PHYS 2210 Exam 2

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1. Suppose a 900 kg satellite is orbiting the earth one earth radius above the surface.

a. Find the force of gravity between the earth and the satellite.

This can be found using

$$F_g = G\frac{M_{satellite}M_{earth}}{r^2} = 0\frac{(900)(5.972 \times 10^{24})}{8.1179282 \times 10^{13}} = 2209.42\ N$$

where r is the distance from the center of the earth to the satellite and G is the gravitational constant.

b. Find the orbital period of the satellite.

This can be found using

$$T = 2\pi\sqrt{\frac{r^3}{GM_{earth}}} = 2\pi\sqrt{\frac{5.1719321 \times 10^{20}}{0(5.972 \times 10^{24})}} = 1.431465 \times 10^4\ seconds$$

c. Find the tangential velocity required to keep the satellite in orbit.

This can be found using

$$v = \sqrt{\frac{GM_{earth}}{r}} = \sqrt{\frac{3.9857606 \times 10^{14}}{`round(r2*r,2)`}} = 5592.9\ \frac{m}{s}$$

d. Find the angular velocity of the satellite.

Angular velocity can be found using $\omega^2 r = \frac{GM_E}{r^2}$ and solving for ω (where ω is the angular velocity) we get

$$\omega = \sqrt{\frac{GM_{earth}}{r^3}} = \sqrt{\frac{3.9857606 \times 10^{14}}{2.0687728 \times 10^{21}}} = 4.39 \times 10^{-4}\ \frac{rad}{s}$$

Side note: We could find the orbital period from here since

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.39 \times 10^{-4}} = 1.431465 \times 10^4$$

which is the same answer we got for part **b**.

2) A painter is standing on 4 kg board which is 12 m long and suspended on each end by a rope. The painter is standing 3 m from the left side, closer to the first rope. The mass of the painter is 90 kg.

a. Find the tension in the first rope.

The system is in equilibrium so the sum of the forces is zero ($\sum F = 0$) AND the sum of the torques is zero ($\sum \tau = 0$).

$$(m_{board}(CM) + m_{painter}(L - x))g + TL$$

where CM is the center of mass of the board and $L - x$ is the distance from the edge of the board (L) that the painter is standing on and g is the acceleration of gravity. So

$$T_1 = g\frac{m_1\frac{L}{2} + m_2(L - x)}{L} = 681.1\ N$$

b. Find the tension in the second rope.

$$T_2 = (m_1 + m_2)9.8 - Tension_1 = 240.1\ N$$

Checking the equilibrium of the system, i.e. $g(m_1 + m_2) = T_1 + T_2$, we can see that

$$9.8(4 + 90) = 681.1 + 240.1 = 921.2\ N$$

c. The painter climbs down from the board, just before the second rope suddenly snaps. Find the instantaneous angular acceleration of the board.

Since $L = I\omega$, where L is the angular momentum, angular acceleration can be found by $\tau = I\alpha$, where α is the angular acceleration, so angular acceleration can be found by $\alpha = \frac{\tau}{I}$.

