

Exercise Solutions for Math 20

Conics (Parabola and Ellipse)

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1.1 Determine the vertex and orientation of the following parabolas.

1.1.a $4y^2 + 4y + x = 2$

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| $\Rightarrow 4y^2 + 4y = -x + 2$ $\Rightarrow y^2 + y = -\frac{x}{4} + \frac{2}{4}$ $\Rightarrow y^2 + y = -\frac{x}{4} + \frac{1}{2}$ | Isolate y . |
| $\Rightarrow y^2 + y + \frac{1}{4} = -\frac{x}{4} + \frac{1}{2} + \frac{1}{4}$ $\Rightarrow (y + \frac{1}{2})^2 = -\frac{x}{4} + \frac{3}{4}$ $\Rightarrow (y + \frac{1}{2})^2 = -\frac{1}{4}(x - 3)$ $\Rightarrow (y + \frac{1}{2})^2 = 4(-\frac{1}{16})(x - 3)$ | Complete the square. |
| \Rightarrow Opening leftwards, $(h, k) = (3, -\frac{1}{2})$ | Final answer. ■ |

1.1.b $x^2 - 6x - 2y = 7$

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|---|--|
| $\Rightarrow x^2 - 6x = 2y + 7$ | Isolate x . |
| $\Rightarrow x^2 - 6x + 9 = 2y + 7 + 9$ $\Rightarrow (x - 3)^2 = 2y + 16$ $\Rightarrow (x - 3)^2 = 2(y + 8)$ $\Rightarrow (x - 3)^2 = 4(\frac{1}{2})(y + 8)$ | Complete the square. |
| \Rightarrow Opening upwards, $(h, k) = (3, -8)$ | Final answer. ■ |

1.1.c $2y^2 - 6y - 9x = 0$

| | |
|--|--|
| $\Rightarrow 2y^2 - 6y = 9x$ $\Rightarrow y^2 - 3y = \frac{9}{2}x$ | Isolate y . |
| $\Rightarrow y^2 - 3y + \frac{9}{4} = \frac{9}{2}x + \frac{9}{4}$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}x + \frac{9}{4}$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{9}{4} \cdot \frac{2}{9})$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{18}{36})$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{1}{2})$ $\Rightarrow (y - \frac{3}{2})^2 = 4(\frac{9}{8})(x + \frac{1}{2})$ | Complete the square. |
| \Rightarrow Opening rightwards, $(h, k) = (-\frac{1}{2}, \frac{3}{2})$ | Final answer. ■ |

1.2 Sketch the graph of the following parabolas.

1.2.a $3y^2 = 8x$

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|-------------------------------------|--|
| $\Rightarrow y^2 = 4(\frac{2}{3})x$ | Rewrite in standard form. |
| \Rightarrow See Figure 1. | Final answer. Graph the parabola. ■ |

