Problem Set 9

Part I

1 a) The time series plot indicates sales of toothpaste is nonstationary. It clearly does not vary about a constant mean (increasing).

b) $w_2 = 239 - 235 = 4$

$w_3 = 244.09 - 239 = 5.09$

c) The differences, w_t, appear to be stationary. Maybe a slight decreasing trend? I would say it looks stationary, varying about a constant mean near 9 or 10.

2 for $z_1 \cdots z_{10}$ $\sum(z_t - \bar{z})^2 = 8{,}762.34$ $\bar{z} = 273.1004$

$$r_2 = \frac{\sum_{t=1}^{8}(z_t - \bar{z})(z_{t+2} - \bar{z})}{\sqrt{\sum_{t=1}^{10}(z_t - \bar{z})^2}} = \frac{3624.018}{8762.34} = .41359$$

3 a) SAC plot for z_t does not indicate stationarity. r_k dies down very gradually

b) SAC plot for w_t seems to indicate stationarity. r_k dies down pretty quickly. Maybe after lag 2 or 3?

c)

k	r_k
1	.64279
2	.32124
3	.24558

$$s_{r_3} = \left(\frac{1 + 2\sum_{k=1}^{2} r_k^2}{89}\right)^{1/2} = \left(\frac{1 + 2[.64279^2 + .32124^2]}{89}\right)^{1/2}$$

$$= .151128$$

$$t_{r_3} = \frac{.24558}{.151128} = \boxed{1.625}$$

d) $r_{11} = r_1 = .64279$

$$r_{22} = \frac{r_2 - r_{11}\cdot r_1}{1 - r_{11}\cdot r_1} = \frac{.32124 - .64279^2}{1 - .64279^2} = -.156673$$

$$r_{21} = r_{11} - r_{22}\cdot r_{11} = .64279 + .15667(.64279) = .743498$$

$$r_{33} = \frac{r_3 - (r_{21}\cdot r_2 + r_{22}\cdot r_1)}{1 - (r_{21}\cdot r_1 + r_{22}\cdot r_2)}$$

$$= \frac{.24558 - (.743498\cdot .32124 - .15667\cdot .64279)}{1 - (.743498\cdot .64279 - .15667\cdot .32124)}$$

$$= .1877$$

$$s_{r_{22}} = \frac{1}{\sqrt{89}} = .106 \Rightarrow t_{r_{22}} = \frac{-.156673}{.106} = -1.478$$

Ignore

4) a) SAC plot follows a damped exponential decay pattern

SPAC plot cuts off abruptly after one lag

$\Rightarrow$ These plots seem to indicate an AR(1) model

b) from SAS output

$\boxed{\hat{\delta} = 3.064672}$ $\boxed{\hat{\phi}_1 = .64774}$

c) $\hat{\mu} = \dfrac{\hat{\delta}}{1-\hat{\phi}_1} = \dfrac{3.064672}{1-.64774} = 8.7$

d) $w_t = \delta + \phi_1 w_{t-1} + u_t$

$(z_t - z_{t-1}) = \delta + \phi_1 (z_{t-1} - z_{t-2}) + u_t$

$z_t = \delta + \phi_1 z_{t-1} + z_{t-1} - \phi_1 z_{t-2} + u_t$

$$\boxed{z_t = \delta + (1+\phi_1) z_{t-1} - \phi_1 z_{t-2} + u_t}$$

e) $\hat{z}_3 = \hat{\delta} + (1+\hat{\phi}_1) z_2 - \hat{\phi}_1 z_1$

$= 3.06464 + (1.64774)(239) - .6477(235)$

$= 244.6556$

$z_3 - \hat{z}_3 = 244.09 - 244.6556 = -.5656$

f) $\hat{z}_{91}(90) = \hat{\delta} + (1+\hat{\phi}_1) z_{90} - \hat{\phi}_1 z_{89}$

$= 3.06464 + 1.64774(1029.480) - .64774(1018.42)$

$= 1039.71$

$\hat{z}_{92}(90) = 3.06464 + 1.64774(\underset{\uparrow\,\hat{z}_{91}(90)}{1039.71}) - .64774(\underset{\uparrow\, z_{90}}{1029.480})$

$= 1049.4$

$\hat{z}_{93}(90) = 3.06464 + 1.64774(\underset{\uparrow\,\hat{z}_{92}(90)}{1049.4}) - .64774(\underset{\uparrow\,\hat{z}_{91}(90)}{1039.71})$

