

Exercise Solutions for Math 20

Graphs of Circular Functions

Nile Jocson <novoseiversia@gmail.com>

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- 1 From the following sine waves, determine the amplitude, period, phase shift, and vertical shift and then sketch one cycle of the graph. Label maximum and minimum points.

1.1 $f(x) = 2\cos(\frac{1}{2}(x + \pi)) - 1$

$\Rightarrow a = 2$

$\Rightarrow b = \frac{1}{2}$

$\Rightarrow c = -\pi$

$\Rightarrow d = -1$

$\Rightarrow M = 1$

$\Rightarrow m = -3$

\Rightarrow See Figure 1.

Final answer. $f(x) = a\cos(b(x - c)) + d$

$\{M, m\} = \pm a - d$

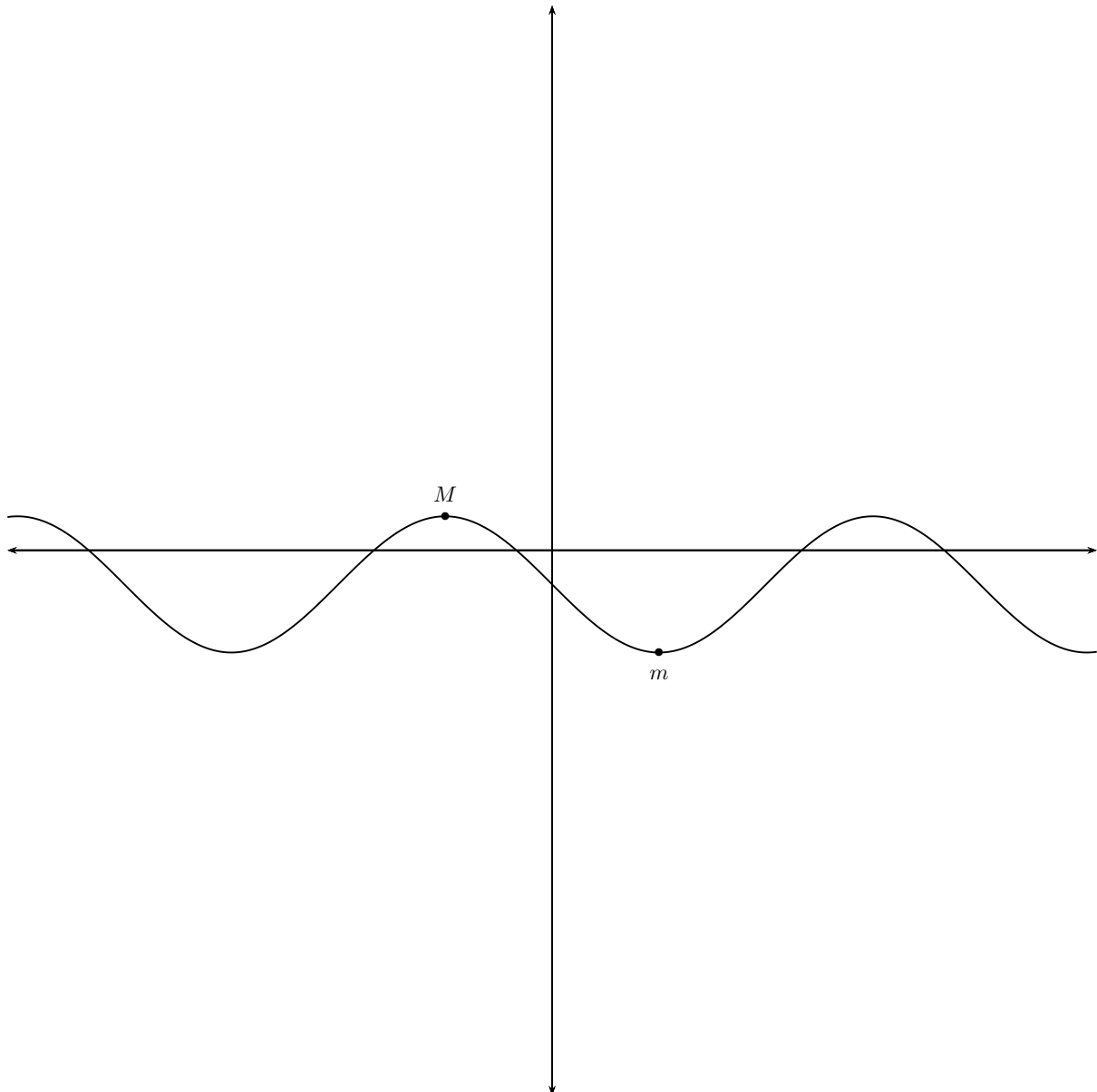


Figure 1. Graph of $f(x) = 2\cos(\frac{1}{2}(x + \pi)) - 1$