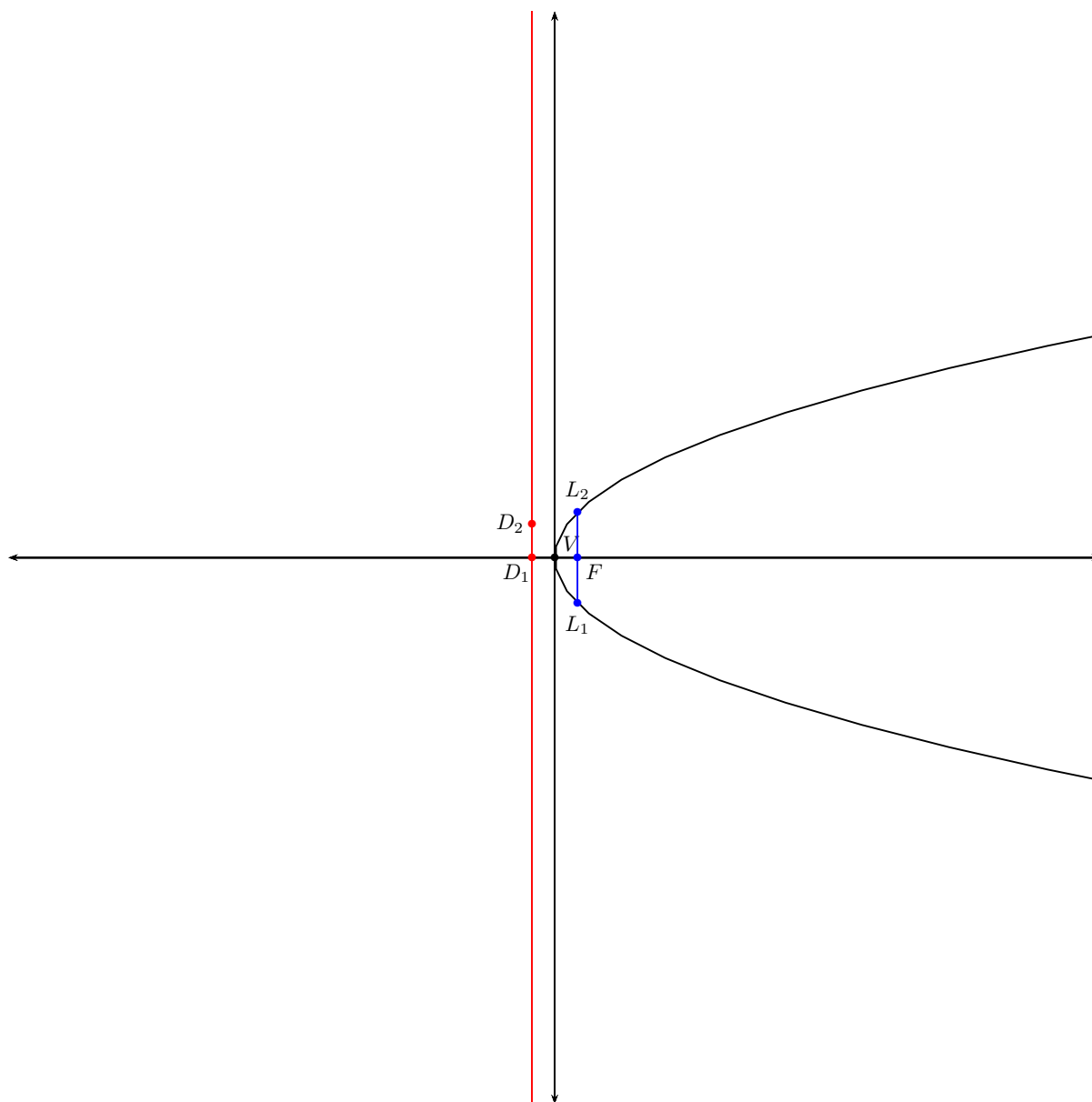


Figure 1. Graph of $y^2 = 4(\frac{2}{3})x$.

1.2.b $x^2 - 8x + 4y = -10$

| | |
|---|---------------------------|
| $\Rightarrow x^2 - 8x = -4y - 10$ | Rewrite in standard form. |
| $\Rightarrow x^2 - 8x + 16 = -4y - 10 + 16$ | Complete the square. |
| $\Rightarrow (x - 4)^2 = -4y + 6$ | |

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$$\Rightarrow (x - 4)^2 = 4(-1)(y - \frac{3}{2})$$

\Rightarrow See Figure 2.

Final answer. Graph the parabola.

