Chapter 8 Homework

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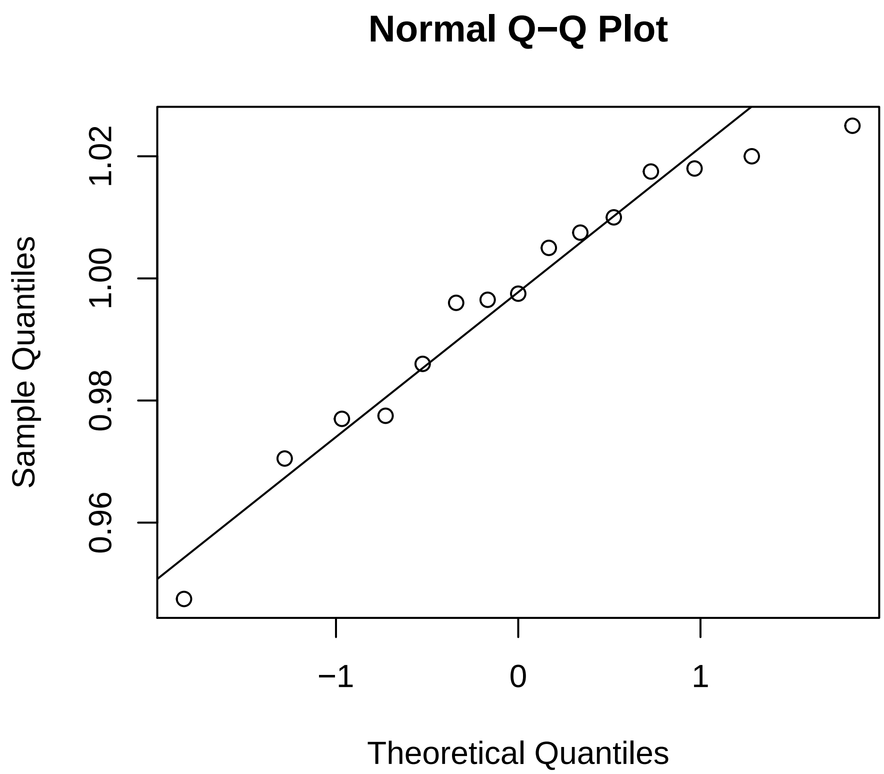
8.11



The weights of 15 1 kg containers have been collected.

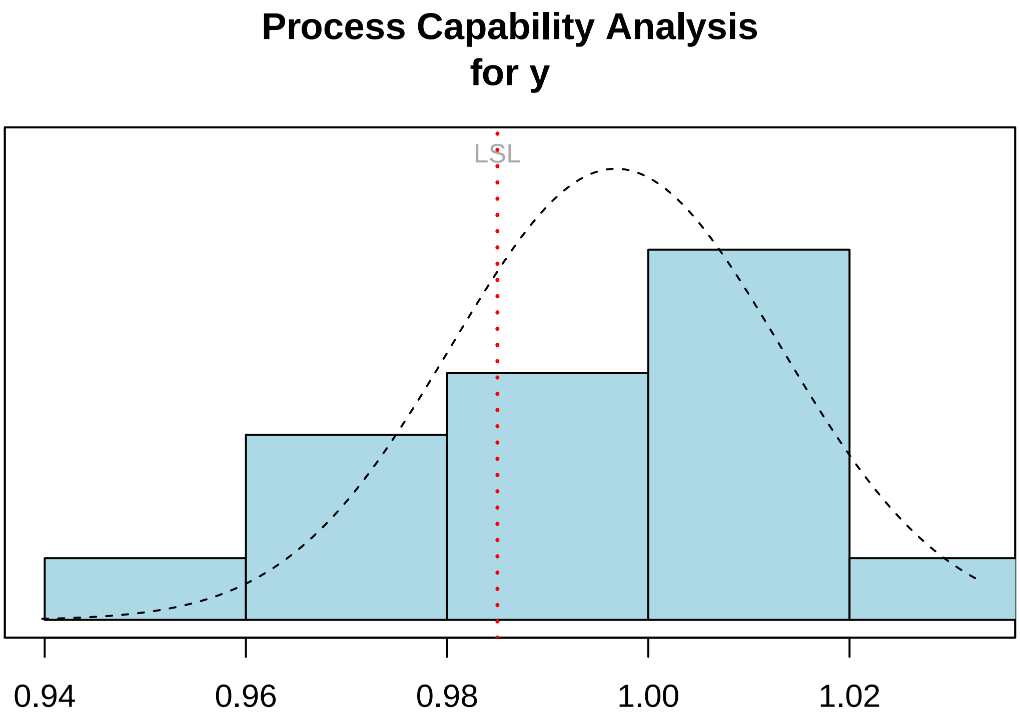
$$\bar{x} = 0.9967667 \quad s = 0.0216662$$