1.2 $f(x) = -\frac{3}{2}\sin(\pi - 2x) + 2$

$\Rightarrow f(x) = -\frac{3}{2}\sin(-2(x - \frac{\pi}{2})) + 2$	Rewrite in standard form.
$\Rightarrow a = -\frac{3}{2}$	Final answer.
$\Rightarrow b = -2$	
$\Rightarrow c = \frac{\pi}{2}$	
$\Rightarrow d = 2$	
$\Rightarrow M = \frac{7}{2}$	
$\Rightarrow m = \frac{1}{2}$	

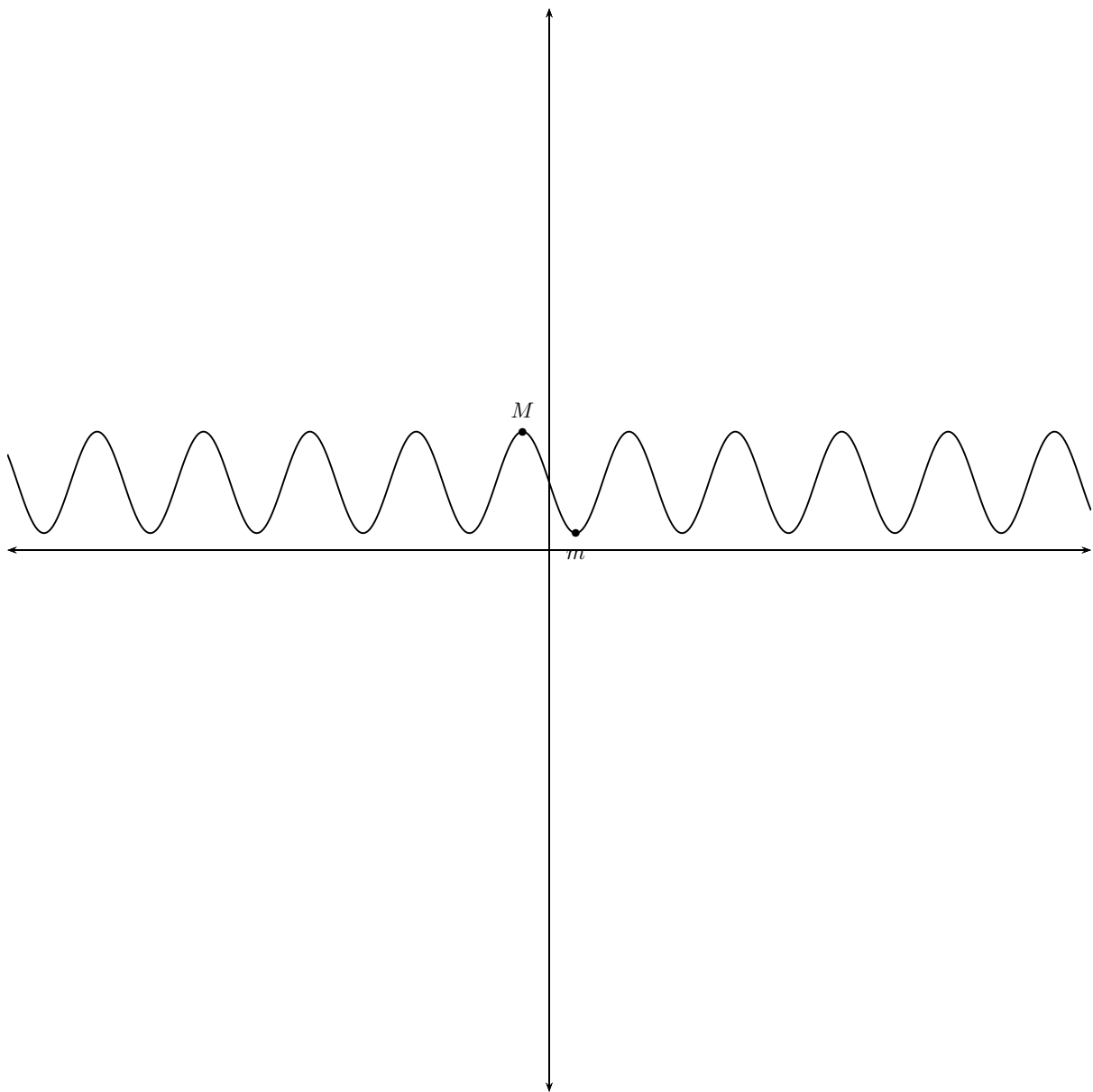


Figure 2. Graph of $f(x) = -\frac{3}{2}\sin(\pi - 2x) + 2$

1.3 $f(x) = 4\sin(4x - 3\pi)$

$\Rightarrow f(x) = 4\sin(4(x - \frac{3\pi}{4}))$	Rewrite in standard form.
$\Rightarrow a = 4$	Final answer.
$\Rightarrow b = 4$	
$\Rightarrow c = \frac{3\pi}{4}$	
$\Rightarrow d = 0$	
$\Rightarrow M = 4$	
$\Rightarrow m = -4$	

