To construct this into a linear constraint

for $(\bar{x} \vee y \vee \bar{z})$

$$(1-x) + y + (1-z) \geq 1$$

ILP

Integer
Linear
Program

for $(x \vee \bar{y} \vee z)$

$$x + (1-y) + z \geq 1$$

for $(x \vee y \vee z)$

$$x + y + z \geq 1$$

for $(\bar{x} \vee \bar{y})$

$$(1-x) + (1-y) \geq 1 \quad \text{ILP}$$

What can we conclude from this exercise

$A \leq B$ $\qquad$ 3Sat $\xrightarrow[\text{Reduced}]{}$ ILP

Optimization Problem $\qquad$ decision problem

MST $\qquad$ answer is $\boxed{\text{yes / no}}$

is there a spanning tree
cost less than C

Decision Problem $\xrightarrow[\text{Reduce}]{}$ optimization Problem?

$\leq_p$ $\qquad$ $\boxed{\text{Alg for Opt.}}$

Opt $\rightarrow$ decision Problem?

$\boxed{\text{Alg for Dec}}$

use Binary Search to find the minimum that outputs "yes"

Problem vs Instance

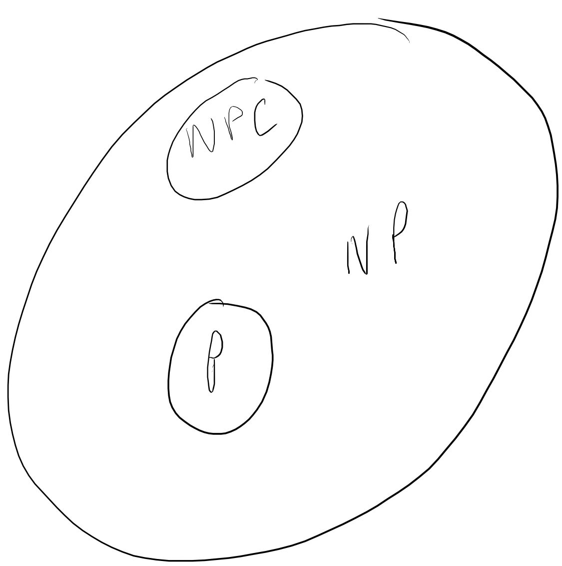
Thursday P NP

P -

NP - given a solution can be verified in Polynomial time

NPC

P is a subset of Np



$P \neq NP$

$P \stackrel{?}{=} NP$

Reduction

3sat

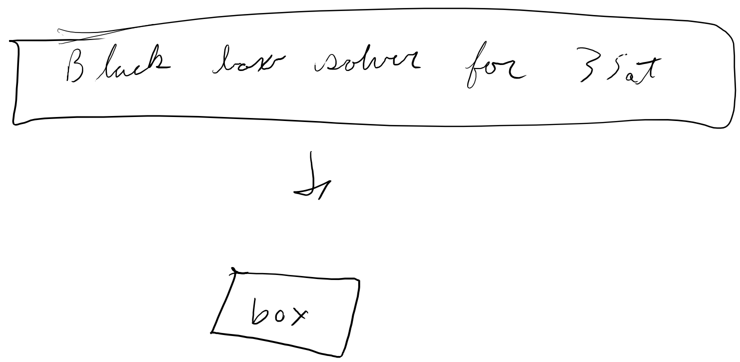
CNF - Conjunctive Normal Form

SAT $\in$ NPC

Reduction Direction

to show 3SAT $\in$ NPC

SAT $\rightarrow$ 3SAT



given a clause of type SAT, shrink it to 3SAT, or size 3 boolean expressions