$= 1058.74$

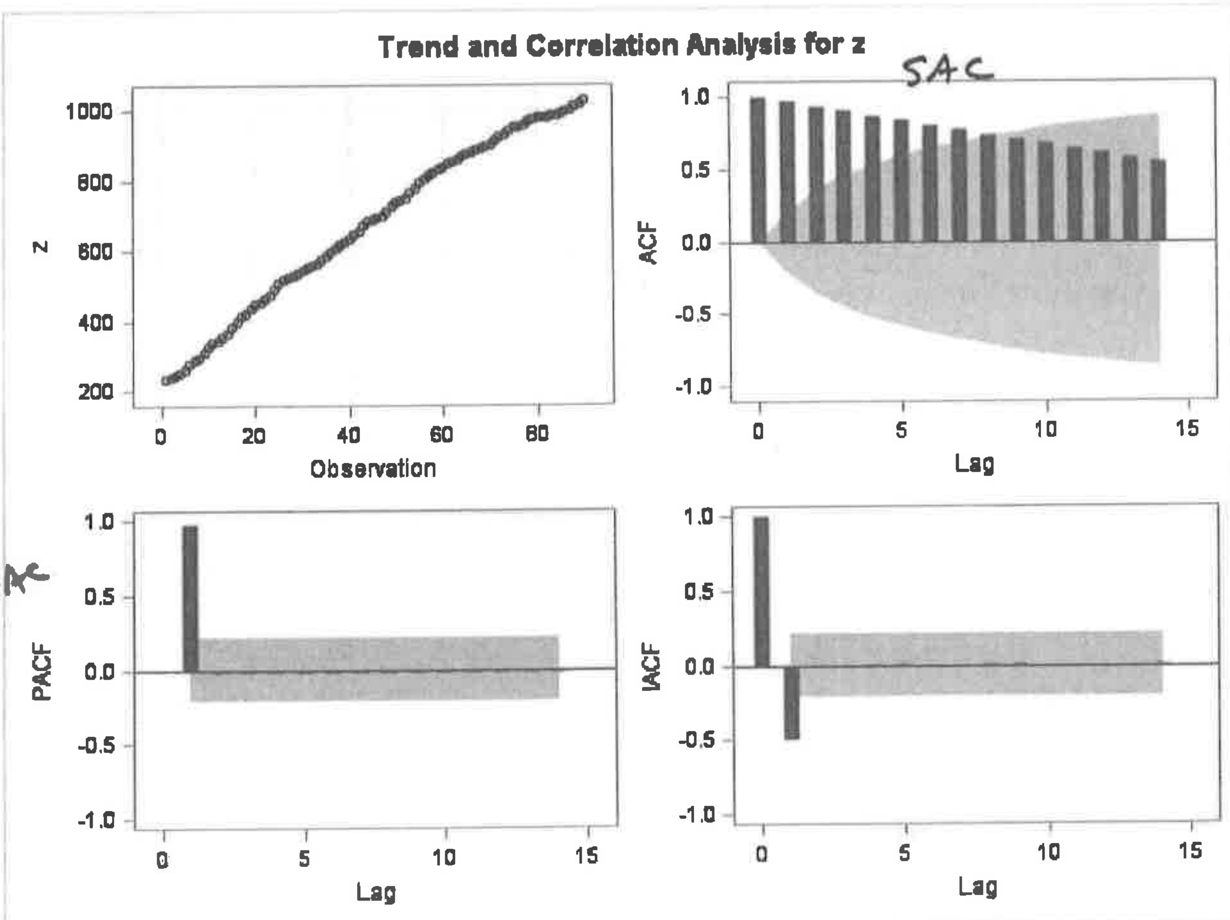
toothpaste original time series

The ARIMA Procedure

Name of Variable = z		
Mean of Working Series	674.2709	$\bar{z}$
Standard Deviation	240.1522	s_z
Number of Observations	90	

Autocorrelation Check for White Noise										
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations						
6	453.48	6	<.0001	0.968	0.937	0.904	0.872	0.839	0.807
12	750.14	12	<.0001	0.774	0.741	0.709	0.676	0.643	0.610

r_k



toothpaste original time series z_t

Obs	LAG	CORR	PARTCORR
1	0	1.00000	1.00000
2	1	0.96846	0.96846
3	2	0.93652	-0.02254
4	3	0.90414	-0.02347
5	4	0.87160	-0.01960
6	5	0.83914	-0.01605
7	6	0.80680	-0.01563
8	7	0.77422	-0.02173
9	8	0.74127	-0.02436
10	9	0.70852	-0.01567
11	10	0.67587	-0.01746
12	11	0.64291	-0.02447
13	12	0.60972	-0.02380
14	13	0.57651	-0.02098
15	14	0.54366	-0.01507
		r_k	r_{kk}