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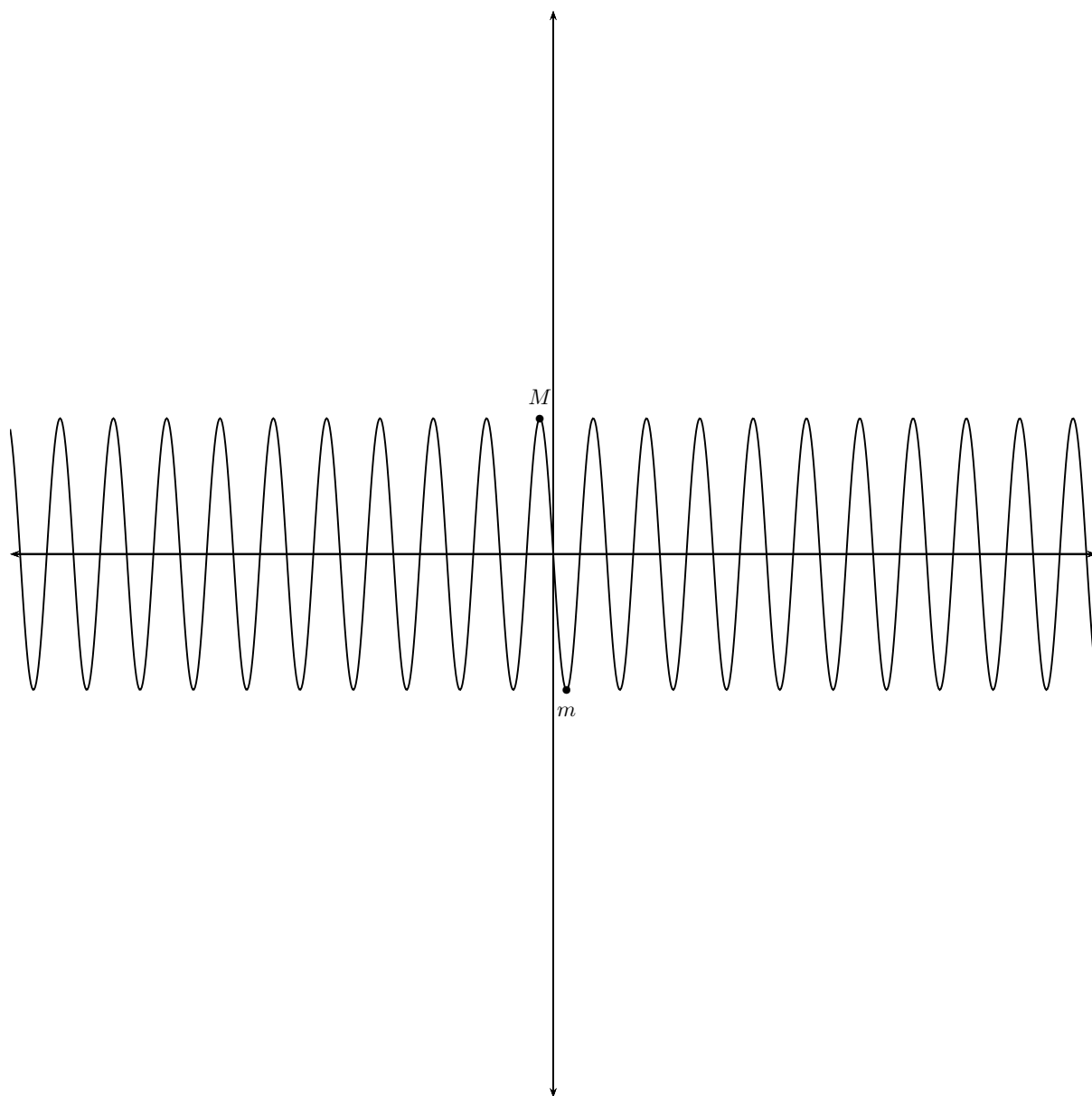


