Exercise Solutions for Math 20

Equations in Quadratic Form and with Radicals and Absolute Values

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Contents

1 Solve for x

1.1 $\sqrt{2x+3} - \sqrt{x-2} = \sqrt{x+1}$

$\Rightarrow \left(\sqrt{2x+3} - \sqrt{x-2}\right)^2 = x+1$ Square both sides.

$\Rightarrow 2x+3-2\sqrt{2x+3}\sqrt{x-2}+x-2 = x+1$

$\Rightarrow 2x+3+x-2-x-1 = 2\sqrt{2x+3}\sqrt{x-2}$

$\Rightarrow 2x = 2\sqrt{2x+3}\sqrt{x-2}$

$\Rightarrow x = \sqrt{2x+3}\sqrt{x-2}$

$\Rightarrow x^2 = (2x+3)(x-2)$ Square both sides.

$\Rightarrow x^2 = 2x^2 - 4x + 3x - 6$

$\Rightarrow x^2 = 2x^2 - x - 6$

$\Rightarrow 2x^2 - x^2 - x - 6 = 0$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x-3)(x+2) = 0$ Factor by grouping.

$\Rightarrow x \subseteq \{-2, 3\}$

$\Rightarrow \sqrt{2(-2)+3} - \sqrt{-2-2} = \sqrt{-2+1}$ Verify $x = -2$

$\Rightarrow \sqrt{-4+3} - \sqrt{-2-2} = \sqrt{-2+1}$

$\Rightarrow \sqrt{-1} - \sqrt{-4} = \sqrt{-1}$

$\Rightarrow i - 2i = i$

$\Rightarrow -i = i$

$\Rightarrow x \neq -2$

$\Rightarrow \sqrt{2(3)+3} - \sqrt{3-2} = \sqrt{3+1}$ Verify $x = 3$

$\Rightarrow \sqrt{6+3} - \sqrt{3-2} = \sqrt{3+1}$

$\Rightarrow \sqrt{9} - \sqrt{1} = \sqrt{4}$

$\Rightarrow 3 - 1 = 2$

$\Rightarrow 2 = 2$

$\Rightarrow x = 3$

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1.2 $1 = x + \sqrt{2x - 3}$

$\Rightarrow 1 - x = \sqrt{2x - 3}$	Isolate the root.
$\Rightarrow (1 - x)^2 = 2x - 3$	Square both sides.
$\Rightarrow 1 - 2x + x^2 = 2x - 3$	
$\Rightarrow 1 - 2x + x^2 - 2x + 3 = 0$	
$\Rightarrow x^2 - 4x + 4 = 0$	
$\Rightarrow (x - 2)^2$	Factor by grouping.
$\Rightarrow x = 2$	
$\Rightarrow 1 = 2 + \sqrt{2(2) - 3}$	Verify $x = 2$
$\Rightarrow 1 = 2 + \sqrt{4 - 3}$	
$\Rightarrow 1 = 2 + \sqrt{1}$	
$\Rightarrow 1 = 2 + 1$	
$\Rightarrow 1 = 3$	
$\Rightarrow x \neq 2$	
$\Rightarrow x \in \emptyset$	Final answer.
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1.3 $\left|\frac{3x-4}{2x+3}\right| = 1$

$\Rightarrow \frac{3x-4}{2x+3} = -1$	$\lvert a\rvert = b \Rightarrow a = \pm b$. Solve for $a = -b$
$\Rightarrow \frac{3x-4}{2x+3} = -\frac{2x+3}{2x+3}$	
$\Rightarrow 3x - 4 = -(2x + 3)$	Eliminate denominator. $x = -\frac{3}{2}$ is an undefined point.
$\Rightarrow 3x - 4 = -2x - 3$	
$\Rightarrow 3x + 2x = -3 + 4$	
$\Rightarrow 5x = 1$	
$\Rightarrow x = \frac{1}{5}$	
$\Rightarrow \frac{3x-4}{2x+3} = 1$	$\lvert a\rvert = b \Rightarrow a = \pm b$. Solve for $a = +b$
$\Rightarrow \frac{3x-4}{2x+3} = \frac{2x+3}{2x+3}$	
$\Rightarrow 3x - 4 = 2x + 3$	Eliminate denominator. $x = -\frac{3}{2}$ is an undefined point.
$\Rightarrow 3x - 2x = 3 + 4$	
$\Rightarrow x = 7$	
$\Rightarrow x \in \{\frac{1}{5}, 7\}$	Final answer.

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