HW 6

1. $\begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ 11 & 2 & 0 & \\ 5 & 3 & 4 & 0 \end{bmatrix}$

a. Single linkage hierarchical procedure

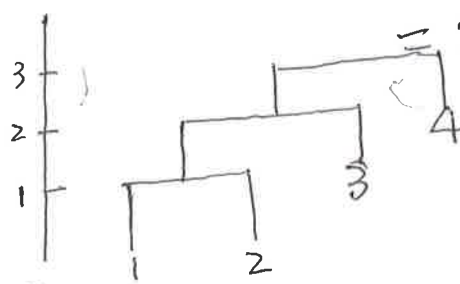
$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & & & \\ 2 & 1 & 0 & & \\ 3 & 11 & 2 & 0 & \\ 4 & 5 & 3 & 4 & 0 \end{array} \Longrightarrow \begin{array}{c|ccc} & 12 & 3 & 4 \\ \hline 12 & 0 & & \\ 3 & 2 & 0 & \\ 4 & 3 & 4 & 0 \end{array}$$

$d_{(12)3} = \min\{d_{13}, d_{23}\} = \min\{11, 2\} = 2$

$d_{(12)4} = \min\{d_{14}, d_{24}\} = \min\{5, 3\} = 3$

$$\begin{array}{c|cc} & 123 & 4 \\ \hline 123 & 0 & \\ 4 & 3 & 0 \end{array}$$

$d_{(123)4} = \min\{d_{(12)4}, d_{34}\} = \min\{3, 4\} = 3$



$\boxed{\text{Dendrogram}}$

b. Complete linkage

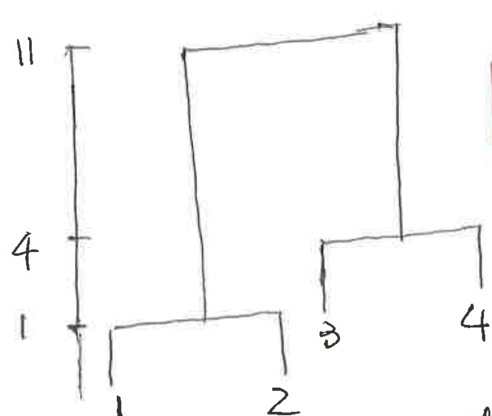
$$\begin{array}{c|ccc} & 12 & 3 & 4 \\ \hline 12 & 0 & & \\ 3 & 11 & 0 & \\ 4 & 5 & 4 & 0 \end{array}$$

$d_{(12)3} = \max\{d_{13}, d_{23}\} = \max\{11, 2\} = 11$

$d_{(12)4} = \max\{d_{14}, d_{24}\} = \max\{5, 3\} = 5$

$d_{(12)(34)} = \max\{d_{(12)3}, d_{(12)4}\} = \max\{11, 5\} = 11$

$$\begin{array}{c|cc} & 12 & 34 \\ \hline 12 & 0 & \\ 34 & 11 & 0 \end{array}$$



$\boxed{\text{Dendrogram}}$

c. Two methods produce the different combination of clusters

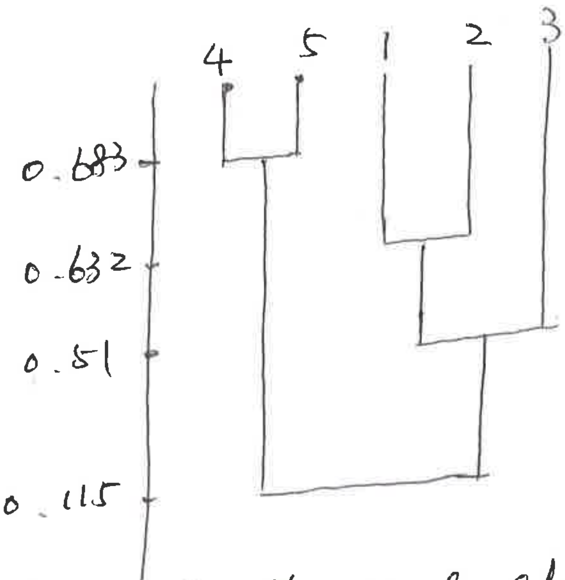
2. correlation matrix

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 1.0 & & & & \\
2 & 0.632 & 1 & & & \\
3 & 0.510 & 0.574 & 1.0 & & \\
4 & 0.115 & 0.322 & 0.182 & 1.0 & \\
5 & 0.154 & 0.213 & 0.146 & 0.683 & 1.0
\end{array}
$$

Complete linkage cluster

$$
\begin{array}{c|cccc}
 & 1 & 2 & 3 & 45 \\
\hline
1 & 1.0 & & & \\
2 & 0.632 & 1 & & \\
3 & 0.510 & 0.574 & 1.0 & \\
45 & 0.115 & 0.213 & 0.146 & 1.0
\end{array}
\Rightarrow
\begin{array}{c|ccc}
 & 12 & 3 & 45 \\
\hline
12 & 1 & & \\
3 & 0.510 & 1. & \\
45 & 0.115 & 0.146 & 1
\end{array}
$$

$$
\begin{array}{c|cc}
 & 123 & 45 \\
\hline
123 & 0 & \\
45 & 0.115 & 1
\end{array}
$$



Both methods arrive at nearly the same clustering.

