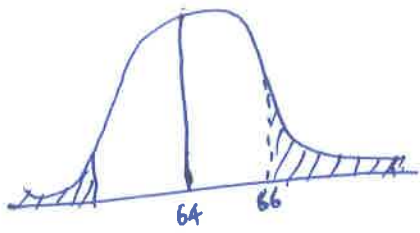


Q1.

a) Population mean $\mu = 100$

b) Sample mean $\bar{x} = 64$

c)

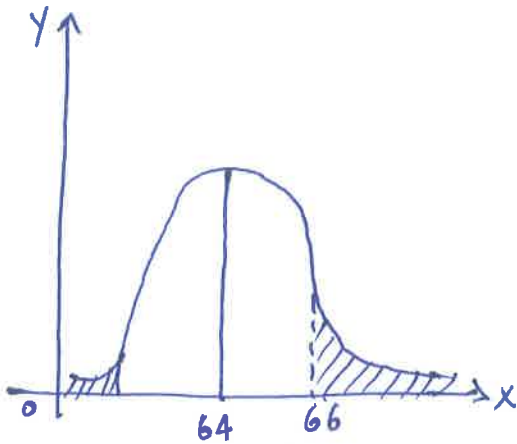


$$66 - 64 = 2$$

$$S.E. = \frac{2}{10} = 0.2$$

* Standard error = $\frac{\sigma}{\sqrt{n}}$

e)



Q2.

a) The prior probability here in the above equation is $P(A)$, the value of the prior probability based on the example is 20%.

b) The posterior probability in the equation above is $P(B|A)$, if a message is spam, the filter marks it as spam with 90% probability.

$$c) P(A|B) = \frac{P(B|A) P(A)}{P(B|A) P(A) + P(B|A^c) P(A^c)}$$

$$P(B|A) = 90\%, P(A) = 20\%$$

$$P(B|A^c) = 5\%, P(A^c) = 80\%$$

$$= \frac{P\left(\frac{90}{100}\right) \left(\frac{20}{100}\right)}{\left(\frac{90}{100}\right) + \left(\frac{20}{100}\right) + \left(\frac{5}{100}\right) \left(\frac{80}{100}\right)}$$

$$= \frac{\frac{9}{50}}{\frac{9}{50} + \frac{4}{50}}$$

$$= \frac{\frac{9}{50}}{\frac{13}{50}} = \frac{9}{13}$$

d] The posterior probability will be higher.



Let $f(x)$ and $g(x)$ be two probability density functions. Let $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$ and $g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-1)^2}{2\sigma^2}}$. Then $f(x) > g(x)$ for $x < 1$ and $f(x) < g(x)$ for $x > 1$.

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$$\frac{f(x)}{g(x)} = \frac{e^{-\frac{x^2}{2\sigma^2}}}{e^{-\frac{(x-1)^2}{2\sigma^2}}} = e^{\frac{(x-1)^2 - x^2}{2\sigma^2}} = e^{\frac{x^2 - 2x + 1 - x^2}{2\sigma^2}} = e^{\frac{-2x + 1}{2\sigma^2}}$$

$$\frac{f(x)}{g(x)} > 1 \Leftrightarrow \frac{-2x + 1}{2\sigma^2} > 0 \Leftrightarrow -2x + 1 > 0 \Leftrightarrow x < \frac{1}{2}$$

$$\frac{f(x)}{g(x)} < 1 \Leftrightarrow \frac{-2x + 1}{2\sigma^2} < 0 \Leftrightarrow -2x + 1 < 0 \Leftrightarrow x > \frac{1}{2}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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