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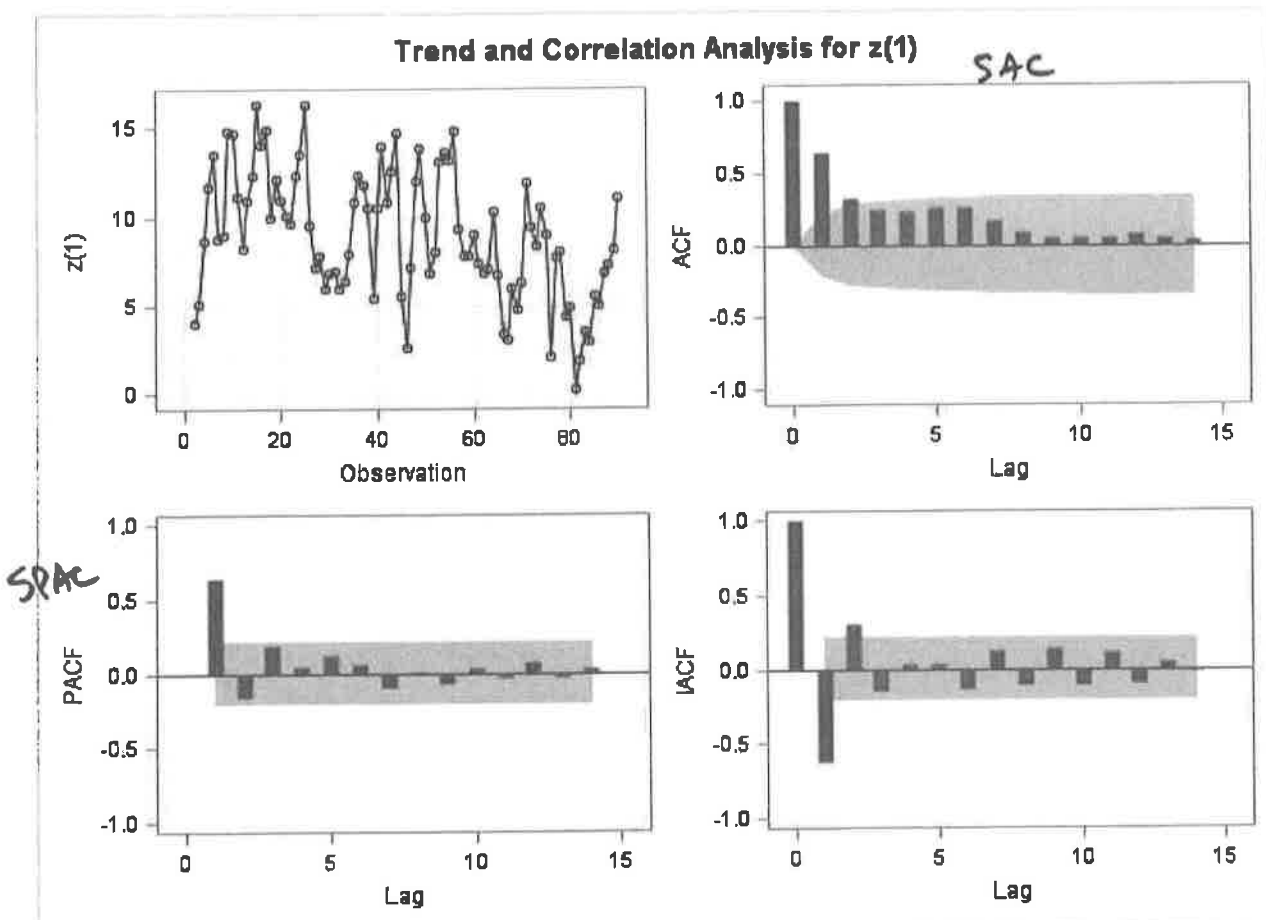
toothpaste 1st differences

The ARIMA Procedure

Name of Variable = z		
Period(s) of Differencing	1	
Mean of Working Series	8.926742	$\bar{w}$
Standard Deviation	3.617174	s_w
Number of Observations	89	
Observation(s) eliminated by differencing	1	

Autocorrelation Check for White Noise

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations r_k					
6	71.67	6	<.0001	0.643	0.321	0.246	0.238	0.256	0.262
12	76.32	12	<.0001	0.168	0.090	0.041	0.042	0.045	0.068



Preliminary Estimation

Initial Autoregressive Estimates	
	Estimate
1	0.64279

Constant Term Estimate	3.188758
White Noise Variance Est	7.678004

Conditional Least Squares Estimation

Iteration	SSE	MU	AR1,1	Constant	Lambda	R Crit
0	681.46	8.92674	0.64279	3.188758	0.00001	1
1	680.86	8.70798	0.64516	3.089906	1E-6	0.029489
2	680.85	8.70279	0.64766	3.066321	1E-7	0.003303
3	680.85	8.69994	0.64774	3.064672	1E-8	0.000384

ARIMA Estimation Optimization Summary

Estimation Method	Conditional Least Squares
Parameters Estimated	2
Termination Criteria	Maximum Relative Change in Estimates
Iteration Stopping Value	0.001
Criteria Value	0.000327
Alternate Criteria	Relative Change in Objective Function
Alternate Criteria Value	1.56E-7
Maximum Absolute Value of Gradient	0.068662
R-Square Change from Last Iteration	0.000384
Objective Function	Sum of Squared Residuals
Objective Function Value	$\boxed{680.848}$ SSE
Marquardt's Lambda Coefficient	1E-8
Numerical Derivative Perturbation Delta	0.001
Iterations	3

Conditional Least Squares Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	8.69994	0.81123	10.72	<.0001	0
AR1,1	$\boxed{0.64774}$	0.08213	7.89	<.0001	1

$\hat{\Phi}_1$

Constant Estimate	$\boxed{3.064672}$	$\hat{\delta}$
Variance Estimate	7.825839	
Std Error Estimate	2.79747	$\to \hat{\sigma}_u$
AIC	437.6596	
SBC	442.6369	
Number of Residuals	89	

