Homework 5

1. (10 pts) Ex. 5.1 Show how the result $\Sigma = \Lambda\Lambda^T + \psi$ arises from the assumptions of uncorrelated factors, independence of the specific variates, and independence of common factors and specific variances. What form does Σ take if the factors are allowed to be correlated?

2. (10 pts) Ex. 5.2 Show that the communalities in a factor analysis model are unaffected by the transformation $\Lambda^* = \Lambda M$, where M is orthonormal matrix.

3. (10 pts) Ex. 5.3 Give a formula for the proportion of variance explained by the jth factor estimated by the principal factor approach.

4. (20 pts) Ex. 5.4 Apply the factor analysis model separately to the life expectancies (life.csv on canvas) of men and women and compare the results.

5. (15 pts) Ex. 5.5 The correlation matrix given below arises from the scores of 220 boys in six school subjects: (1) French, (2) English, (3) History, (4) Arithmetic, (5) Algebra, and (6) Geometry. Find the two-factor solution from a maximum likelihood factor analysis. By plotting the derived loadings, find an orthogonal rotation that allows easier interpretation of the results.

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\mathbf{R} = \begin{array}{l} \text{French} \\ \text{English} \\ \text{History} \\ \text{Arithmetic} \\ \text{Algebra} \\ \text{Geometry} \end{array} \left(\begin{array}{llllll} 1.00 & & & & & \\ 0.44 & 1.00 & & & & \\ 0.41 & 0.35 & 1.00 & & & \\ 0.29 & 0.35 & 0.16 & 1.00 & & \\ 0.33 & 0.32 & 0.19 & 0.59 & 1.00 & \\ 0.25 & 0.33 & 0.18 & 0.47 & 0.46 & 1.00 \end{array} \right).
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6. (15 pts) Ex. 5.6 The matrix below shows the correlations between ratings on nine statements about pain made by 123 people suffering from extreme pain. Each statement was scored on a scale from 1 to 6, ranging from agreement to disagreement. The nine pain statements were as follows:

   1) Whether or not I am in pain in the future depends on the skills of the doctors.
   2) Whenever I am in pain, it is usually because of something I have done or not done,
   3) Whether or not I am in pain depends on what the doctors do for me.
   4) I cannot get any help for my pain unless I go to seek medical advice.
   5) When I am in pain I know that it is because I have not been taking proper exercise or eating the right food.
   6) People's pain results from their own carelessness.
   7) I am directly responsible for my pain,
   8) Relief from pain is chief controlled by the doctors.
   9) People who are never 0 in pain are just plain lucky.

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\left(\begin{array}{lllllllll} 1.00 & & & & & & & & \\ -0.04 & 1.00 & & & & & & & \\ 0.61 & -0.07 & 1.00 & & & & & & \\ 0.45 & -0.12 & 0.59 & 1.00 & & & & & \\ 0.03 & 0.49 & 0.03 & -0.08 & 1.00 & & & & \\ -0.29 & 0.43 & -0.13 & -0.21 & 0.47 & 1.00 & & & \\ -0.30 & 0.30 & -0.24 & -0.19 & 0.41 & 0.63 & 1.00 & & \\ 0.45 & -0.31 & 0.59 & 0.63 & -0.14 & -0.13 & -0.26 & 1.00 & \\ 0.30 & -0.17 & 0.32 & 0.37 & -0.24 & -0.15 & -0.29 & 0.40 & 1.00 \end{array} \right)
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   (a) (5 pts) Apply maximum likelihood factor analysis, and use the test described in the chapter to select the necessary number of common factors.

   (b) (10 pts) Rotate the factor solution selected using an orthogonal procedure, and interpret the results.