

## Assignment 2 (Closed Book)

Time: 45 minutes

Total Marks: 20

Q1.

A company claims that its new relaxation app reduces the average resting heart rate of its regular users.

Historically, the resting heart rate of similar adults is modeled as normal with a mean of 72 bpm and a standard deviation of 10 bpm.

A student takes a simple random sample of 64 regular users of the app and finds that their average resting heart rate is 69 bpm.

Answer the following:

- Identify the population mean and the sample mean in this context. [1 mark]
- State the null hypothesis and the alternative hypothesis. [2 marks]
- Compute the standard error of the sample mean. Explain in one sentence what the standard error means. [2 marks]
- Compute the z-score for the observed sample mean. [2 marks]
- Draw a rough sketch of the relevant distribution, mark the historical mean and observed sample mean on the relevant axis, and shade the area corresponding to the p-value for this test. [3 marks]

Q2.

A mobile provider uses an automatic filter to mark incoming SMS messages as "spam" or "not spam".

- 20% of all incoming messages are actually spam.
- If a message is spam, the filter marks it as "spam" with probability 0.9.
- If a message is not spam, the filter still marks it as "spam" with probability 0.05.

Let's say, we can use Bayes' rule (given below) to find the probability of spam given marked as spam.

$$P(A | B) = \frac{P(B | A) P(A)}{P(B | A) P(A) + P(B | A^c) P(A^c)}$$

where  $P(A^c)$  is probability of not A (complementary probability).

- a) What is the prior probability here in the equation above? What is the value of the prior probability based on the example above? [2 marks]
- b) What is the posterior probability in the equation above? Explain posterior probability in one sentence in the context of the example above. [2 marks]
- c) Write down the expression from the equation given above that gives the total probability i.e. the probability that the filter marks a message as spam. Provide the values you will use from the given example to calculate total probability. [3 marks]
- d) In the above example, will the posterior probability be higher or lower than the prior (no calculations required)? Justify in one sentence. [3 marks]