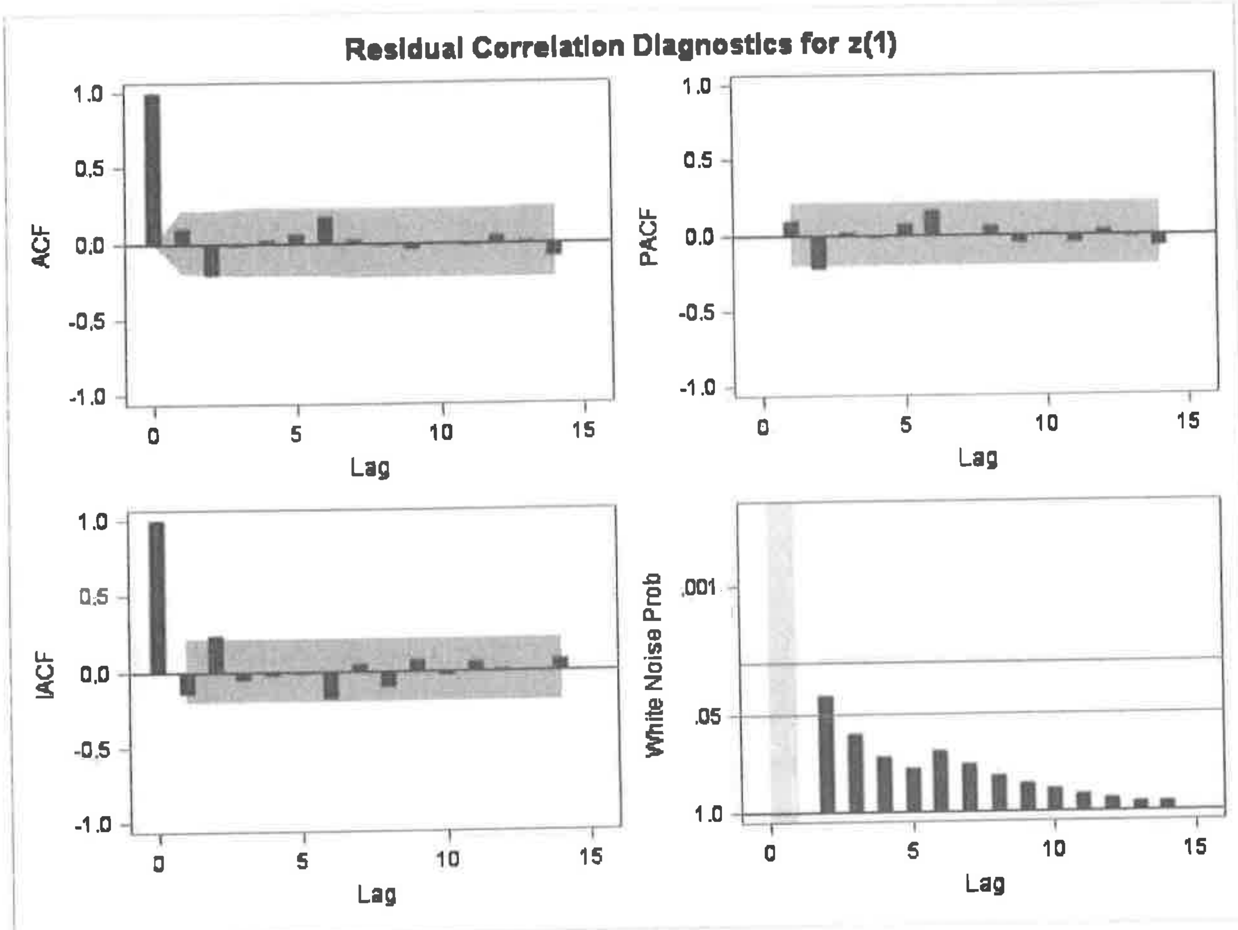
*** AIC and SBC do not include log determinant.**

