

Sin(x) = x Fixed Point Iteration

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First I define a function called `fixed_plot` where I combine fixed point iteration and plotting (I'm a visual guy).

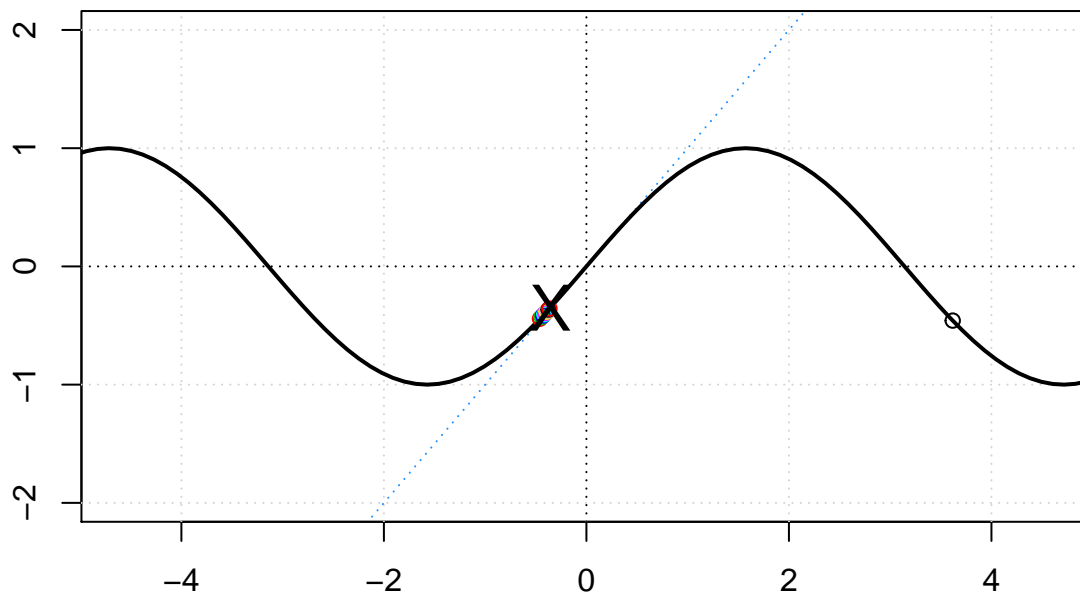
Notice that I have multiplied $\sin(x)$ by $\frac{x^3}{6}$ which makes

$$f(x) = \sin(x) \frac{x^3}{6}$$

which you can easily identify as the second term in the Taylor series expansion of $\sin(x)$.

So, this little “fudge function” makes my function converge super fast now, but ONLY if my starting point is less than $|3.6|$. If I choose any number greater than that my iteration diverges, and rapidly. Here's some examples with only 5 iterations.

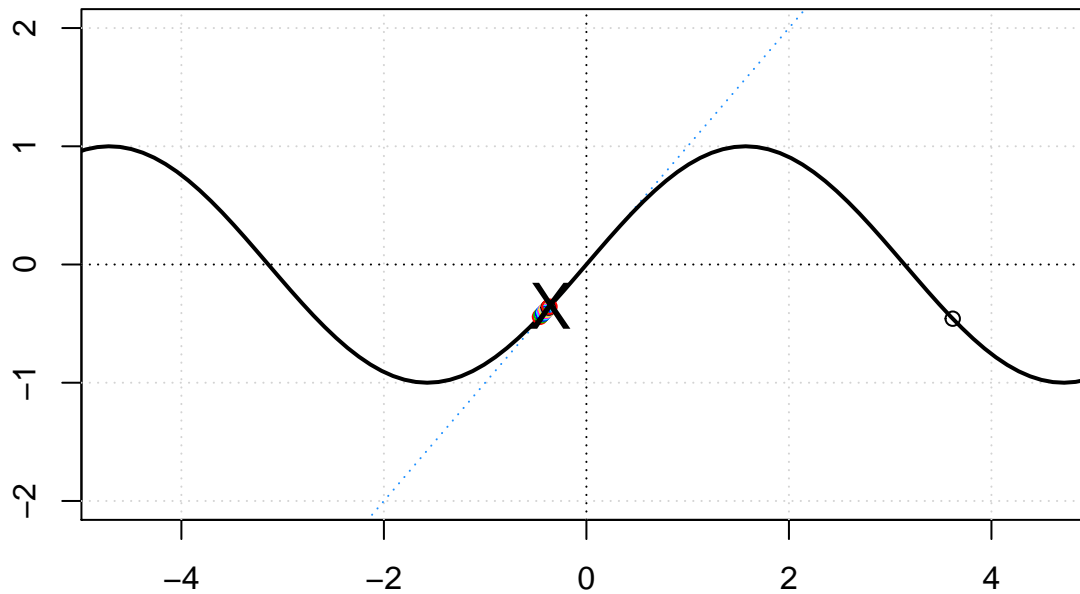
```
fixed_plot(3.6178, FUN = sin, n = 10)
```



	x0	ratio	errors
[1,]	-0.4584118	-0.1267101	
[2,]	-0.4425244	0.9653426	
[3,]	-0.4282221	0.9676801	
[4,]	-0.4152541	0.9697166	
[5,]	-0.4034224	0.9715074	
[6,]	-0.3925683	0.9730949	
[7,]	-0.3825626	0.9745122	
[8,]	-0.3732991	0.9757855	
[9,]	-0.3646893	0.9769359	
[10,]	-0.3566590	0.9779806	

I'm at **zero** super fast as you can see. Now try a starting guess greater than that.

```
fixed_plot(3.61781, FUN = sin, n = 10)
```



	x0	ratio	errors
[1,]	-0.4584207	-0.1267122	
[2,]	-0.4425324	0.9653413	
[3,]	-0.4282293	0.9676790	
[4,]	-0.4152606	0.9697156	
[5,]	-0.4034284	0.9715066	
[6,]	-0.3925738	0.9730941	
[7,]	-0.3825677	0.9745115	
[8,]	-0.3733038	0.9757849	
[9,]	-0.3646937	0.9769353	
[10,]	-0.3566631	0.9779800	

And we diverge super fast.

What's going on here?