = 3.02

7.55. $\bar{u} = 4$, UCL = 9

7.57. (a) $\bar{c} = 0.533$, UCL = 1.993, LCL = 0.
(b) $\alpha = 0.104$.
(c) $\beta = 0.271$. (d) $\text{ARL}_1 = 1.372 \approx 2$.

7.59. (a) $\bar{c} = 4$, UCL = 10, LCL = 0. (b) $\alpha = 0.03$.

7.61. $n > L^2 / \bar{c}$

7.65. The variable NYRSB can be thought of as an "inspection unit," representing an identical "area of opportunity" for each "sample." The "process characteristic" to be controlled is the rate of CAT scans. A u chart which monitors the average number of CAT scans per NYRSB is appropriate.

CHAPTER 8

8.1. $\hat{C}_p = 1.17$, $\hat{C}_{pk} = 1.13$

8.3. $\hat{C}_p = 5.48$, $\hat{C}_{pk} = 4.34$, $\hat{C}_{pkm} = 0.43$

8.5. (a) $\hat{C}_p = 2.98$. (b) $\hat{C}_{pk} = 1.49$.
(c) $\hat{p}_{\text{actual}} = 0.000004$, $\hat{p}_{\text{potential}} = 0.000000$.

8.7. (a) $\hat{C}_p = 0.75$. (b) $\hat{C}_{pk} = 0.71$.

(c) $\hat{C}_{pkm} = 0.70$.
(d) $\hat{p}_{\text{Actual}} = 0.025348$, $\hat{p}_{\text{Potential}} = 0.024220$

8.9. Process A: $\hat{C}_p = \hat{C}_{pk} = \hat{C}_{pm} = 1.045$, $= 0.001726$
Process B: $\hat{C}_p = 3.133$, $\hat{C}_{pk} = 1.566$,
$\hat{C}_{pm} = 0.652$, $\hat{p} = 0.000001$

8.11. $6\hat{\sigma} = 0.1350$

8.13. $6\hat{\sigma} = 0.05514$

8.15. (a) $6\hat{\sigma} = 73.2$. (b) $C_{pu} = 0.58$, $\hat{p} = 0.041047$.

8.17. No. Data is not normally distributed.

8.19. $1.26 \le C_p$, $\alpha = 0.12$

8.21. (a) $\hat{C}_{pk} = 0.42$
(b) $0.2957 \le C_{pk} \le 0.5443$

8.23. $\hat{\sigma}_{\text{process}} = 4$

8.25. (a) R chart indicates operator has no difficulty making consistent measurements.
(b) $\hat{\sigma}^2_{\text{total}} = 4.717$, $\hat{\sigma}^2_{\text{product}} = 1.695$. (c) 62.5%.
(d) $P/T = 0.272$.

8.27. (a) $6\hat{\sigma}_{\text{gage}} = 8.154$. (b) R chart indicates operator has difficulty using gage.

8.31. $\hat{p} = 0.4330$

8.33. $\hat{\mu}_{\text{Weight}} = 48$, $\hat{\sigma}_{\text{Weight}} = 0.04252$

8.35. $\mu_I \cong \mu_E/(\mu_{R_1} + \mu_{R_2})$

$$\sigma_I^2 \cong \sigma_E^2/(\mu_{R_1} + \mu_{R_2})^2 + \left(\mu_E^2/(\mu_{R_1} + \mu_{R_2})^2\right)$$
$$(\sigma_{R_1}^2 + \sigma_{R_2}^2)$$

8.37. $C \sim N(0.006, 0.000005)$. Pr{positive clearance} = 0.9964.

8.39. UTL = 323.55

8.41. UTL = 372.08

8.43. (a) $0.1257 \le x \le 0.1271$. (b) $0.1263 \le \bar{x} \le 0.1265$.

CHAPTER 9

9.1. (a) $K = 12.5$, $H = 125$. Process is out of control on upper side after observation 7.
(b) $\hat{\sigma} = 34.43$

9.3. (a) $K = 12.5$, $H = 62.5$. Process is out of control on upper side after observation 7.
(b) Process is out of control on lower side at sample 6 and upper at sample 15.