---

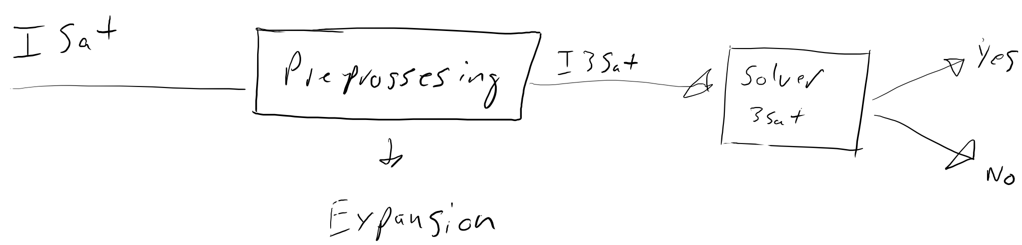
$(a_1 \vee a_2 \vee \dots a_k)$ $k > 3$

$(a_1 \vee a_2 \vee y_1)$ $(\overline{y}_1 \vee a_3 \vee y_2)$ $(\overline{y}_2 \vee a_4 \; y_3)$ $\dots$ $(\overline{y}_{k-3} \vee a_{k-1} \vee a_k)$

if the new construct is satisfied, at least one of $a_1, a_2, \dots, a_k$ has to be True

if $a_1, a_2, \dots, a_k$ is true, then at least one of them is true $\Rightarrow a_i == \text{true}$

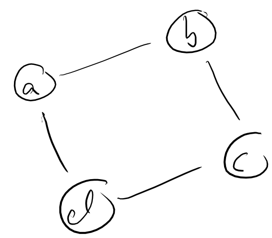
$(a_1 \vee a_2 \vee y_1)(\overline{y}_1 \vee a_3 \vee y_2) \dots (\overline{y}_{i-2} \vee a_i \vee y_{i-1})(y_{i-1} \vee a_{i-1} \vee a_i)$



Independent set

V = nodes
E = edges

given a graph $G = (V, E)$ an independent set is a set of nodes $S \subseteq V$ such that no edge in E connect to any two nodes in S



$\{a\}$ yes

$\{a, c\}$ yes

$\{a, b, c\}$ no

---

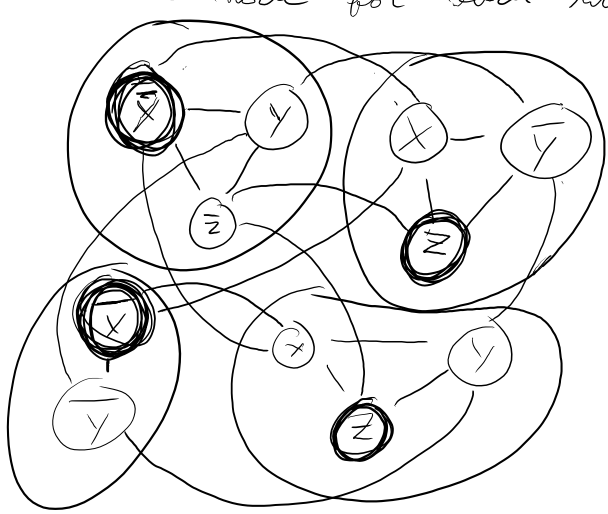
given a graph G and an integer g is there an independent set while size is greater than or equal to g

1: in NP: yes

2: reduce 3sat → IS

$$(\bar{x} \vee y \vee \bar{z})(x \vee \bar{y} \vee z)(x \vee y \vee z)(\bar{x} \vee \bar{y})$$

Create a node for each literal

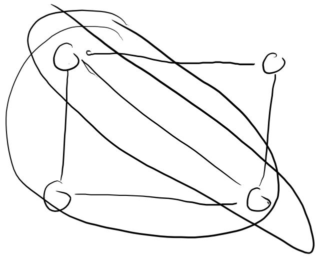
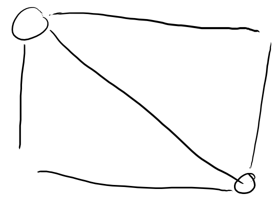