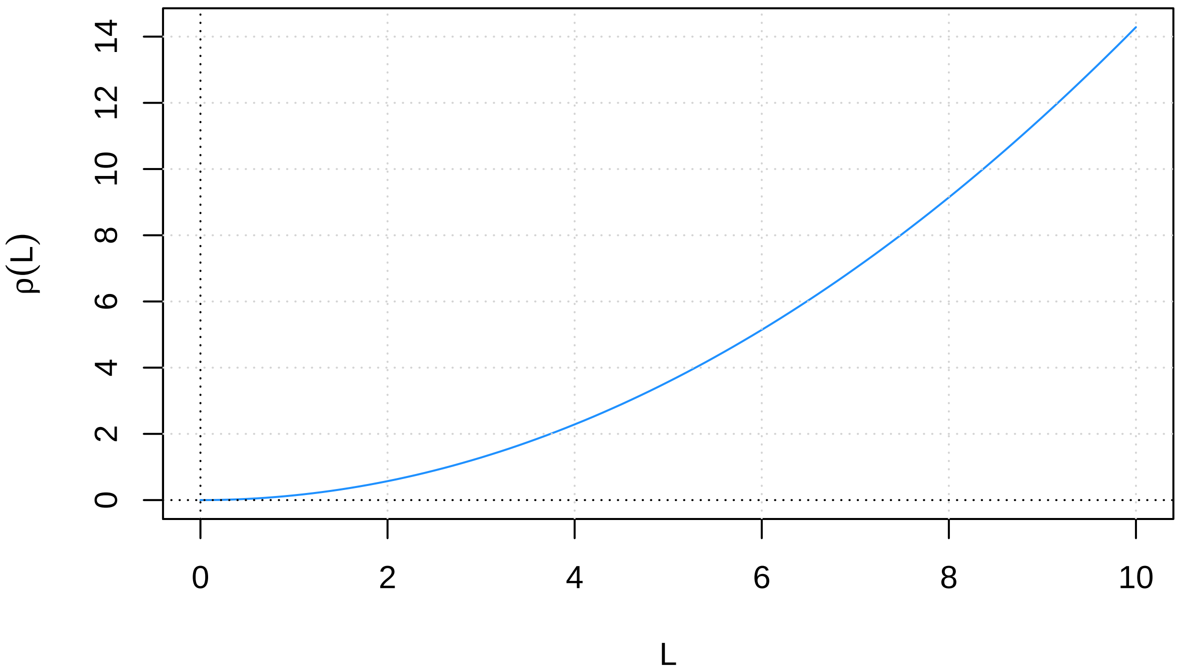
So we need to find I and τ. Using $I = \frac{Ma^2}{3} = 192$ since the pivot point is now 12 meters from where the rope snaps and $\tau = mg\frac{L}{2} = 235.2$.

$$\alpha = \frac{235.2}{192} = 1.225\ \frac{rad}{s}$$

3) A certain thin (one-dimensional) rod has a length of L and a non-uniform density which is given by the following function:

$$\rho(L) = \frac{L^2}{7}$$

We can picture the density of the rod as a function of L. A plot of this function is pictured here for reference for lengths from 0 to 10.



We can see that the density takes on what appears to be an exponential function, meaning that the density of the rod increases exponentially with its length.

a. Find the rod's center of mass as a function of L.

The mass of our object depends on it's length.

$$m = \int_0^L \frac{1}{7}x^2 dx = \frac{1}{21}L^3 \ kg$$

Now, the center or mass, or $\bar{x}$, can be defined as

$$\bar{x} = \int_0^L \frac{\rho(x)dx}{m} x$$

where I have already found m above, I focus on the top piece.

$$\bar{x} = \frac{\int_0^L \rho(x)xdx}{m} = \frac{1}{m}\int_0^L \frac{x^3}{7}dx = \frac{1}{28m}L^4$$

Plugging in for m we get

$$\bar{x} = \frac{21L^4}{28L^3} = \frac{3}{4}L$$

b. Find the moment of inertia for the rod if it is rotated about its midpoint($\frac{L}{2}$).

In general the moment of inertia (I) can be defined as

$$I = mr^2$$

where m is the objects mass and r is the distance of the object from the axis of rotation. In part a we found that $m = \frac{1}{21}L^3$. For a solid object we have to take all the particles that make up it's mass into account (you didn't make this easy). Using (since it's one-dimensional linear density)

$$dI = x^2 dm$$

I will attempt to find the moment of inertia of our non-uniform rod. Recall that $\rho(x) = \frac{x^2}{7}$ (our density function) and $dm = \rho(x)dx$.

$$dI = \frac{x^4}{7}dx$$

$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} dI = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{x^4}{7}dx = \frac{1}{35}[\frac{L^5}{2^5} - (-\frac{L^5}{2^5})] = \frac{1}{560}L^5$$

Note: This is about 45% smaller than if we were to use the formula for the moment of inertia for a rod of uniform density($\frac{1}{12}MR^2$), which would end up being approx. $\frac{1}{252}L^5$.

4) A large metal ball with a uniform density is dropped into a lake. The ball has a radius of 12 cm and (beginning at rest) sinks 40 m to the bottom in 3.6 s.

a. How much water pressure does the ball experience at the bottom of the lake?

