

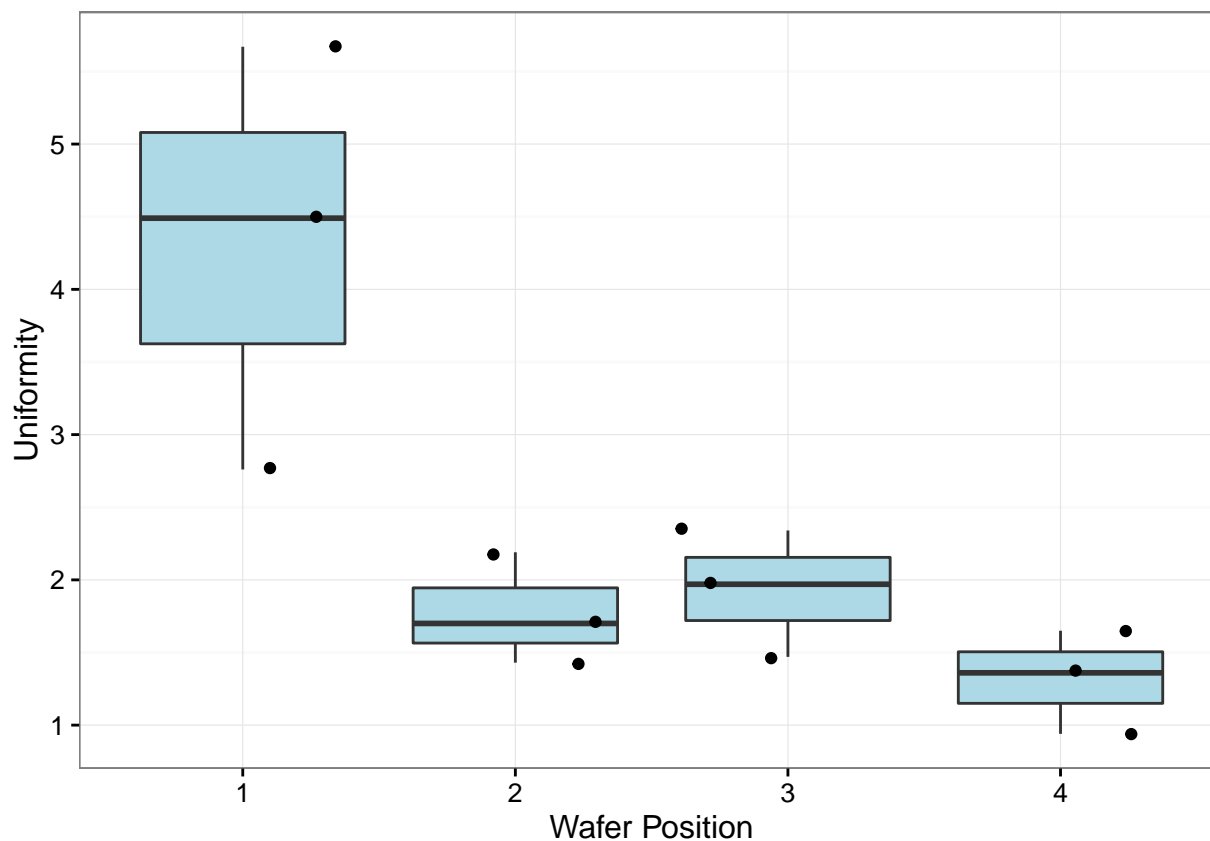
Homework 9

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1) Random effects, 1 factor, 4 level experiment

First I'd like to take a look at a plot of the data



It looks like there is an effect from wafer position on uniformity. Using the methods of random effects we will estimate this.

- There is a difference in *uniformity* due to *wafer position* with an F statistic of 8.290013, which is statistically significant.
- The variability due to wafer position is equal to 1.5848. This can be found from the ANOVA table from fitting the model

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij} \begin{cases} i = 1, 2, 3, 4 \\ j = 1, 2, 3 \end{cases}$$

and then calculating

$$\hat{\sigma}_{\tau}^2 = \frac{MS_{Treatments} - MS_E}{n}$$

- The random error component is 0.6522. This is the same as MSE from the fixed effects model.
- Total variability is found by simply adding the variance components above, $\hat{\sigma}_\tau^2 + \hat{\sigma}^2 = 1.5848 + 0.6522 = 2.237$.
- E) Total variability due to wafer position is $\frac{1.5848}{1.5848+0.6522} \times 100$ percent, 70.85%.

