Engineering Tripos Part IIB

4A7: Aircraft Aerodynamics and Design

Aviation and the Environment

Bill Dawes

(with thanks to Dr Chez Hall)

Lecture Slides

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UNIVERSITY OF CAMBRIDGE

Course Overview

Emissions from passenger aircraft are a significant and growing contributor to climate change. CO_2 is not the only problem.

- Aircraft are designed primarily to satisfy market demand.
- Operations are constrained by established air traffic systems.
- New technology is focused on minimising fuel burn and noise.

What if aircraft were optimised to reduce radiative forcing?

- How would the design range and payload change?
- How much benefit is available from technology developments?
- Should aircraft be flown at different profiles of altitude and speed?

The idea for this course is to present the key theories for emissions, engine performance and airframe aerodynamics and to combine these into a method that can be used to investigate these important issues.

Coursework: Aircraft Environmental Impact

- The coursework consists of two case studies, based on the simple modelling presented in lectures, to study the emissions trade-offs associated with, for example:
  - aircraft design range and payload
  - cruise altitude
  - engine overall pressure ratio.
- It is intended that the case studies will be a spreadsheet, Matlab, Python, or a simple Fortran code.
- Full details are given later.

Outline

1. Atmospheric Modelling and Environmental Impact

Objectives

i. Outline the motivation to reduce aviation's environmental impact

ii. Describe how key pollutants: CO_2, NO_x and Contrails are generated by aircraft and how their impact varies with altitude and location

iii. Understand the key pollution metrics, such as emissions Index, EI, emission per payload-range, and Radiative Forcing, RF.

iv. Review how to calculate the variation of properties of the ISA

International action on climate change

The 21st United Nations Framework Convention on Climate Change (UNFCCC) conference in Paris in December 2015 produced, a ‘climate agreement’ signed by 195 member states.

The governments agreed:

1. a long-term goal of keeping global average temperatures to below 2°C above pre-industrial levels;
2. to aim to limit the increase to 1.5°C, to minimise climate change impacts;
3. on the need for global emissions to peak as soon as possible,
4. to undertake rapid reductions in line with the best available science.

The recognition that it is the mean global temperature rise that matters (rather than, say, atmospheric CO_2 concentration alone) is significant. This has important consequences for aviation.

Aviation Regulation and Future Outlook

To date, regulation has been focused on noise and local emissions

- ICAO (International Civil Aviation Organisation)
  - Sets noise standards in ICAO Appendix 16, Chapters 1-4
- CAEP (Committee on Aviation Environmental Protection)
  - A sub-committee of ICAO that formulates emission standards
  - Considering aviation regulations for climate change
- IPCC
  - Intergovernmental Panel on Climate Change
- ACARE
  - Advisory Council for Aeronautics Research in Europe

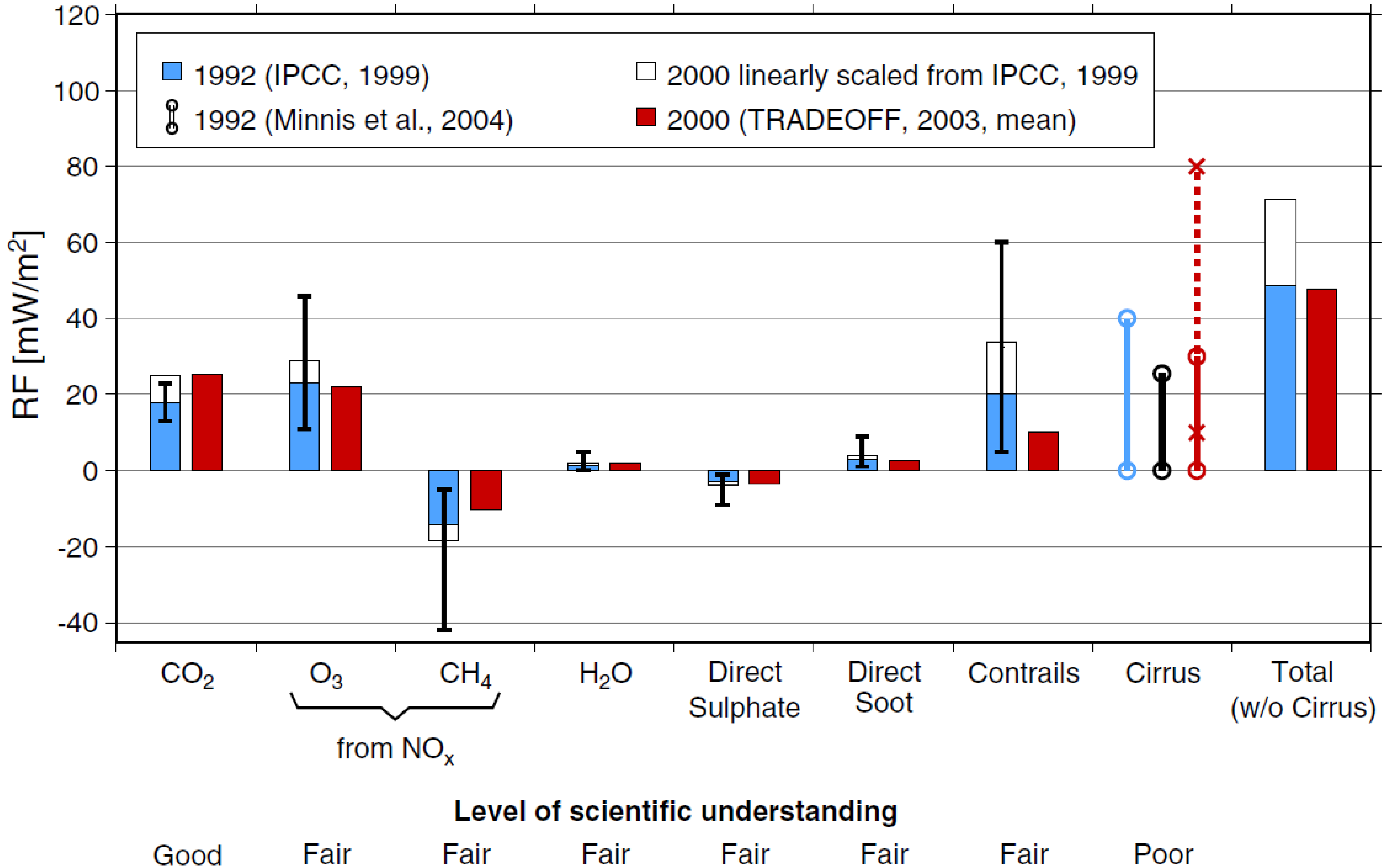
There are no agreed means to control aviation's impact on global temperature rise, although ambitious goals (e.g. ACARE 2020) have been established to reduce future aircraft fuel burn and NO_x emissions.

The key challenge is that air transport is growing rapidly and is set to continue to grow, leading to increased climate impact.

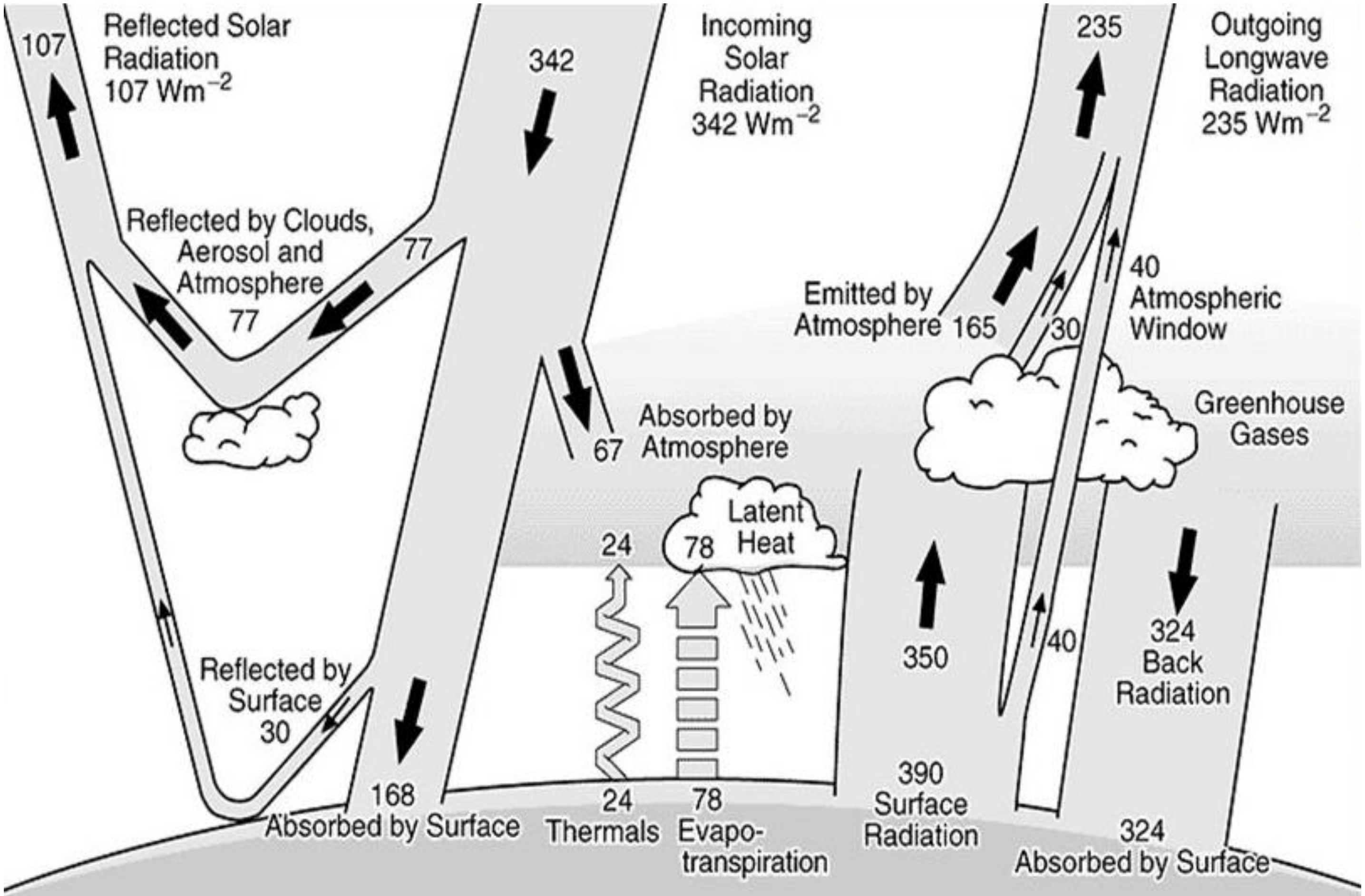
The main pollutants from aircraft

- Carbon Dioxide, CO_2
  - CO_2 emission depends only on the type and amount of fuel burned
  - remains in the atmosphere for centuries, slowly removed through natural carbon ‘sinks’ (plants, soil and the oceans)
  - In 2015, 247 Mtonne of jet fuel were burned by aviation, releasing ~780 Mtonne of CO_2 into the atmosphere. Significant and rising!
- Nitrous Oxides, NO_x
  - a mixture of nitric oxide, NO, and nitrogen dioxide, NO_2.
  - a result of chemical reactions at high temperature and pressure between N_2 and O_2 in the combustor of a gas turbine.
  - not itself a greenhouse gas, but it changes the concentrations of two important atmospheric greenhouse gases, namely ozone, O_3 and methane CH_4, through a series of chemical reactions.
- Water Vapour, H_2O
  - contributes to contrail formation (see later).

Radiative Forcing from Aircraft



Atmospheric energy balance



Estimate of the Earth's annual and global mean energy balance.
[IPCC (2001), Kiehl and Trenberth (1997)]

Contrail formation

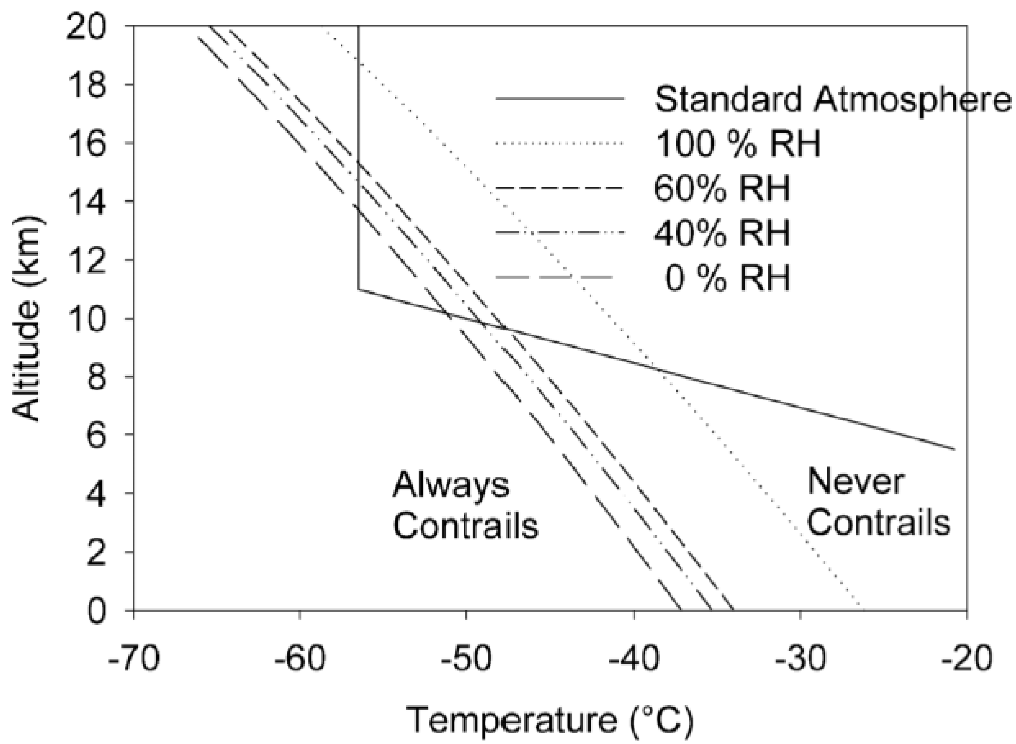
Aircraft contrails over Northern Europe

- Contrails are fields of ice crystals, formed by an interaction between the aircraft and the atmosphere.
- A critical ambient air temperature exists below which the water vapour in the engine exhaust freezes out to form visible ice crystals.
- If the surrounding atmosphere is ‘super-saturated with respect to ice’ the exhaust particulates initiate nucleation leading to persistent contrail cirrus

Contrails…

- This recent publication is dedicated to Contrails

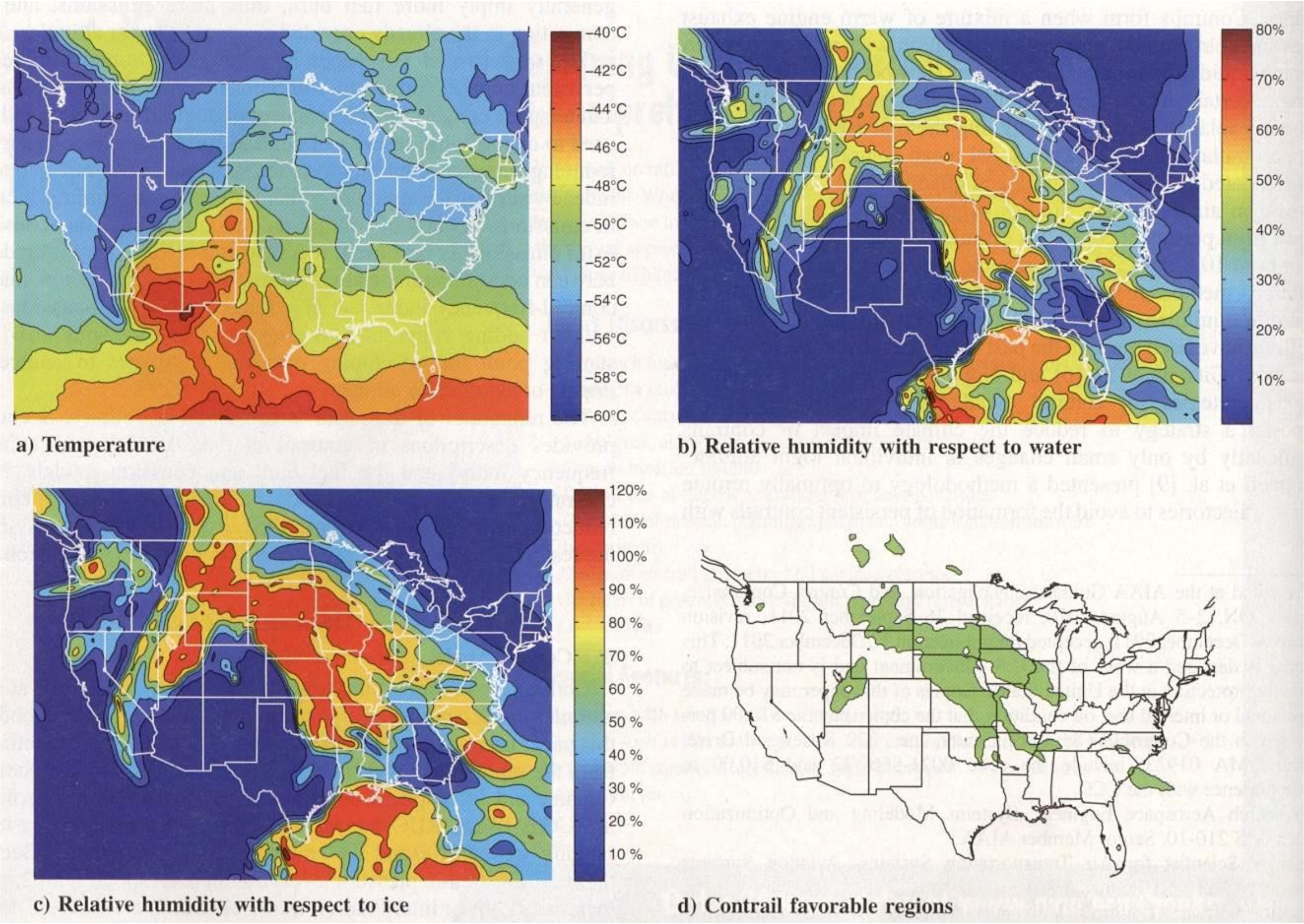
The conditions required for contrails



Contrail Production with Altitude, Temperature and Relative Humidity
[U. Schumann, C. R. Physique 6 (2005)]

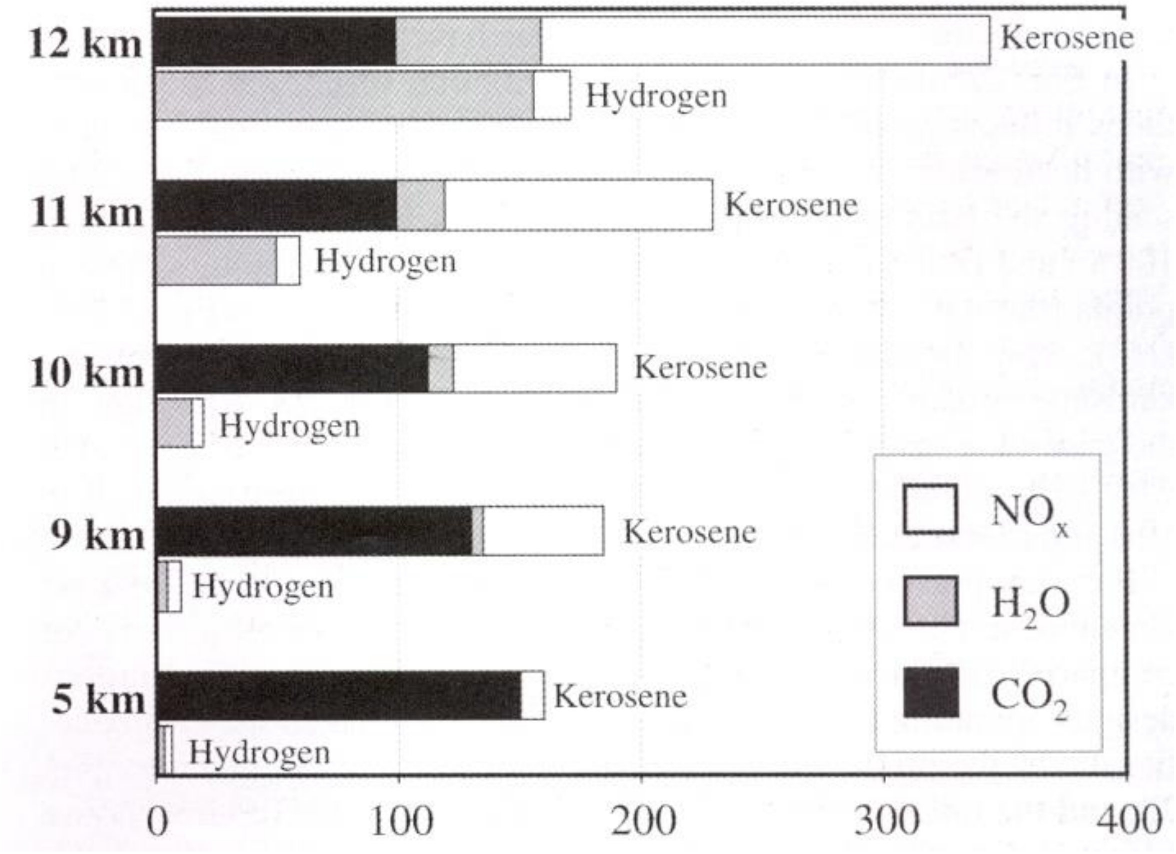
- **Contrail** formation can be avoided only by flying in warmer and dryer air.
- **Contrail cirrus** could be reduced by avoiding flights in ice-supersaturated regions e.g. by increasing flight altitude into the lower stratosphere.

