Prepared for submission to JHEP

Small Cosmological Constants

James Halverson,[a] Cody Long,[a] Brent Nelson,[a] Fabian Ruehle[b]

[a] *Department of Physics, Northeastern University, Boston, MA 02115, USA*

[b] *Rudolf Peierls Centre for Theoretical Physics, Oxford University, 1 Keble Road, Oxford, OX1 3NP, UK*

Abstract:

Note: this being in JHEP format is just for convenience, not saying this is necessarily for JHEP.

Contents

1 Introduction

2 Bousso-Polchinski Problem Setup

2.1 The Bousso-Polchinski Model

$$\Lambda = \Lambda_0 + g_{ij} N_i N_j \tag{2.1}$$

$$Vol(L) = \frac{(\pi L)^{k/2}}{(k/2)!\,\sqrt{detg}} \tag{2.2}$$

$$\delta Vol = \frac{k}{2\Lambda_0}\, Vol(|\Lambda_0|)\, \epsilon \tag{2.3}$$

For simplicity we set $M_{\rm pl} = 1$ and then $\Lambda_0 = -1$. Having many vacua with cosmological constants in the BP shell, and also having the volume approximate the number of lattice points in the shell, requires $\delta Vol >> 1$. We will henceforth refer to this as shell volume condition (SVC). In this language, one critical observation of Bousso-Polchinski is that at large k the SVC condition is automatically satisfied and there are an exponentially large number of vacua with small cosmological constant.

However, our applications will begin for simplicity at small k, and therefore we must analyze conditions under which the SVC condition is satisfied. Specifically, for any fixed value of k we would like to know regimes in which $det\, g$ and ϵ are such that they allow for an exponentially large number of vacua with cosmological constant of order ϵ. To do this it is useful to express the problem in terms of a parameter that has mass dimension one, μ

such that $\mu^4 := (det\, g)^{1/k}$, where g has mass dimension $[g] = 4$ because in our convention the flux vectors N_i are dimensionless. This yields

$$Vol(k,\mu) = \frac{\pi^{k/2}}{\mu^{2k}(k/2)!}, \qquad \delta Vol(k,\mu) = \frac{kVol(k,\mu)}{2}\,\epsilon, \tag{2.4}$$

and contours for $log_{10}(\delta Vol(k,\mu,\epsilon))$ are presented in Figure 1. There we see, at large k, the observation of BP that the number of vacua is exponentially large for many values of μ and ϵ. At small k such as $k = 5, 10$, we see a $\delta Vol = 1$ contour that cuts across the plot, signifying the absence of vacua with $O(\epsilon)$ cosmological constant above that contour. This means that at small fixed (k,ϵ) we must evaluate whether it is reasonable in a possible metric ansatz to actually obtain solutions to the problem; we will find that is is the case, and choose physically reasonable parameters for the metric ansatz accordingly.

2.2 Bousso-Polchinski with Wishart Metrics

Any positive definite $k \times k$ metric g may be written as

$$g = A^\dagger A \tag{2.5}$$

where A is a $Q \times k$ matrix. We choose g to be from the Wishart distribution by requiring that A is real with entries drawn from a Gaussian distribution with standard deviation σ. Throughout, we will choose $Q = k$ for simplicity.

We would like to ask how the requirement of many vacua via the SVC may be satisfied in the Wishart distribution. There the dependence of the general shell volume on the triple $(k, \epsilon, detg)$ may be recast into a dependence on (k, ϵ, σ). This may be done by using the fact that

$$E[log\, det\, g] = \Psi_k(k/2) + k\, ln(2) + 2k\, ln(\sigma), \tag{2.6}$$

and therefore $E[det\, g] \propto \sigma^{2k}$ and $E[\delta Vol] \propto \sigma^{-k}$. This scaling of $E[det\, g]$ with σ is easily verified in simple numerical experiments. ten line code commented out in tex file $\Psi_k(z)$ is the multivariate digamma function,

$$\sum_{i=1}^{k} \Psi\left(z + \frac{1-i}{2}\right), \tag{2.7}$$

where $\Psi(z)$ is the digamma polygamma? check all this again function.

In sampling Wishart metrics, σ must be chosen to be sufficiently small for fixed ϵ, k so that the SVC is satisfies. Defining σ_t to be the threshold value of σ for which $E[\delta Vol] = 1$, a short calculation then gives

$$\sigma_t = exp\left[\frac{2ln\left(\frac{k\pi^{k/2}\epsilon}{2(k/2)!}\right) - \Psi_k(k/2) - k\, ln(2)}{2k}\right], \tag{2.8}$$

and satisfying the SVC requires that $\sigma << \sigma_t$. However, it is not immediately clear how much smaller σ must be.