2. correlation matrix

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1.0 & & & & \\ 0.632 & 1 & & & \\ 0.510 & 0.574 & 1 & & \\ 0.115 & 0.322 & 0.182 & 1 & \\ 0.154 & 0.213 & 0.146 & 0.683 & 1 \end{bmatrix} \end{array}$$

$$d_{(45)1} = \max\{d_{41}\ d_{51}\} = \max\{0.115, 0.154\} = 0.154$$

$$d_{(45)2} = \max\{d_{42}.\ d_{52}\} = \max\{0.322, 0.213\} = 0.322$$

$$d_{(45)3} = \max\{d_{43}\ d_{53}\} = \max\{0.182, 0.146\} = 0.182$$

Single linkage cluster.

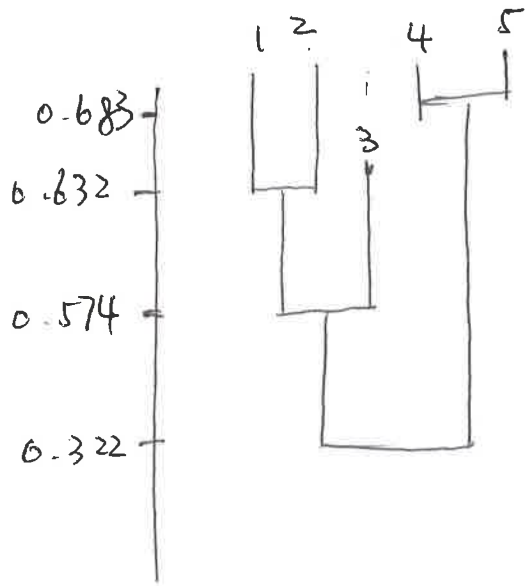
$$\begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 45 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 45 \end{array} & \begin{bmatrix} 1 & & & \\ 0.632 & 1 & & \\ 0.510 & 0.574 & 1 & \\ 0.154 & 0.322 & 0.182 & 1 \end{bmatrix} \end{array}$$

$$d_{(12)3} = \max\{d_{13}, d_{23}\} = \max\{0.510.\ 0.574\} = 0.574$$

$$d_{(12)(45)} = \max\{d_{1(45)}\ d_{2(45)}\} = \max\{0.154.\ 0.322\} = 0.322$$

$$\begin{array}{c|c} & \begin{array}{ccc} 12 & 3 & 45 \end{array} \\ \begin{array}{c} 12 \\ 3 \\ 45 \end{array} & \begin{bmatrix} 1 & & \\ 0.574 & 1 & \\ 0.322 & 0.182 & 1 \end{bmatrix} \end{array}$$

$$\begin{array}{c|c} & \begin{array}{cc} 123 & 45 \end{array} \\ \begin{array}{c} 123 \\ 45 \end{array} & \begin{bmatrix} 1 & \\ 0.322 & 1 \end{bmatrix} \end{array}$$



3.

	X_1	X_2
A	5	4
B	1	-2
C	-1	1
D	3	1

$k=2$.

	$\bar{X}_1$	$\bar{X}_2$
(AB)	3	1
(CD)	1	1

1° if A is not moved: $d^2(A,(AB)) = (5-3)^2 + (4-1)^2 = 4+9 = 13$

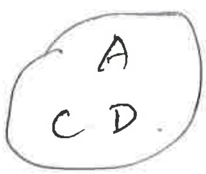
$d^2(A.(CD)) = (5-1)^2 + (4-1)^2 = 16+9 = 25$

2° If A is moved: $d^2(A.B) = (5-1)^2 + (4+2)^2 = 16+36 = 52$.

$\bar{X}_{ACD} = (\frac{7}{3}, 2)$ $\quad d^2(A.\ ACD) = (5-\frac{7}{3})^2 + (4-2)^2 = 100/9 = 11.1$

$\Rightarrow$ A is moved.

if B is Not move. $d^2(B.B) = 0$



if B is moved, then it is back to initial groups

B is not moved.

If C is not moved. $d^2(C.\ ACD) = (-1-\frac{7}{3})^2 + (1-2)^2 = 12.1$

$d^2(C.B) = (-1-1)^2 + (1+2)^2 = 13$

If C is moved $\quad d^2(C\ (BC)) = (-1-0)^2 + (1+\frac{1}{2})^2 = 3.25$

$\bar{X}_{BC} = (0\ \ -\frac{1}{2})$ $\quad d^2(C\ (AD)) = (-1-4)^2 + (1-\frac{5}{2})^2 = 27.25$

$X_{AD} = (4\ \ \frac{5}{2})$ $\quad \Rightarrow$ C is moved.

If D is Not moved.

$d^2(D,(AD)) = (3-4)^2 + (1-\frac{5}{2})^2 = 3.25$

$d^2(D.(BC)) = (3-0)^2 + (1+\frac{1}{2})^2$

$d^2(D.A) = (3-5)^2 + (1-4)^2 = 13$

If D is moved. $\quad d^2(D.\ (BCD)) = 5$

D is Not moved