Example of contrail favourable regions



Atmospheric data and contrail favourable regions at 34,000 ft
[AIAA J. Aircraft, Vol. 49, pp1367-, Sept. 2012]

The effect of altitude (1)



Relative net greenhouse effect (Green [2002])

The effect of altitude (2)

- The effects of pollutants vary with season, altitude and location in ways which are poorly understood
- The previous figure (from Green [2002]) roughly quantifies the relative net greenhouse effect of NOx, CO_2 & H_20 with altitude
- It shows how for kerosene CO_2 is much worse than NOx at lower altitude but comparable at typical aircraft cruise levels

altitude (km)	CO_2	NOx
5	147	10
9	126	47
10	110	63
11	100	105
12	100	126

Relative net greenhouse effect (tabulated)

The effect of altitude – other data (3)

- The effects of pollutants with altitude are poorly understood…
- Data from Svensson *et al* [2004] described in terms of Global Warming Potential, are rather different from Green [2002] with much stronger penalty for NOx at “airplane” altitudes

ELSEVIER
Available online at www.sciencedirect.com
SCIENCE DIRECT
Aerospace Science and Technology 8 (2004) 307–320
Aerospace Science and Technology
www.elsevier.com/locate/aescte

Reduced environmental impact by lowered cruise altitude for liquid hydrogen-fuelled aircraft

Minskad miljöpåverkan genom sänkt marsch yghöjd för vätgasdrivna ygplan

Fredrik Svensson [a,*], Anders Hasselrot [a], Jana Moldanova [b]

[a] Swedish Defence Research Agency, FOI, Aeronautics Division, FFA, Department of Aviation Environmental Research, SE-172 90 Stockholm, Sweden
[b] IVL Swedish Environmental Research Institute, Box 470 86, SE-402 58 Göteborg, Sweden

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Available online 17 March 2004

Table A.3
GWP figures for CO_2, H_2O and NO_x versus altitude

Altitude (km)	GWP (CO_2)	GWP (H_2O)	GWP (NO_x)
0	1	0.00	−7.1
1	1	0.00	−7.1
2	1	0.00	−7.1
3	1	0.00	−4.3
4	1	0.00	−1.5
5	1	0.00	6.5
6	1	0.00	14.5
7	1	0.00	37.5
8	1	0.00	60.5
9	1	0.00	64.7
10	1	0.24	68.9
11	1	0.34	57.7
12	1	0.43	46.5
13	1	0.53	25.6
14	1	0.62	4.6
15	1	0.72	0.6

Pollutant metrics

i. Emission Index, EI

$$\text{Emission Index, } EI = \frac{\text{Mass of pollutant}}{\text{Mass of fuel}}$$

e.g. EI_{CO_2} measured in gCO_2 per kg fuel, $EI_{CO_2} = m_{CO_2} / m_f$

ii. Fuel burn per payload-range: $m_f / (s m_p)$

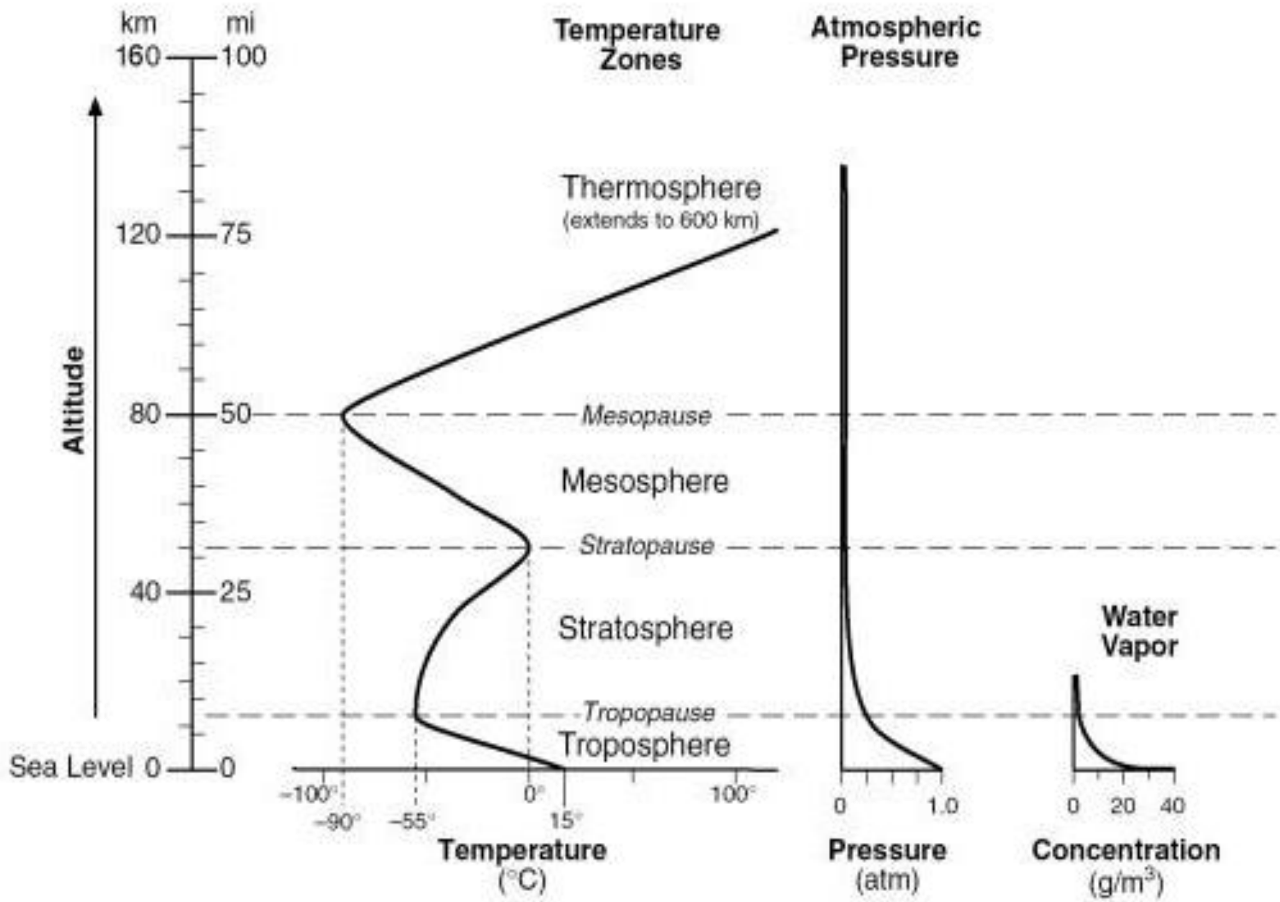
iii. Pollutant emission per passenger-range:

$$\frac{m_{CO_2}}{s N_p} = \frac{m_f}{s m_p} \times EI_{CO_2} \times 100 \text{ kg/pass}$$

1 passenger ~100kg (a person plus luggage)

Radiative Forcing, RF, expressed in W/m^2, represents the impact of pollutants on the atmospheric energy balance. Mapping EI to RF is currently very uncertain.

Atmospheric Modelling



Variation of atmospheric properties with altitude

The International Standard Atmosphere (1)

For the International Standard Atmosphere (ISA), conditions at sea level are defined to be:

$$T_{sl} = 288.15 \text{ K} \quad p_{sl} = 101.325 \text{ kPa} \quad \rho_{sl} = 1.225 \text{ kg/m}^3 \quad a_{sl} = 340.3 \text{ m/s}$$

The atmosphere MUST satisfy the hydrostatic pressure gradient: $\mathrm{d}p/\mathrm{d}h = -\rho g$

and the ideal gas law: $p = \rho RT$

Between sea level and the Tropopause, at (h = 11 km):

$T = 288.15 - 6.5h$ K, where altitude, h, is in km.

For altitudes between 11 km and 20 km:

$T = T_T = 216.65$ K (approx. –56.5 °C)

The International Standard Atmosphere (2)

Solving the equation: $$\frac{dp}{dh} = -\frac{p}{RT}g$$

Below the Tropopause:

$$\frac{dp}{p} = -\frac{gdh}{RT} = \frac{g}{R \times 0.0065}\frac{dT}{T}$$

$$\frac{p}{p_{sl}} = \left(\frac{T}{T_{sl}}\right)^{5.256}, \quad \frac{\rho}{\rho_{sl}} = \left(\frac{T}{T_{sl}}\right)^{4.256}$$

$$\frac{a}{a_{sl}} = \left(\frac{T}{T_{sl}}\right)^{0.5}$$

Above the Tropopause:

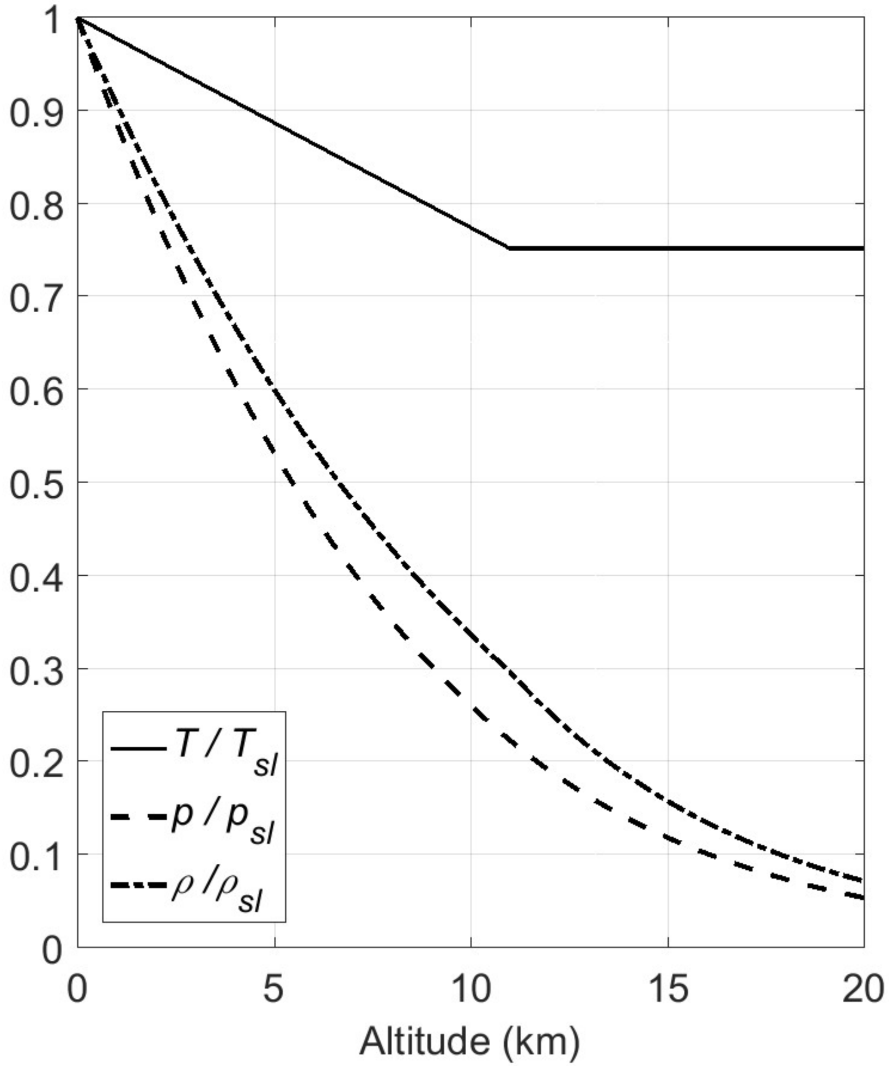
$$\frac{dp}{p} = -\frac{gdh}{RT_T} = \frac{g}{R \times 216.65}dh$$

$$\frac{p}{p_T} = \exp\left(-0.1577\left(h-11\right)\right)$$

$$\frac{\rho}{\rho_T} = \exp\left(-0.1577\left(h-11\right)\right)$$

The International Standard Atmosphere (3)

Plot of the results:



h (km)	T (°C)	a/a_0	p/p_0	ρ/ρ_0
0	15.0	1.0000	1.0000	1.0000
1	8.5	0.9887	0.8870	0.9075
2	2.0	0.9772	0.7846	0.8216
3	−4.5	0.9656	0.6919	0.7421
4	−11.0	0.9538	0.6083	0.6687
5	−17.5	0.9419	0.5331	0.6009
6	−24.0	0.9299	0.4656	0.5385
7	−30.5	0.9177	0.4052	0.4812
8	−37.0	0.9053	0.3513	0.4287
9	−43.5	0.8927	0.3034	0.3807
10	−50.0	0.8800	0.2609	0.3369
11	−56.5	0.8671	0.2234	0.2971
12	−56.5	0.8671	0.1908	0.2537
13	−56.5	0.8671	0.1629	0.2167
14	−56.5	0.8671	0.1392	0.1851
15	−56.5	0.8671	0.1189	0.1581
16	−56.5	0.8671	0.1015	0.1350
17	−56.5	0.8671	0.0867	0.1153
18	−56.5	0.8671	0.0741	0.0985
19	−56.5	0.8671	0.0633	0.0841
20	−56.5	0.8671	0.0540	0.0719

ISA Properties (page 32 of Databook)

2. Modelling the Airframe

<u>Objectives</u>

i. Determine the typical weight breakdown of an aircraft

ii. Understand the sources of aircraft drag for a modern transonic aircraft and their impact on the Lift/Drag, or L/D, variation.

iii. Derive a simplified model of L/D and relations for the optimum flight conditions and L/D as a function of equivalent air speed.

Aircraft Weight Breakdown

- The sum of the aircraft empty weight, payload weight and fuel weight must sum to the takeoff weight:

$$\frac{w_E}{w_{TO}} + \frac{w_P}{w_{TO}} + \frac{w_F}{w_{TO}} = 1$$

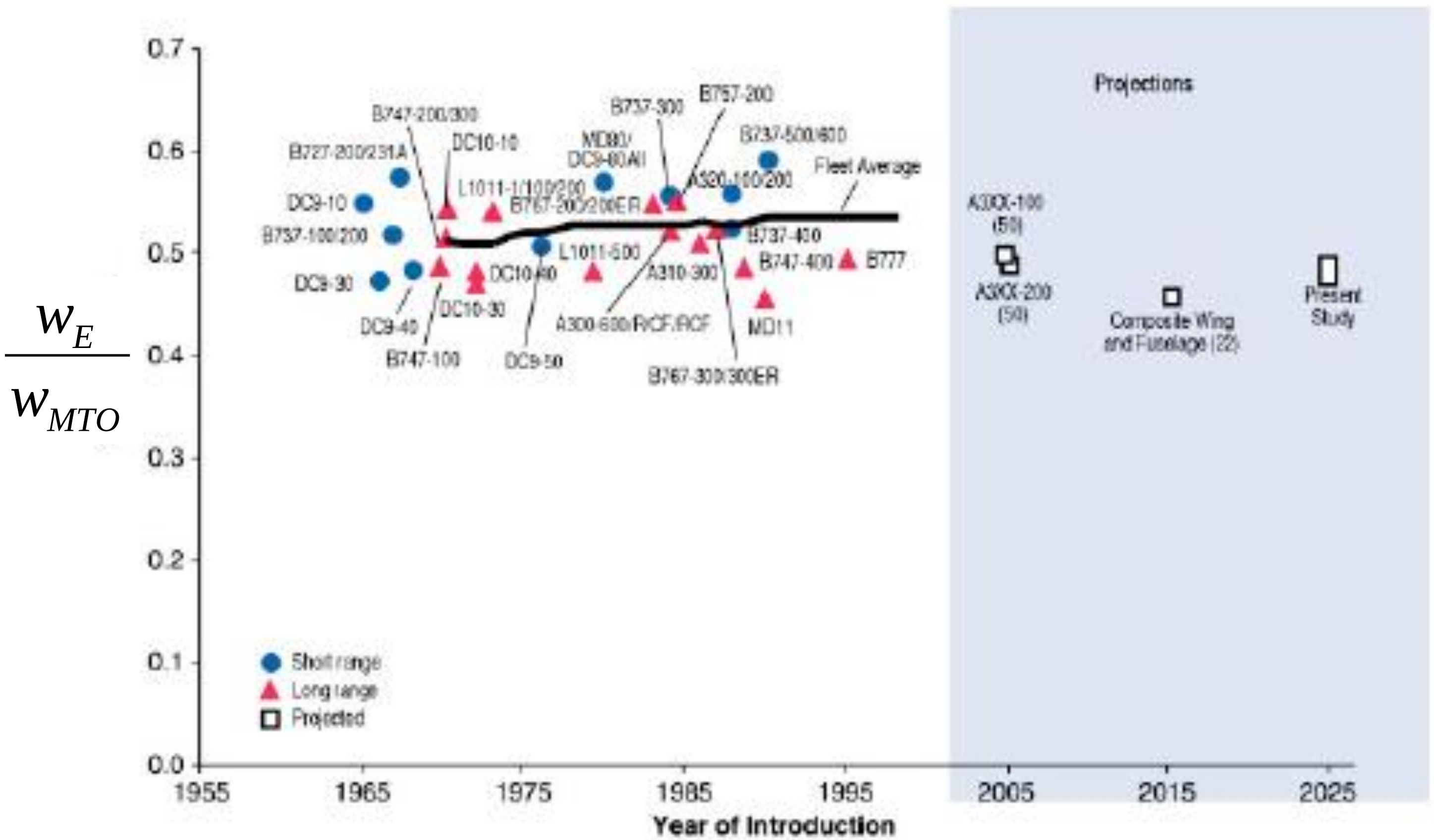
- The empty weight is the weight of the aircraft without any fuel, passengers, cargo or baggage. It consists of two components:

(i) items related to the related to the maximum takeoff weight, w_{MTO}. This includes the wing and horizontal tail structure, the engines and the landing gear.

(ii) items related to the maximum payload, w_{MP} . This includes the fuselage and vertical tail structures, air conditioning, pressurisation, electrical systems, passenger accommodation, cabin furnishings, etc.

