$x = 11u + 4v \quad y = 4u + 11v$

1) $x^2 - y^2 = 105$

$D = ac \quad u^2 - Dv^2 = 1$

$D = 105$ $\quad (1, 0)$

$u^2 - 105v^2 = 1 \quad ($

$$\boxed{\begin{array}{c}(11,4)\\(13,8)\\(19,16)\end{array}}$$

2a)

1	5	9	13	17	21	25	33	37	41
45	49	53	57	61	65	69	73	77	81
85	89	93	97	101	105	109	113	117	121
125	129	133	137	141	145	149	153	157	161
165	169	173	177	181	185	189	193	197	201
205	209	213	217	221	225				

first 15 S-Primes

$\{5, 9, 13, 17, 21, 33, 37, 41, 49, 53, 57, 61, 69, 73, 77\}$

b) first 15 S-Composites:

$\{25, 45, 65, 81, 85, 105, 117, 125, 145, 153, 165, 169, 185, 189, 205\}$

c) $\boxed{441} = 21 \cdot 21 = 49 \cdot 9$

d) $\boxed{22869}$ $\quad 33^2 21 \quad 1\,21 \cdot 21 \cdot 9 \quad 33 \cdot 77 \cdot 9$

3) Primes from {601 ... 800} =
{601, 607, 613, 617, 619, 631, 641,
647, 653, 659, 661, 673, 677, 683,
691, 701, 709, 719, 727, 733, 739, 743,
751, 757, 761, 769, 773, 787, 797}

4a) $12 = 2^2 \cdot 3$

2	3	5	7	11	13	17	19	23	29	31	37	41
1												
1 ·	1											

b) $123 = 3 \cdot 41$

2	3	5	7	11	13	17	19	23	29	31	37	41
×	1	×	$<\sqrt{41}$ ⟶									

c) $1234 = 2 \cdot 617$

2	3	5	7	11	13	17	19	23	29	31	37	41
1	×	×	×	×	>	×	×	$>\sqrt{617}$ ⟶				

d) $12345 = 3 \cdot 5 \cdot 823$

2	3	5	7	11	13	17	19	23	29	31	37	41
×	1											
	×	1										
		×	×	×	×	×	×	×	$>\sqrt{823}$ ⟶			

e) $123456 = 2^6 \cdot 3 \cdot 643$

2	3	5	7	11	13	17	19	23	29	31	37	41
|												
|												
|												
|												
|												
|												
×	|											
	×	×	×	×	×	×	×	×	$> \sqrt{643} \longrightarrow$			