An edge is a conflict on this graph

$x=0$ ~~$y=1$~~ $z=1$

We ignore y to build the independent set given a graph $G(V,E)$

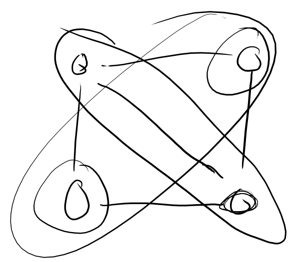
Vertex Cover, cover every edge

Trivial vertex cover = V, every node

is there a vertex cover size $\leq v$

1. show in NP
2. Reduction



$\bar{V} =$

$G, 8$ for IS

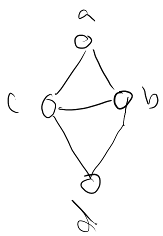
$\downarrow$

$G, |v| - 8$ for VC

!I

Tuesday, dead Week

Sat $\to$ 3sat $\to$ Independent Set $\to$ Vertex cover



$\underline{g}$

$\{a, d\}$ independent set

$\underbrace{\{c, b\}}_{\leq |v| - g}$ vertex cover

To prove reduction is correct for $\underbrace{A \to B}_{\text{decision problems}}$

$\to$ if B has a 'yes' answer, then A must have a 'yes' answer

$\to$ if A has a 'yes' answer then B must have a 'yes' answer

Visa versa with 'no' answers

for example, if A has a solution for an independent set, then B has a viable solution for a vertex cover

---

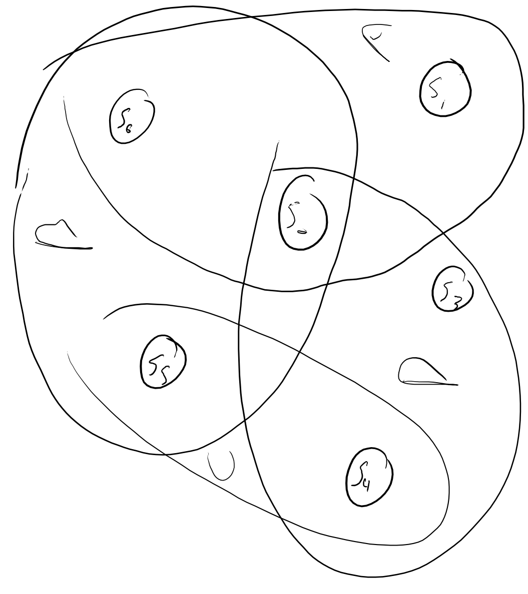
for IS $\to$ Vc reduction the above is true because

Set cover

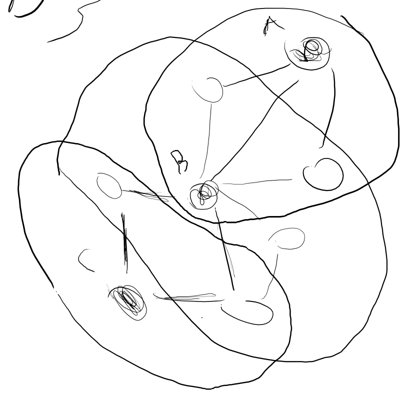
given a set of elements U, and a collection of sets

$S_1, S_2, \ldots, S_m \subset U$.

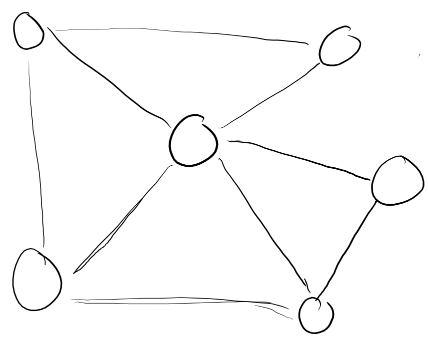
A set cover is a subset of these sets whose union is U

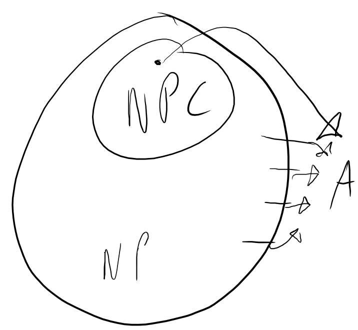


$\{A, B, D\}$



edges are the grouping





1. Show it's in NP
2. Show it's NP-Hard

CLIQUE

a set of vertices in a graph such that every pair of vertices are connected by an edge

given a graph, is there a clique of size $\geq k$

clique $\leq$ IS