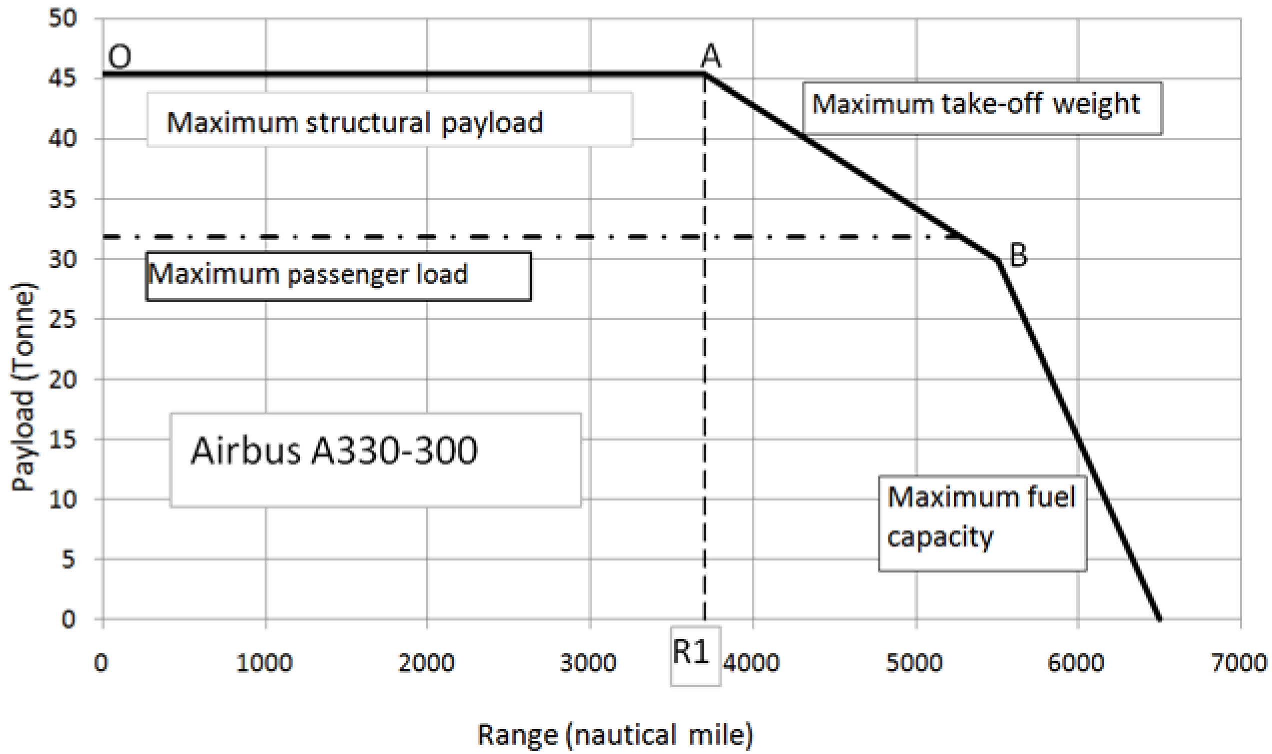
$$\frac{w_E}{w_{MTO}} = c_1 + c_2 \frac{w_{MP}}{w_{MTO}} \qquad \text{Typically, } c_1 \approx 0.3 \text{ and } c_2 \approx 1.0$$

Aircraft Empty Weight Fraction



Variation of w_E/w_{MTO} for a wide range of aircraft [Lee et. al (2001)]

Aircraft Payload-Range Diagram



Example payload versus range diagram for the A330-300

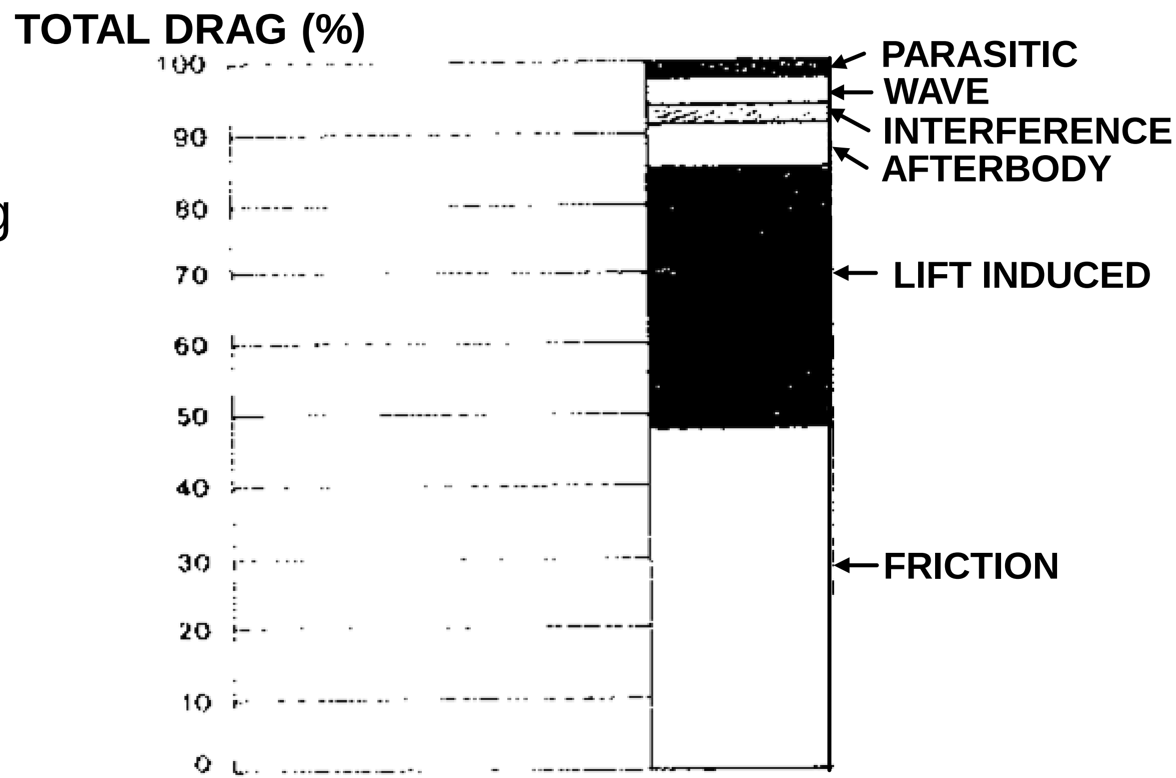
- Note that payload isn't necessarily just passengers and their baggage. Significant freight can also be carried under the cabin floor.

Aircraft Drag Breakdown

- Aircraft drag
  = viscous + vortex drag

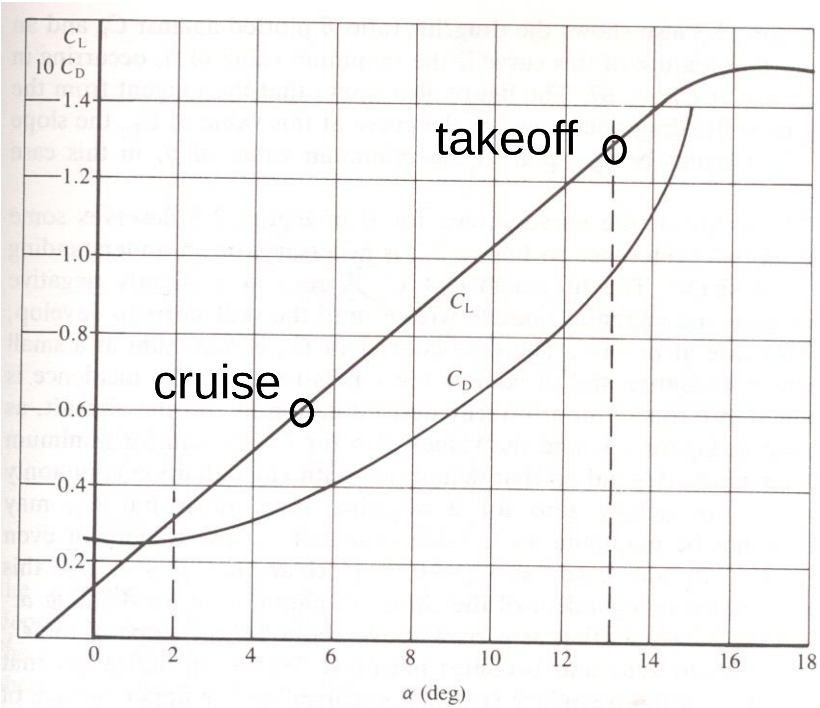
$$C_D = C_{D0} + C_{Dv}$$

$$C_{Dv} = \frac{KC_L^2}{\pi AR}$$

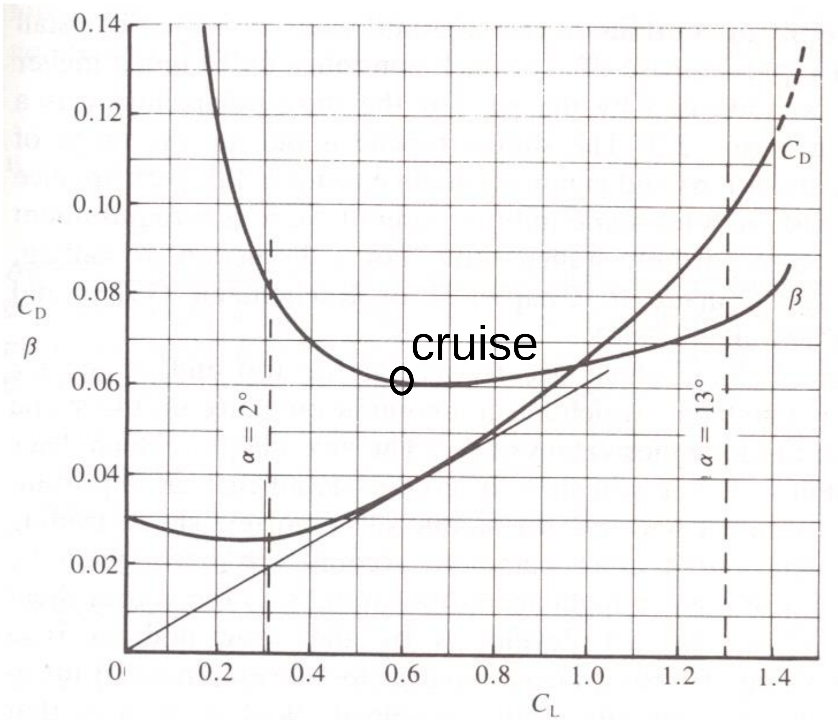


Drag breakdown of a transonic aircraft at cruise

Aircraft Lift and Drag Coefficients

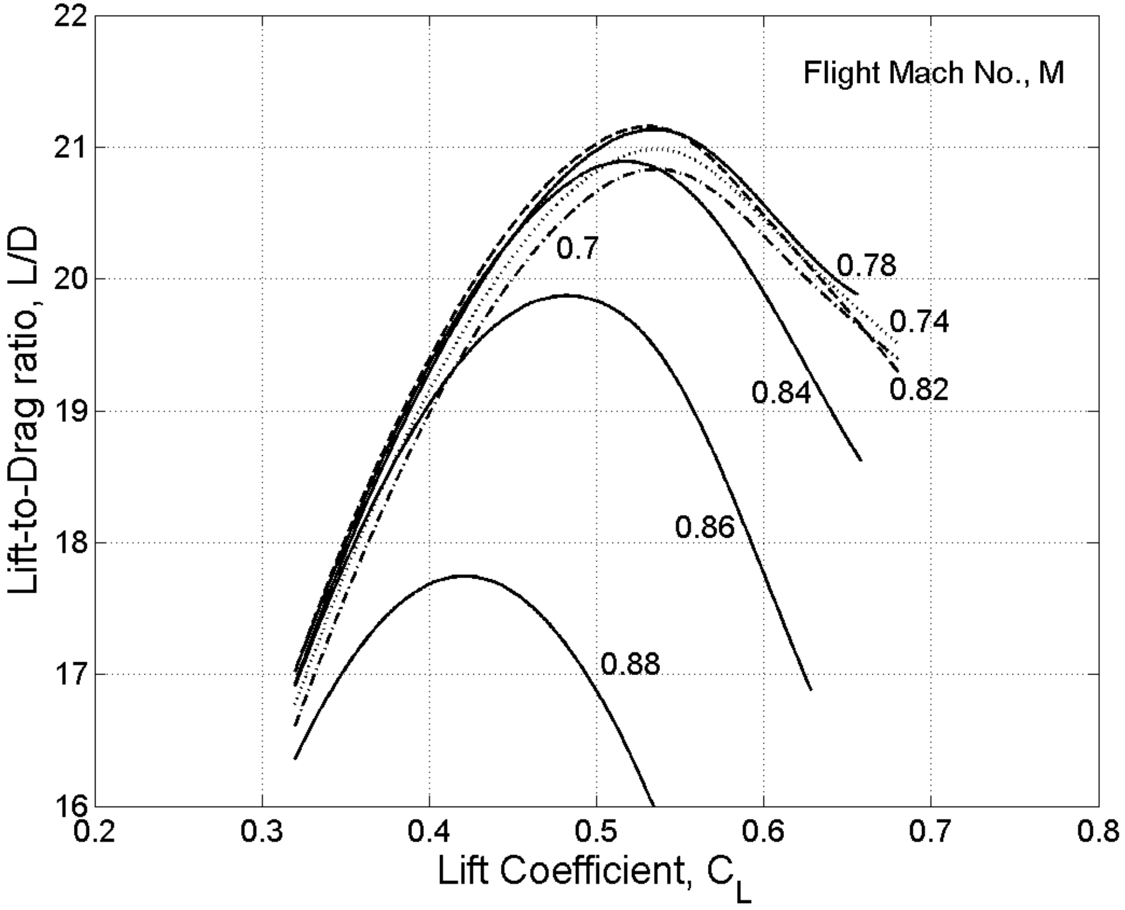


Typical C_L & C_D vs angle of attack



Corresponding variation of $\beta = 1/(L/D)$ with C_L

Modern Aircraft L/D variation



Variation of L/D with C_L and M for the Boeing 787-8 aircraft at 35000 ft

- Note the *transonic drag onset* once M exceeds ~ 0.85.

Parabolic drag model

- A simple model for aircraft drag is:

$$C_D = K_1 + K_2 C_L^2$$

- For this, L/D can be written as:

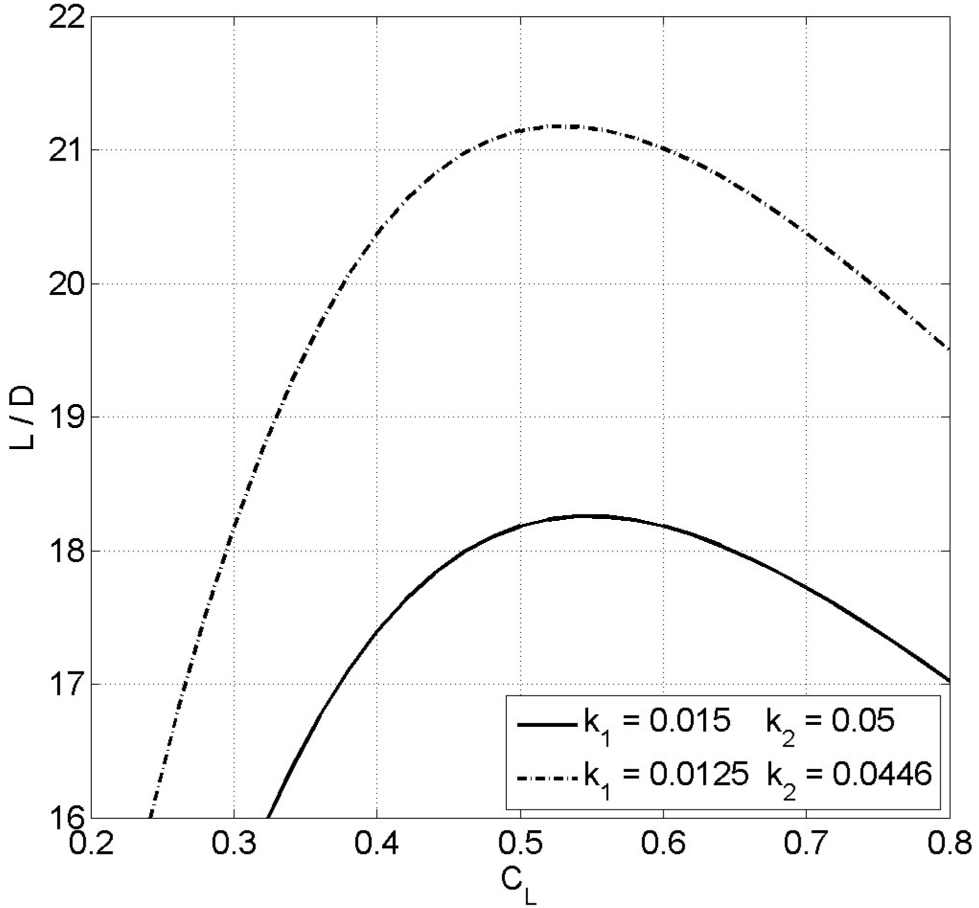
$$\frac{C_L}{C_D} = \frac{C_L}{K_1 + K_2 C_L^2} \qquad \frac{1}{L/D} = \beta = \frac{K_1}{C_L} + K_2 C_L$$

- The maximum L/D occurs at:

$$\frac{\partial \beta}{\partial C_L} = 0\,, \quad C_L^* = \sqrt{\frac{K_1}{K_2}}\,, \quad \left(\frac{L}{D}\right)_{\max} = \frac{1}{2\sqrt{K_1 K_2}}\,, \quad \beta^* = 2\sqrt{K_1 K_2}$$

- This gives the lift coefficient for minimum cruise thrust requirement.
- Note that this gives no account of transonic wave drag.

Parabolic drag model (2)



Parabolic L/D variations for 2 sets of constants

Equivalent Air Speed (EAS)

- Aircraft are controlled using Equivalent Air Speed (EAS), V_e, which is related to True Air Speed (TAS), V via the lift coefficient:

$$C_L = \frac{W}{0.5\rho V^2 S} = \frac{W}{0.5\rho_0 V_e^2 S}$$

- Hence EAS and TAS are given by:

$$\rho V^2 = \rho_0 V_e^2 \,, \quad \sigma = \rho / \rho_0 \quad \text{and} \quad V_e = \sigma^{0.5} V$$

- The optimum value of EAS for maximum L/D occurs when:

$$C_L = C_L^* = \sqrt{K_1/K_2} \qquad \therefore V_e^* = \left(\frac{W}{0.5\rho_0 S}\right)^{0.5} \left(\frac{K_2}{K_1}\right)^{0.25}$$

Constant Speed Ratio Flight

- The lift coefficient and Mach number of an aircraft during flight is controlled via the <u>speed ratio</u>, *v:*

$$v = V_e / V_e^*$$

- The inverse L/D equation can be re-written as:

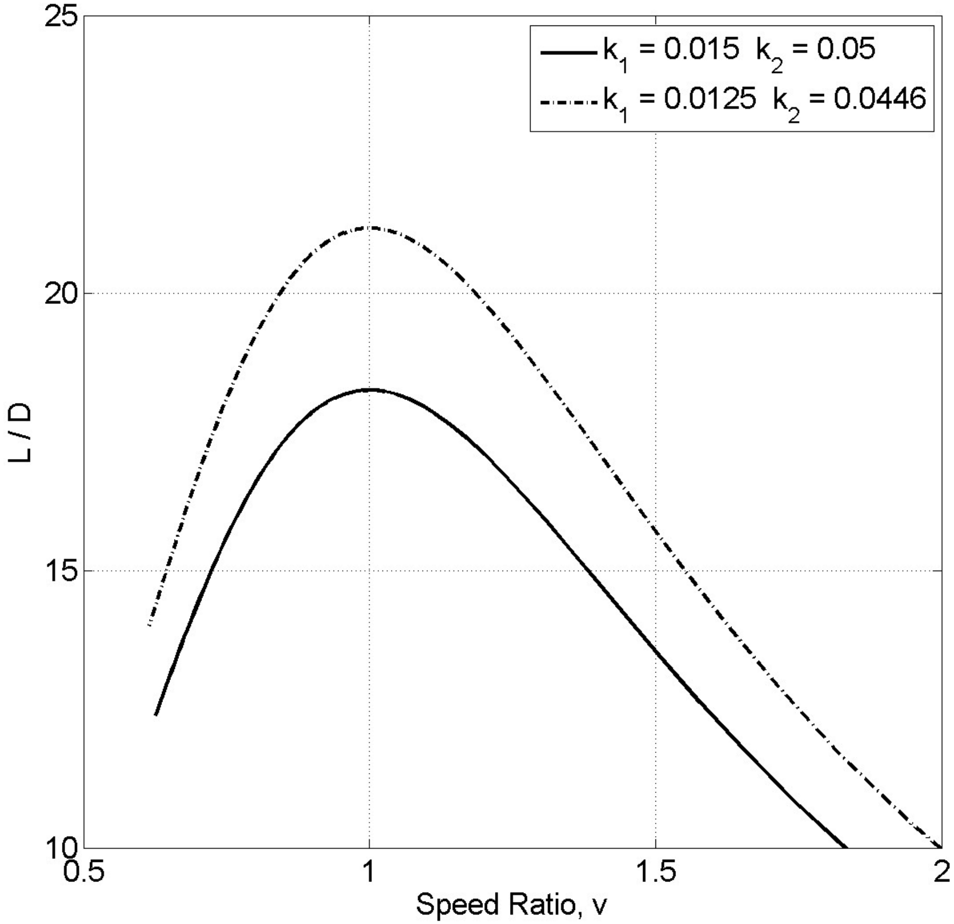
$$\frac{1}{L/D} = \beta = \frac{K_1}{C_L} + K_2 C_L = \frac{0.5\rho_0 V_e^{*2} S K_1}{W} v^2 + \frac{K_2 W}{0.5\rho_0 V_e^{*2} S} \frac{1}{v^2}$$

- This can be recast as:

$$\beta = \frac{1}{2}\beta^* \left(v^2 + 1/v^2 \right)$$

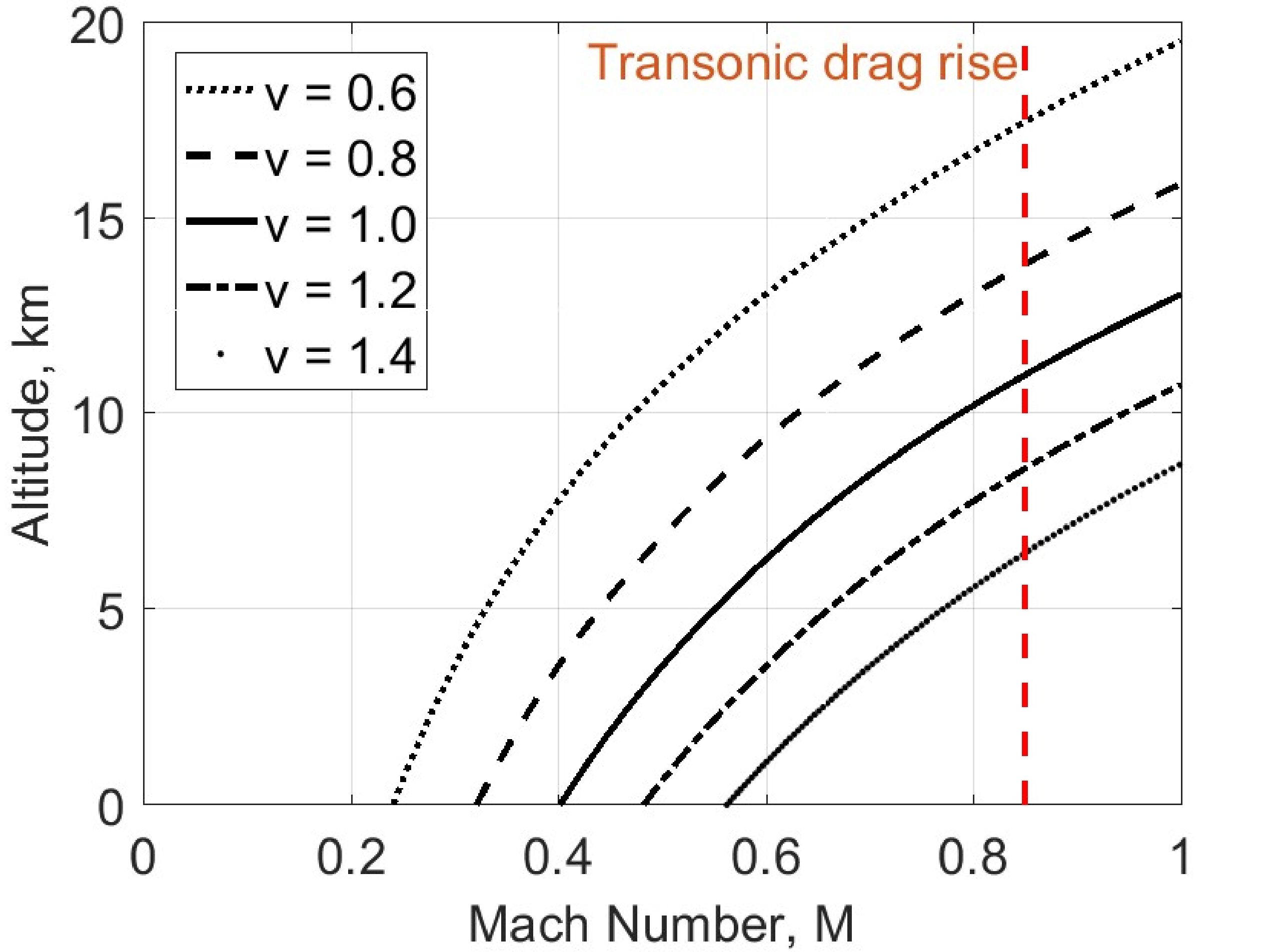
- During flight, aircraft weight reduces and to maintain lift coefficient the plane climbs, reducing the ambient density.
- This "cruise climb" takes place at constant *v*
- Note that Mach number increases with altitude at constant *v*, so that transonic drag may become significant during cruise climb.