f) $1234567 = 127 \cdot 9721$

...	127	131	...
×	|		
	×	$> \sqrt{9721} \longrightarrow$	

g) $12345678 = 2 \cdot 3^2 \cdot 47 \cdot 14593$

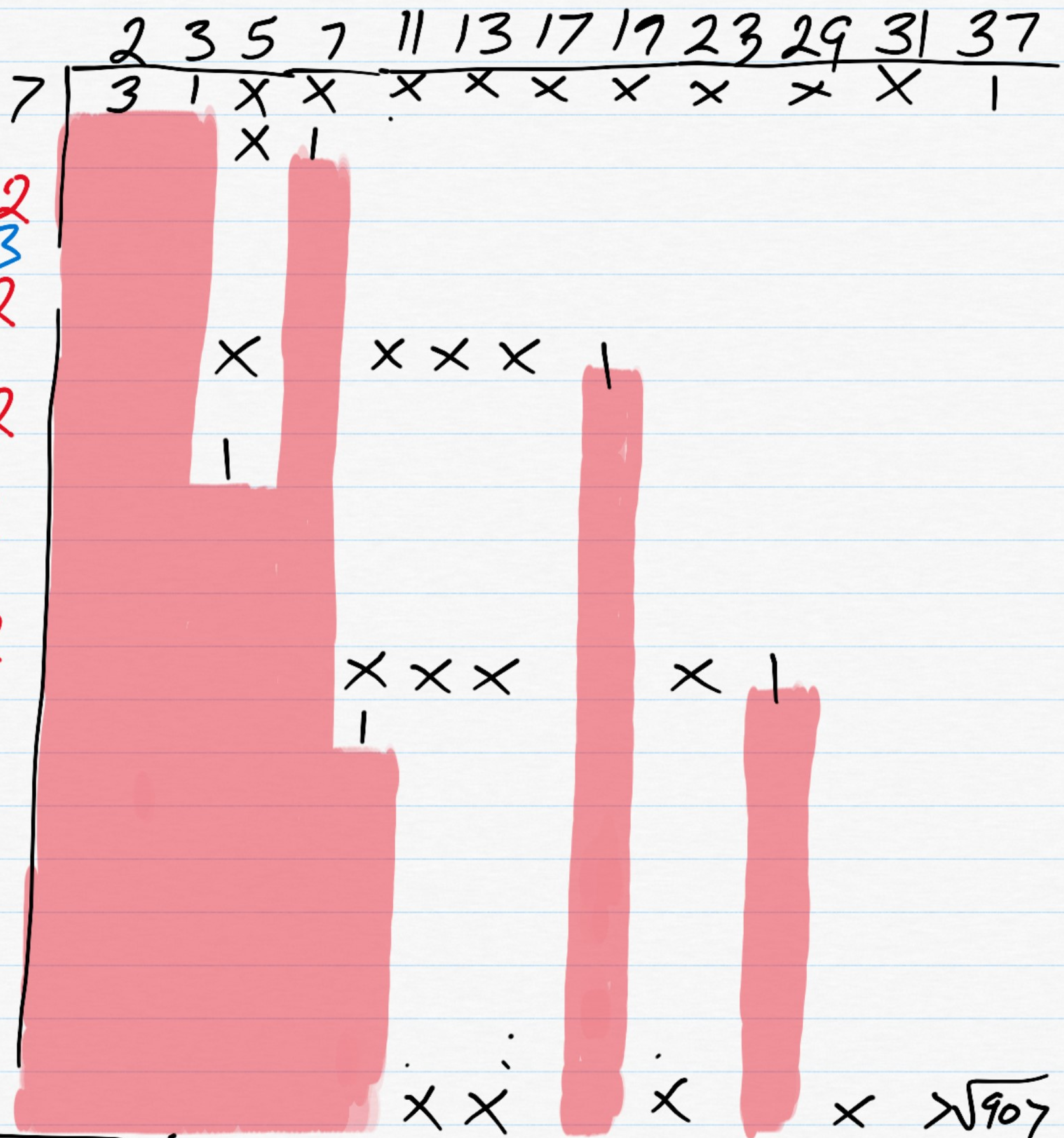
2	3	5	7	11	13	17	19	23	29	31	37	41	47
|													
×	|												
	|												
	×	×	×	×	×	×	×	×	×	×	×	×	|

h) $123444321 = 3 \cdot 7 \cdot 11^2 \cdot 13 \cdot 37 \cdot 101$

2	3	5	7	11	13	17	19	23	29	31	37	41	...	101
×	|													
	×	×	|										
			×	|									
				|									
				×	|								
					×	×	×	×	×	×	|		...	|

5a) use trial division to determine if a number is prime. If it's not - use the Sieve to eliminate future non-primes. See 5b. for example

b) $888 = 2^3 \cdot 3 \cdot 37$
$889 = 7 \cdot 127$
~~890~~ multiple of 2
~~891~~ multiple of 3
~~892~~ multiple of 2
$893 = 19 \cdot 47$
~~894~~ multiple of 2
$895 = 5 \cdot 179$
~~896~~ multiple of 2
~~897~~ multiple of 3
~~898~~ multiple of 2
$899 = 29 \cdot 31$
~~900~~ multiple of 2
$901 = 3^4 \cdot 11$
~~902~~ multiple of 2
~~903~~ multiple of 3
~~904~~ multiple of 2
~~905~~ = multiple of 5
~~906~~ multiple of 2
907 = Prime!



32 Divisions

6) find all values $\{3x+7y=n \mid x,y \in \mathbb{N}\}$

	3	6	9	12	15	17	21	24	27
7	10	13	16	19	22	25	28	31	34
14	17	20	23	26	29	32	35	38	41
21	24	27	30	33	36	39	42	45	48
28	31	34	37	40	43	46	49	52	55
31	34	37	40	43	46	49	52	55	58

There is a triangle kind of pattern to the order of valid numbers in this set.

	#3		#7	
odd	1	x	1	= 10 [1,1]
	2	x	1	= 13 [1,1]
even	3	x	1	= 16 [1,2]
odd	1	x	2	= 17 [1,2]
	4	x	1	= 19 [1,2]
even	2	x	2	= 20 [1,2]
	5	x	1	= 22 [1,3]
	3	x	2	= 23 [1,3]
odd	1	x	3	= 24 [1,3]
				...

The number of 3's forms an even/odd pattern while the number of 7's forms a pattern that repeats twice before adding one to the sequence size