1. (30 pts) Ex. 4.1 Consider 51 objects O_1, \ldots, O_{51} assumed to be arranged along a straight line with the jth object being located at a point with coordinate j. Define the similarity s_{ij} between object i and object j as

$$s_{ij} = \begin{cases} 9 & \text{if } i = j \\ 8 & \text{if } 1 \le |i - j| \le 3 \\ 7 & \text{if } 4 \le |i - j| \le 6 \\ \dots \\ 1 & \text{if } 22 \le |i - j| \le 24 \\ 0 & \text{if } |i - j| \ge 25. \end{cases}$$

Convert these similarities into dissimilarities δ_{ij} by using $\delta_{ij} = \sqrt{s_{ii} + s_{jj} - 2s_{ij}}$, and then apply classical multidimensional scaling to the resulting dissimilarity matrix. Explain the shape of the derived two-dimensional solution.

2. (20 pts) The Table in the below summarizes data collected during a survey in which subjects were asked to compare a set of eight legal offenses, and to say for each one how unlike it was, in terms of seriousness, from the others. Each entry in the table shows the percentage of respondents who judged that the two offenses are very dissimilar. Find a two-dimensional scaling solution and try to interpret the dimensions underlying the subjects' judgements.

Dissimilarity Matrix for a Set of Eight Legal Offenses organized in table

| Offense | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------|------|------|------|------|------|------|----|---|
| 1 | 0 | | | | | | | |
| 2 | 21.1 | 0 | | | | | | |
| 3 | 71.2 | 54.1 | 0 | | | | | |
| 4 | 36.4 | 36.4 | 36.4 | 0 | | | | |
| 5 | 52.1 | 54.1 | 52.1 | 0.7 | 0 | | | |
| 6 | 89.9 | 75.2 | 36.4 | 54.1 | 53 | 0 | | |
| 7 | 53 | 73 | 75.2 | 52.1 | 36.4 | 88.3 | 0 | |
| 8 | 90.1 | 93.2 | 71.2 | 63.4 | 52.1 | 36.4 | 73 | 0 |

Offenses: (1) assault and battery, (2) rape, (3) embezzlement, (4) perjury, (5) libel, (6) burglary, (7) prostitution, (8) receiving stolen goods.

- 3. (20 pts) Ex. 4.3 In the data *garden flowers* data in Table 4.7 (from Kaufman and Rousseeuw 1990), the dissimilarity matrix of 18 species of garden flowers is shown. Use some form of multidimensional scaling to investigate which species share common properties.
- 4. (10 pts) Consider the *voting data* problem outlined in Section 4.5.1. Carry out a classical scaling of the data and show that the solution. Compare your solution to the nonmetric scaling solution given in Section 4.51.