Exercise Solutions for Math 20

Factoring Polynomials and Simplifying Rational Expressions

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1 Factor the following completely.

1.1 $16x^4 - 1$

$\Rightarrow (4x^2 - 1)(4x^2 + 1)$	Factor using difference of two squares.
$\Rightarrow (2x-1)(2x+1)(4x^2+1)$	Factor using difference of two squares.
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1.2 $8j^3 - 125k^6$

$$\Rightarrow (2j-5k^2)(4j^2+10jk^2+25k^4)$$
 Factor using difference of two cubes.

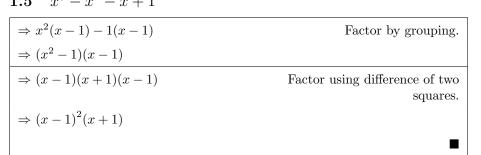
1.3 $s^2 + 7s + 10$

$\Rightarrow (s+2)(s+5)$	Factor by grouping.
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1.4 $4n^2 - 12n + 9$

$\Rightarrow 4n^2 - 6n - 6n + 9$	Factor by grouping.
$\Rightarrow 2n(2n-3) - 3(2n-3)$	
$\Rightarrow (2n-3)^2$	

1.5 $x^3 - x^2 - x + 1$



1.6 $48 - 13q - q^2$

$\Rightarrow -q^2 - 13q + 48$	Rewrite in standard form.
$\Rightarrow -(q^2 + 13q - 48)$	
$\Rightarrow -(q-3)(q+16)$	Factor by grouping.
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2 Reduce the following rational expressions to lowest terms.

2.1 $\frac{a^2-b^2}{a^3-b^3}$

$\Rightarrow \frac{(a-b)(a+b)}{a^3-b^3}$	Factor using difference of two
	squares.
$\Rightarrow \frac{(a-b)(a+b)}{(a-b)(a^2+ab+b^2)}$	Factor using difference of two cubes.
$\Rightarrow \frac{a+b}{a^2+ab+b^2}$	
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2.2 $\frac{x^3-x^2y+xy^2-y^3}{x^6+y^6}$

$$\Rightarrow \frac{x^2(x-y)+y^2(x-y)}{x^6+y^6}$$
 Factor by grouping.
$$\Rightarrow \frac{(x^2+y^2)(x-y)}{x^6+y^6}$$

$$\Rightarrow \frac{(x^2+y^2)(x-y)}{(x^2+y^2)(x^4-x^2y^2+y^4)}$$
 Factor using difference of two cubes.
$$\Rightarrow \frac{x-y}{x^4-x^2y^2+y^4}$$

3 Perform the following operations and simplify.

3.1 $\left(\frac{x}{x^2-1} - \frac{3}{x+1}\right) \div \frac{2x^2-x-3}{x^3-1}$

$$\Rightarrow \left(\frac{x}{(x-1)(x+1)} - \frac{3}{x+1}\right) \div \cdots \qquad \text{Factor using difference of two squares.}$$

$$\Rightarrow \left(\frac{x}{(x-1)(x+1)} - \frac{3(x-1)}{(x-1)(x+1)}\right) \div \cdots \qquad \text{LCM} = (x-1)(x+1)$$

$$\Rightarrow \frac{x-3(x-1)}{(x-1)(x+1)} \div \cdots$$

$$\Rightarrow \frac{x-3x+3}{(x-1)(x+1)} \div \cdots$$

$$\Rightarrow \frac{-2x+3}{(x-1)(x+1)} \div \cdots$$

$$\Rightarrow \cdots \div \frac{2x^2+2x-3x-3}{x^3-1} \qquad \text{Factor by grouping.}$$

$$\Rightarrow \cdots \div \frac{2x(x+1)-3(x+1)}{x^3-1}$$

$$\Rightarrow \cdots \div \frac{(2x-3)(x+1)}{x^3-1} \qquad \text{Factor using difference of two cubes.}$$

$$\Rightarrow \frac{-2x+3}{(x-1)(x+1)} \cdot \frac{(x-1)(x^2+x+1)}{(2x-3)(x+1)} \qquad a \div b = a \cdot \frac{1}{b}$$

$$\Rightarrow \frac{(-2x+3)(x^2+x+1)}{(2x-3)(x+1)^2}$$

$$\Rightarrow \frac{-(2x-3)(x^2+x+1)}{(2x-3)(x+1)^2}$$

$$\Rightarrow \frac{-(2x-3)(x^2+x+1)}{(2x-3)(x+1)^2}$$

3.2
$$\left(\frac{x}{x+y} + \frac{y}{x-y}\right) \cdot \frac{x^2 - xy}{x^4 - y^4} \div \frac{x}{x^2 + 2xy + y^2}$$