9.5. Process is in control. $\text{ARL}_0 = 370.84$

9.7. (a) $\hat{\sigma} = 12.16$. (b) Process is out of control on upper side after reading 2.

9.9. (a) $\hat{\sigma} = 5.95$. (b) Process is out of control on lower side at start, then upper after observation 9.

9.11. Process is out of control on upper side after observation 7.

9.13. $\text{ARL}_0 = 215.23$, $\text{ARL}_1 = 25.02$

9.15. $K = 54.35$, $H = 543.5$. Process is out of control virtually from the first sample.

9.17. EWMA chart: CL = 1050, UCL = 1065.49, LCL = 1034.51.
Process exceeds upper control limit at sample 10.

9.19. EWMA chart: CL = 8.02, UCL = 8.07, LCL = 7.97. Process is in control.

9.21. EWMA chart: CL = 950, UCL = 957.53, LCL = 942.47.
Process is out of control at samples 8, 12, and 13.

9.23. EWMA chart: CL = 175, UCL = 177.3, LCL = 172.70.
Process is out of control.

9.25. EWMA chart: CL = 3200, UCL = 3203.69, LCL = 3196.31.
Process is out of control from the first sample.

9.27. MA chart: CL = 8.02, UCL = 8.087, LCL = 7.953. Process is in control.

9.29. MA chart: CL = 1050, UCL = 1080.62, LCL = 1019.38.
Process is out of control at sample 10.

9.35. $k = 0.5L$

CHAPTER 10

10.1. $\bar{x}$ chart: CL = 0.55, UCL = 4.44, LCL = –3.34
R chart: CL = 3.8, UCL = 9.78, LCL = 0

10.5. $\bar{x}$ chart: CL = 52.988, UCL = 55.379, LCL = 50.596
R chart: CL = 2.338, UCL = 6.017, LCL = 0

10.7. (a) $\bar{x}$ chart: CL = 52.988, UCL = 58.727, LCL = 47.248
R chart: CL = 2.158, UCL = 7.050, LCL = 0
(c) $\bar{x}$ chart: CL = 52.988, UCL = 55.159, LCL = 49.816
s chart: CL = 1.948, UCL = 4.415, LCL = 0

10.9. (a) UCL = 44.503, LCL = 35.497
(b) UCL = 43.609, LCL = 36.391
(c) UCL = 43.239, LCL = 36.761

10.13. $\bar{x}$ chart: CL = 50, UCL = 65.848, LCL = 34.152

10.15. (a) $\hat{\sigma} = 4.000$. (b) $\hat{p} = 0.1056$. (c) UCL = 619.35, LCL = 600.65.

10.17. $\mu_0 = 0$, $\delta = 1\sigma$, $k = 0.5$, $h = 5$, UCL = 97.9, LCL = (97.9, no FIR.
No observations exceed the control limit.

10.19. $\alpha = 0.1$, $\lambda = 0.9238$, $\hat{\sigma} = 15.93$.
Observation 16 exceeds UCL.

10.21. $\mu_0 = 0$, $\delta = 1\sigma$, $k = 0.5$, $h = 5$, UCL = 22.79, LCL = (22.79, no FIR.
No observations exceed the control limits.

10.23. $\alpha = 0.1$, $\lambda = 0.7055$, = 3.227.
Observations 8, 56 and 90 exceed control limits.

10.25. $\mu_0 = 0$, $\delta = 1\sigma$, $k = 0.5$, $h = 5$, UCL = 36.69, LCL = (36.69, no FIR.
No observations exceed the control limit.

10.27. $\alpha = 0.1$, $\lambda = 0.150$, $\hat{\sigma} = 7.349$.
Several observations exceed the control limits.