$$process\ capability = \hat{\mu} \pm 3\hat{\sigma} = 0.9967667 \pm 0.0649986$$

and the normal distribution appears to be an appropriate model for the data.

8.12

Using the data from above, we are given a lower specification of 0.985 kg.



From the looks of this plot, our process does not appear to be very capable of staying inside the lsl.

$$\hat{C}_{pl} = \frac{\hat{\mu} - LSL}{3\hat{\sigma}} = \frac{0.9967667 - 0.985}{0.0649986} = 0.1810296$$

And estimating the percent of fallout

$$Z = \frac{LSL - \hat{\mu}}{\hat{\sigma}} = \frac{0.985 - 0.9967667}{0.0216662} = -0.5430889$$

and then calculating the probability of Z under the standard normal distribution and multiplying by 100 we would have the estimated percent fallout, 29.35%.

8.24

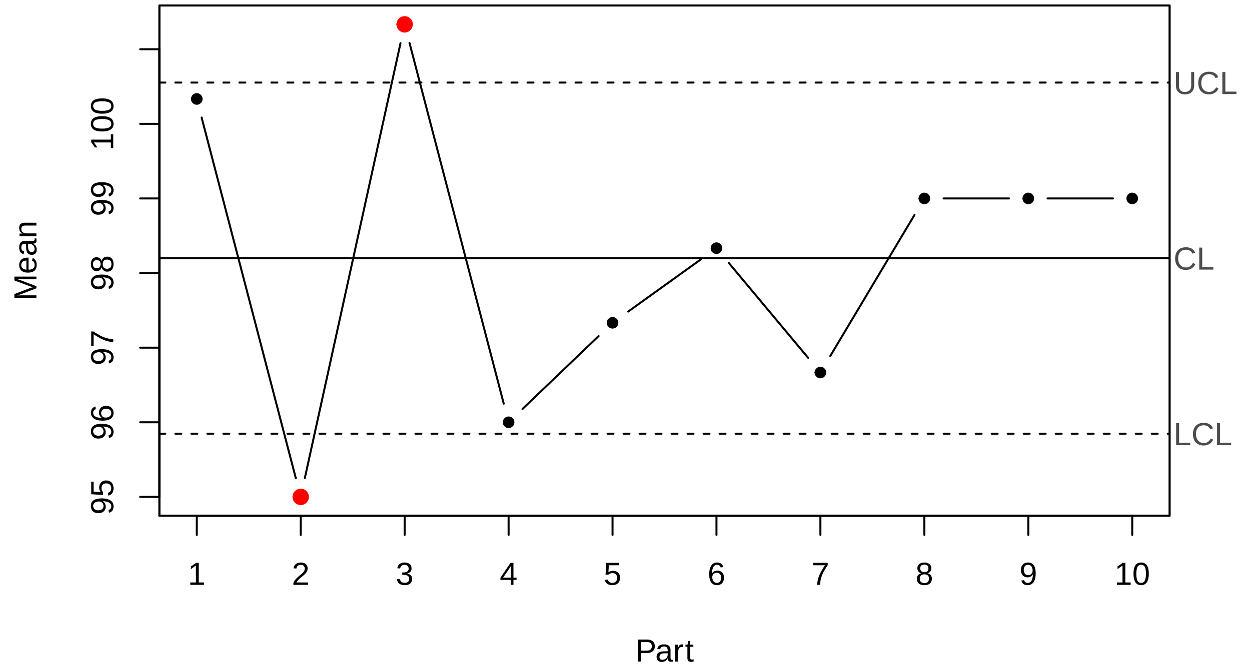
First we note that there is not an interaction term in our model. We will only be estimating total variability from part and system. This study did not vary the operator so we will not have a measure of operator interaction with measurement system.