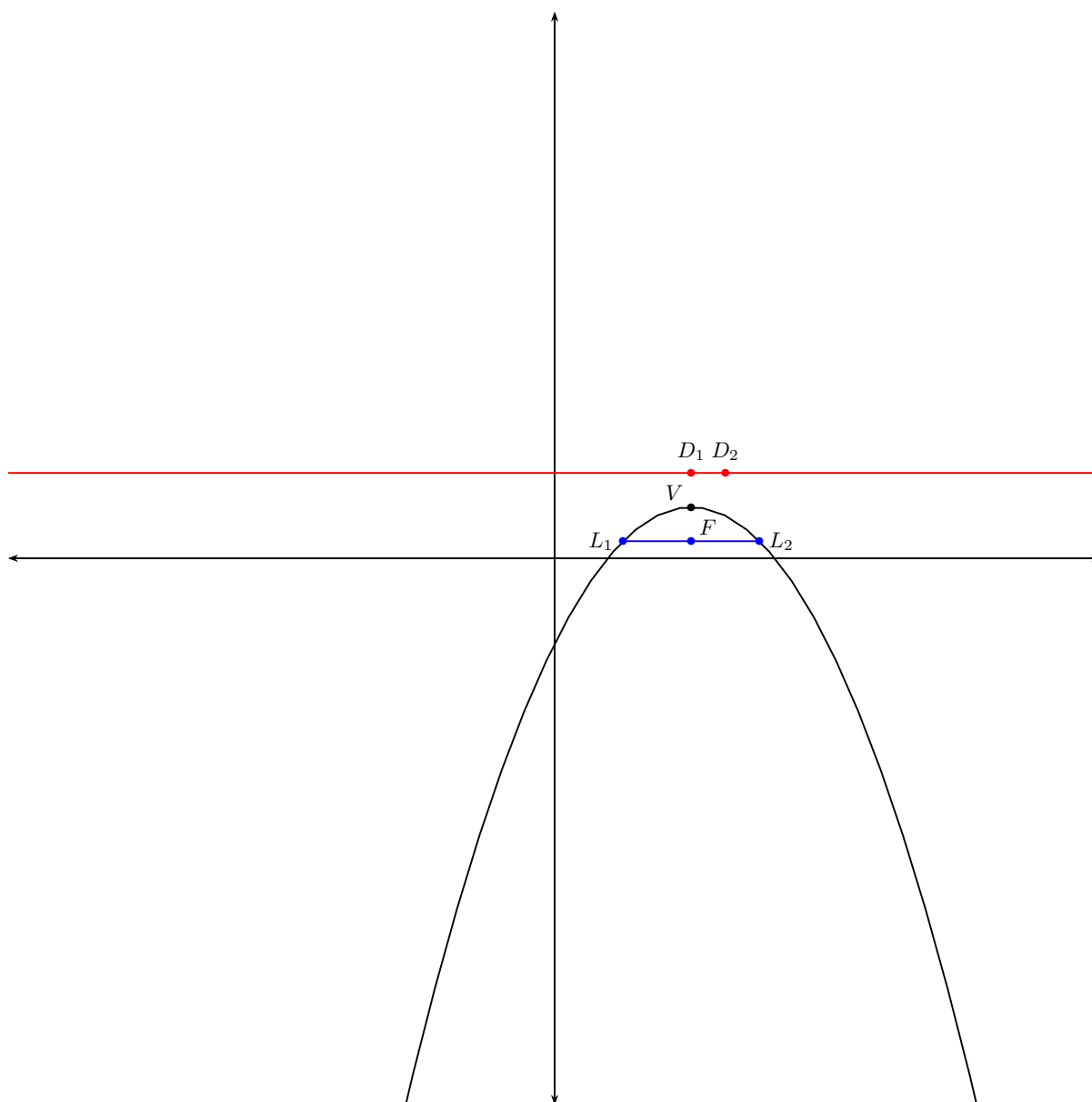


Figure 2. Graph of $(x - 4)^2 = 4(-1)(y - \frac{3}{2})$.

1.3 Sketch the graph of $y = -x^2 + 6x - 8$. Label the vertex, x- and y-intercept(s).

$$\Rightarrow -x^2 + 6x = y + 8$$

$$\Rightarrow x^2 - 6x = -y - 8$$

$$\Rightarrow x^2 - 6x + 9 = -y - 8 + 9$$

$$\Rightarrow (x - 3)^2 = -y + 1$$

Rewrite in standard form.

Complete the square.

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| | |
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| $\Rightarrow (x - 3)^2 = 4(-\frac{1}{4})(y - 1)$ | |
| $\Rightarrow (x - 3)^2 = 4(-\frac{1}{4})(-1)$ | Find the x-intercepts. |
| $\Rightarrow (x - 3)^2 = 4(\frac{1}{4})$ | |
| $\Rightarrow (x - 3)^2 = 1$ | |
| $\Rightarrow x = \pm 1 + 3$ | |
| $\Rightarrow x = 1 + 3, x = -1 + 3$ | |
| $\Rightarrow x_i \in \{2, 4\}$ | |
| $\Rightarrow (0 - 3)^2 = 4(-\frac{1}{4})(y - 1)$ | Find the y-intercepts. |
| $\Rightarrow (-3)^2 = 4(-\frac{1}{4})(y - 1)$ | |
| $\Rightarrow 9 = -(y - 1)$ | |
| $\Rightarrow 9 = -y + 1$ | |
| $\Rightarrow y = 1 - 9$ | |
| $\Rightarrow y_i = -8$ | |
| \Rightarrow See Figure 3. | Final answer. Graph the parabola. ■ |