10.29. (a) $r_1 = 0.49$
(b) I chart: CL = 28.57, UCL = 37.11, LCL = 20.03
(c) $\mu_0 = 28.569$, $\delta = 1\sigma$, $k = 0.5$, $h = 5$, UCL = 14.24, LCL = (14.24, no FIR.
Several observations are out of control on both lower and upper sides.
(d) EWMA chart: $\lambda = 0.15$, $L = 2.7$, CL = 28.569, UCL = 30.759, LCL = 26.380
(e) Moving CL EWMA chart: $\alpha = 0.1$, $\lambda = 0.150$, = 2.85.
A few observations are beyond the lower control limit.
(f) $\xi = 20.5017$, $\phi_1 = 0.7193$, $\phi_2 = -0.4349$.
Set up an I and MR chart for residuals.
I chart: CL = –0.04, UCL = 9.60, LCL = –9.68

10.31. (a) $E(L)$ = \$4.12/hr. (b) $E(L)$ = \$4.98/hr.
(c) $n = 5$, $k_{opt} = 3.080$, $h_{opt} = 1.368$, $\alpha = 0.00207$, $1 - \beta = 0.918$, $E(L)$ = \$4.01392/hr

10.33. (a) $E(L)$ = \$16.17/hr. (b) $E(L)$ = \$10.39762/hr.

CHAPTER 11

11.1. $UCL_{\text{Phase 2}} = 14.186$, $LCL_{\text{Phase 2}} = 0$

11.3. $UCL_{\text{Phase 2}} = 13.186$

11.5. (a) $UCL_{\text{Phase 2}} = 23.882$, $LCL_{\text{Phase 2}} = 0$.
(b) $UCL_{\text{chi-square}} = 18.548$.

11.7. (a) $UCL_{\text{Phase 2}} = 39.326$.
(b) $UCL_{\text{chi-square}} = 25.188$. (c) $m = 988$.

11.9. Assume $\alpha = 0.01$. $UCL_{\text{Phase 1}} = 32.638$, $UCL_{\text{Phase 2}} = 35.360$

11.11. (a) $\mathbf{\Sigma} = \begin{bmatrix} 1 & 0.8 & 0.8 \\ 0.8 & 1 & 0.8 \\ 0.8 & 0.8 & 1 \end{bmatrix}$
(b) $UCL_{\text{chi-square}} = 7.815$.
(c) $T^2 = 11.154$. (d) $d_1 = 0.043$, $d_2 = 8.376$, $d_3 = 6.154$.
(e) $T^2 = 21.800$. (f) $d_1 = 16.800$, $d_2 = 16.800$, $d_3 = 17.356$.

11.13. $\mathbf{\Sigma} = \begin{bmatrix} 4.440 & -0.016 & 5.395 \\ -0.016 & 0.001 & -0.014 \\ 5.395 & -0.014 & 27.599 \end{bmatrix}$

11.15. $\lambda = 0.1$ with UCL = H = 12.73, ARL_1 is between 7.22 and 12.17.

11.17. $\lambda = 0.2$ with UCL = H = 9.65. ARL_1 is between 5.49 and 10.20.

11.19. Significant variables for y_1 are x_1, x_3, x_4, x_8, and x_9.
Control limits for y_1 model I chart: CL = 0, UCL = 2.105, LCL = –2.105
Control limits for y_1 model MR chart: CL = 0.791, UCL = 2.586, LCL = 0
Significant variables for y_2 are x_1, x_3, x_4, x_8, and x_9.
Control limits for y_2 model I chart: CL = 0, UCL = 6.52, LCL = –6.52
Control limits for y_2 model MR chart: CL = 2.45, UCL = 8.02, LCL = 0

11.21. (a) $\mathbf{z}_1$ = {0.29168, 0.29428, 0.19734, 0.83902, 3.20488, 0.20327, –0.99211, –1.70241, –0.14246, –0.99498, 0.94470, –1.21950, 2.60867, –0.12378, –1.10423, –0.27825, –2.65608, 2.36528, 0.41131, –2.14662}

CHAPTER 12

12.3. Process is adjusted at observations 3, 4, 7, and 29.

12.5. $m = 1$, $Var_1 = 147.11$, $Var_1/Var_1 = 1.000$
$m = 2$, $Var_2 = 175.72$, $Var_2/Var_1 = 1.195$
$m = 3$, $Var_3 = 147.47$, $Var_3/Var_1 = 1.002$
$m = 4$, $Var_4 = 179.02$, $Var_4/Var_1 = 1.217$
$m = 5$, $Var_5 = 136.60$, $Var_5/Var_1 = 0.929$, . . .
Variogram stabilizes near 1.5
$r_1 = 0.44$, $r_2 = 0.33$, $r_3 = 0.44$, $r_4 = 0.32$, $r_5 = 0.30$, . . .
Sample ACF slowly decays.

