Homework 9

Cody Frisby

April 11, 2016

Here, I did all the Calculations for the ANOVA table by "hand". **Note**: *all R code will be at the end of this document.*

First, I display the contrasts matrix

	Al	Bl	Aq	Bq	AlBl	AlBq	AqBl	AqBq
00	-1	-1	1	1	1	-1	-1	1
01	-1	0	1	-2	0	2	0	-2
02	-1	1	1	1	-1	-1	1	1
10	0	-1	-2	1	0	0	2	-2
11	0	0	-2	-2	0	0	0	4
12	0	1	-2	1	0	0	-2	-2
20	1	-1	1	1	-1	1	-1	1
21	1	0	1	-2	0	-2	0	-2
22	1	1	1	1	1	1	1	1

and with the sum of each treatment levels' combinations

	Al	Bl	Aq	Bq	AlBl	AlBq	AqBl	AqBq	y
00	-1	-1	1	1	1	-1	-1	1	1718
01	-1	0	1	-2	0	2	0	-2	3262
02	-1	1	1	1	-1	-1	1	1	4158
10	0	-1	-2	1	0	0	2	-2	1659
11	0	0	-2	-2	0	0	0	4	3105
12	0	1	-2	1	0	0	-2	-2	3939
20	1	-1	1	1	-1	1	-1	1	1720
21	1	0	1	-2	0	-2	0	-2	3164
22	1	1	1	1	1	1	1	1	2660

Now, we can estimate the effect of each term in our model by multiplying y by each column of this matrix, taking the sum, and dividing by $n = 3$. The result is

Al	-531.3333
Bl	1886.6667
Aq	-241.3333
Bq	-1069.3333
AlBl	-500.0000
AlBq	-433.3333
AqBl	-393.3333
AqBq	-457.3333

Now, for the Sums of Squares for each term in the model we calculate

$$\frac{n(contrasts)^2}{\sum c_i^2}$$

which results in

Al	141157.556
Bl	1779755.556
Aq	9706.963
Bq	190578.963
AlBl	187500.000
AlBq	46944.444
AqBl	38677.778
AqBq	17429.481

Before I forget, the sum of squares for A, B, and AB is

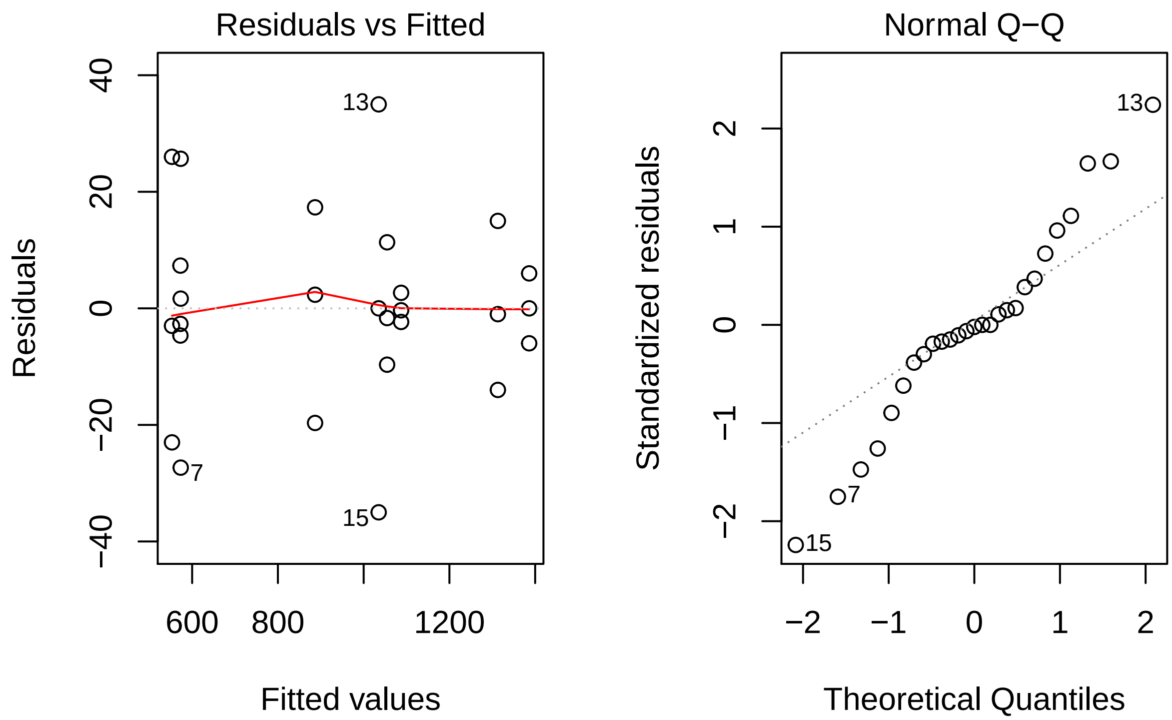
ssa	ssb	ssab	sse	sst
150865	1970335	290552	6579	2418330