Constant Speed Ratio Flight (2)



Relation between L/D and speed ratio, v

Variation of Mach Number with Altitude



Relation between Altitude and Mach Number for constant *v*

3. Modelling the Engine

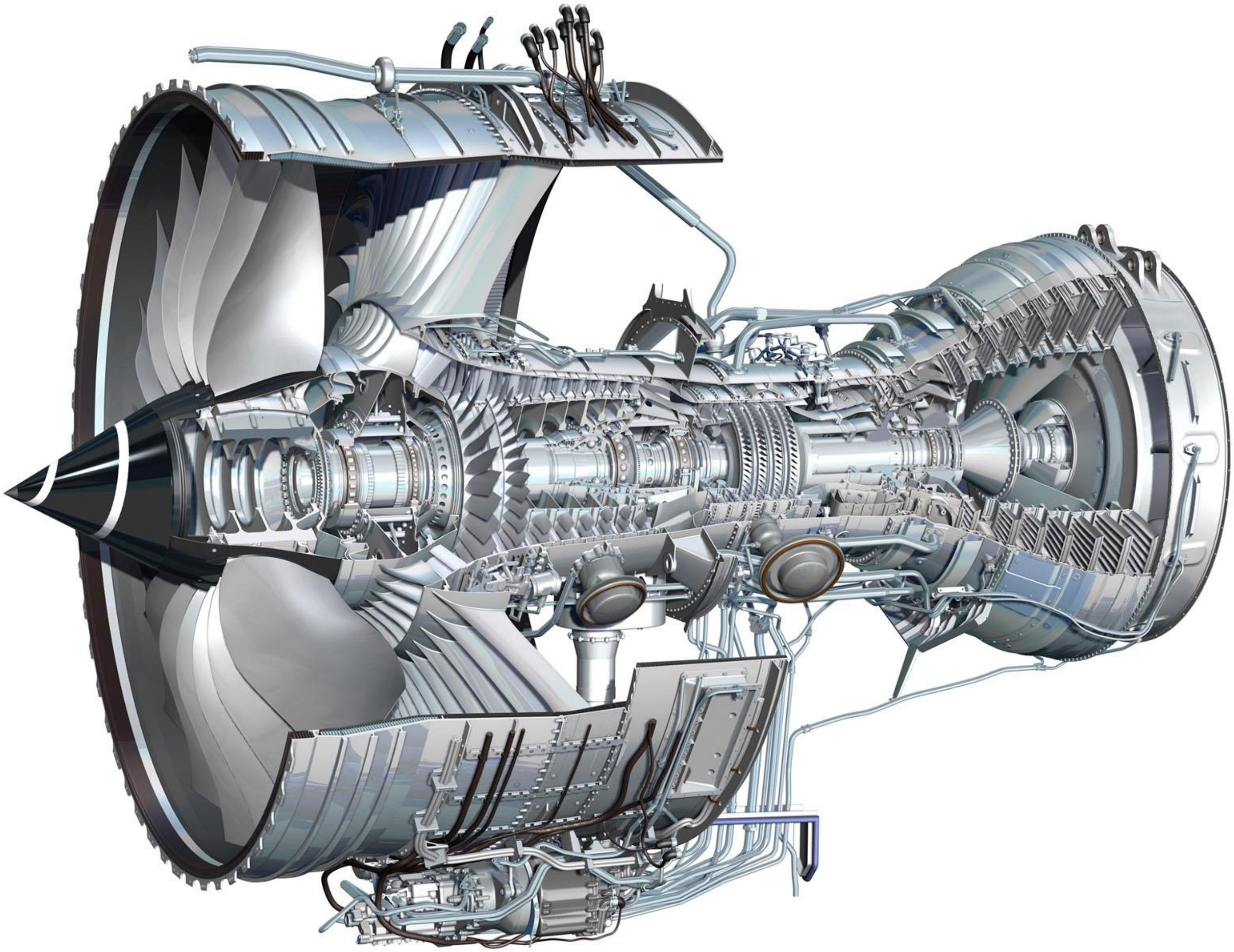
Objectives

i. Determine models for the engine propulsive and thermal efficiency based on gas turbine thermodynamics

ii. Understand the practical challenges and thermodynamic limits on the engine efficiency measures.

iii. Calculate NOx emissions based on the engine cycle

Modern High Bypass Ratio Civil Engine



Trent 1000 (BPR=11, fan tip diameter 2.85 m) www.rolls-royce.com

Thermal and Propulsive Efficiencies

- The engine thermal efficiency is

$$\eta_{th} = \frac{Increase\ in\ jet\ kinetic\ energy}{Thermal\ energy} = \frac{\dot{m}_{air}\left(V_j^2 - V^2\right)/2}{\dot{m}_{fuel} LCV}$$

- The engine propulsive efficiency is

$$\eta_{prop} = \frac{Power\ propelling\ aircraft}{Increase\ in\ jet\ kinetic\ energy} = \frac{VF_N}{\dot{m}_{air}\left(V_j^2 - V^2\right)/2} = \frac{V\dot{m}_{air}\left(V_j - V\right)}{\dot{m}_{air}\left(V_j^2 - V^2\right)/2}$$

- This simplifies to

$$\eta_{prop} = \frac{2V}{V_j + V}$$

Specific Fuel Consumption and Overall Efficency

- The engine overall efficiency is

$$\eta_0 = \frac{Power\ propelling\ aircraft}{Thermal\ energy} = \frac{VF_N}{\dot{m}_{fuel} LCV}$$

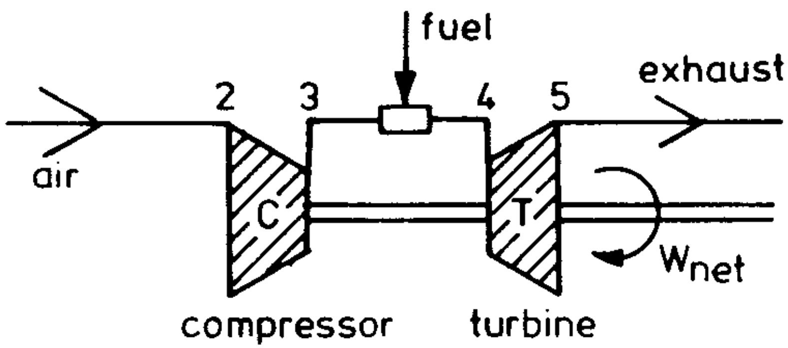
$$\eta_0 = \eta_{prop} \times \eta_{th} = \frac{VF_N}{\Delta KE} \times \frac{\Delta KE}{\dot{m}_{fuel} LCV} = \eta_{prop} \times \eta_{cycle} \times \eta_{tr}$$

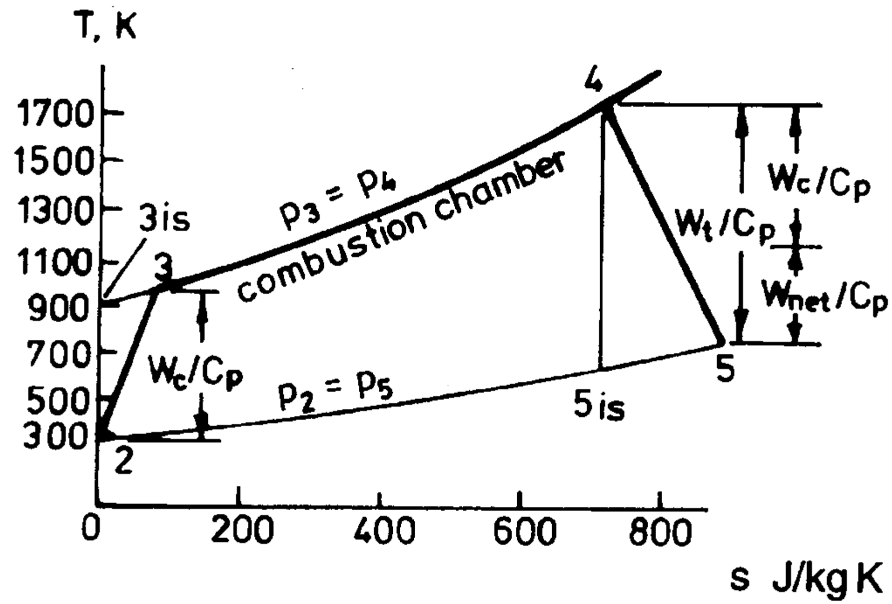
Where the transfer efficiency, η_{tr} , is how much of the core work output is transferred to kinetic energy of the jet. It is typically ≈0.90 for a modern turbofan

- The engine specific fuel consumption is then:

$$sfc = \frac{\dot{m}_{fuel}}{F_N} = \frac{\dot{m}_{fuel} LCV}{VF_N} \times \frac{V}{LCV} = \frac{1}{\eta_0} \frac{V}{LCV}$$

Simple Gas Turbine (Core) Cycle





Combustion of fuel modelled as a heat input. Constant C_p and γ assumed.

Heat Input: $\dot{Q} = \dot{m}C_p\left(T_{04} - T_{03}\right)$

Turbine Work: $\dot{W}_t = \dot{m}C_p\left(T_{04} - T_{05}\right)$

Compressor Work: $\dot{W}_c = \dot{m}C_p\left(T_{03} - T_{02}\right)$

Calculation of performance parameters

Net Work Output: $\dot{W}_{net} = \dot{W}_t - \dot{W}_c = \dot{m}C_p\left[\left(T_{04} - T_{05}\right) - \left(T_{03} - T_{02}\right)\right]$

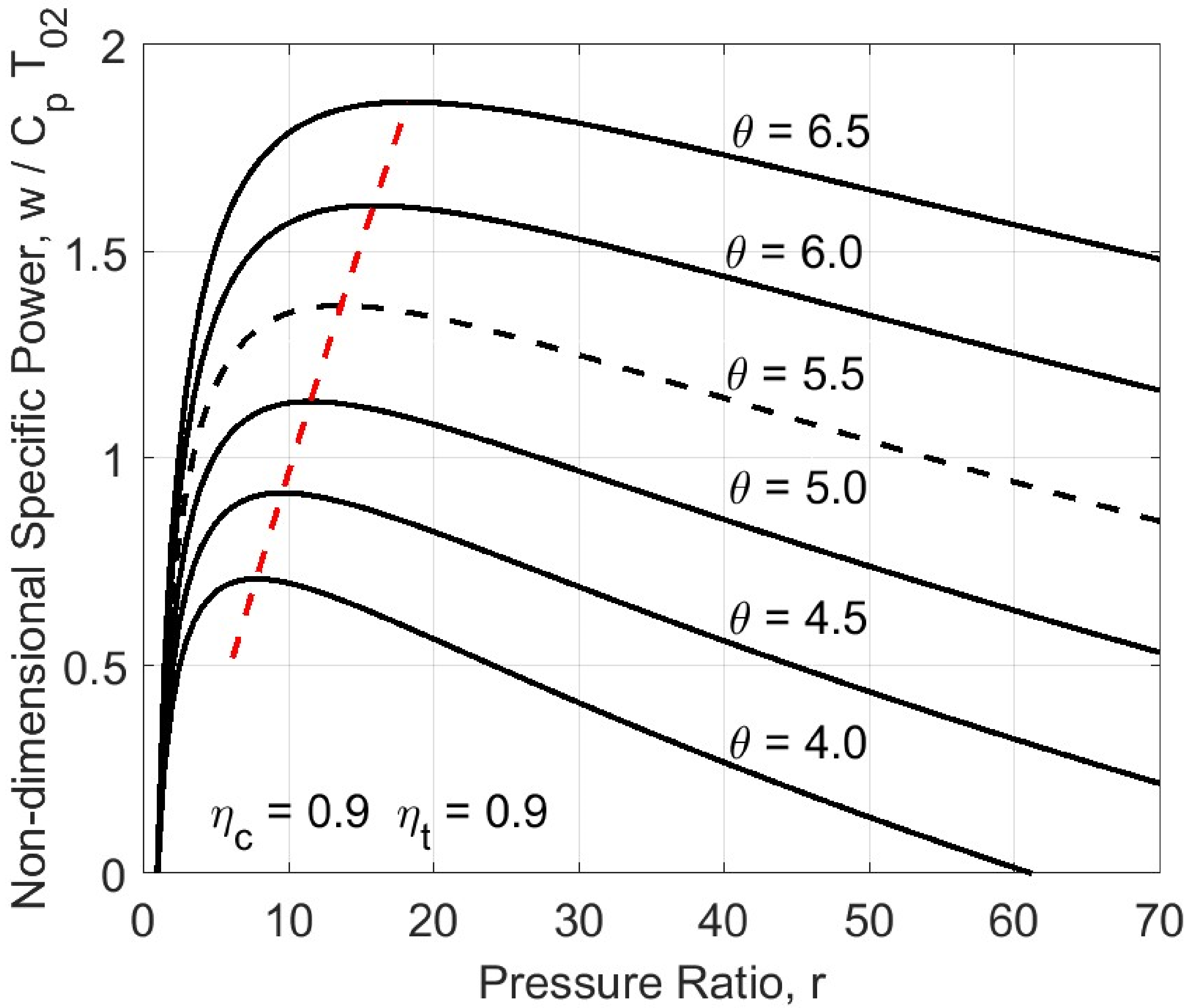
$$\Rightarrow \dot{W}_{net} = \dot{m}C_p T_{02}\left[\theta\left(1 - 1/r^{(\gamma-1)/\gamma}\right)\eta_t - \left(r^{(\gamma-1)/\gamma} - 1\right)/\eta_c\right]$$

where, $r = p_{03}/p_{02}$, $\theta = T_{04}/T_{02}$

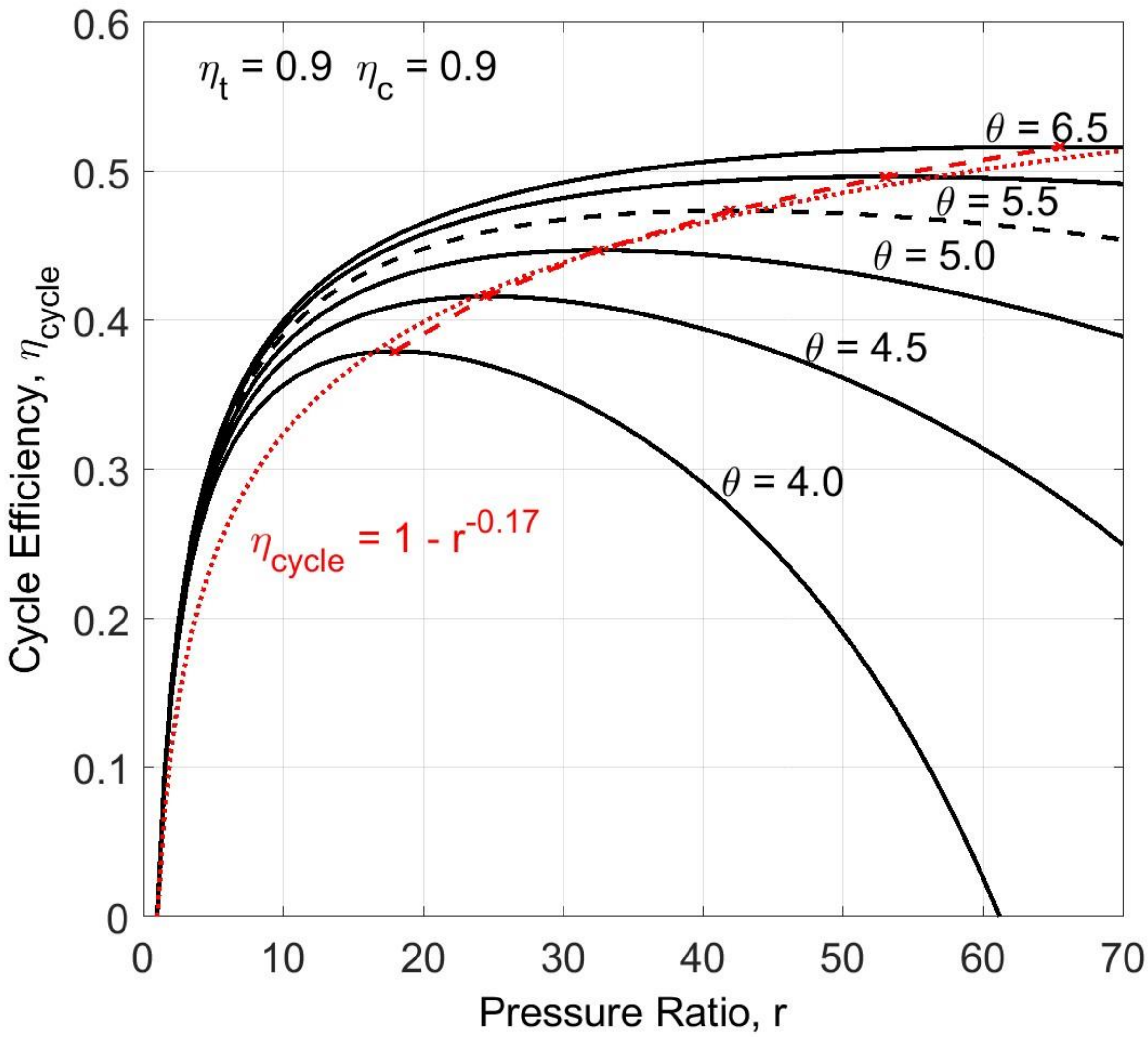
Cycle efficiency: $\eta_{cycle} = \dfrac{\dot{W}_{net}}{\dot{Q}} = \dfrac{\dot{m}C_p\left[\left(T_{04} - T_{05}\right) - \left(T_{03} - T_{02}\right)\right]}{\dot{m}C_p\left(T_{04} - T_{03}\right)}$

$$\Rightarrow \eta_{cycle} = \frac{\theta\left(1 - 1/r^{(\gamma-1)/\gamma}\right)\eta_t - \left(r^{(\gamma-1)/\gamma} - 1\right)/\eta_c}{\theta - 1 - \left(r^{(\gamma-1)/\gamma} - 1\right)/\eta_c}$$

Variation of engine work output



Variation of core cycle efficiency



Limits on core cycle efficiency

- Maximum turbine entry temperature during cruise limited by turbine blade life and cooling considerations.

$$\therefore \theta \leq 6$$

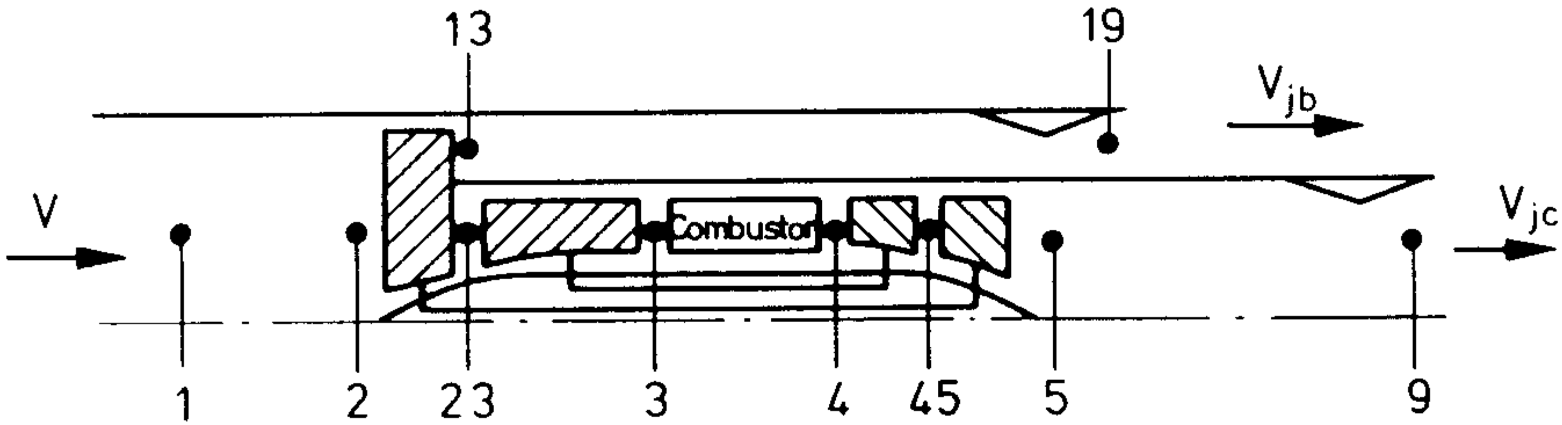
- Maximum compressor exit temperature limited by material limits and number of compressor stages required.

$$\therefore r \leq 50$$

- Compressor and Turbine component efficiencies limited by boundary layers, tip clearances, 3D flows, leakages, cooling, etc.