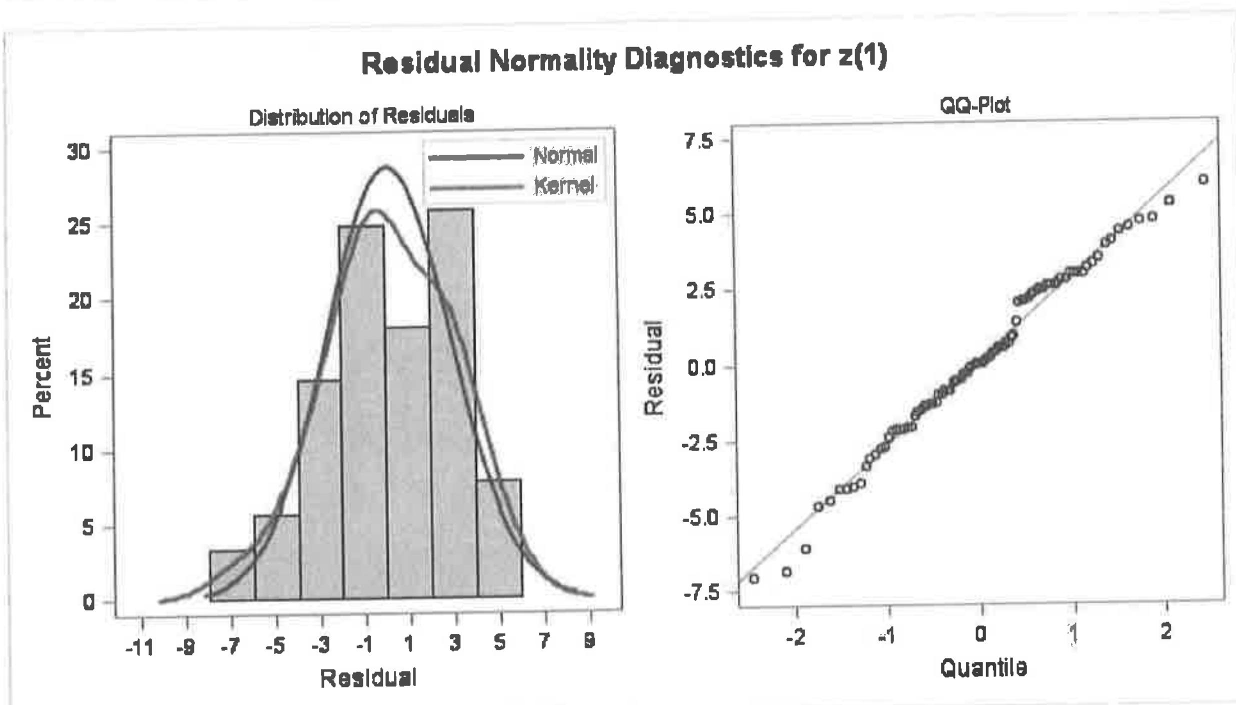
Correlations of Parameter Estimates

Parameter	MU	AR1,1
MU	1.000	-0.053
AR1,1	-0.053	1.000

Autocorrelation Check of Residuals

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	8.02	5	0.1550	0.104	-0.202	-0.022	0.024	0.064	0.168
12	8.63	11	0.6562	0.016	-0.015	-0.048	-0.004	-0.013	0.054
18	13.29	17	0.7164	0.010	-0.095	0.092	0.106	-0.101	0.056
24	21.64	23	0.5418	0.193	0.008	-0.013	-0.116	-0.099	0.097





Model for variable z

Estimated Mean	8.699938
Period(s) of Differencing	1

Autoregressive Factors	
Factor 1:	1 - 0.64774 B**(1)

toothpaste 1st differences w_t

Obs	LAG	CORR	PARTCORR
1	0	1.00000	1.00000
2	1	0.64279	0.64279
3	2	0.32124	-0.15667
4	3	0.24558	0.18771
5	4	0.23751	0.04019
6	5	0.25555	0.12233
7	6	0.26173	0.05707
8	7	0.16819	-0.08918
9	8	0.08980	0.00569
10	9	0.04144	-0.06552
11	10	0.04230	0.03286
12	11	0.04489	-0.02993
13	12	0.06817	0.06615
14	13	0.05113	-0.03135
15	14	0.03741	0.03979
		r_k	r_{kk}

Part II

5 a) The time series plot seems to indicate stationarity. It varies about a constant mean near 35

The SAC dies down quickly (after lag 2?) This is an indication of stationarity.

The SPAC appears to be damped exponential decay w/ oscillation. This is also an indication of stationarity.

b) Based on observations in part (a), I would recommend an MA(2). The SAC "cuts off" after lag 2, and the SPAC "dies off" relative to the SAC.

$$\boxed{z_t = \mu + u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2}}$$

note: I would include a constant because the mean appears to be nonzero

viscosity original series

Obs	z	time
1	39.9	1
2	31.9	2
3	37.5	3
4	31.7	4
5	37.7	5
6	30.3	6
7	38.7	7
8	35.3	8
9	34.9	9
10	36.4	10
11	35.6	11
12	30.5	12
13	34.7	13
14	28.4	14
15	34.1	15
16	31.9	16
17	35.6	17
18	35.2	18
19	31.3	19
20	38.3	20
21	30.0	21
22	36.5	22
23	32.3	23
24	38.4	24
25	41.3	25
26	32.5	26
27	37.5	27
28	36.2	28
29	36.1	29
30	35.5	30
31	37.9	31
32	32.3	32

33	36.0	33
34	34.5	34
35	32.1	35
36	29.2	36
37	39.2	37
38	32.6	38
39	35.4	39
40	38.4	40
41	31.4	41
42	39.3	42
43	32.4	43
44	35.1	44
45	33.3	45
46	37.3	46
47	34.4	47
48	30.4	48
49	38.2	49
50	28.7	50
51	36.3	51
52	32.1	52
53	34.0	53
54	34.5	54
55	34.4	55
56	36.2	56
57	39.1	57
58	32.6	58
59	38.6	59
60	38.5	60
61	30.5	61
62	40.1	62
63	32.9	63
64	36.2	64
65	32.3	65
66	37.1	66
67	30.1	67

	40.3	68
69	36.5	69
70	32.9	70
71	35.1	71
72	41.1	72
73	25.9	73
74	41.3	74
75	32.8	75
76	38.0	76
77	36.5	77
78	37.2	78
79	36.4	79
80	37.2	80
81	34.2	81
82	37.0	82
83	35.4	83
84	34.4	84
85	35.2	85
86	37.1	86
87	32.3	87
88	36.9	88
89	34.8	89
90	35.8	90
91	36.1	91
92	36.7	92
93	36.6	93
94	35.1	94
95	37.8	95
96	33.9	96
97	37.2	97
98	34.3	98
99	38.3	99
100	33.9	100
101	33.8	101
102	40.2	102

	35.3	103
104	38.8	104
105	39.0	105
106	32.2	106
107	38.8	107
108	34.3	108
109	30.8	109
110	35.9	110
111	31.4	111
112	33.0	112
113	34.6	113
114	36.4	114
115	33.1	115
116	39.4	116
117	35.4	117
118	34.4	118
119	36.9	119
120	32.8	120
121	35.2	121
122	34.6	122
123	36.4	123
124	35.8	124
125	35.8	125
126	31.7	126
127	37.0	127
128	28.7	128
129	38.0	129
130	32.2	130
131	33.5	131
132	36.3	132
133	37.1	133
134	30.5	134
135	36.8	135
136	37.7	136
137	33.2	137

