## 8.9 CIs for $\sigma^2$

Many applications require an estimate of $\sigma^2$

→ manufacturing

→ risk in stock mkt, etc.

What is a good pt. estimate for $\sigma^2$?

In 8.3 we showed $E(s^2) = \sigma^2$, $s^2 = \dfrac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$

$s^2$ is an unbiased pt. estimator for $\sigma^2$, so now we need to find a pivotal quantity relating $s^2$ and $\sigma^2$...

from 7.3, we know that if $y_1 \cdots y_n$ is a r.s. from $Y \sim N(\mu, \sigma)$ then

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{(n-1)}$$

Prob. stmt:

$$P\left[\chi^2_{1-\alpha/2} \le \frac{(n-1)s^2}{\sigma^2} \le \chi^2_{\alpha/2}\right] = 1-\alpha$$

manipulating the inequalities...

Confidence stmt:

$$C\left[\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \le \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right] = 1-\alpha$$

i.e. $100(1-\alpha)\%$ CI for $\sigma^2$: $\boxed{\left(\dfrac{(n-1)s^2}{\chi^2_{\alpha/2}}, \dfrac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right)}$

e.g. 8.95 maximum noise level for trucks: 83 decibels (dB)

How to apply the limit?

1. All trucks must satisfy limit
2. Mean of truck fleet's noise level is under limit.

If option 2 is taken, then $\sigma^2$ will also be important because if $\sigma^2$ is large, a large no. of trucks will still exceed the limit even if $\mu < 83$.

R.S. of 6 heavy trucks:

85.4 86.8 86.1 85.3 84.8 86.0

Find a 90% CI for $\sigma^2$:

$n=6$ $s^2 = .50267$ $\chi^2_{.95,5} = 1.145$ $\chi^2_{.05,5} = 11.071$

$$\left( \frac{(6-1)(.50267)}{11.071}, \frac{(6-1)(.50267)}{1.145} \right) = \boxed{(.227, 2.195)}$$

90% CI for $\sigma$: $\left(\sqrt{.227}, \sqrt{2.195}\right) = \boxed{(.476, 1.48)}$

i.e. We are 95% confident $\sigma < 1.48$

note: This normality assumption is not robust.

$\boxed{\text{simulation}}$