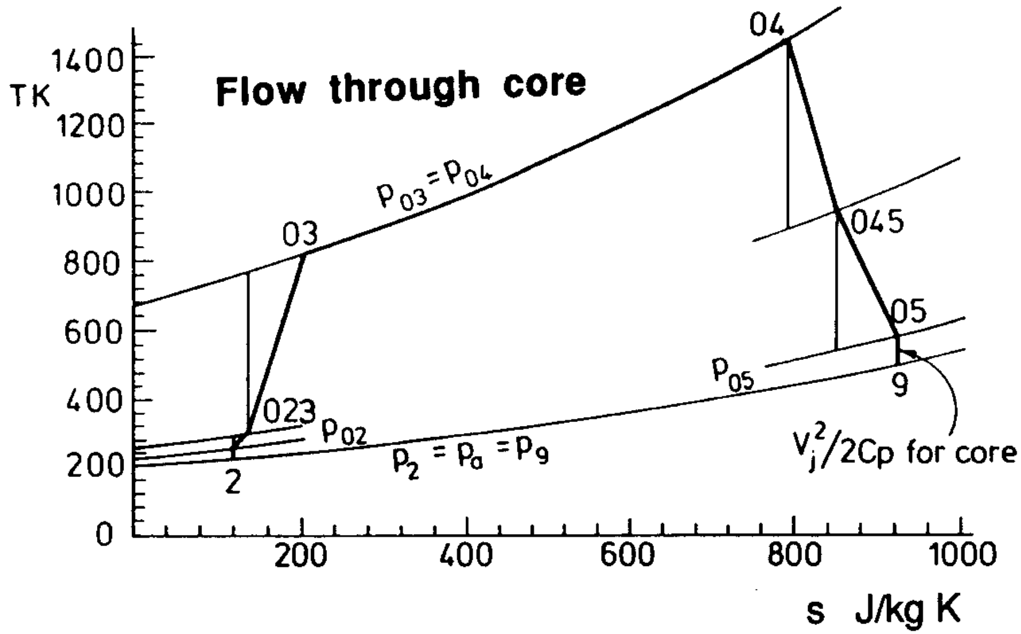
$$\therefore \eta_c , \eta_t \leq 0.92$$

Consider the core within a turbofan engine



- Core thermodynamics are the same for a turbofan as the simple gas turbine
- Assume uniform jet velocity

$$V_j = V_{jb} = V_{jc}$$

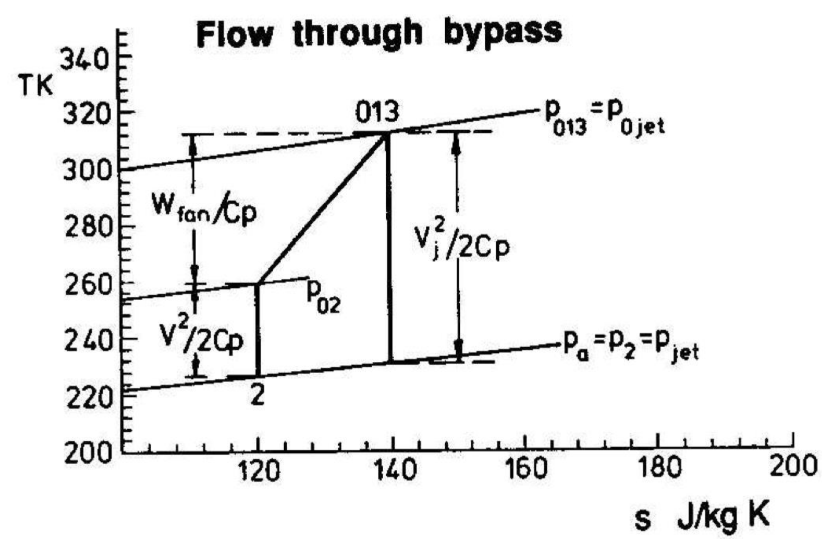


Calculation of Propulsive Efficiency

- Fan Pressure Ratio (FPR), not bypass ratio, determines propulsive efficiency

$$FPR = \frac{p_{013}}{p_{02}} = \frac{p_{013}/p_a}{p_{02}/p_a} = \left[\frac{1+0.5(\gamma-1)M_j^2}{1+0.5(\gamma-1)M^2}\right]^{\frac{\gamma}{\gamma-}}$$

$$\therefore M_j^2 = \frac{2}{\gamma-1}\left[\left(FPR\times p_{02}/p_a\right)^{(\gamma-1)/\gamma} - 1\right.$$

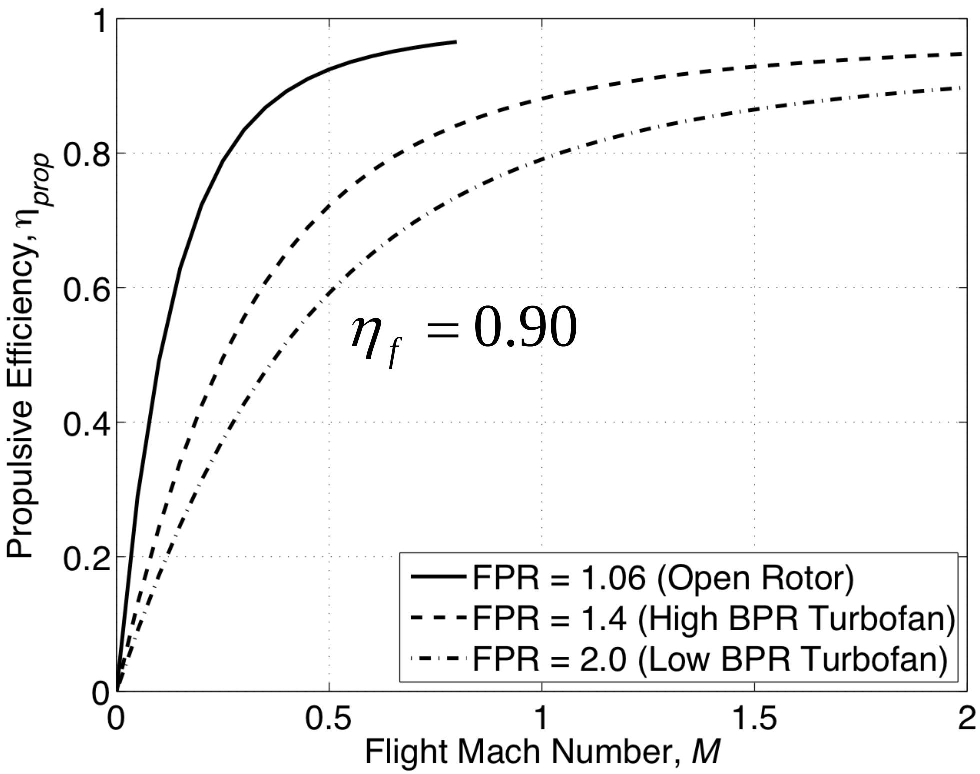


$$\frac{T_j}{T_a} = \frac{T_{02}/T_a \times T_{013}/T_{02}}{T_{013}/T_j} = \frac{1+0.5(\gamma-1)M^2}{1+0.5(\gamma-1)M_j^2}FPR^{\frac{}{\gamma\eta_f}}$$

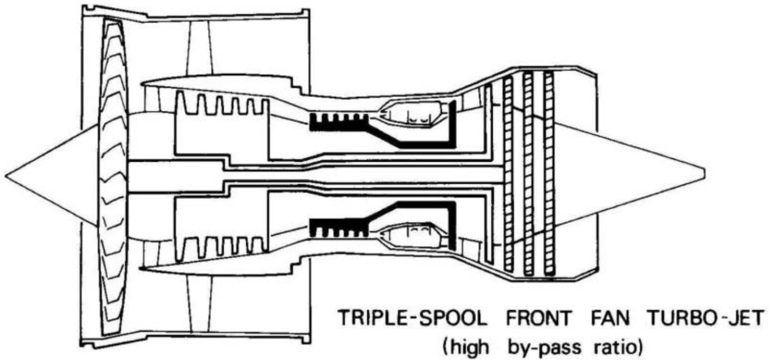
- Propulsive efficiency,

$$\eta_{prop} = \frac{2V}{V+V_j} = \frac{2}{1+V_j/V} = \underline{2\left(1+\frac{M_j}{M}\sqrt{\frac{T_j}{T}}\right)^{-1}}$$

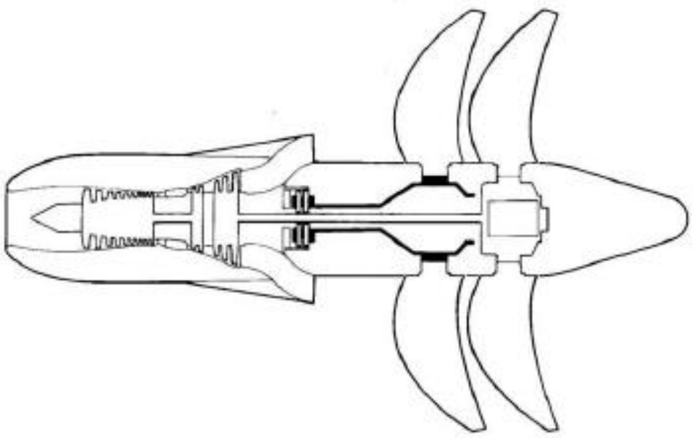
Effect of FPR and M on Propulsive Efficiency



Relation between propulsive efficiency, FPR and Flight Mach Number



Turbofan



Open Rotor

Limits on propulsive efficiency

- Decreasing the engine FPR increases the engine diameter, increasing the engine weight and nacelle drag:

$$w_{eng} \propto d^{2.4}$$

- As FPR decreases, a turbofan engine fan runs into operability problems (rotating stall), unless a variable area nozzle or variable pitch is used. For a conventional turbofan,

$$FPR \geq 1.35$$

- The fan efficiency is limited by the best aerodynamic design and mechanical constraints (birdstrike, flutter, structural requirements, etc)

$$\therefore \eta_f \leq 0.94$$

Determining NOx Emission

- Various oxides of nitrogen form in the combustion chamber - named NOx as a family

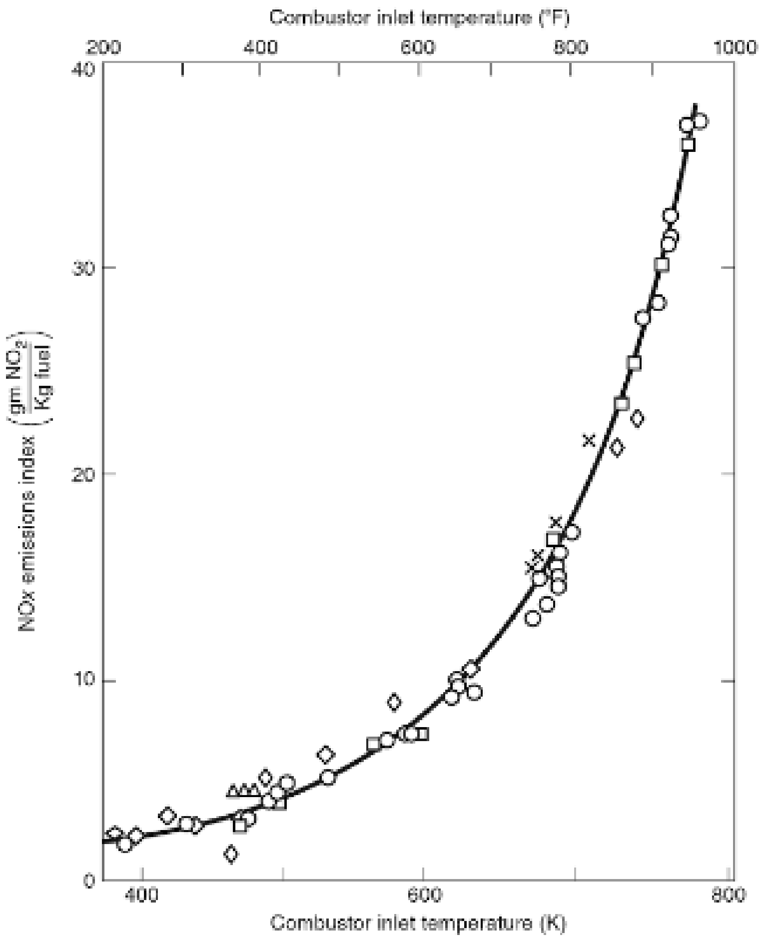
- Basic correlation given by:

$$EI_{NO_X} = 0.011445 e^{0.00676593 \times T_{03}}$$

in g NOx / kg air

where T_{03} is the compressor exit temperature (K)

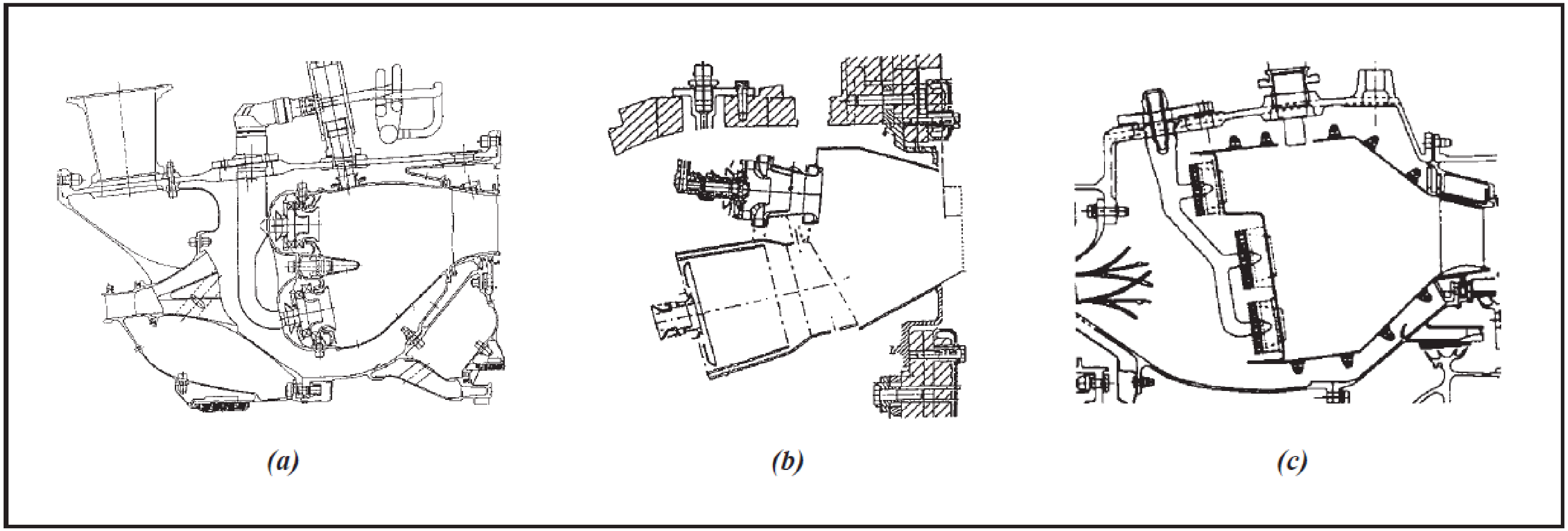
$$\frac{T_{03}}{T_{02}} = 1 + \left(r^{(\gamma-1)/\gamma} - 1 \right) \Big/ \eta_c$$



Correlation of NOx with T03 (Lipfert [1972])

Technology to reduce NOx

- Latest combustors aim to (i) reduce mean temperatures
  (ii) eliminate spatial non-uniformities
- Modern designs use staged devices with lean pre-mixed burn (up to 2× stoichiometric, which with a bypass ratio of 10, implies an overall engine air-fuel ratio of ~300)



Staged combustors: (a) General Electric, (b) Snecma, (c) Pratt and Whitney, from Green [2002]

4. Fuel Burn and the Breguet Range Equation

Objectives

i. Review the basic Breguet Range Equation

ii. Show how the Range Equation is modified by aircraft takeoff, climb and descent operations.

iii. Consider how the airframe aerodynamics, engine efficiency and aircraft empty weight feed into the Fuel Burn

iv. Relate fuel consumption to pollutant emissions

The Breguet Range Equation

- Start from the definition of Specific Fuel Consumption:

$$sfc = \frac{Fuel\ mass\ \ flow}{Net\ Thrust} = \frac{\dot{m}_{fuel}}{F_N}$$

- As fuel is burnt to provide thrust the aircraft becomes lighter and less lift is needed. Thus less drag is produced so less thrust is required and fuel is burnt more slowly:

$$\frac{\mathrm{dw}}{\mathrm{dt}} = -\mathrm{g}\dot{\mathrm{m}}_\mathrm{f} = -\mathrm{g\,sfc\,F_N} = -\mathrm{g\,sfc} \times \mathrm{drag} = -\mathrm{g\,sfc} \times \frac{\mathrm{lift}}{\mathrm{L/D}} = -\frac{\mathrm{g\,sfc}}{\mathrm{L/D}} \times \mathrm{w}$$

$$\frac{\mathrm{dw}}{\mathrm{w}} = -\frac{\mathrm{g\,sfc}}{\mathrm{L/D}} \times \mathrm{dt} = -\frac{\mathrm{g\,sfc}}{\mathrm{L/D}} \times \frac{\mathrm{ds}}{\mathrm{V}}$$

$$[\ln \mathrm{w}]_{\mathrm{start}}^{\mathrm{end}} = -\frac{\mathrm{g\,sfc}}{\mathrm{V\,L/D}} \times [\mathrm{s}]$$

$$\text{Distance travelled, s } = \frac{\mathrm{V\,L/D}}{\mathrm{g\,sfc}} \times \ln\left(\frac{\mathrm{w_{start}}}{\mathrm{w_{end}}}\right)$$

Total Fuel Burn Fraction

Distance travelled, s $= \dfrac{V\,L/D}{g\,sfc} \times \ln\left(\dfrac{w_{start}}{w_{end}}\right)$

Re-writing the above in terms of fuel and takeoff weight and *H*:

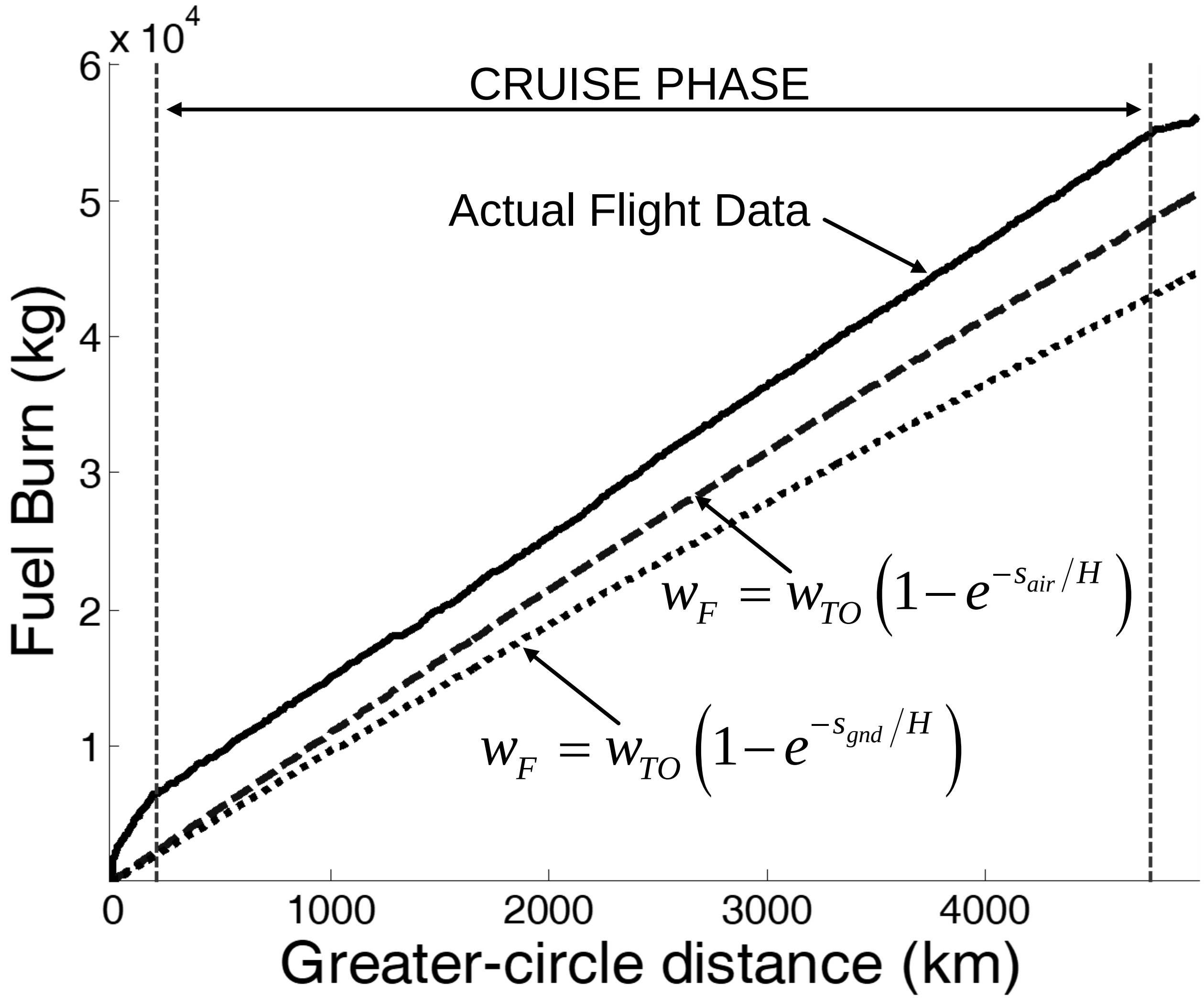
$$s = H \ln\left(\frac{w_{TO}}{w_{TO} - w_F}\right)$$