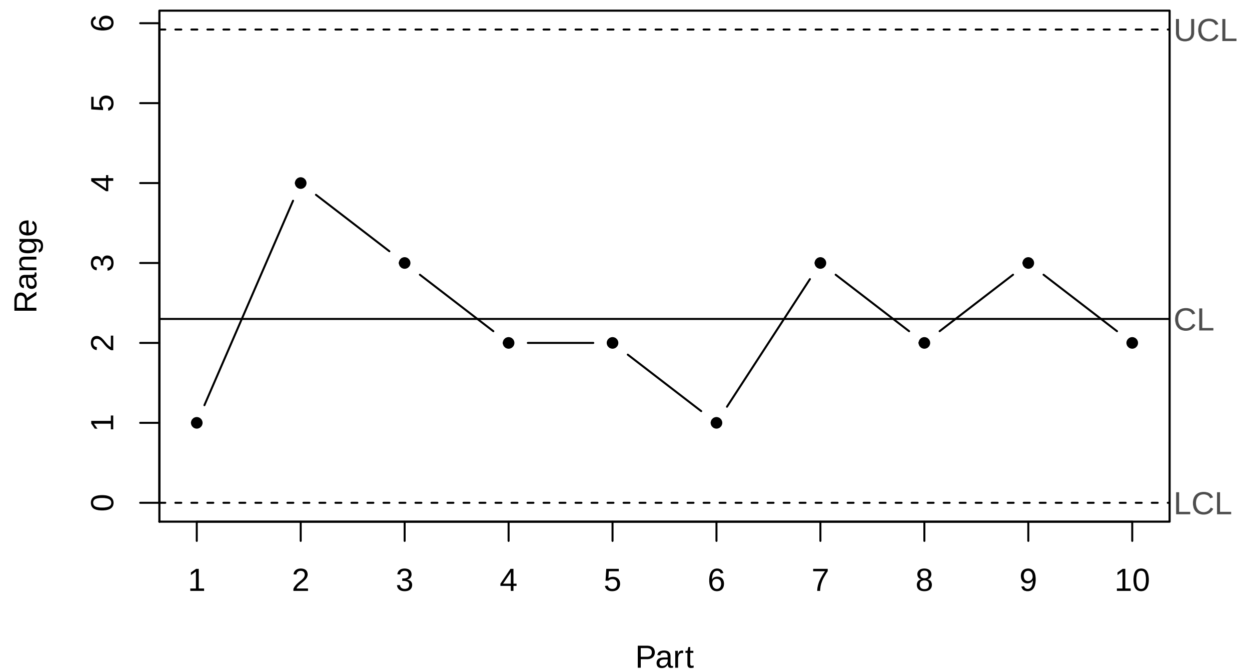
$$\hat{\sigma}_{Gauge} = \frac{\bar{R}}{d_2} = \frac{2.8}{1.128} = 2.4822695$$

It appears that the new gauge performs worse relative to the older one. The variance from the new gauge is larger than the old one.

$$P/T = \frac{k\hat{\sigma}_{Gauge}}{USL - LSL} = 0.4964539$$

8.25





From the range chart it can be seen that there is decent repeat ability between parts. Here, the operator has measured each part 3 times.

