Exercise Solutions for Math 20

Lines and Circles

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1.1 Find the value of k such that the lines with equations $3x + 2y - 4 = 0$ and $kx - 3y + 8$ are:

1.1.a Parallel.

$\Rightarrow 2y = -3x + 4$ $\Rightarrow y = -\frac{3}{2}x + 4$	Rewrite the first equation in slope-intercept form.
$\Rightarrow -3y = -kx - 8$ $\Rightarrow 3y = kx + 8$ $\Rightarrow y = \frac{k}{3}x + \frac{8}{3}$	Rewrite the second equation in slope-intercept form.
$\Rightarrow \frac{k}{3} = -\frac{3}{2}$	Parallel slopes are equal.
$\Rightarrow k = -\frac{9}{2}$	Final answer. ∎

1.1.b Perpendicular.

$\Rightarrow \frac{k}{3} = -\frac{1}{-\frac{3}{2}}$ $\Rightarrow \frac{k}{3} = \frac{2}{3}$	Perpendicular slopes are the negative reciprocal of each other.
$\Rightarrow k = 2$	Final answer. ∎

1.2 Line l is perpendicular to the line segment with endpoints $P(-4, 7)$ and $Q(2, -3)$. If l passes through the midpoint of the line segment $\overline{PQ}$, find an equation for l in slope-intercept form.

$\Rightarrow m = \frac{-3-7}{2+4}$ $\Rightarrow m = \frac{-10}{6}$ $\Rightarrow m = -\frac{5}{3}$	Find the slope of $\overline{PQ}$.
$\Rightarrow M = (\frac{-4+2}{2}, \frac{7-3}{2})$ $\Rightarrow M = (\frac{-2}{2}, \frac{4}{2})$ $\Rightarrow M = (-1, 2)$	Find the midpoint of $\overline{PQ}$.
$\Rightarrow y - 2 = -\frac{5}{3}(x + 1)$ $\Rightarrow y - 2 = -\frac{5}{3}x - \frac{5}{3}$ $\Rightarrow y = -\frac{5}{3}x - \frac{5}{3} + 2$ $\Rightarrow y = -\frac{5}{3}x - \frac{5}{3} + \frac{6}{3}$	Use the point-slope formula.
$\Rightarrow y = -\frac{5}{3}x - \frac{1}{3}$	Final answer. ∎

1.3 Find a general equation of the line that is parallel to the line with equation $3x - y + 1 = 0$ and whose x-intercept is also the x-intercept of the line with equation $2x - 3y + 6 = 0$

$\Rightarrow -y = -3x - 1$	Find the slope of the first equation.
$\Rightarrow y = 3x + 1$	
$\Rightarrow m = 3$	
$\Rightarrow 2x - 3(0) + 6 = 0$	Find the x-intercept of the second equation.
$\Rightarrow 2x + 6 = 0$	
$\Rightarrow 2x = -6$	
$\Rightarrow x = -3$	
$\Rightarrow y - 0 = 3(x + 3)$	Use the point-slope formula.
$\Rightarrow y = 3x + 9$	Final answer.
	∎

1.4 From the following equations, determine the center and radius of the circle if it exists.

1.4.a $(x + 1)^2 + (y + 3)^2 = 5$

$\Rightarrow (h, k) = (-1, -3), r = \sqrt{5}$	Final answer.
	∎

1.4.b $x^2 + y^2 + 8x + 7 = 0$

$\Rightarrow x^2 + y^2 + 8x = -7$	Isolate the constants.
$\Rightarrow x^2 + 8x + 16 + y^2 = -7 + 16$	Complete the square.
$\Rightarrow (x + 4)^2 + y^2 = 9$	
$\Rightarrow (x + 4)^2 + y^2 = 3^2$	
$\Rightarrow (h, k) = (-4, 0), r = 3$	Final answer.
	∎

1.4.c $x^2 + y^2 - 6x + 13 = 0$

$\Rightarrow x^2 + y^2 - 6x = -13$	Isolate the constants.
$\Rightarrow x^2 - 6x + 9 + y^2 = -13 + 9$	Complete the square.
$\Rightarrow (x - 3)^2 + y^2 = -4$	
$\Rightarrow \emptyset$	Final answer. Since the radius is imaginary, the circle does not exist.
	∎