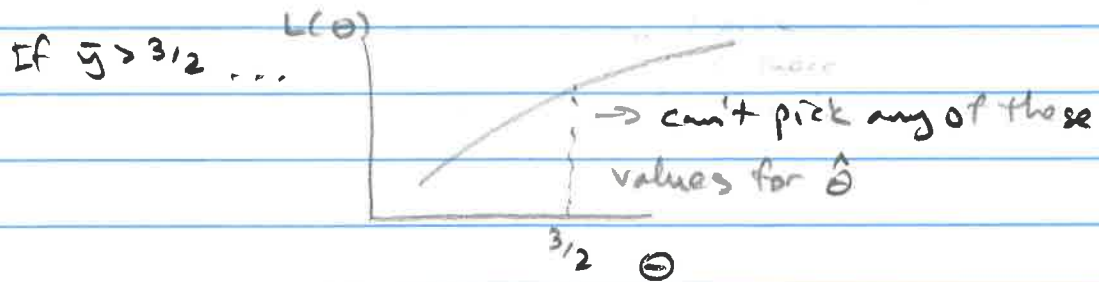


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So to maximize $L(\theta)$, choose the largest possible value for $\hat{\theta} = 3/2$



λ will now be a piecewise fct \dots

$$\lambda = \begin{cases} \frac{e^{-n(3/2)} \sum y_i}{\prod_{i=1}^n y_i!} & , \text{ if } \bar{y} < 3/2 \\ \frac{e^{-n\bar{y}} \sum y_i}{\prod_{i=1}^n y_i!} & , \text{ if } \bar{y} \geq 3/2 \end{cases}$$

\Rightarrow note: This tells us if $\bar{y} \geq 3/2$, there is no reason to conclude $\theta < 3/2$

We concern ourselves with the case $\bar{y} < 3/2$

$$\lambda = e^{-n(3/2 - \bar{y})} \cdot \left(\frac{3}{2\bar{y}}\right)^{n\bar{y}}$$

note: λ is a fct of \bar{y} . If we can determine if $\lambda(\bar{y})$ is increasing or decreasing when $\bar{y} < 3/2$, we can use \bar{y} as a test statistic.

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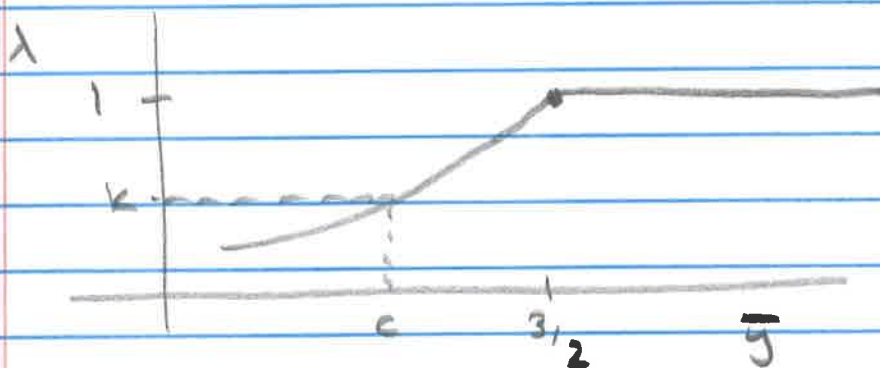
$$\ln \lambda = -n(3/2 - \bar{y}) + n\bar{y} \cdot \ln\left(\frac{3}{2\bar{y}}\right)$$

$$\frac{\partial \ln \lambda}{\partial \bar{y}} = n + (n\bar{y}) \cdot \frac{1}{\left(\frac{3}{2\bar{y}}\right)} \cdot \left(-\frac{3}{2\bar{y}^2}\right) + n \cdot \ln\left(\frac{3}{2\bar{y}}\right)$$

$$= n - n + n \ln\left(\frac{3}{2\bar{y}}\right)$$

$$= n \ln\left(\frac{3}{2\bar{y}}\right) > 0 \quad \text{if } \bar{y} < 3/2$$

i.e. λ is an increasing fct of \bar{y} when $\bar{y} < 3/2$



Reject H_0 for $\lambda \geq k$ is equivalent to rejecting H_0 for $\bar{y} \leq c$ or $\sum y_i \leq n \cdot c$

under H_0 $\sum y_i \sim \text{POT}(n\theta)$

note: Since $\sum y_i$ is discrete, we may not match α exactly