12.7. In each control scheme, adjustments are made after each observation following observation 2. There is no difference in results; variance for each procedure is the same.

12.9. (b) Average is closer to target (44.4 vs. 46.262), and variance is smaller (223.51 vs. 78.32).
(c) Average is closer to target (47.833) and variance is smaller (56.40).

CHAPTER 13

13.1. Glass effect: $F_0 = 273.79$, P value = 0.000
Phosphor effect: $F_0 = 8.84$, P value = 0.004
Glass × Phosphor interaction: $F_0 = 1.26$, P value = 0.318

13.3. Normality assumption is reasonable.
Constant variance assumption is reasonable.

13.5. Plots of residuals versus factors A and C show unequal scatter. Residuals versus predicted indicates that variance not constant. Residuals are approximately normally distributed.

13.7. Largest effect is factor A.

13.9. Block 1: (1), *ab*, *ac*, *bc*, *ad*, *bd*, *cd*, *ae*, *be*, *ce*, *de*, *abcd*, *abce*, *abde*, *acde*, *bcde*
Block 2: *a*, *b*, *c*, *d*, *e*, *abc*, *abd*, *acd*, *bcd*, *abe*, *ace*, *bce*, *ade*, *bde*, *cde*, *abcde*

13.11. (b) $I = ACE = BDE = ABCD$, $A = CE = BCD = ABDE$, $B = DE = ACD = ABCE$, $C = AE = ABD = BCDE$, $D = BE = ABC = ACDE$, $E = AC = BD = BCDE$, $AB = CD = ADE = BCE$, $AD = BC = ABE = CDE$
(c) A = –1.525, B = –5.175, C = 2.275, D = –0.675, E = 2.275, AB = 1.825, AD = –1.275
(d) With only main effect B: $F_0 = 8.88$, P value = 0.025.
(e) Residuals plots are satisfactory.

13.13. (a) Main Effects: $F_0 = 1.70$, P value = 0.234
2-Way Interactions: $F_0 = 0.46$, P value = 0.822
Curvature: $F_0 = 16.60$, P value = 0.004
Lack of fit: $F_0 = 0.25$, P value = 0.915

13.15. (a) A = 47.7, B = –0.50, C = 80.6, D = –2.40, AB = 1.10, AC = 72.80, AD = –2.00
(b) Model with C, AC, A:
Main Effects: $F_0 = 1710.43$, P value = 0.000
2-Way Interactions: $F_0 = 2066.89$, P value = 0.000
Curvature: $F_0 = 1.11$, P value = 0.327

CHAPTER 14

14.1. (b) $\Delta x = 1$, $\Delta x_2 = 0.6$

14.3. (a) CCD with $k = 2$ and $\alpha = 1.5$. The design is not rotatable.
(b) $y = 160.868 - 58.294x_1 + 2.412x_2$
$- 10.855x_1^2 + 6.923x_2^2 - 0.750x_1x_2$
(c) $x_1 = +1.5$, $x_2 = -0.22$
(d) Temp = 825, Time = 26.7

14.5. (a) CCD with $k = 2$ and $\alpha = 1.4$. The design is rotatable.
(b) $y = 13.727 + 0.298x_1 - 0.407x_2$
$- 0.125x_1^2 - 0.079x_2^2 + 0.055x_1x_2$
From the plots and the optimizer, setting x_1 in a range from 0 to +1.4 and setting x_2 between –1 and –1.4 will maximize viscosity.

14.7. (a) The design is resolution IV with $A = BCD$, $B = ACD$, $C = ABD$, $D = ABC$, $E = ABCDE$, $AB = CD$, $AC = BD$, $AD = BC$, $AE = BCDE$, $BE = ACDE$, $CE = ABDE$, $DE = ABCE$, $ABE = CDE$, $ACE = BDE$, $ADE = BCE$.
(b) Factors A, B, D, E and interaction BE affect mean free height.
(c) Factors A, B, D and interactions CE and ADE affect standard deviation of free height.
(e) A 2^{5-1}, resolution V design can be generated with $E = \pm ABCD$.