Figure 3. Graph of $f(x) = 4\sin(4x - 3\pi)$

1.4 $f(x) = -2\cos(\frac{x}{2} - \frac{\pi}{4}) + 1$

$\Rightarrow f(x) = -2\cos(\frac{1}{2}(x - \frac{\pi}{2})) + 1$	Rewrite in standard form.
$\Rightarrow a = -2$	Final answer.
$\Rightarrow b = \frac{1}{2}$	
$\Rightarrow c = \frac{\pi}{2}$	
$\Rightarrow d = 1$	
$\Rightarrow M = 3$	
$\Rightarrow m = -1$	

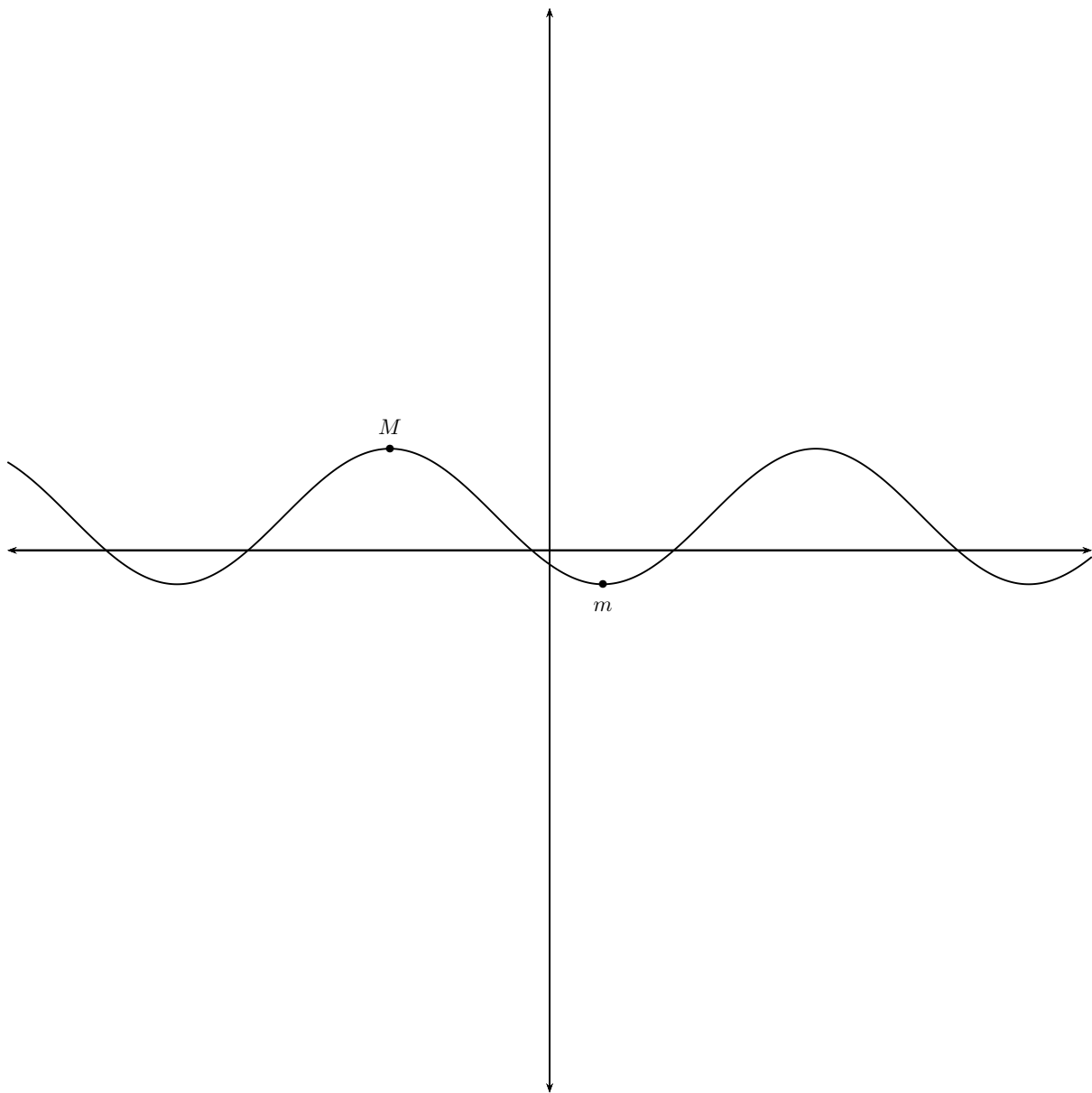


Figure 3. Graph of $f(x) = -2\cos(\frac{x}{2} - \frac{\pi}{4}) + 1$