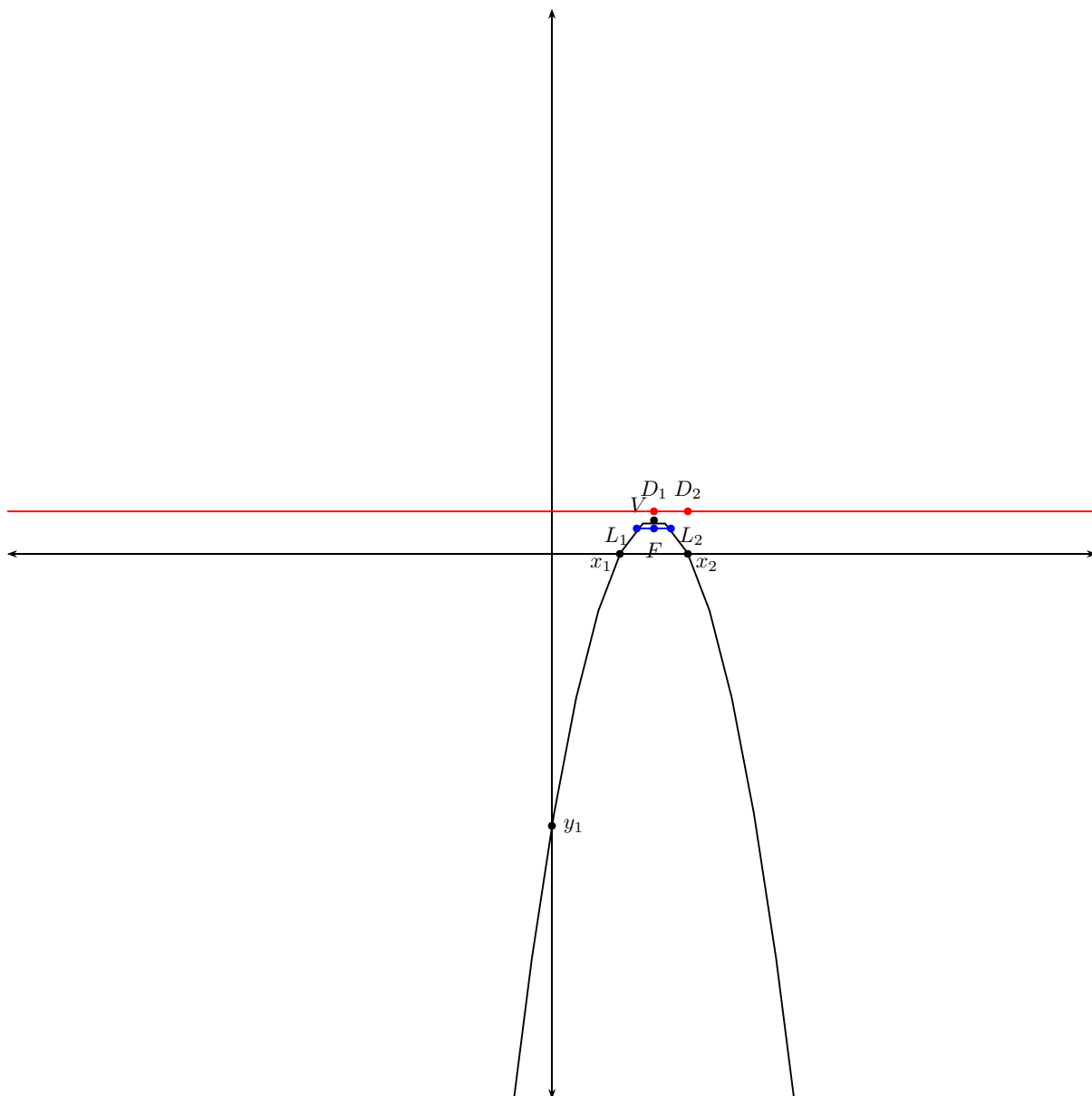


Figure 3. Graph of $(x - 3)^2 = 4(-\frac{1}{4})(y - 1)$ with x- and y-intercepts.