14.9. Mean Free Height = 7.63 + 0.12A – 0.081B
Variance of Free Height = $(0.046)^2 + (-0.12 + 0.077\text{B})^2 + 0.02$
One solution with mean Free Height ≅ 7.50 and minimum standard deviation of Free Height is A = –0.42 and B = 0.99.

14.11. (a) Recommended operating conditions are temperature = +1.4109 and pressure = (1.4142, to achieve predicted filtration time of 36.7.
(b) Recommended operating conditions are temperature = +1.3415 and pressure = (0.0785, to achieve predicted filtration time of 46.0.

CHAPTER 15

15.1. Two points on OC curve are $P_a\{p = 0.007\} = 0.95190$ and $P_a\{p = 0.080\} = 0.08271$.

15.3. (a) Two points on OC curve are $P_a\{d = 35\} = 0.95271$ and $P_a\{d = 375\} = 0.10133$.

(b) Two points on OC curve are $P_a\{p = 0.0070\} = 0.9519$ and $P_a\{p = 0.0750\} = 0.1025$.
(c) Difference in curves is small. Either is appropriate.

15.5. $n = 80$, $c = 7$

15.7. Different sample sizes offer different levels of protection. Consumer is protected from an LTPD = 0.05 by $P_a\{N = 5000\} = 0.00046$ or $P_a\{N = 10{,}000\} = 0.00000$, but pays for high probability of rejecting acceptable lots (i.e., for $p = 0.025$, $P_a\{N = 5000\} = 0.294$ while $P_a\{N = 10{,}000\} = 0.182$).

15.9. AOQL = 0.0234

15.11. (a) Two points on OC curve are $P_a\{p = 0.016\}= 0.95397$ and $P_a\{p = 0.105\}= 0.09255$.
(b) $p = 0.103$
(d) $n = 20$, $c = 0$. This OC curve is much steeper.
(e) For $c = 2$, Pr{reject} = 0.00206, ATI = 60. For $c = 0$, Pr{reject} = 0.09539, ATI = 495

15.13. (a) Constants for limit lines are: $k = 1.0414$, $h_1= 0.9389$, $h_2 = 1.2054$, and $s = 0.0397$.
(b) Three points on OC curve are $P_a\{p_1 = 0.01\} = 1 - \alpha = 0.95$, $P_a\{p_2 = 0.10\} = \beta = 0.10$, and $P_a\{p = s = 0.0397\} = 0.5621$.

15.15. $AOQ = [P_a \times p \times (N - n)] / [N - P_a \times (n\,p) - (1 - P_a) \times (N\,p)]$

15.17. Normal: sample size code letter = H, $n = 50$, Ac = 1, Re = 2
Tightened: sample size code letter = J, $n = 80$, Ac = 1, Re = 2
Reduced: sample size code letter = H, $n = 20$, Ac = 0, Re = 2

15.19. (a) Sample size code letter = L
Normal: $n = 200$, Ac = 3, Re = 4
Tightened: $n = 200$, Ac = 2, Re = 3
Reduced: $n = 80$, Ac = 1, Re = 4

15.21. (a) Minimum cost sampling effort that meets quality requirements is $50{,}001 \le N \le 100{,}000$, $n = 65$, $c = 3$. (b) ATI = 82

CHAPTER 16

16.1. (a) $n = 35$, $k = 1.7$. (b) $Z_{LSL} = 2.857 > 1.7$, so accept lot.
(c) From nomograph, $P_a\{p = 0.05\} \approx 0.38$

16.3. $AOQ = P_a \times p \times (N - n) / N$, $\rightarrow$ $ATI = n + (1 - P_a) \times (N - n)$

16.5. From MIL-STD-105E, $n = 200$ for normal and tightened and $n = 80$ for reduced. Sample sizes required by MIL-STD-414 are considerably smaller than those for MIL-STD-105E.