	35.2	138
139	35.7	139
140	36.0	140
141	34.0	141
142	40.3	142
143	37.0	143
144	40.2	144
145	34.4	145
146	38.5	146
147	35.2	147
148	35.6	148
149	31.9	149
150	35.2	150

viscosity original series

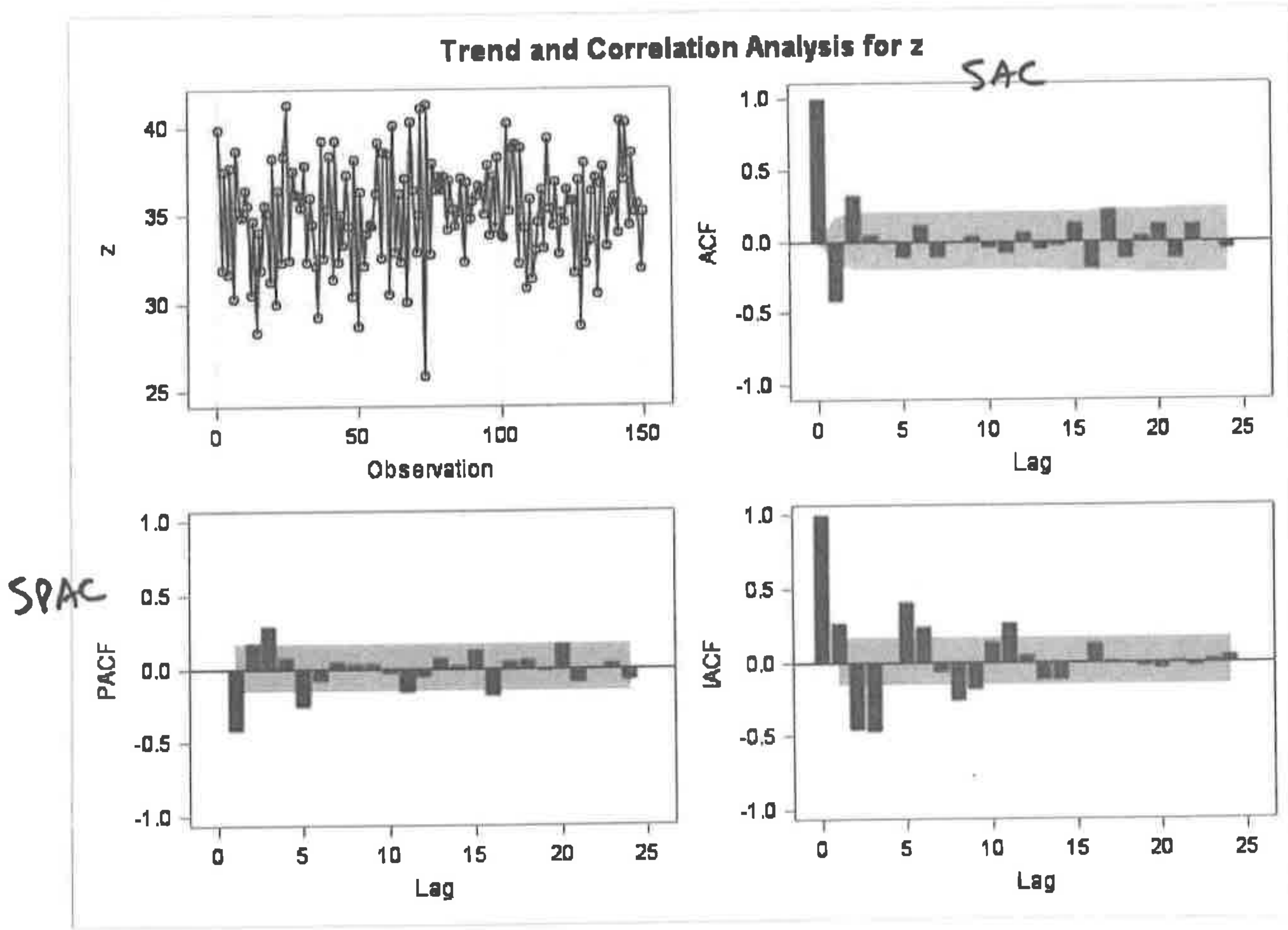
The ARIMA Procedure

Name of Variable = z	
Mean of Working Series	35.20133
Standard Deviation	2.922008
Number of Observations	150

$\bar{z}$

s_z

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations r_k					
6	46.31	6	<.0001	-0.415	0.319	0.049	0.004	-0.114	0.109
12	50.46	12	<.0001	-0.110	0.000	0.037	-0.042	-0.083	0.059
18	71.24	18	<.0001	-0.063	-0.033	0.124	-0.193	0.221	-0.124
24	79.60	24	<.0001	0.032	0.116	-0.121	0.117	-0.012	-0.064



viscosity original series z_t

Obs	LAG	CORR	PARTCORR
1	0	1.00000	1.00000
2	1	-0.41512	-0.41512
3	2	0.31871	0.17686
4	3	0.04888	0.28836
5	4	0.00381	0.07473
6	5	-0.11424	-0.26723
7	6	0.10860	-0.08259
8	7	-0.11007	0.04994
9	8	0.00005	0.03011
10	9	0.03652	0.03461
11	10	-0.04234	-0.02604
12	11	-0.08306	-0.16437
13	12	0.05918	-0.05476
14	13	-0.06273	0.06789
15	14	-0.03316	0.02677
16	15	0.12410	0.11839
17	16	-0.19263	-0.18489
18	17	0.22120	0.05006
19	18	-0.12365	0.05416
20	19	0.03219	-0.02427
21	20	0.11645	0.16749
22	21	-0.12123	-0.09680
23	22	0.11728	-0.00590
24	23	-0.01150	0.03003
25	24	-0.06361	-0.08578
		r_k	r_{kk}

Part III

6 a) • time series plot of the original series seems to have some drift. This is an indication of a non-stationary process.