, where the Range Parameter, $H = \dfrac{V\,L/D}{g\,\mathrm{sfc}} = \dfrac{LCV\eta_o\,L/D}{g}$

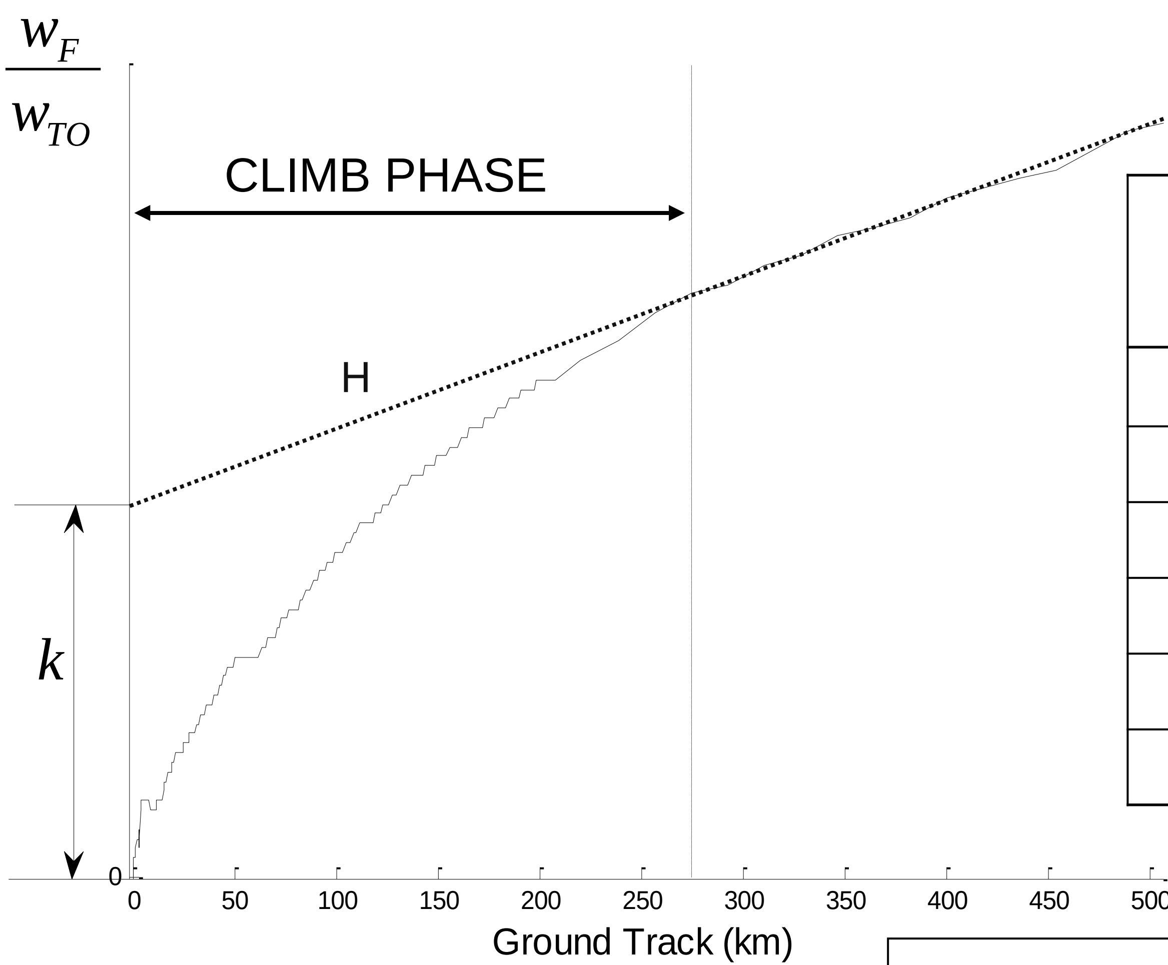
$$\boxed{\therefore \frac{w_F}{w_{TO}} = 1 - e^{\frac{-s}{H}}}$$

This gives the total fuel fraction based on the Breguet Range Equation

Measured Aircraft Fuel Burn vs Breguet Eqn.



The effect of Takeoff and Climb

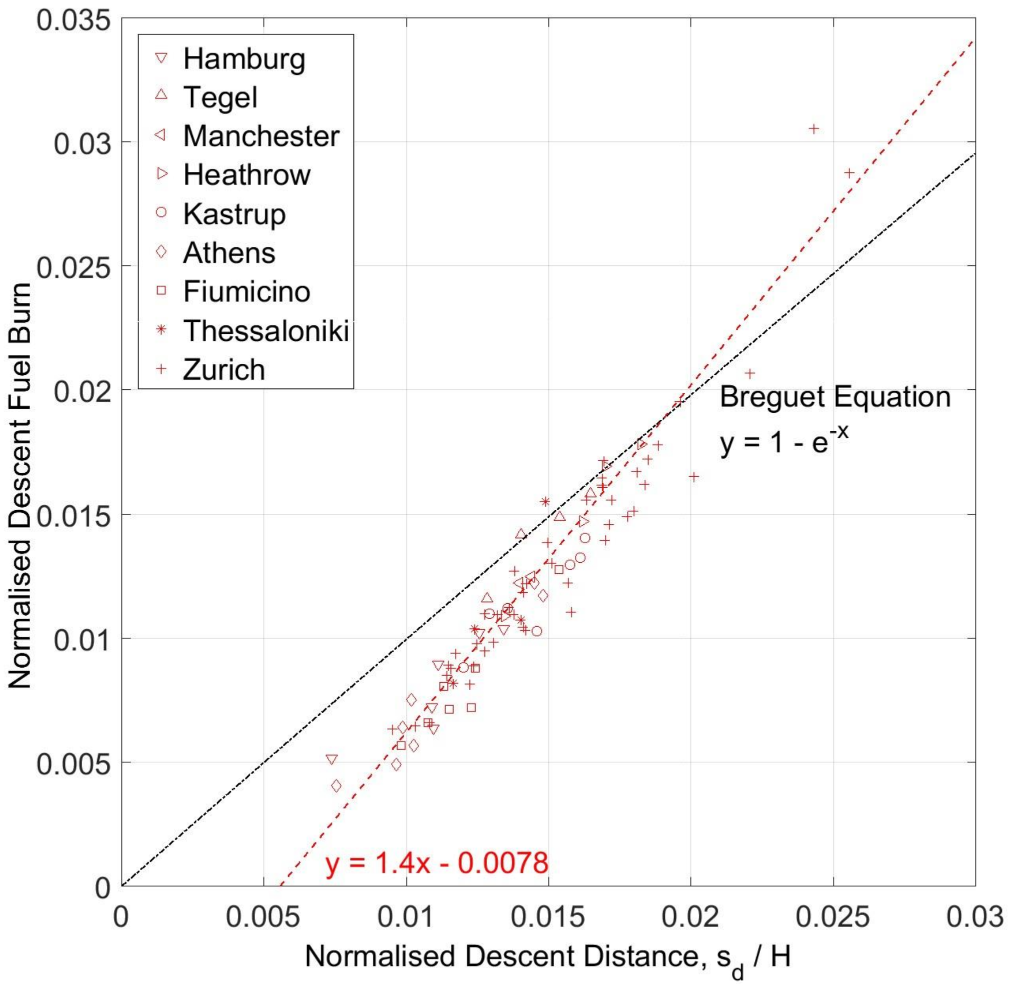


Aircraft	Average k (%)
A320	1.60
A330	1.52
A340	1.55
B757	1.61
B767	1.53
B777	1.31

This gives an adjusted Fuel Burn Equation:

$$\frac{w_F}{w_{TO}} = 1 - e^{\frac{-s}{H}} + k \quad , \text{where } k \cong 0.015$$

The effect of Descent and Approach



- Overall, recovery of GPE roughly balanced by Descent Operational Inefficiency

Fuel Burn per payload-range

- Fuel burn per payload-range is a measure of transport effectiveness that can be converted to pollution emission per passenger–km and usefully compared.

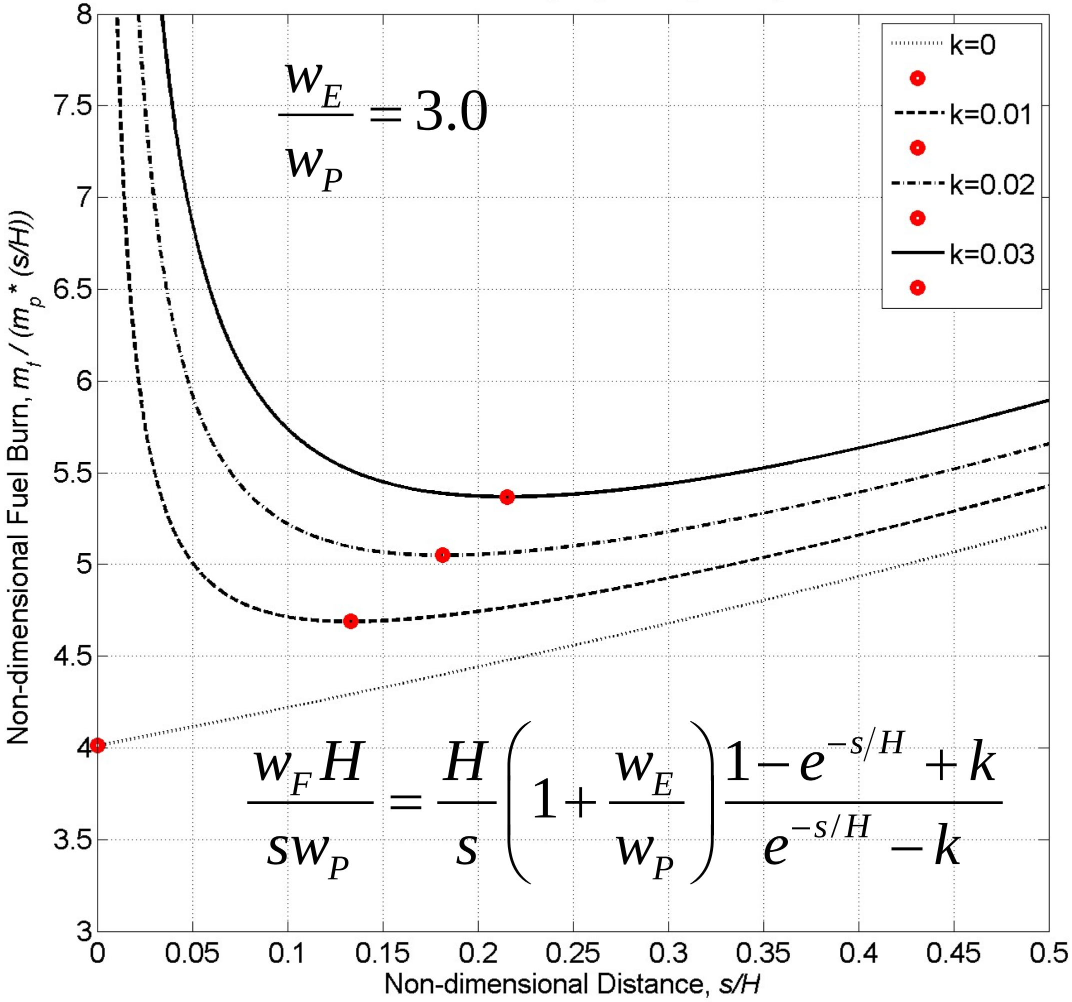
$$\frac{w_F}{w_{TO}} = 1 - e^{\frac{-s}{H}} + k \quad \text{, where } k \cong 0.015$$

$$\frac{w_F}{s w_P} = \frac{w_F}{s w_{TO}}\left(\frac{w_{TO}}{w_P}\right) = \frac{w_F}{s w_{TO}}\left(\frac{w_E + w_P + w_F}{w_P}\right)$$

$$\Rightarrow \frac{w_F}{s w_P}\left(1 - \frac{w_F}{w_{TO}}\right) = \frac{w_F}{s w_{TO}}\left(1 + \frac{w_E}{w_P}\right)$$

$$\therefore \frac{w_F}{s w_P} = \frac{w_F}{s w_{TO}}\left(1 + \frac{w_E}{w_P}\right) \Big/ \left(1 - \frac{w_F}{w_{TO}}\right) = \underline{\frac{1}{s}\left(1 + \frac{w_E}{w_P}\right)\frac{1 - e^{-s/H} + k}{e^{-s/H} - k}}$$

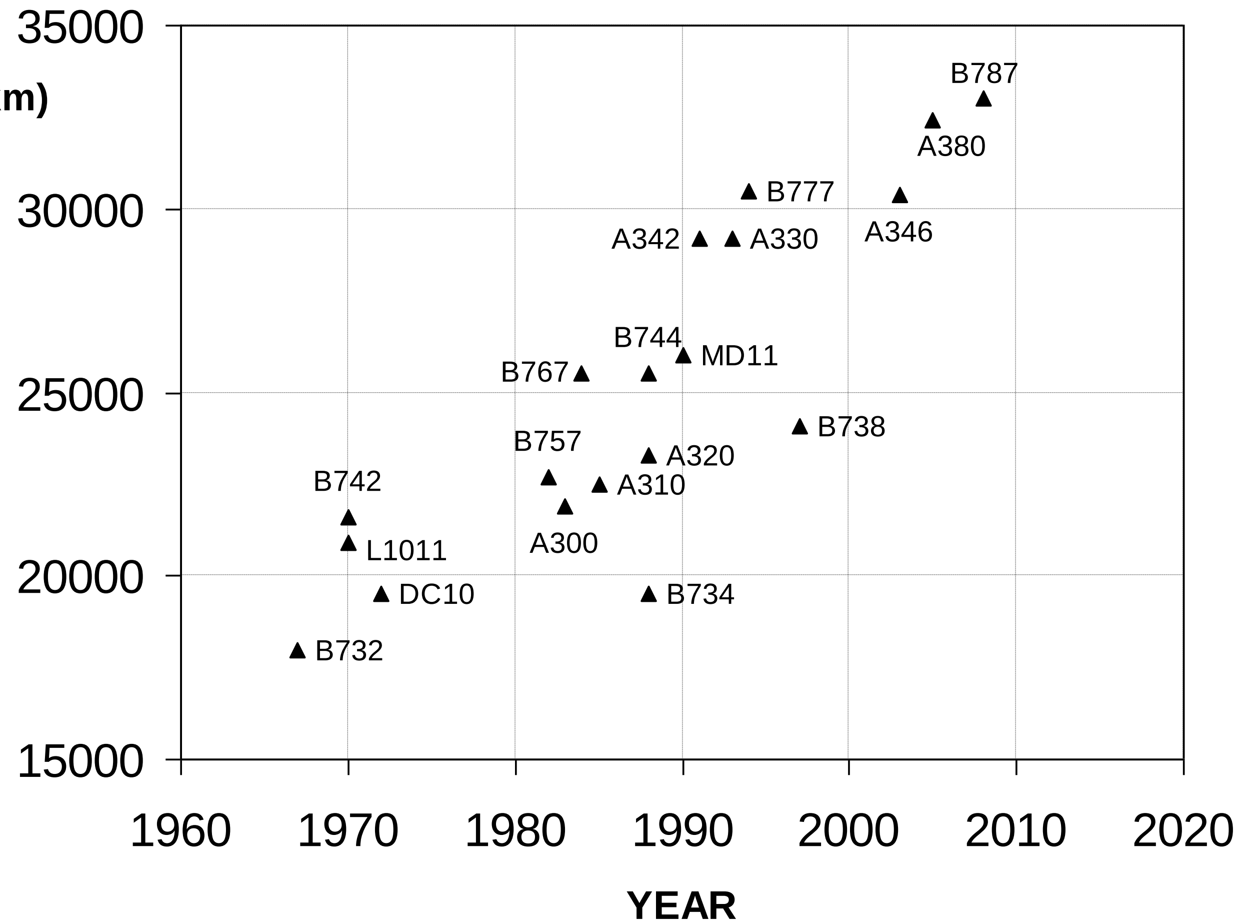
Minimum Fuel Burn per payload-range



- Fuel burn increases with *k* and w_E/w_P, and decreases with *H*
- Optimum range for minimum fuel burn increases with *H* and *k*

Historical Variation of the Range Parameter, *H*

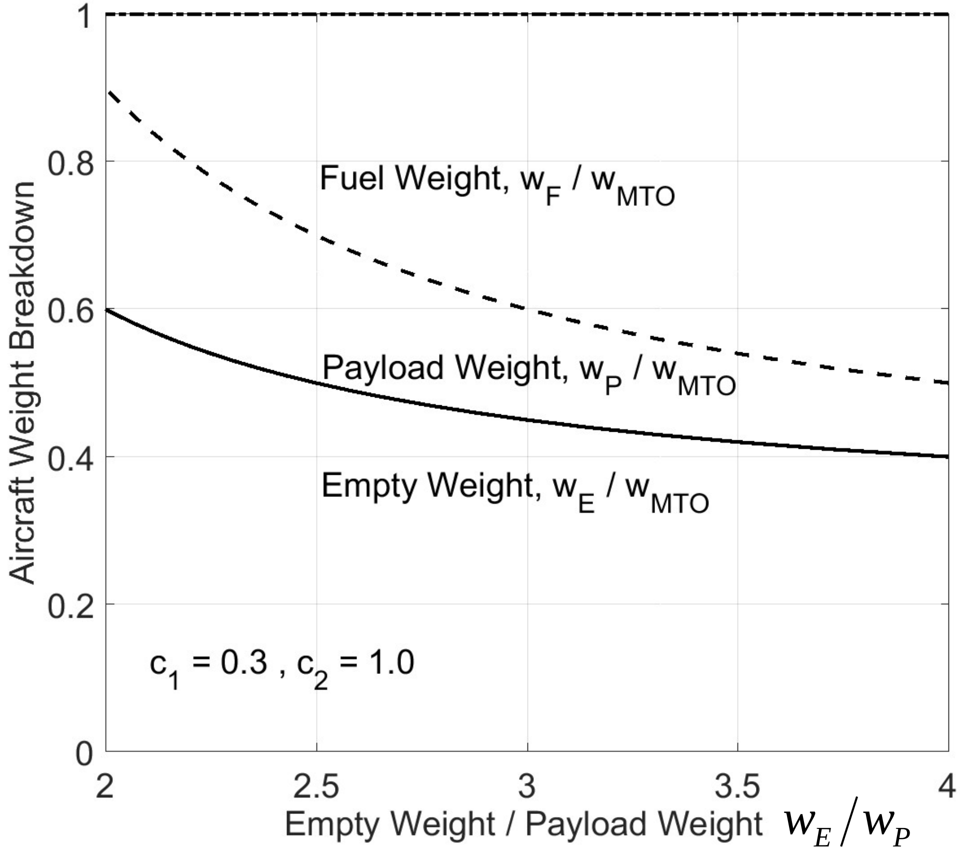
$$H = \frac{\mathrm{V\,L/D}}{g\ \mathrm{sfc}}\ \textbf{(km)}$$



- Calculate *H* using engine efficiencies and *L/D* polar: $H = \frac{LCV}{g}\eta_{prop}\eta_{cycle}\eta_{tr}\ \mathrm{L/D}$

Variation of w_e / w_p

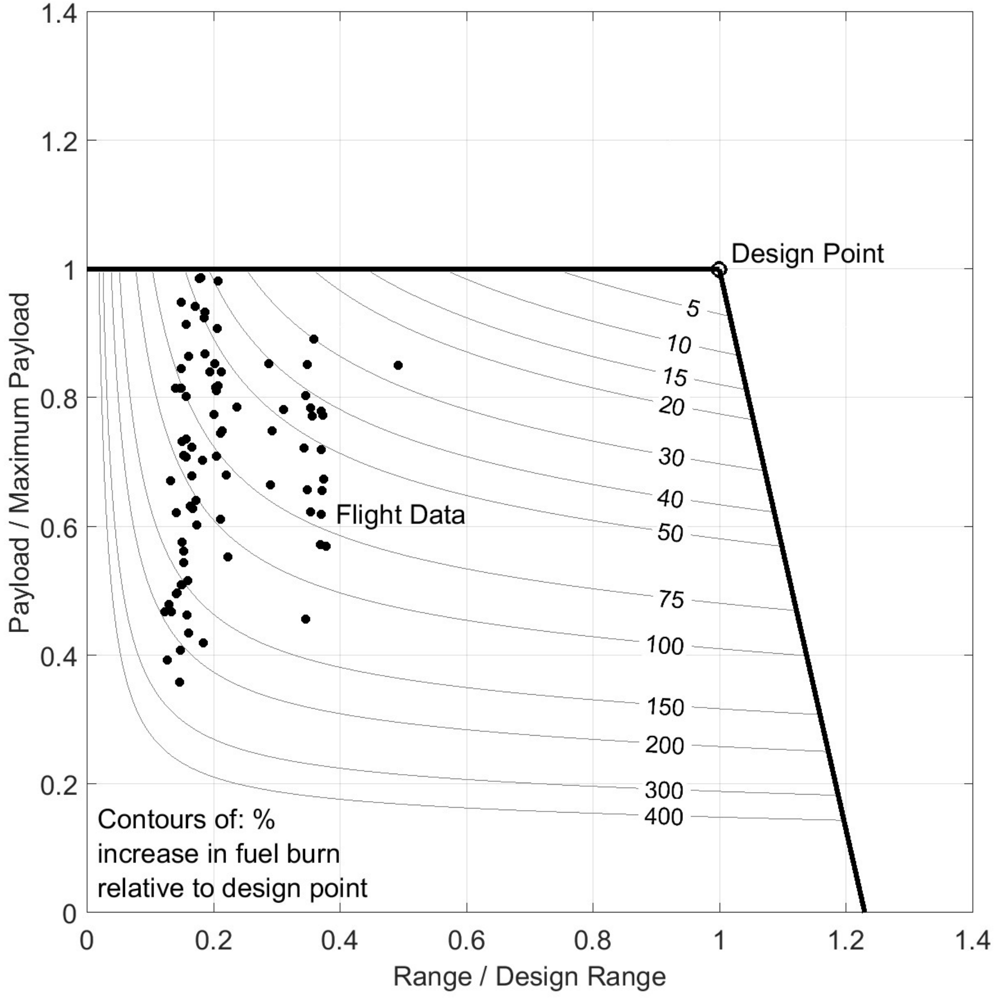
- Reducing w_E/w_P is possible through increased use of lightweight materials and through designing for higher payloads, however empty weight and thus maximum takeoff weight also increase with maxiumum payload.