16.7. Assume inspection level IV. Sample size code letter = O, $n = 100$, $k_{\text{normal}} = 2.00$, $k_{\text{tightened}} = 2.14$. $Z_{LSL} = 3.000 > 2.00$, so accept lot.

16.9. (a) From nomograph for variables: $n = 30$, $k = 1.8$
(b) Assume inspection level IV. Sample size code letter = M
Normal: $n = 50$, $M = 1.00$
Tightened: $n = 50$, $M = 1.71$
Reduced: $n = 20$, $M = 4.09$
σ known permits smaller sample sizes than σ unknown.
(c) From nomograph for attributes: $n = 60$, $c = 2$
Variables sampling is more economic when σ is known.
(d) Assume inspection level II. Sample size code letter = L
Normal: $n = 200$, Ac = 5, Re = 6
Tightened: $n = 200$, Ac = 3, Re = 4
Reduced: $n = 80$, Ac = 2, Re = 5
Much larger samples are required for this plan than others.

16.11. (a) Three points on OC curve are $P_a\{p = 0.001\} = 0.9685$, $P_a\{p = 0.015\} = 0.9531$, and $P_a\{p = 0.070\} = 0.0981$.
(b) ATI = 976
(c) $P_a\{p = 0.001\} = 0.9967$, ATI = 131
(d) $P_a\{p = 0.001\} = 0.9958$, ATI = 158

16.13. $i = 4$, $P_a\{p = 0.02\} = 0.9526$

16.15. For $f = 1/2$, $i = 140$: $u = 155.915$, $v = 1333.3$, AFI = 0.5523, $P_a\{p = 0.0015\} = 0.8953$
For $f = 1/10$, $i = 550$: $u = 855.530$, $v = 6666.7$, AFI = 0.2024, $P_a\{p = 0.0015\} = 0.8863$
For $f = 1/100$, $i = 1302$: $u = 4040.00$, $v = 66666.7$, AFI = 0.0666, $P_a\{p = 0.0015\} = 0.9429$

16.17. For $f = 1/5$, $i = 38$: AFI = 0.5165, $P_a\{p = 0.0375\} = 0.6043$
For $f = 1/25$, $i = 86$: AFI = 0.5272, $P_a\{p = 0.0375\} = 0.4925$

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SPC
Calculations for Control Limits

Notation:

UCL—Upper Control Limit	$\bar{x}$ —Average of Measurements
LCL—Lower Control Limit	$\bar{\bar{x}}$ —Average of Averages
CL —Center Line	R —Range
n —Sample Size	$\bar{R}$ —Average of Ranges
PCR—Process Capability Ratio	USL—Upper Specification Limit
$\hat{\sigma}$ —Process Standard Deviation	LSL—Lower Specification Limit

Variables Data ($\bar{x}$ and R Control Charts)

$\bar{x}$ Control Chart

UCL = $\bar{\bar{x}} + A_2\bar{R}$

LCL = $\bar{\bar{x}} - A_2\bar{R}$

CL = $\bar{\bar{x}}$

R Control Chart

UCL = $\bar{R}\,D_4$

LCL = $\bar{R}\,D_3$

CL = $\bar{R}$

Capability Study

$C_p = (\text{USL} - \text{LSL})/(6\hat{\sigma})$; where $\hat{\sigma} = \bar{R}/d_2$