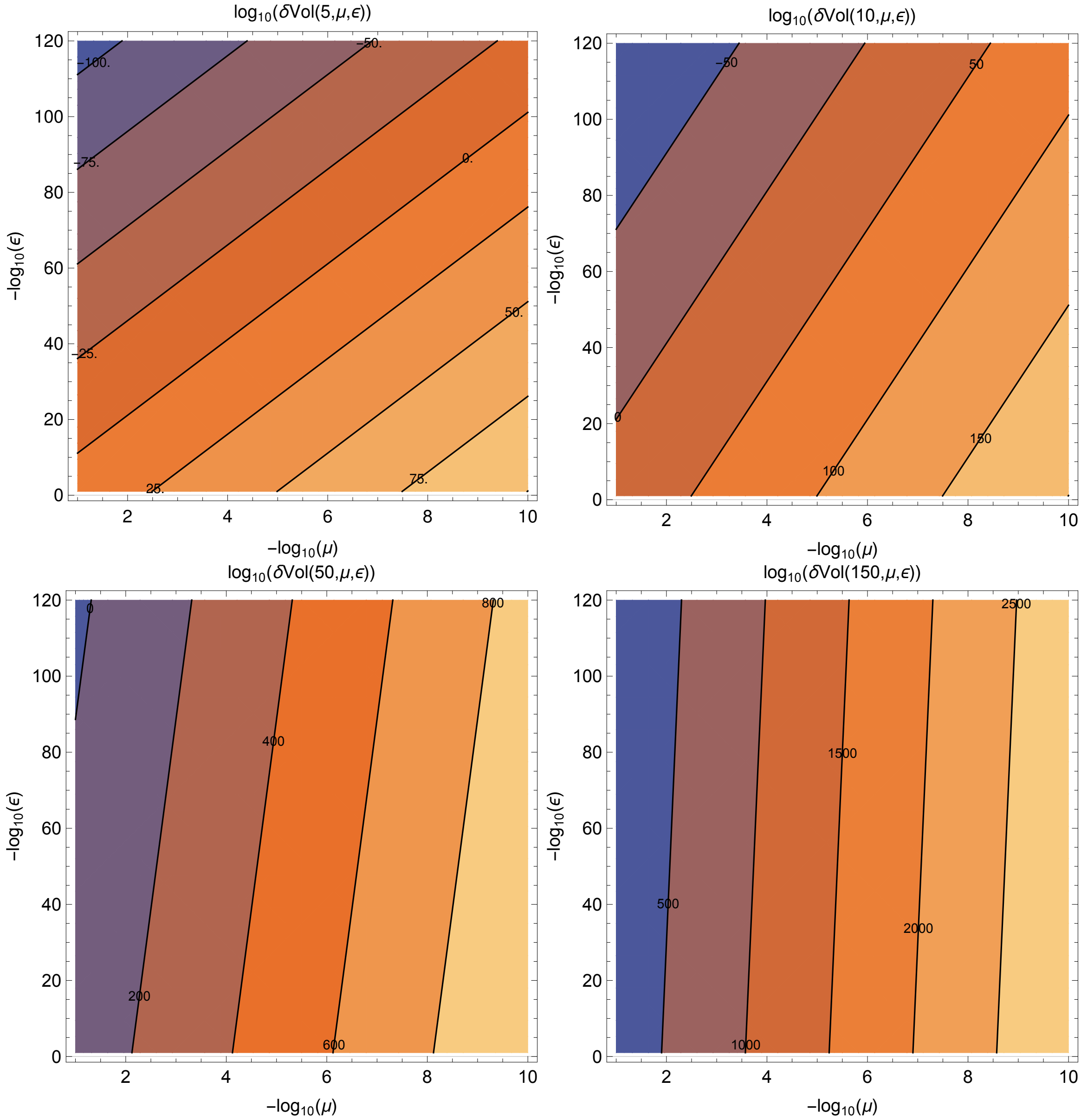

Figure 1. Shell volumes for $k = (5, 10, 50, 150)$ as a function of (μ, ϵ), beginning with the upper left and moving to the bottom right. Do you guys know a way to use the same gradient colors across all plots? Only relevant if these make the paper.

Instead, we may choose σ for fixed (k, ϵ) such that there are an exponentially large number of vacua with $O(\epsilon)$ cosmological constants. We may do this by comparing to figure 1 and determining σ from μ. A short calculation gives $\sigma = \mu^2 F(k)$ where

$$F(k) = \sqrt{\frac{1}{2} e^{-\Psi_k(k/2)/k}}. \tag{2.9}$$

However, since $F(5) = 1.1$ and $F(500) = .10$, the functional dependence on k is mild enough that $\sigma \sim \mu^2$ is a good approximation in the regime $k \in (5, 100)$ that we are interested in.

3 Small Cosmological Constants

3.1 Reinforcement Learning

3.2 Genetic Algorithms

3.3 Control Experiments

4 Discussion