Variation of weight breakdown for simple weight model

Effect of Flight Mission on Fuel Burn

- Need to fly as close to the design range and payload as possible:



A320 payload-range diagram with contours of fuel burn per pass-km

Effect of Fuel Type

	kerosene	hydrogen	ratio
specific volume (m^3/kg)	0.124 $\times10^{-2}$	1.42 $\times10^{-2}$	11.3
calorific value, LCV (J/kg = m)	42.7 $\times10^{6}$	115.3 $\times10^{6}$	2.7
stoichiometric air (kg air/kg fuel)	15.1	34.2	2.3
"heat content" (J/kg air)	2.83 $\times10^{6}$	3.38 $\times10^{6}$	1.2

- *H* for hydrogen is 2.7 times bigger than for kerosene (for fixed η & *L/D*)
- Unfortunately, the volume ratio for hydrogen is 11.3× bigger – which implies a factor of $\sim 11.3^{2/3} = 5\times$ more wetted area $\Rightarrow$ much more drag
- Hydrogen needs 2.3× more air – implying the engine intake diameter would need to be $\sim 2.3^{1/2} = 1.5\times$ bigger
- However, hydrogen combustion produces zero CO_2 !

Relating fuel burn to pollutant emissions (1)

- For Carbon Dioxide, the Emission Index only depends on the fuel.
- Assuming an "average" chemical formula of C_8H_{18} for kerosene, the stoichiometric combustion equation is:

$$C_8H_{18} + 12.5\left[O_2 + \frac{79}{21}N_2^*\right] \rightarrow 8CO_2 + 9H_2O + 47.02N_2^*$$

- Stoichiometric Air-to-Fuel ratio:

$$\frac{m_{air}}{m_{fuel}} = \frac{12.5\times M_{O_2} + 47.02\times M_{N_2^*}}{8\times M_C + 18\times M_H} = \frac{12.5\times 32 + 47.02\times 28.15}{8\times 12 + 18\times 1} = \underline{15.1}$$

- Emissions Index for CO_2:

$$EI_{CO_2} = \frac{8\times M_{CO_2}}{8\times M_C + 18\times M_H} = \frac{8\times 44}{8\times 12 + 18\times 1} = \underline{3088\ \mathrm{gCO_2/kg\ fuel}}$$

Relating fuel burn to pollutant emissions (2)

Example

- What is the CO_2 and NO_X emission per pass-km for an aircraft with H=25 000 km, m_E = 50 tonne, flying 9000 km with 150 passengers? Assume T_{03} = 850 K and an air/fuel ratio that is 2 x stoichiometric.

- Fuel burn per payload-km (assuming 100 kg per passenger):

$$\frac{m_F}{s m_P} = \frac{1}{s}\left(1+\frac{m_E}{m_P}\right)\frac{1-e^{-s/H}+k}{e^{-s/H}-k} = \frac{1}{9000}\left(1+\frac{50000}{150\times 100}\right)\frac{1-e^{-9/25}+0.015}{e^{-9/25}-0.015}$$

$$\therefore \frac{m_F}{s m_P} = 2.24\times 10^{-4} \text{ kg / kg km}$$

- From previous slide: $EI_{CO_2} = 3088 \text{ gCO}_2\text{/kg fuel}$

- From correlation on slide 47: $EI_{NO_X} = 3.60 \text{ gNO}_X\text{/kg air}$

Relating fuel burn to pollutant emissions (3)

- It is now possible to calculate the emissions per passenger-km:

$$\frac{m_{CO_2}}{sN_P} = \frac{m_F}{sm_P} \times EI_{CO_2} \times m_{pass} = 2.24\times10^{-4} \times 3088 \times 100$$

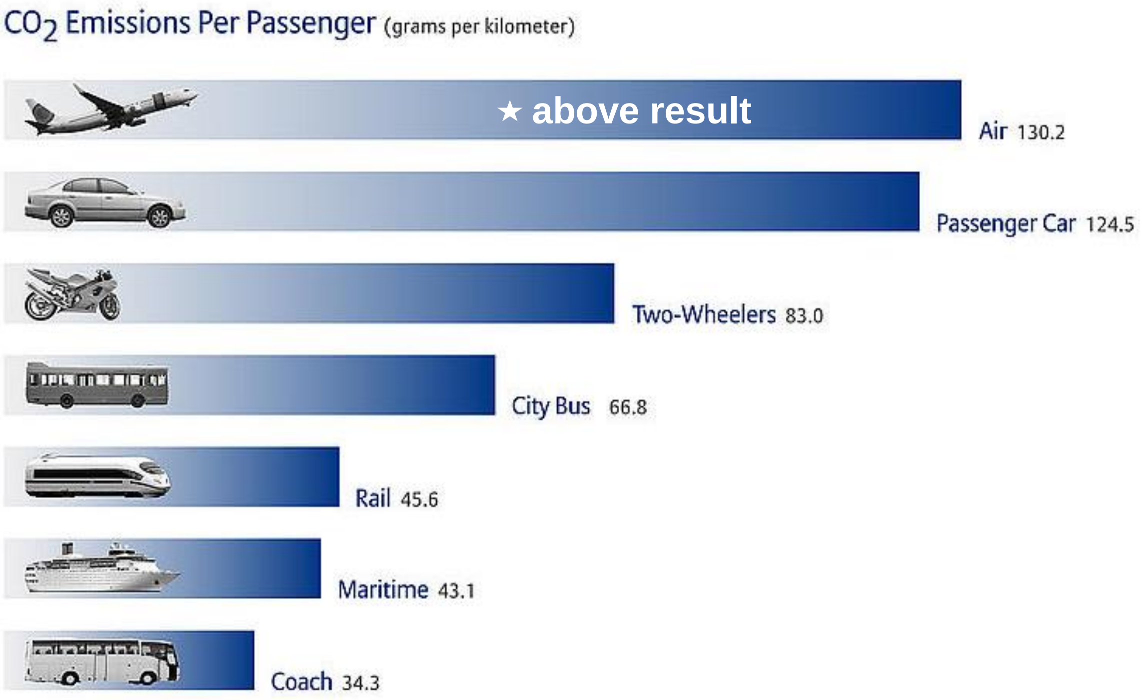
$$= \underline{69\ \mathrm{gCO_2}\ /\ \mathrm{pass\ km}}$$

$$\frac{m_{NO_X}}{sN_P} = EI_{NO_X} \times \frac{m_F}{sm_P} \times \frac{m_{air}}{m_F} \times m_{pass} = 3.6\times 2.24\times10^{-4} \times (15.1\times 2)\times 100$$

$$= \underline{2.44\ \mathrm{gNO}_X\ /\ \mathrm{pass\ km}}$$

- Remember that the “source” of pollution for NO_x is the air that flows through the combustor.

Comparison with other transport



Average CO_2 emissions of transport. Data Source: European Environment Agency. Chart available from www.knowledge.allianz.com/en/media/graphics

5. Combining the Modelling for the Coursework

Objectives

i. Provide the aircraft and modelling reference data

ii. Describe a typical program/spreadsheet work flow

iii. Detail the report aims, deadlines and marking criteria

iv. Give an example

Reference data - 1

*Modern Efficient Airplane	
Max. number of passengers	240
Range at max. payload	12,000 km
Maximum payload, w_{MP}	40 tonnes
Empty weight, w_E	106 tonnes
Fuel capacity at max. payload	74 tonnes
Max. take-off weight, w_{MTO}	220 tonnes
Cruise TAS, V	256 m/s
Cruise Mach number, M	0.85
Initial cruise altitude	31,000 ft (9.5 km)
Cruise L/D	21
Wing area	315 m^2

*Weights based on B787-8 adjusted for c_1=0.3, c_2=1.0 and thus a slightly reduced w_E and w_{TO}.

Reference data - 2

ISA & Engine Parameters		
Sea level:		T_a=288 K, p_a=101 kPa
Cruise:	M=0.85, 31000 ft (9.5 km)	T_a=227 K, p_a=28.7 kPa
	Stagnation conditions:	T_{o2}=259 K, p_{o2}=45.7 kPa
Engine at cruise:		
	Overall Pressure Ratio, $r = p_{03}/p_{02}$	45
	Turbine Entry Temperature Ratio, $\theta = T_{04}/T_{02}$	6
	Turbine and Compressor Efficiencies, η_c , η_t	0.9
	Fan Pressure Ratio, *FPR*	1.45
	Fan Efficiency, η_f	0.92
	Transfer Efficiency, η_{tr}	0.9

Reference data - 3

Modelling constants	
Breguet Range equation	
Fuel burn offset, k	0.015
Aircraft weight correlation	
c_1	0.3
c_2	1.0
Parabolic drag law constants	
K_1	0.0125
K_2	0.0446

Program or spreadsheet work flow

1. Basic Data
   - i. Weights w_E, w_P, w_F and wing area S
   - ii. ISA reference data, engine thermodynamic cycle data, modelling data
   - iii. Optimum L/D from: $\beta^* = 2\sqrt{K_1 K_2}$
2. General calculations, for any altitude, *h*
   - i. Generate look-up table of ISA density, temperatures and pressures
   - ii. Set up values of OPR, flight altitude, range, payload of interest
   - iii. Determine engine core cycle efficiency from Slide 39
3. Split flight into multiple stages
   - i. Determine current wing loading of aircraft from *mg*/*S*
   - ii. Calculate Optimum EAS for current weight (slide30), using: $V_e^* = \left(\frac{W}{0.5\rho_0 S}\right)^{0.5}\left(\frac{K_2}{K_1}\right)^{0.25}$
   - iii. Calculate actual EAS, from selected EAS ratio, $v = V_e / V_e^*$
   - iv. Hence determine TAS and Mach number (check less than transonic drag onset)
   - v. Calculate engine propulsive efficiency (Slide 44) and L/D from: $\beta = \frac{1}{2}\beta^*\left(v^2 + 1/v^2\right)$
   - vi. Hence, determine range parameter, *H*, and fuel burn for current stage:

$$H = \eta_{prop}\eta_{cycle}\eta_{tr}\ \mathrm{L/D} \times LCV/g \qquad w_{start}/w_{end} = e^{s_{stage}/H}$$

Decrement stage fuel

Integrate the range equation in 5-10 stages
holding parameters constant for each stage

4. Complete Pollution Analysis: Fuel burn per pass-km, CO_2 and NO_x per pass-km, etc.

Coursework Overview

- Airplanes & engines and their flight paths have been developed to satisfy commercial objectives which have led to some focus on fuel burn but not on their environmental impact.

- The motivation for the coursework is to ask the question: **How could the environmental impact be improved by operating aircraft differently or by applying changes to the technology?**

- The basis of the Coursework is to create a code based on Slide 69 and use it, and the information in this Lecture Course, to help answer this question

- The following slide outlines the Coursework aims, reporting expectations and deadlines. The possible coursework case studies are then discussed and two simplified examples presented.

Coursework Reporting and Deadlines

- The objective is to use a code or spreadsheet based on Slide 69 to investigate the effects of two of the following on environmental impact:
  1. Cruise altitude
  2. Engine overall pressure ratio
  3. Aircraft design range and payload

- This Lecture Course contains all the data needed
- The start date is **Wednesday November 24th** and the report must be uploaded to Moodle just over two weeks later – **4pm on Wednesday December 8th 2021**
- The report should consist of no more than ten pages and need not contain a copy of your code or spreadsheet (but this can be added in an Appendix).
- Marks will be awarded approximately equally for
  - the structure & content of the report
  - on the imaginative use of tables & graphs
  - on the clarity of the discussion of the results, findings & observations
- The marks available represent 50% of the total available for 4A7 – ie. 30/60

Coursework Case Studies

- The Coursework addresses the impact on the environment of:
  - **cruise altitude**

    This is a complex trade off between optimum design altitude for fuel economy (which places aircraft for cruise at about 35,000ft) and optimum altitude for environmental impact which is uncertain, especially due to contrails (but which would probably place aircraft lower).
  - **engine overall pressure ratio**

    Engine cycle efficiency is increased with increased engine OPR – this reduces the fuel burn and hence the CO_2 output – but combustion inlet and exit temperatures increase – and this increases the NOx production – which is better from an environmental perspective?
  - **aircraft design range and payload**

    The payload and range capability of an aircraft are driven by market demand and the need to serve multiple routes, but this leads to inefficient operation when flying shorter routes (see slide 59) and the optimum range of an aircraft is around $0.2 \times H$ (slide 56). Should we fly in shorter range aircraft that can carry lots of passengers?

Example: Overall Pressure Ratio (1)

- We will use the fuel burn equation, $w_F / w_{TO} = 1 - e^{-s/H} + 0.015$
- Consider our reference airplane first with OPR=40 then with OPR=20
  - Use w_{TO}=220 tonnes and a payload of 240 passengers flying a range of 12 000 km
- At an altitude of 9.5 km and engine **OPR=40**
  - Assume a range parameter H = 33 000 km
  - The fuel burn is $w_f = 220 \times \left(1 - e^{-12/33} + 0.015\right) = 70.37 \text{ tonne}$
  - with EI_{CO_2}=3088 gCO_2/kg, $\frac{m_{CO_2}}{sN_P} = \frac{m_F}{sN_P} \times EI_{CO_2} = \frac{70.37 \times 1000}{12000 \times 240} \times 3088 = \underline{75.4 \text{ gCO}_2\text{/pass-km}}$
  - ISA T_a=227K, Mach=0.85 and T_{02}=259K
  - Thus the compressor exit temperature is (with OPR=**40**, γ=1.4 & η_c=0.90)

$$T_{03} = T_{02}\left(1 + \left(r^{(\gamma-1)/\gamma} - 1\right)\Big/\eta_c\right) = 259\left(1 + \left(40^{2/7} - 1\right)\Big/0.9\right) = 797K$$

  - From the correlation on Slide 47,

$$EI_{NO_X} = 0.011445 e^{0.00676593 \times T_{03}} = 2.51 \text{ g/kg air}$$

  - With an air/fuel ratio of 2xstoichiometric

$$\frac{m_{NO_X}}{sN_P} = EI_{NO_X} \times \frac{m_F}{sN_P} \times \frac{m_{air}}{m_F} = 2.51 \times \frac{70.37 \times 1000}{12000 \times 240} \times \left(15.1 \times 2\right)$$

$$= \underline{1.85 \text{ gNO}_\text{x} \text{ / pass km}}$$

Example: Overall Pressure Ratio (2)

- At an altitude of 9.5 km and engine **OPR=20**
  - The range parameter will be scaled by the core efficiency (Slide 41)

$$H_{r=20} = H_{r=40} \times \left(1-20^{-0.17}\right) / \left(1-40^{-0.17}\right) = 28270 \text{ km}$$

  - The fuel burn is now $w_f = 220 \times \left(1 - e^{-12/28.27} + 0.015\right) = 79.4 \text{ tonne}$
  - with EI_{CO_2}=3088 gCO_2/kg, $\frac{m_{CO_2}}{sN_P} = \frac{m_F}{sN_P} \times EI_{CO_2} = \frac{79.4 \times 1000}{12000 \times 240} \times 3088 = \underline{85.2 \text{ gCO}_2\text{/pass-km}}$
  - ISA T_a=227K, Mach=0.85 and T_{02}=259K
  - The new compressor exit temperature is (with OPR=**20**, γ=1.4 & η_c=0.90)

$$T_{03} = T_{02}\left(1+\left(r^{(\gamma-1)/\gamma}-1\right)/\eta_c\right) = 259\left(1+\left(20^{2/7}-1\right)/0.9\right) = 649K$$

  - From the correlation on Slide 47,

$$EI_{NO_X} = 0.011445 e^{0.00676593 \times T_{03}} = 0.92 \text{ g/kg air}$$

  - With an air/fuel ratio of 2xstoichiometric

$$\frac{m_{NO_X}}{sN_P} = EI_{NO_X} \times \frac{m_F}{sN_P} \times \frac{m_{air}}{m_F} = 13.9 \times \frac{79.4 \times 1000}{12000 \times 240} \times (15.1 \times 2)$$

$$= \underline{0.77 \text{ gNO}_\text{x} \text{ / pass km}}$$

So, a lower engine OPR gives higher CO_2 emissions, but much lower NO_x emissions and easier for the engine in terms of the compressor and turbine cooling technology!

Example: Aircraft Design Range

- Consider our reference airplane flying 12 000 km in two stages rather than one
  - Use w_E=106 tonnes and a payload of 240 passengers (with no freight).
  - We will use the fuel burn per payload-range equation on Slide 55.

- For a single flight
  - Assume a range parameter H = 33 000 km
  - The fuel burn per payload-km is given by:

$$\frac{w_F}{s w_P} = \frac{1}{s}\left(1+\frac{w_E}{w_P}\right)\frac{1-e^{-s/H}+k}{e^{-s/H}-k} = \frac{1}{12000}\left(1+\frac{106000}{240\times 100}\right)\frac{1-e^{-12/33}+0.015}{e^{-12/33}-0.015} = \underline{2.123\times 10^{-4}\ \text{kg fuel/kg km}}$$

- For a flight with 2 stages, each of 6 000 km
  - Assume the empty weight is the same for each flight.
  - The fuel burn per payload-km is given by:

$$\frac{w_F}{s w_P} = \frac{1}{s}\left(1+\frac{w_E}{w_P}\right)\frac{1-e^{-s/H}+k}{e^{-s/H}-k} = \frac{1}{6000}\left(1+\frac{106000}{240\times 100}\right)\frac{1-e^{-6/33}+0.015}{e^{-6/33}-0.015} = \underline{1.998\times 10^{-4}\ \text{kg fuel/kg km}}$$

So, the reduction in fuel burn is ~6%, but this neglects the potential additional benefit to the empty weight (and w_{MTO}) – the aircraft could be redesigned to have a lower range.