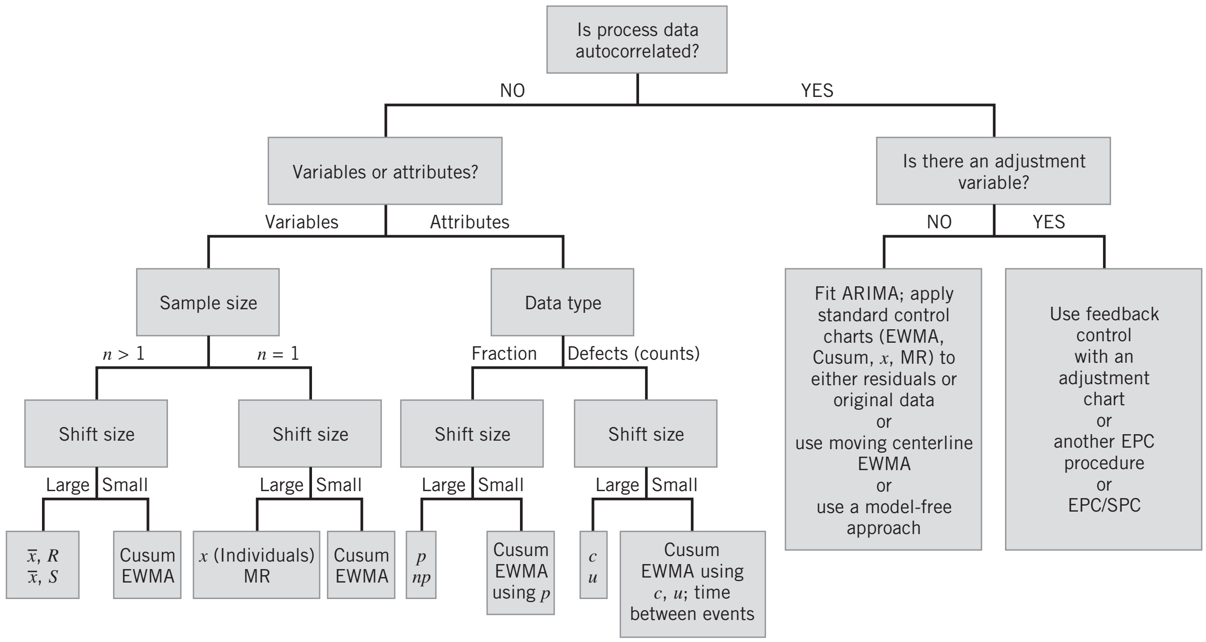
n	A_2	D_3	D_4	d_2
2	1.880	0.000	3.267	1.128
3	1.023	0.000	2.574	1.693
4	0.729	0.000	2.282	2.059
5	0.577	0.000	2.114	2.326
6	0.483	0.000	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078

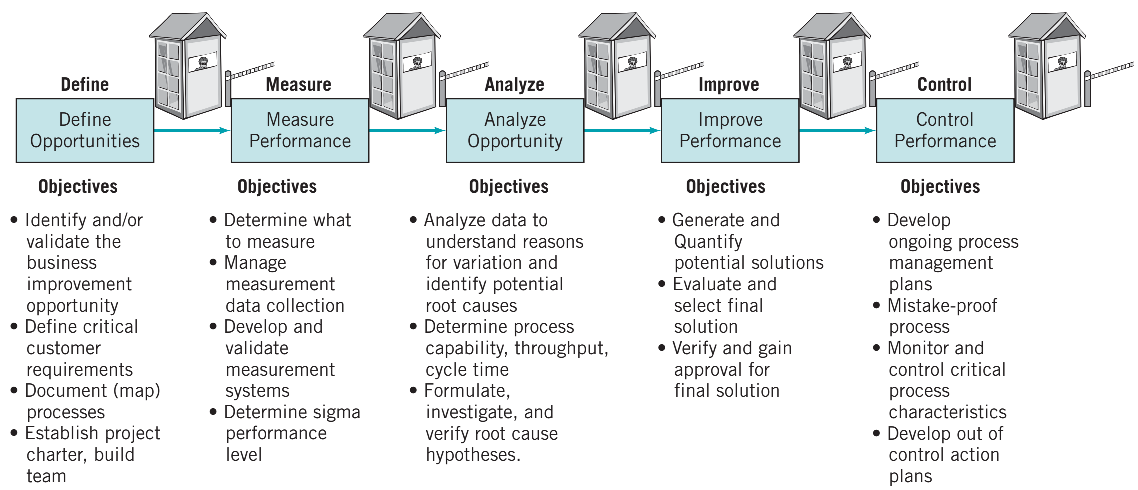
Attribute Data (p, np, c, and u Control Charts)

