

Exercise Solutions for Math 20

Linear Inequalities System, Nonlinear Systems

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1.1 Solve algebraically for the solution sets of the following systems of equations.

1.1.a

$$\begin{cases} y = x^2 - 2x - 8 \\ 4x + 3y + 3 = 0 \end{cases}$$

$\Rightarrow 3y = -4x - 3$ $\Rightarrow y = -\frac{4}{3}x - 1$	Rewrite in terms of y .
$\Rightarrow x^2 - 2x - 8 = -\frac{4}{3}x - 1$ $\Rightarrow x^2 - 2x + \frac{4}{3}x - 8 + 1 = 0$ $\Rightarrow x^2 - \frac{6}{3}x + \frac{4}{3}x - 7 = 0$ $\Rightarrow x^2 - \frac{2}{3}x - 7 = 0$ $\Rightarrow 3x^2 - 2x - 21 = 0$ $\Rightarrow 3x^2 - 9x + 7x - 21 = 0$ $\Rightarrow 3x(x - 3) + 7(x - 3) = 0$ $\Rightarrow (3x + 7)(x - 3) = 0$ $\Rightarrow x \in \{-\frac{7}{3}, 3\}$	Solve for x . Factor by grouping.
$\Rightarrow 4(-\frac{7}{3}) + 3y + 3 = 0$ $\Rightarrow -\frac{28}{3} + 3y + 3 = 0$ $\Rightarrow -\frac{28}{3} + 3y + \frac{9}{3} = 0$ $\Rightarrow 3y - \frac{19}{3} = 0$ $\Rightarrow 3y = \frac{19}{3}$ $\Rightarrow y = \frac{19}{9}$	Solve for $y, x = -\frac{7}{3}$
$\Rightarrow 4(3) + 3y + 3 = 0$ $\Rightarrow 12 + 3y + 3 = 0$ $\Rightarrow 3y + 15 = 0$ $\Rightarrow 3y = -15$ $\Rightarrow y = -5$	Solve for $y, x = 3$
$\Rightarrow (x, y) \in \{(-\frac{7}{3}, \frac{19}{9}), (3, -5)\}$	Final answer. ■

1.1.b

$$\begin{cases} 10x^2 - xy + 4y^2 = 28 \\ 2x^2 - 3xy - 2y^2 = 0 \end{cases}$$

$\Rightarrow 2x^2 - 3yx - 2y^2 = 0$ $\Rightarrow \frac{3y \pm \sqrt{(3y)^2 - 4(2)(-2y^2)}}{4}$ $\Rightarrow \frac{3y \pm \sqrt{9y^2 + 16y^2}}{4}$ $\Rightarrow \frac{3y \pm \sqrt{25y^2}}{4}$	Rewrite in terms of x . Use the quadratic formula.
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$\Rightarrow \frac{3y \pm 5y}{4}$ $\Rightarrow \frac{3y+5y}{4}, \frac{3y-5y}{4}$ $\Rightarrow \frac{8y}{4}, \frac{-2y}{4}$ $\Rightarrow x \in \{-\frac{1}{2}y, 2y\}$	
$\Rightarrow 10(-\frac{1}{2}y)^2 - (-\frac{1}{2}y)y + 4y^2 = 28$ $\Rightarrow 10(\frac{1}{4}y^2) + \frac{1}{2}y^2 + 4y^2 = 28$ $\Rightarrow \frac{5}{2}y^2 + \frac{1}{2}y^2 + 4y^2 = 28$ $\Rightarrow \frac{5}{2}y^2 + \frac{1}{2}y^2 + \frac{8}{2}y^2 = 28$ $\Rightarrow \frac{14}{2}y^2 = 28$ $\Rightarrow 7y^2 = 28$ $\Rightarrow y^2 = 4$ $\Rightarrow y = \pm 2$	Solve for $y, x = -\frac{1}{2}y$.
$\Rightarrow x = -\frac{1}{2}(-2)$ $\Rightarrow x = 1$ $\Rightarrow x = -\frac{1}{2}(2)$ $\Rightarrow x = -1$	Solve for $x, y = -2$ Solve for $x, y = 2$
$\Rightarrow 10(2y)^2 - (2y)y + 4y^2 = 28$ $\Rightarrow 10(4y^2) - 2y^2 + 4y^2 = 28$ $\Rightarrow 40y^2 - 2y^2 + 4y^2 = 28$ $\Rightarrow 42y^2 = 28$ $\Rightarrow y^2 = \frac{28}{42}$ $\Rightarrow y^2 = \frac{2}{3}$ $\Rightarrow y = \pm \sqrt{\frac{2}{3}}$	Solve for $y, x = 2y$.
$\Rightarrow x = -2\sqrt{\frac{2}{3}}$ $\Rightarrow x = 2\sqrt{\frac{2}{3}}$	Solve for $x, y = -\sqrt{\frac{2}{3}}$ Solve for $x, y = \sqrt{\frac{2}{3}}$
$\Rightarrow (x, y) = \{(1, -2), (-1, 2), (-2\sqrt{\frac{2}{3}}, -\sqrt{\frac{2}{3}}), (2\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}})\}$	Final answer. ■