The variance components are

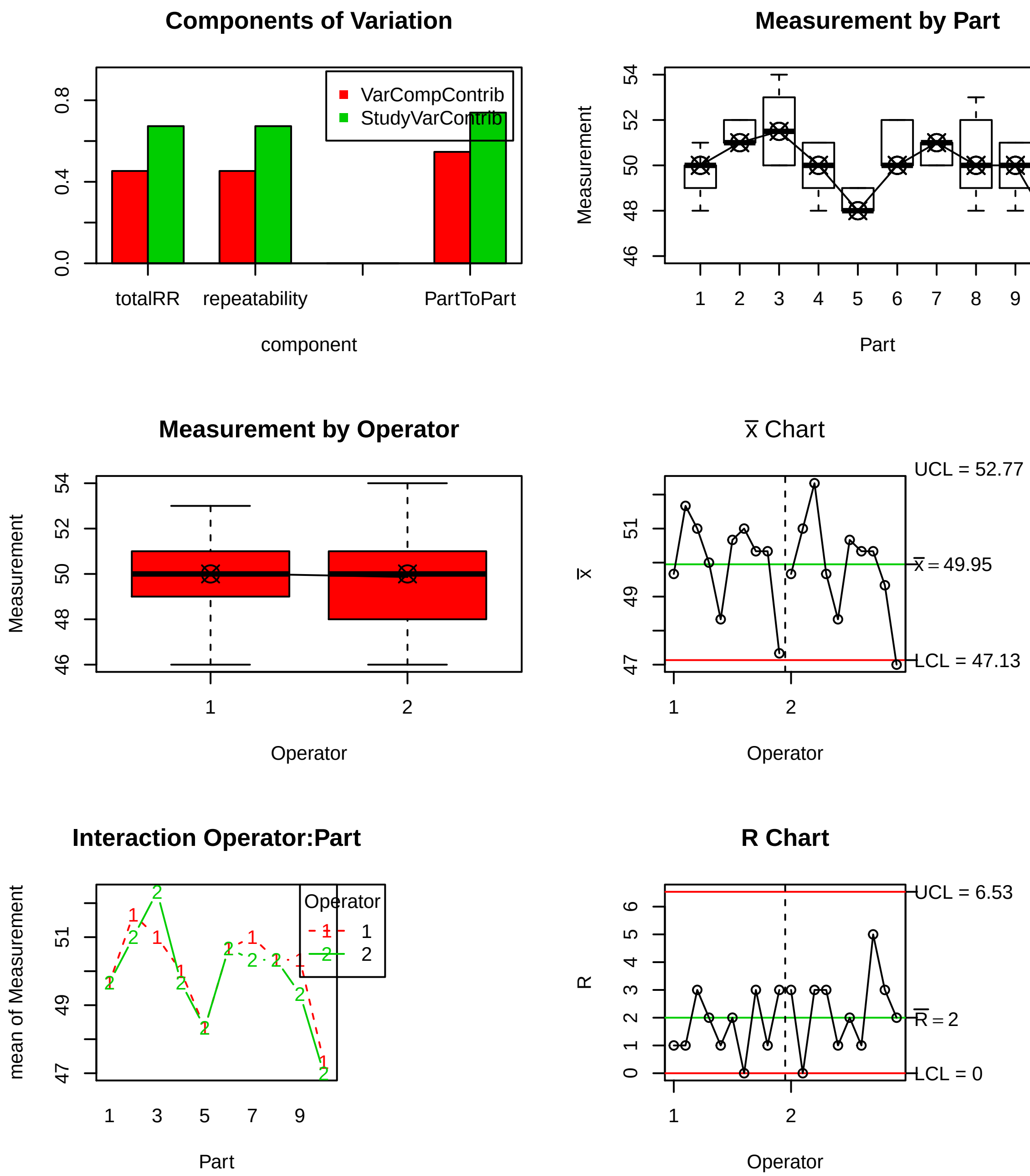
$$\hat{\sigma}^2_{total} = 4.9432099, \quad \hat{\sigma}^2_{part} = 3.2765432, \quad \hat{\sigma}^2 = 1.6666667$$

and $\hat{\sigma}^2$ can be interpreted as the repeat ability of the gauge and the measurement error. The percent of the total variation due to the parts is 66.28%. The values for the calculations for the variance components can be taken from the ANOVA table. I display one here for convenience.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
part	9	103.46667	11.496296	6.897778	0.0001685
Residuals	20	33.33333	1.666667	NA	NA