Control Chart Formulas

	***p* (fraction)**	***np* (number of nonconforming)**	***c* (count of nonconformances)**	***u* (count of nonconformances/unit)**
CL	$\bar{p}$	$n\bar{p}$	$\bar{c}$	$\bar{u}$
UCL	$\bar{p}+3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p}+3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c}+3\sqrt{\bar{c}}$	$\bar{u}+3\sqrt{\frac{\bar{u}}{n}}$
LCL	$\bar{p}-3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p}-3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c}-3\sqrt{\bar{c}}$	$\bar{u}-3\sqrt{\frac{\bar{u}}{n}}$
Notes	If n varies, use $\bar{n}$ or individual n_i	n must be a constant	n must be a constant	If n varies, use $\bar{n}$ or individual n_i

Guide to Univariate Process Monitoring and Control





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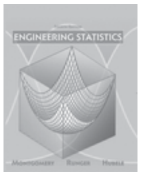

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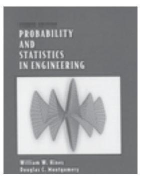

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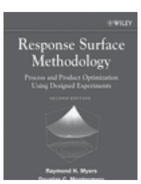

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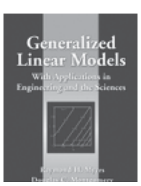

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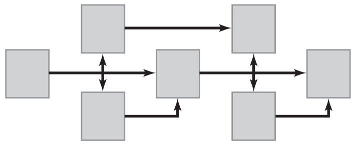
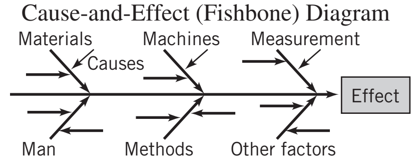
Quality Improvement Tools

Process Flow Diagram



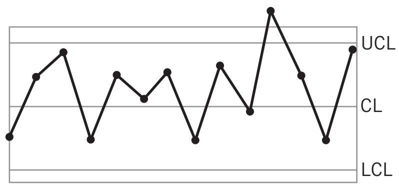
- Expresses detailed knowledge of the process
- Identifies process flow and interaction among the process steps
- Identifies potential control points

Cause-and-Effect (Fishbone) Diagram



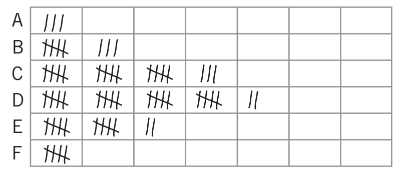
- All contributing factors and their relationships are displayed
- Identifies problem areas where data can be collected and analyzed

Control Chart



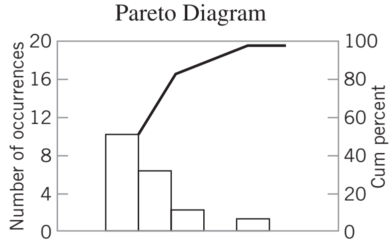
- Helps reduce variability
- Monitors performance over time
- Allows process corrections to prevent rejections
- Trends and out-of-control conditions are immediately detected

Check Sheet



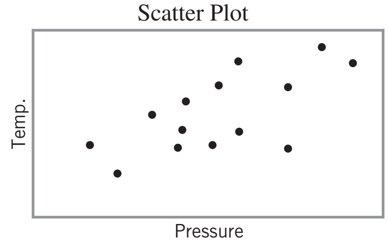
- Simplifies data collection and analysis
- Spots problem areas by frequency of location, type, or cause

Pareto Diagram



- Identifies most significant problems to be worked first
- Historically 80% of the problems are due to 20% of the factors
- Shows the vital few

Scatter Plot

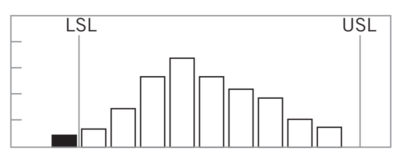


- Identifies the relationship between two variables
- A positive, negative, or no relationship can be easily detected

Design of Experiments (DOX)

- Useful in process development and troubleshooting
- Identifies magnitude and direction of important process variable effects
- Greatly reduces the number of runs required with a process experiment
- Identifies interaction among process variables
- Useful in engineering design and development
- Focuses on optimizing system performance

Histogram



- The shape shows the nature of the distribution of the data
- The central tendency (average) and variability are easily seen
- Specification limits can be used to display the capability of the process