- Additional Counting Problems
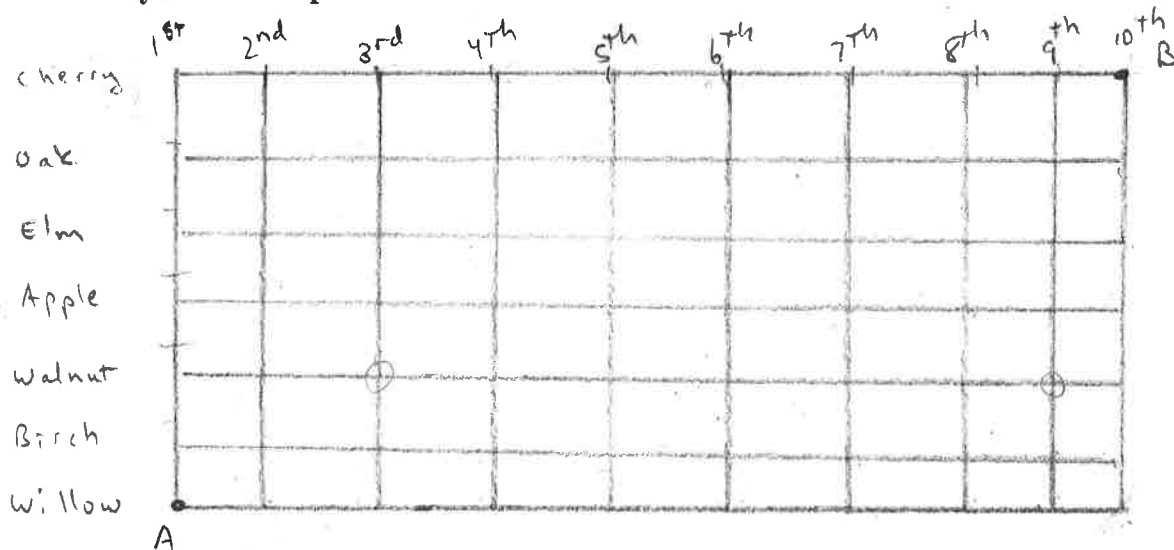
1. A postman wants to know how many routes he can take from a substation A
    to the main post office B by taking the shortest route possible. Assume the
    postman can drive only in the direction of North or East at each block. How
    many routes are possible if:

a) there are no restrictions?
b) he must travel along Apple between 4th and 7th streets?
c) Walnut is being repaved and he can only cross Walnut at the intersection of
    3rd or 9th streets?

2. Twenty identical basketballs are to be divided among four schools. How
    many divisions are possible with no restrictions? How many divisions are
    possible if each school must receive at least three basketballs?

3. A band coordinator has 30 flags to distribute among 9 bands in the district.
    In how many ways can she distribute the flags if each school must receive at
    least two flags?

4. Suppose 5 bad light bulbs get mixed up with 30 good ones and that you start
    testing the bulbs one by one until you have found all five bad bulbs. What is
    the probability that you will find the fourth bad bulb on the 15th bulb you
    test?

5. If a 9-card hand is dealt from a deck of 52 cards, find the <u>number of hands</u> containing:

a) three of a kind and two pairs only.
b) a straight of 9 cards. (Aces can be high or low)
c) three pairs only.

6. How many different 7-place license plates can be formed if:

a) the first three places are letters and the last four are numbers (assume sampling with replacement)?
b) in part (a), the four numbers must be in increasing order?
c) Repeat part (b) but now assume sampling without replacement.

7. A bookshelf contains thirteen books: four science books, four English books, two math books and three history books. How many arrangements are possible for the thirteen distinguishable books if:

a) there are no restrictions?
b) books from the same category must be side by side?
c) the four English books cannot be side by side as a group of four?

8. You are given the following numbers: 1,2,3,4,6,7,and 9. How many numbers of four digits or less can be formed if: (note, the number may be repeated)

a) the number is greater than 3000?
b) the number is less than 500?
c) the number is even and between 4000 and 7000?

Additional Counting Problems — Solutions

① a) $\binom{15}{6}$

b) $\binom{6}{3}^2$

c) $\binom{3}{1}\binom{10}{7} + \binom{9}{1}\binom{4}{1}$

② no restrictions:

20 stars, 3 bars $\qquad \binom{23}{3}$

each must receive at least three:

8 stars, 3 bars $\qquad \binom{11}{3}$

③ 12 stars, 8 bars

$\binom{20}{8}$

④ 5 bad
30 good

$$\frac{\binom{30}{11}\cdot\binom{5}{3}}{\binom{35}{14}} \cdot \frac{2}{21}$$

⑤

a) $\binom{13}{1}\binom{4}{3}\cdot\binom{12}{2}\binom{4}{2}^2\cdot\binom{10}{2}\binom{4}{1}^2$

b) $\binom{6}{1}\binom{4}{1}^9 - 6\cdot4$ $\qquad$ (not including straight flushes)

c) $\binom{13}{3}\binom{4}{2}^3\cdot\binom{10}{3}\binom{4}{1}^3$

⑥ a) $26^3 \, 10^4$

b) $26^3 \cdot \binom{10}{4}$

c) $26\cdot25\cdot24\cdot\binom{10}{4}$

⑦ a) $13!$

b) $4!\,4!\,4!\,2!\,3!$

c) $13! - \binom{10}{1}4!\cdot9!$

⑧ a) $5\cdot7^3$

b) $4\cdot7^2$

c) $2\cdot7^2\cdot3$