• time series plot of differenced series looks much more stationary. It appears to vary about a constant mean of 0.

• SAC for original series follows damped exponential decay. It's not too bad in terms of stationarity

• SAC cuts off after lag 1 for differenced series.

Based on both the time series plot and the SAC plots, the differenced series seems to be the better choice.

b) To me, the SPAC "cuts off" at lag 1 more abruptly than the SAC. Relative to the SPAC, the SAC appears to follow exponential decay with oscillation. I would use an AR(1).

$$\boxed{w_t = \phi_1 w_{t-1} + u_t} \rightarrow \boxed{z_t = (1+\phi_1) z_{t-1} - \phi_1 z_{t-2} + u_t}$$

note: I do not include a constant term because the differenced series varies about a mean of 0.

Shampoo original series

Obs	z	time
1	339	1
2	319	2
3	352	3
4	330	4
5	378	5
6	392	6
7	390	7
8	395	8
9	386	9
10	383	10
11	396	11
12	396	12
13	412	13
14	387	14
15	382	15
16	423	16
17	386	17
18	420	18
19	417	19
20	474	20
21	450	21
22	444	22
23	456	23
24	449	24
25	428	25
26	444	26
27	389	27
28	447	28
29	395	29
30	417	30

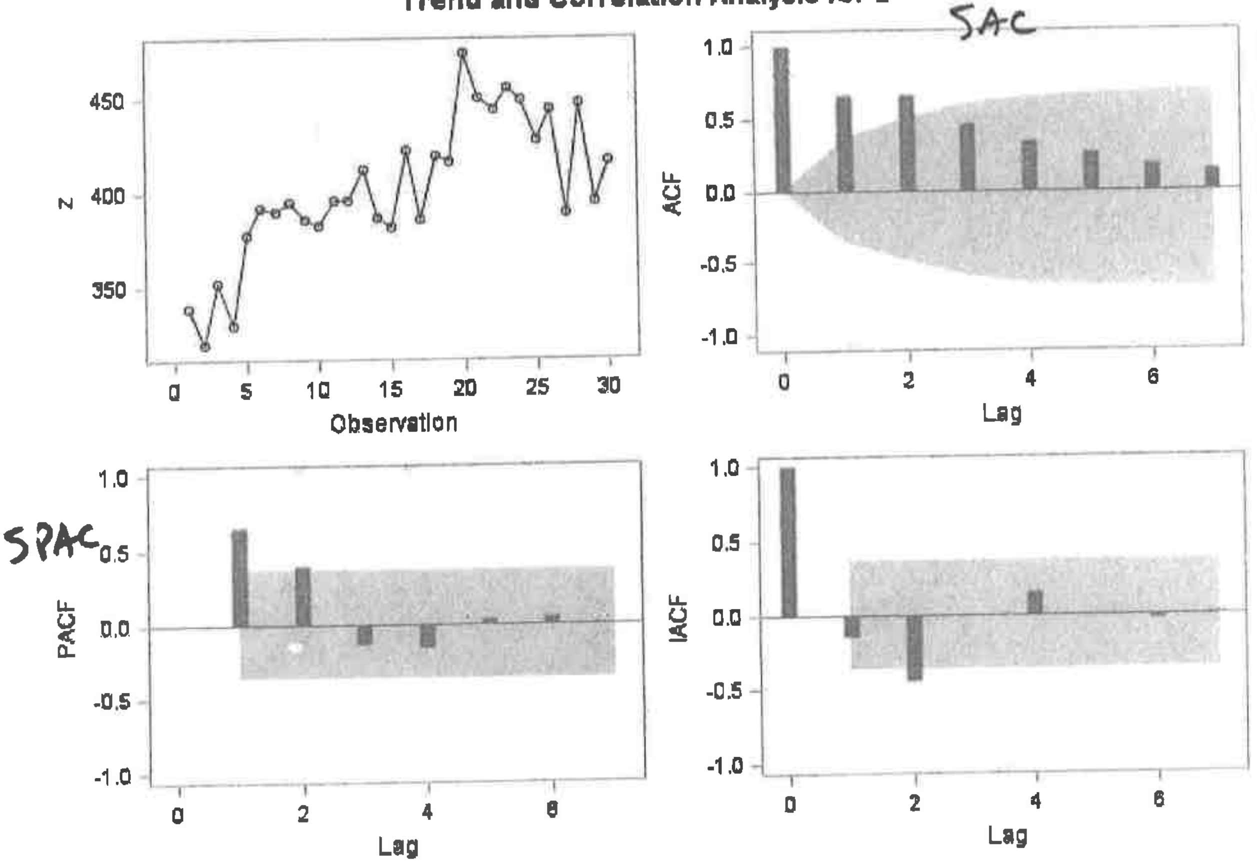
Shampoo original series

The ARIMA Procedure

Name of Variable = z		
Mean of Working Series	402.5333	$\bar{z}$
Standard Deviation	37.12477	s_z
Number of Observations	30	