Using

$$Pressure = \frac{Force}{Area}$$

we can then rewrite this as

$$P = \frac{\rho g h \pi r^2}{\pi r^2} = \rho g h$$

and applying it to our problem we get (assuming the density of the lake water is $1000\frac{kg}{m^3}$)

$$P = 1000\frac{kg}{m^3}(9.8)\frac{m}{s^2}(40)m = 3.92 \times 10^5 Pa$$

b. What is the ball made of? (Hint: Find its density).

Density (ρ) is equal to the volume of an object divided by it's mass.

Ignoring the drag force, the net force (F_{net}) of the system is

$$F_{net} = W_{ball} - F_b = 9.8\rho_{ball}V - 9.8\rho_{water}V$$

where F_b is the buoyant force and V is the volume of the ball.

In order to solve this we need to know the acceleration of the ball (we could also use this equality to solve for a if we knew the density of the ball and water). Using the kinematic equation, $x = \frac{1}{2}at^2$, I can solve for a since I know x and t.

$$a = \frac{2x}{t^2} = \frac{80}{3.6^2} = 6.17 \ \frac{m}{s^2}$$

Doing some more rearranging I get

$$\rho_{ball} = \frac{9.8(1000)\frac{kg}{m^3}}{9.8 - 6.1728395} = 2701.84 \ \frac{kg}{m^3}$$

The ball is made of aluminum (Al), which has a density of $2.70\frac{g}{cm^3}$.

5) Astronauts use springs to work out in space because there isn't enough gravity to provide significant resistance to keep their muscles from atrophying. Suppose an astronaut is attached to a horizontal spring and begins oscillating back-and-forth once every 2 seconds. Back on Earth, a 20 kg mass was hung vertically on this same spring, causing it to stretch 21 cm.

a. What is the mass of the astronaut?

First, the spring constant is

$$F = -kx$$

$$k = \frac{mg}{x} = \frac{(20kg)(9.8)\frac{m}{s^2}}{0.21m} = 933.33 \ \frac{N}{m}$$

and now mass can now be found with period, $T = 2s$. The period is defined as

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and so solving for m we get

$$m = \frac{kT^2}{4\pi^2} = \frac{933.33(4)}{4\pi^2} = 94.5664381 \ kg$$

6) Suppose a certain musical instrument consists of a pipe which is open at one end and closed at the other. Music is produced as sound waves move through the pipe, creating standing wave patterns. The pipe has a length of 80 cm.

a. Find the wavelength of the first, third, and fifth harmonics (show your work).

Using

$$f_n = \frac{nv}{4L}$$

where f_n is the nth frequency and v is the speed of sound (I'll use $343\frac{m}{s}$) and L is the length of the pipe, in this case $0.8 \ m$. Wavelength, (λ), can be found using

$$\lambda_n = \frac{4L}{n}$$

$$\lambda_1 = \frac{4(0.80)}{1} = 3.2 \ m$$

$$\lambda_3 = \frac{4(0.80)}{3} = 1.07\ m$$

$$\lambda_5 = \frac{4(0.80)}{5} = 0.64\ m$$

b. Find the frequency of the first, third, and fifth harmonics (show your work).

$$f_1 = \frac{1(343)\frac{m}{s}}{4(0.80)m} = 107.19\ Hz$$

$$f_3 = \frac{3(343)\frac{m}{s}}{4(0.80)m} = 321.56\ Hz$$

$$f_5 = \frac{5(343)\frac{m}{s}}{4(0.80)m} = 535.94\ Hz$$

c. Suppose you were riding your bike at 18 m/s towards the instrument as the fifth harmonic was being played. What frequency would you hear?

We can use what's known as the Doppler effect to solve this one. This is stated in the equation

$$f_L = \frac{\nu + v_L}{\nu + v_s} f_s$$

where ν is the speed of sound, f_L is the frequency as heard by the listener, v_L is the velocity of the listener, f_s is the frequency of the source, and v_s is the velocity of the source.

$$f_L = \frac{(343 + 18)\frac{m}{s}}{(343 + 0)\frac{m}{s}} 535.94\ Hz = 564.06\ Hz$$

This is slightly higher than the original pitch which is what we would expect.