6. Future Directions

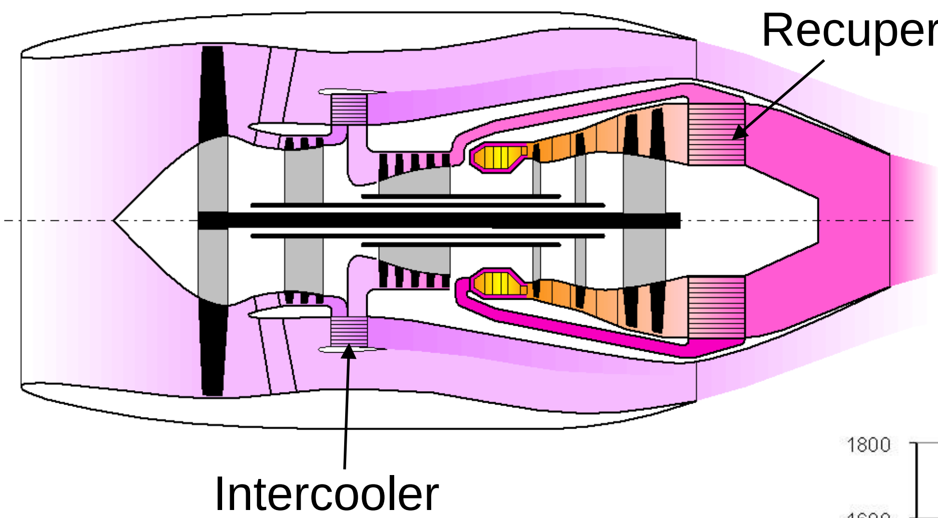
<u>Objectives</u>

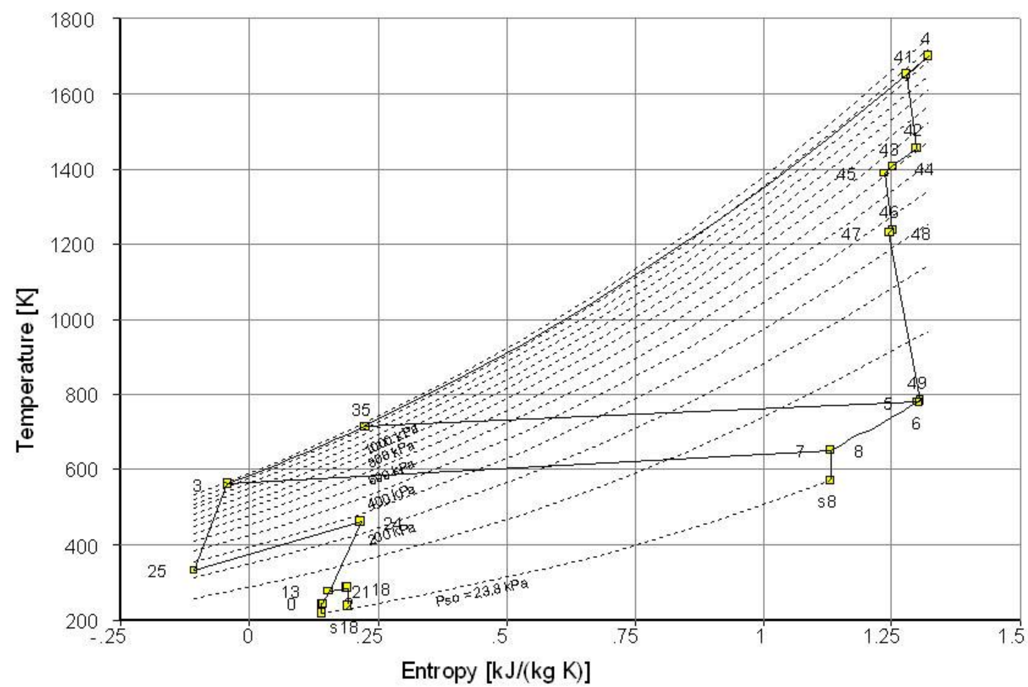
i. Describe some technology for increased η_{th} and η_{prop}

ii. Describe some technology for increased *L/D*

iii. Discuss the potential of improving aircraft operations

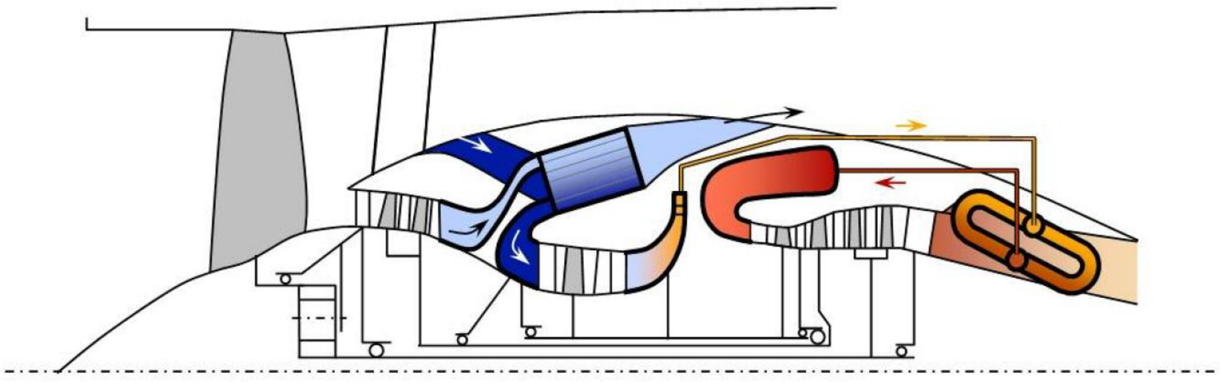
Increasing η_{th} – Intercooled, Recuperated Cycles



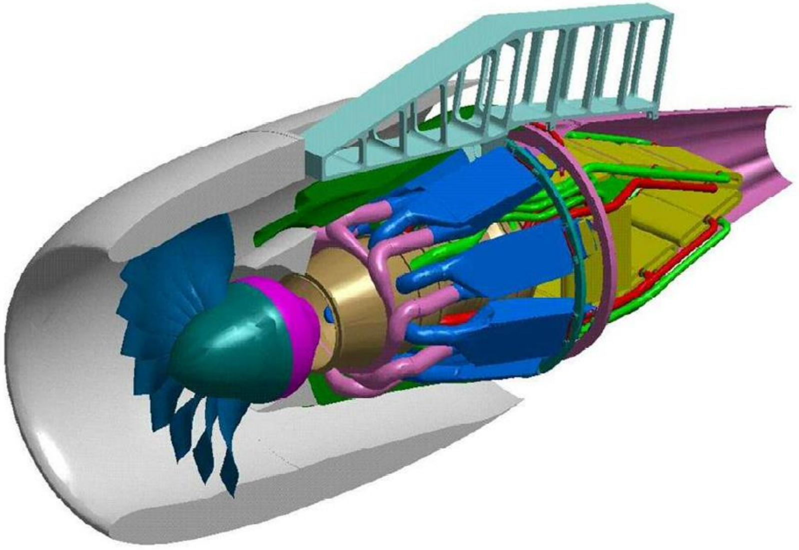


- Intercooling reduces compressor work and drops compressor exit temperature.
- Recuperator increases mean temperature of heat addition.

Intercooled, Recuperated Cycles(2)



- Difficulty is the integration, weight and pressure drops of effective heat exchangers
- Concept is low NOx, but requires lighter heat exchanger materials and configurations.



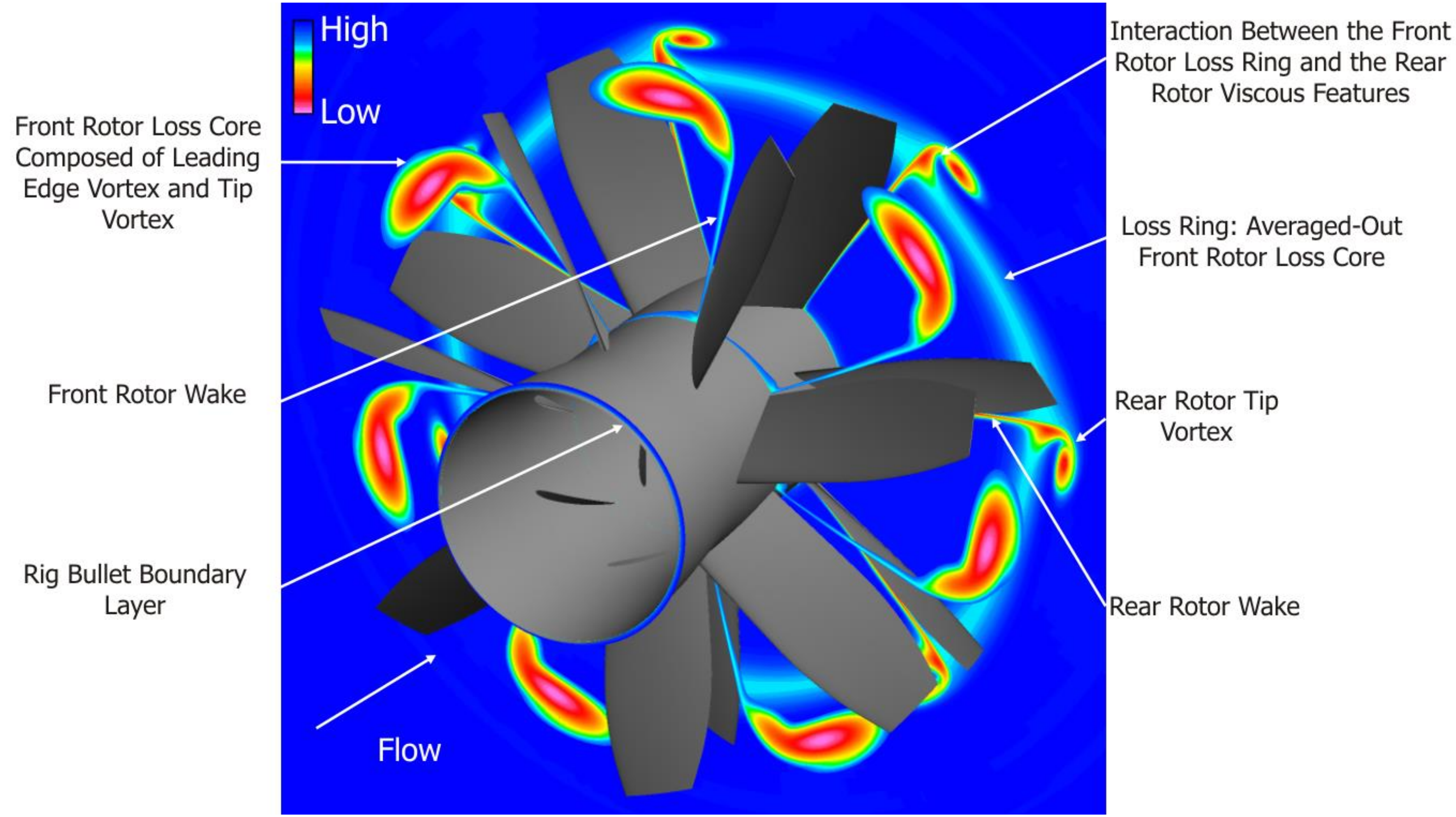
Images from: http://www.newac.eu/uploads/media

Increasing η_{prop} - Open rotor engines

- Large savings in fuel burn (very low FPR, no duct, no downstream swirl).
- Noise emission a big challenge (no liners, lots of sources)
- New engine installations needed (rear mounted or high wing).

Source: FlightGlobal, Earth2Tech

Open rotor flow-field at take-off



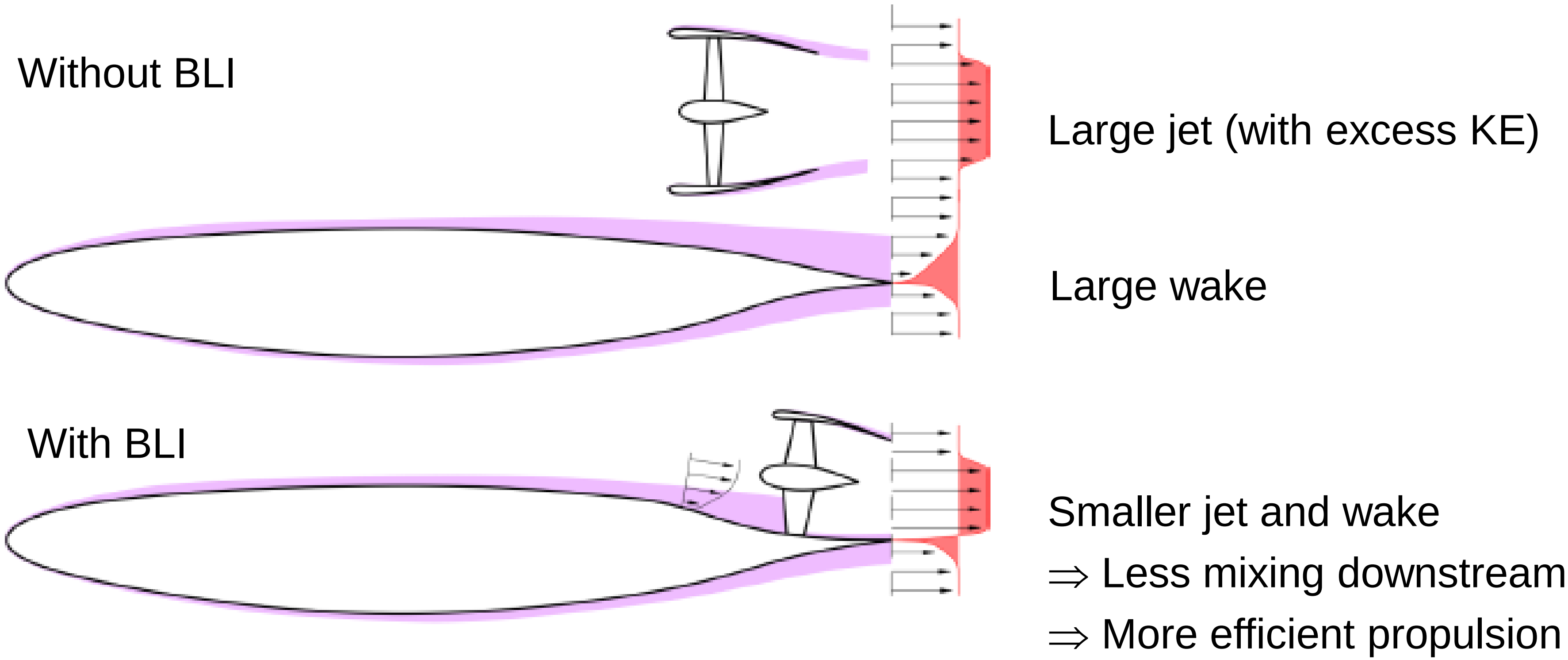
Contours of Entropy at Take-Off (A. Zachariadis)

Increasing L/D

- By eliminating the non-lifting fuselage and simply having a flying wing the L/D ratio can be improved by of order 20% even without laminar flow.

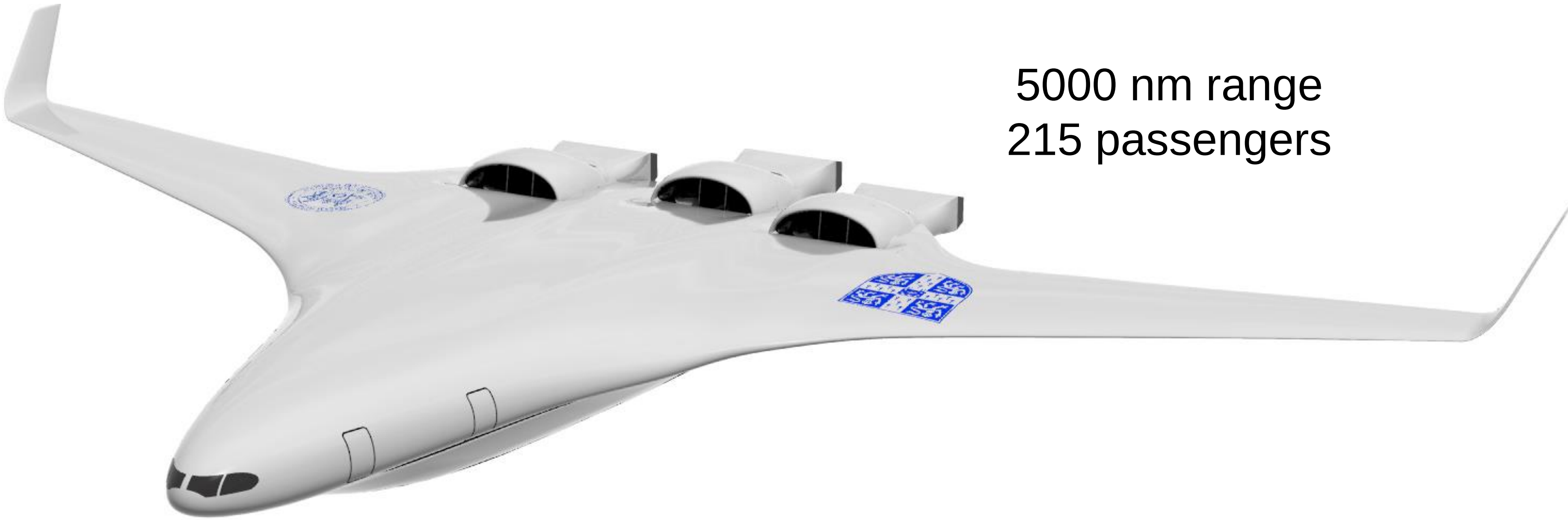
Blended Wing Body (BWB) concept used in the Silent Aircraft Initiative

Boundary Layer Ingestion



- Boundary layer Ingestion (BLI) means taking vehicle boundary layer fluid into a propulsor to improve performance.
- It has the potential to give significant reductions in fuel burn.

Silent Aircraft Final Design: SAX-40



5000 nm range
215 passengers

Fuel Burn of 124 pax-miles per gallon (~100 for B787)

Noise of 63 dBA outside airport perimeter

209 cumulative EPNdB at certification points

(ICAO Chapter 4 Requirement is 284.5 EPNdB)

Layout of the Silent Aircraft engine

Image courtesy of SAI and Steve Cross

Lots of novel technology with the potential to reduce noise and fuel burn.

Propulsive Fuselage Concepts

Propulsive Fuselage Aircraft
Image credit: DisPURSAL project

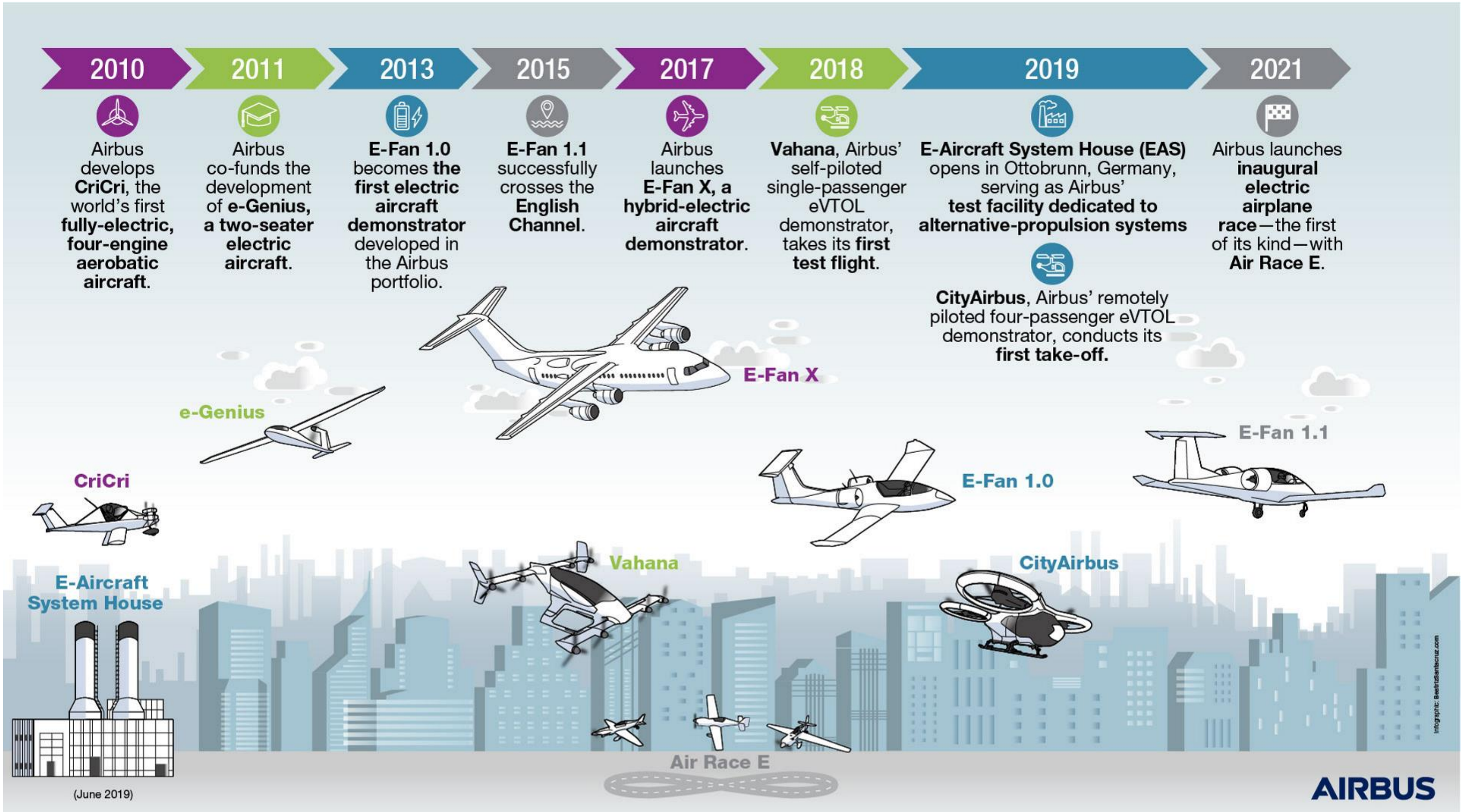
NOVA Aircraft Configuration
Image credit: ONERA

The Starc-ABL Turbo-Electric Aircraft
Image credit: NASA

CENTRELINE - EU Horizon 2020 Project
Image credit: Bauhaus-Luftfahrt

Electric Aircraft (1)

https://www.airbus.com/innovation/zero-emission/electric-flight.html



Electric Aircraft (2)

https://www.airbus.com/innovation/zero-emission/electric-flight.html

Vahana

CityAirbus

Low Emissions Flight Operations

1. Fly at close to $v = 1$ throughout flight?
2. Operate as close to design w_p and optimum s as possible.
3. Use multiple hops for long haul flights.
4. Apply consistent, minimum inefficiency descent trajectories.
5. Adopt formation flying for busy routes?
6. Cruise to avoid persistent contrail formation?

Additional Notes