1a) $42x-30y=12$ $\gcd(42,60)=6$ $6|12$

$x=x_1-\frac{kb}{D}$ $y=y_1+\frac{ka}{D}$

$x=1-\frac{k(-30)}{6}$ $y=1+\frac{k(42)}{6}$

$x=1+\frac{30k}{6}$ $y=1+\frac{42k}{6}$

$x=1+5k$ $y=1+7k$

b) $42x-30y=6$ $(x_1,y_1)=(3,4)$

$x=3-\frac{k(-30)}{6}$ $y=4+\frac{k(42)}{6}$

$x=3+5k$ $y=4+7k$

c) $954x+234y=126$ $\gcd(954,234)=18$

$x=13k+7$ $y=-53k-28$

d) $954x+234y=128$

$\gcd(954,234)=18$ $18\nmid 128$

no solution

e) $300894x+284262y=462$

$x=43071t-1316$ $y=1393-4559t$

2) $\$x.y \mid x,y<100$ and I'll assume $x,y \geq 0$

$2(100x+y=c)$

$-\quad x+100y=2c+47$

$199x-98y=-47$

$x=98t-1551 \quad t=16 \quad x=17$

$y=199t-3149 \qquad\qquad y=35$

$100(17)+35=1735$

$17+3500=1735*2+47$

$c=100(98t-1551)+(199t-3149)$

$c=9800t-1551$

$+\ 199t-3149$

$\boxed{c=9999t-4700 \mid t \geq 16}$

3a) $\boxed{11 \cdot 17 \cdot 23 \cdot 31}$

b) $\boxed{13331 \text{ is Prime}}$

c) $\boxed{37^2 \cdot 73^5 \cdot 101}$

d) $\boxed{3907^4 \cdot 5147^5}$

e) $\boxed{3^{12} \cdot 5^{13} \cdot 37^2 \cdot 59^{10}}$

2	
3	4
6	6
9	8
12	10
15	12
18	14
21	16
24	18
27	20
	22

4) $\frac{n}{2}=2^a 3^b$

$n=2^{a+1} 3^b \qquad n=2^a 3^{b+1}$

$a \equiv 1 \bmod 2 \quad b \equiv 0 \bmod 2$

$a \equiv 0 \bmod 3 \quad b \equiv 1 \bmod 3$

$a=3 \qquad b=16$

$\boxed{2^3 3^{16}}$

4b) $\boxed{2^{15} \cdot 3^{10} \cdot 5^6}$

$a \equiv 1 \bmod 2 \quad a \equiv 0 \bmod 3 \quad a \equiv 0 \bmod 5 \quad a=15$

$b \equiv 0 \bmod 2 \quad b \equiv 1 \bmod 3 \quad b \equiv 0 \bmod 5 \quad b=10$

$c \equiv 0 \bmod 2 \quad c \equiv 0 \bmod 3 \quad c \equiv 1 \bmod 5 \quad c=6$

5) 1, 2, 3 are Square Free, 4 is not.

there is no Sequence $\in \mathbb{N}$ longer than 3 without a multiple of Four

1 2 3 4 5 6 7 8 9 10 11 12 ...

longest has square ...

3

6) $\sqrt[3]{A^2 \cdot \frac{2}{3}} \in \mathbb{N}$

$n = \sqrt[3]{\frac{a^2 \, 2}{3}}$ $\quad n^3 = \frac{2a^2}{3}$ $\quad \frac{3n^3}{2} = a^2$ $\quad 2 \mid n$, $3 \mid a$

$\frac{3(6)^3}{2} = 18^2$

18

7a)

0	1	2	3	4	5	6
M	T	W	H	F	Sa	Su
☆	+150 →		*			

$0 + 150 \bmod 7 \equiv 3$

Thursday

b) 9 $\quad 12345 \bmod 12 \equiv 9$

8) $13^{650} \pmod{151}$

512 256 128 64 32 16 8 4 2 1

1 0 1 0 0 0 0 1 0 1 0

$650_{10} = 1010000101_2$

$13^{650} = 13^{512} \cdot 13^{128} \cdot 13^{8} \cdot 13^{2}$

$18 \cdot 31 \cdot 21 \cdot 144 \pmod{151} \equiv 118$

118

13	13
13^2	18
13^4	22
13^8	31
13^{16}	55
13^{32}	5
13^{64}	25
13^{128}	21
13^{256}	139
13^{512}	144

4a) $231x \equiv 1 \pmod{479}$ $\gcd(231, 479) = 1$

$x = 336$

b) 110

10a) 15

b) 3

c) 3

d) 2

e) 4

$6x \equiv 3$ gcd = 4