2) Two factor, mixed level random experiment

- The model for this experiment is

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1, 2, 3 \\ j = 1, 2, 3, 4 \\ k = 1, 2 \end{cases}$$

where τ is the effect from *operator* and β is the effect from machine, both being random effects and we assume all terms in the model are $N(0, \sigma_i^2)$.

- Here is a table of operator and machine F statistics and p values for operator and machine.

	MS	F value	p
operator	80.167	10.769344	0.0103425
machine	4.153	0.557899	0.6619101
operator:machine	7.444	1.963080	0.1507304

- Operator appears statistically significant while machine does not appear to be significant.
- First, the variance component for operator is

$$\hat{\sigma}_\tau^2 = \frac{80.167 - 7.444}{4(2)} = 9.090375$$

the variance component for machine is

$$\hat{\sigma}_\beta^2 = \frac{4.153 - 7.444}{3(2)} = -0.5485$$

and the variance component for the interaction of operator and machine is

$$\hat{\sigma}_{\tau\beta}^2 = \frac{7.444 - 3.792}{2} = 1.826$$

- The Anova table for the reduced model is

SS	MS	Fvalue	Pvalue
160.333	80.167	16.0037042	0.0001014
12.458	4.153	0.8290616	0.4950769

sse	mse
90.167	5.009278

- We now need to recalculate the variance components since we've changed the model.

$$\hat{\sigma}_{\tau}^2 = \frac{80.167 - 5.0092778}{4(2)} = 9.3947153$$

and for machine

$$\hat{\sigma}_{\beta}^2 = \frac{4.153 - 5.0092778}{3(2)} = -0.142713$$

and we conclude that all the variance is due to operator, being that the variance component for machine is negative so we assume it is zero or that we need a different model.

- The estimate for the variance of the gauge can be thought of as the sum of the variance components $\hat{\sigma}^2$ and $\hat{\sigma}_{\tau}^2$ which is $5.0092778 + 9.3947153 = 14.4039931$. We can conclude that gauge is terrible. All the variation is due to operator. There is no reproducibility or reliability from this gauge. Perhaps part should have been a variable in the study where we sampled from a range of different parts that this gauge is capable of measuring.

3) Two-Factor Mixed Model

- Our model is as follows

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1, 2 \\ j = 1, 2, \dots, 10 \\ k = 1, 2, 3 \end{cases}$$

where τ_i is the specified operator and β_j is the part.

- Here is a table of operator and machine F statistics and p values for operator and machine.

	MS	F value	p
operator	0.417	0.6926910	0.4267823
part	11.002	7.3346667	0.0000032
operator:part	0.602	0.4013333	0.9269563

- It appears that the effect of parts is statistically significant. Operators is not.
- The variance components are $\sigma_{(\beta)}^2$, $\sigma_{\tau\beta}^2$, and of course σ^2 and the estimates for each are, for parts

$$\hat{\sigma}_{\beta}^2 = \frac{11.002 - 1.5}{2(3)} = 1.5836667$$

and for operator:part interaction

$$\hat{\sigma}_{\tau\beta}^2 = \frac{0.602 - 1.5}{3} = -0.2993333$$

and

$$\hat{\sigma}^2 = 1.5$$

so we can see that the variance component for the interaction of part and operator is negative so we assume it to be zero. We should drop the interaction term. Most of the variability is from **part**.

- Dropping the interaction term from our model yields

Source	SS	Df	MS	F_0	P-Value
Operator	0.417	1	0.417	0.31235	0.5793574
Parts	99.017	9	11.002	8.2409465	8.3723286×10^{-7}
Error	65.417	49	1.3350408		
Total	164.851	59			

and we conclude that **part** is statistically significant while **operator** is not.