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	44.36	6	<.0001	0.656	0.656	0.456	0.338	0.253	0.181

Trend and Correlation Analysis for z



Shampoo original series z_t

Obs	LAG	CORR	PARTCORR
1	0	1.00000	1.00000
2	1	0.65589	0.65589
3	2	0.65618	0.39659
4	3	0.45568	-0.13357
5	4	0.33760	-0.15734
6	5	0.25334	0.03571
7	6	0.18067	0.04685
8	7	0.13470	-0.00540

r_k r_{kk}

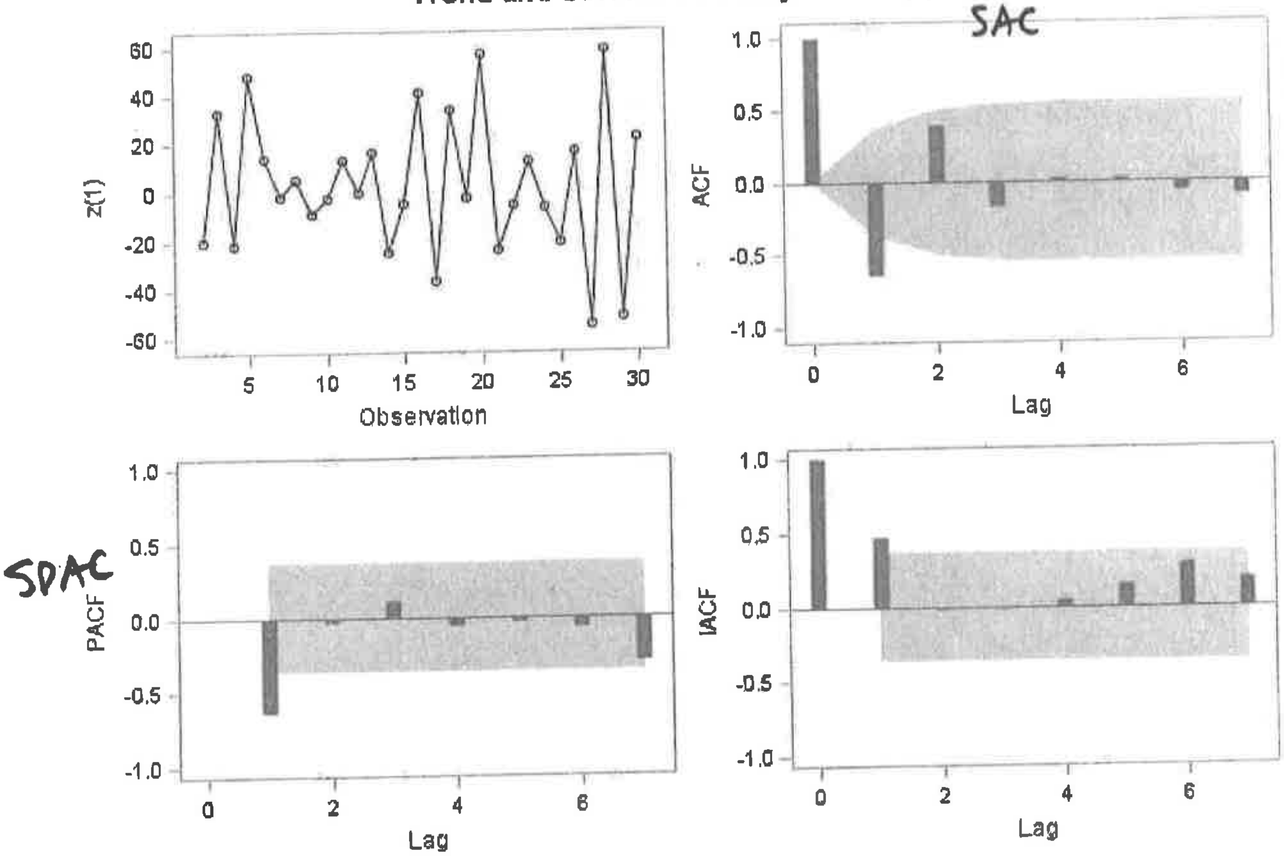
Shampoo 1st differences series

The ARIMA Procedure

Name of Variable = z		
Period(s) of Differencing	1	
Mean of Working Series	2.689655	$\bar{\omega}$
Standard Deviation	28.76792	S_w
Number of Observations	29	
Observation(s) eliminated by differencing	1	

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	19.59	6	0.0033	-0.642	0.391	-0.173	0.027	0.021	-0.072

Trend and Correlation Analysis for z(1) $\rightarrow w$



Shampoo 1st differences series W_t

Obs	LAG	CORR	PARTCORR
1	0	1.00000	1.00000
2	1	-0.64181	-0.64181
3	2	0.39066	-0.03615
4	3	-0.17276	0.10868
5	4	0.02702	-0.05436
6	5	0.02121	-0.03079
7	6	-0.07238	-0.06471
8	7	-0.10028	-0.30602

r_k r_{kk}