8.26

```
##
## AnOVa Table -  crossed Design
##               Df Sum Sq Mean Sq F value   Pr(>F)
## Operator       1   0.42   0.417   0.278    0.601
## Part           9  99.02  11.002   7.335 3.22e-06 ***
## Operator:Part  9   5.42   0.602   0.401    0.927
## Residuals     40  60.00   1.500
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## ----------
## AnOVa Table Without Interaction -  crossed Design
##             Df Sum Sq Mean Sq F value   Pr(>F)
## Operator     1   0.42   0.417   0.312    0.579
## Part         9  99.02  11.002   8.241 2.46e-07 ***
## Residuals   49  65.42   1.335
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## ----------
##
## Gage R&R
##                   VarComp VarCompContrib Stdev StudyVar StudyVarContrib
## totalRR              1.34          0.453  1.16     6.93           0.673
##  repeatability       1.34          0.453  1.16     6.93           0.673
##  reproducibility     0.00          0.000  0.00     0.00           0.000
##    Operator          0.00          0.000  0.00     0.00           0.000
##    Operator:Part     0.00          0.000  0.00     0.00           0.000
## Part to Part         1.61          0.547  1.27     7.62           0.739
## totalVar             2.95          1.000  1.72    10.30           1.000
##
## ---
##  * Contrib equals Contribution in %
##  **Number of Distinct Categories (truncated signal-to-noise-ratio) = 1
```



A model for this study could be represented by

$$y_{ijk} = \mu + P_i + O_j + (PO)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1, 2, ..., 10 \\ j = 1, 2 \\ k = 1, 2, 3 \end{cases}$$

Where P_i is the ith part and O_j is the jth operator. Here I display the ANOVA table from the fitted model.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
part	9	99.0166667	11.0018519	7.3345679	0.0000032
operator	1	0.4166667	0.4166667	0.2777778	0.6010725
part:operator	9	5.4166667	0.6018519	0.4012346	0.9270089
Residuals	40	60.0000000	1.5000000	NA	NA

- First, the variance component for part is

$$\hat{\sigma}_P^2 = \frac{11.0018519 - 0.6018519}{2(3)} = 1.7333333$$

the variance component for operator is

$$\hat{\sigma}_O^2 = \frac{0.4166667 - 0.6018519}{10(3)} = 0$$

and the variance component for the interaction of operator and part is

$$\hat{\sigma}_{PO}^2 = \frac{0.6018519 - 1.5}{3} = 0$$

where we round up to zero if the variance component is less than zero.

The variance component for repeat ability $= \sigma^2 = 1.5$. The variance component for reproducibility can be thought of

$$\sigma^2_{reproducability} = \sigma^2_O + \sigma^2_{PO} = 0$$

The standard deviation of measurement error is simply

$$\sqrt{MSE} = 1.2247449$$

Since operator and operator:part components are not significant in our model, we could consider a reduced model from which we would then estimate the variance components from the reduced model.

If the spec limits are 50 ± 10 then we simply plug into the formula

$$P/T = \frac{k\hat{\sigma}_{Gauge}}{USL - LSL} = \frac{6(1.5)}{60 - 40} = 0.45$$

8.28

A model for this study could be represented by

$$y_{ijk} = \mu + P_i + O_j + (PO)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1, 2, ..., 10 \\ j = 1, 2 \\ k = 1, 2, 3 \end{cases}$$

Where P_i is the ith part and O_j is the jth operator. Here I display the ANOVA table from the fitted model.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
part	19	1185.425000	62.3907895	62.915082	0.0000000
operator	2	2.616667	1.3083333	1.319328	0.2749569
part:operator	38	27.050000	0.7118421	0.717824	0.8614345
Residuals	60	59.500000	0.9916667	NA	NA

- First, the variance component for part is

$$\hat{\sigma}_P^2 = \frac{62.3907895 - 0.7118421}{3(2)} = 10.2798246$$

the variance component for operator is

$$\hat{\sigma}_O^2 = \frac{1.3083333 - 0.7118421}{20(2)} = 0.0149123$$

and the variance component for the interaction of operator and part is

$$\hat{\sigma}^2_{PO} = \frac{0.7118421 - 0.9916667}{2} = 0$$

where we round up to zero if the variance component is less than zero.

The variance component for repeat ability $= \sigma^2 = 0.9916667$. The variance component for reproducibility can be thought of

$$\sigma^2_{reproducability} = \sigma^2_O + \sigma^2_{PO} = 0.0149123$$

The standard deviation of measurement error is simply

$$\sqrt{MSE} = 0.9958246$$

If the spec limits are [60, 6] then we simply plug into the formula

$$P/T = \frac{k\hat{\sigma}_{Gauge}}{USL - LSL} = \frac{6(1.0065789)}{60 - 6} = 0.1118421$$