The remaining ANOVA table is similar to previous methods used in this class, where mean squares are $\frac{SS}{df}$ and the F statistic is $\frac{MS_i}{MS_E}$. Here, I finish filling out the ANOVA table

Source	Sum Sq	Df	Mean Sq	F value	Pr(>F)
glass	1.4115756×10^5	1	1.4115756×10^5	386.1844158	<0.000
temp	1.7797556×10^6	1	1.7797556×10^6	4869.1255446	<0.000
$glass^2$	9706.962963	1	9706.962963	26.5566927	<0.000
$temp^2$	1.9057896×10^5	1	1.9057896×10^5	521.3934543	<0.000
temp:glass	1.875×10^5	1	1.875×10^5	512.9699058	<0.000
$glass : temp^2$	4.6944444×10^4	1	4.6944444×10^4	128.4324653	<0.000
$glass^2 : temp$	3.8677778×10^4	1	3.8677778×10^4	105.8161921	<0.000
$glass^2 : temp^2$	1.7429481×10^4	1	1.7429481×10^4	47.6842639	<0.000
Error	6579.3333333	18	365.5185185		
Total	2.4183301×10^6	26			

All effects have a significant effect on *light output* and are statistically significant, with **glass** having the most significant effect. Theere is also some *curvature* to the main effects **glass** and **temp**

Now, I take a look at the residual plots of the model



The normality assumption may not be valid. We may need to reexamine the model or possibly perform a transformation.

R code:

```
####### Exploring 3^2 full factorial models using R ########
df <- read.csv("~/Documents/STAT4100/data/hw9.csv")
df.c <- df
library(dplyr)
y <- df$y
y.i <- df.c %>%
  group_by(glass) %>%
  summarise_each(funs(sum), y)
y.j <- df.c %>%
  group_by(temp) %>%
  summarise_each(funs(sum), y)
yij. <- df.c %>%
  group_by(glass, temp) %>%
  summarise_each(funs(sum), y)
y.. <- sum(y)
a <- 3; b <- 3; n <- 3; N <- a*b*n
ssa <- (1/(b*n))*sum(y.i$y^2) - y..^2/N
ssb <- (1/(a*n))*sum(y.j$y^2) - y..^2/N
sssub <- (1/n)*sum(yij.$y^2) - y..^2/N
ssab <- sssub - ssa - ssb
sst <- sum(y^2) - y..^2/N
sse <- sst - ssab - ssa - ssb
#round(c(ssa,ssb,ssab,sse,sst))
# now to add squared terms
```

```r
row.sums <- df %>%
  group_by(glass,temp) %>%
  summarise_each(funs(sum), y)
y <- row.sums$y
id <- c("00", "01", "02", "10", "11", "12", "20", "21", "22")
#data.frame(id, y) # these are our summaries..
#... now to develop the contrasts matrix
Al <- c(rep(-1,3), rep(0,3), rep(1,3))
Bl <- rep(c(-1,0,1),3)
Aq <- c(rep(1,3), rep(-2,3), rep(1,3))
Bq <- rep(c(1,-2,1), 3)
AlBl <- Al*Bl; AlBq <- Al*Bq; AqBl <- Aq*Bl; AqBq <- Aq*Bq
dm <- cbind(Al,Bl,Aq,Bq,AlBl,AlBq,AqBl,AqBq)
rownames(dm) <- id
m.effects <- apply(dm,2, FUN = function(d) sum(d*y)/3) # effects
# now for the sum of squares...
C <- apply(dm, 2, function(x) sum(x^2)) #contrasts
ss <- 3*(m.effects^2)/C # sum of squares
ss2 <- round(cbind(ssa,ssb,ssab,sse,sst))
sse <- sst - sum(ss); mse <- sse/18
par(mfrow=c(1,2))
plot(fit.full, which = c(1,2))
```