# Exercise Solutions for Math 20

Factoring Polynomials and Simplifying Rational Expressions

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# 1 Factor the following completely.

## 1.1 $16x^4 - 1$

| | |
|---|---|
| $\Rightarrow (4x^2 - 1)(4x^2 + 1)$ | Factor using difference of two squares. |
| $\Rightarrow (2x - 1)(2x + 1)(4x^2 + 1)$ | Factor using difference of two squares. |

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## 1.2 $8j^3 - 125k^6$

| | |
|---|---|
| $\Rightarrow (2j - 5k^2)(4j^2 + 10jk^2 + 25k^4)$ | Factor using difference of two cubes. |

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## 1.3 $s^2 + 7s + 10$

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|---|---|
| $\Rightarrow (s + 2)(s + 5)$ | Factor by grouping. |

■

## 1.4 $4n^2 - 12n + 9$

| | |
|---|---|
| $\Rightarrow 4n^2 - 6n - 6n + 9$ | Factor by grouping. |
| $\Rightarrow 2n(2n - 3) - 3(2n - 3)$ | |
| $\Rightarrow (2n - 3)^2$ | |

■

## 1.5 $x^3 - x^2 - x + 1$

| | |
|---|---|
| $\Rightarrow x^2(x - 1) - 1(x - 1)$ | Factor by grouping. |
| $\Rightarrow (x^2 - 1)(x - 1)$ | |
| $\Rightarrow (x - 1)(x + 1)(x - 1)$ | Factor using difference of two squares. |
| $\Rightarrow (x - 1)^2(x + 1)$ | |

■

## 1.6 $48 - 13q - q^2$

| | |
|---|---|
| $\Rightarrow -q^2 - 13q + 48$ | Rewrite in standard form. |
| $\Rightarrow -(q^2 + 13q - 48)$ | |
| $\Rightarrow -(q - 3)(q + 16)$ | Factor by grouping. |

■

## 2 Reduce the following rational expressions to lowest terms.

### 2.1 $\frac{a^2-b^2}{a^3-b^3}$

| | |
|---|---|
| $\Rightarrow \frac{(a-b)(a+b)}{a^3-b^3}$ | Factor using difference of two squares. |
| $\Rightarrow \frac{(a-b)(a+b)}{(a-b)(a^2+ab+b^2)}$ | Factor using difference of two cubes. |
| $\Rightarrow \frac{a+b}{a^2+ab+b^2}$ | |
| ■ | |

### 2.2 $\frac{x^3-x^2y+xy^2-y^3}{x^6+y^6}$

| | |
|---|---|
| $\Rightarrow \frac{x^2(x-y)+y^2(x-y)}{x^6+y^6}$ | Factor by grouping. |
| $\Rightarrow \frac{(x^2+y^2)(x-y)}{x^6+y^6}$ | |
| $\Rightarrow \frac{(x^2+y^2)(x-y)}{(x^2+y^2)(x^4-x^2y^22+y^4)}$ | Factor using difference of two cubes. |
| $\Rightarrow \frac{x-y}{x^4-x^2y^2+y^4}$ | |
| ■ | |

# 3 Perform the following operations and simplify.

## 3.1 $\left(\frac{x}{x^2-1} - \frac{3}{x+1}\right) \div \frac{2x^2-x-3}{x^3-1}$

| | |
|---|---|
| $\Rightarrow \left(\frac{x}{(x-1)(x+1)} - \frac{3}{x+1}\right) \div \frac{2x^2-x-3}{x^3-1}$ | Factor using difference of two squares. |
| $\Rightarrow \left(\frac{x}{(x-1)(x+1)} - \frac{3(x-1)}{(x-1)(x+1)}\right) \div \frac{2x^2-x-3}{x^3-1}$<br>$\Rightarrow \frac{x-3(x-1)}{(x-1)(x+1)} \div \frac{2x^2-x-3}{x^3-1}$<br>$\Rightarrow \frac{x-3x+3}{(x-1)(x+1)} \div \frac{2x^2-x-3}{x^3-1}$<br>$\Rightarrow \frac{-2x+3}{(x-1)(x+1)} \div \frac{2x^2-x-3}{x^3-1}$ | LCM $= (x-1)(x+1)$ |
| $\Rightarrow \frac{-2x+3}{(x-1)(x+1)} \div \frac{2x^2+2x-3x-3}{x^3-1}$<br>$\Rightarrow \frac{-2x+3}{(x-1)(x+1)} \div \frac{2x(x+1)-3(x+1)}{x^3-1}$<br>$\Rightarrow \frac{-2x+3}{(x-1)(x+1)} \div \frac{(2x-3)(x+1)}{x^3-1}$ | Factor by grouping. |
| $\Rightarrow \frac{-2x+3}{(x-1)(x+1)} \div \frac{(2x-3)(x+1)}{(x-1)(x^2+x+1)}$ | Factor using difference of two cubes. |
| $\Rightarrow \frac{-2x+3}{(x-1)(x+1)} \cdot \frac{(x-1)(x^2+x+1)}{(2x-3)(x+1)}$<br>$\Rightarrow \frac{(-2x+3)(x^2+x+1)}{(2x-3)(x+1)^2}$<br>$\Rightarrow \frac{-(2x-3)(x^2+x+1)}{(2x-3)(x+1)^2}$<br>$\Rightarrow -\frac{x^2+x+1}{(x+1)^2}$<br>■ | $a \div b = a \cdot \frac{1}{b}$ |

## 3.2 $\left(\frac{x}{x+y} + \frac{y}{x-y}\right) \cdot \frac{x^2-xy}{x^4-y^4} \div \frac{x}{x^2+2xy+y^2}$

| | |
|---|---|
| $\Rightarrow \left(\frac{x(x-y)}{(x-y)(x+y)} + \frac{y(x+y)}{(x-y)(x+y)}\right) \cdot \frac{x^2-xy}{x^4-y^4} \div \frac{x}{x^2+2xy+y^2}$<br>$\Rightarrow \frac{x(x-y)+y(x+y)}{(x-y)(x+y)} \cdot \frac{x^2-xy}{x^4-y^4} \div \frac{x}{x^2+2xy+y^2}$<br>$\Rightarrow \frac{x^2-xy+xy+y^2}{(x-y)(x+y)} \cdot \frac{x^2-xy}{x^4-y^4} \div \frac{x}{x^2+2xy+y^2}$<br>$\Rightarrow \frac{x^2+y^2}{(x-y)(x+y)} \cdot \frac{x^2-xy}{x^4-y^4} \div \frac{x}{x^2+2xy+y^2}$ | LCM $= (x-y)(x+y)$ |
| $\Rightarrow \frac{x^2+y^2}{(x-y)(x+y)} \cdot \frac{x^2-xy}{(x^2-y^2)(x^2+y^2)} \div \frac{x}{x^2+2xy+y^2}$ | Factor using difference of two squares. |
| $\Rightarrow \frac{x^2+y^2}{(x-y)(x+y)} \cdot \frac{x^2-xy}{(x-y)(x+y)(x^2+y^2)} \div \frac{x}{x^2+2xy+y^2}$<br>$\Rightarrow \frac{x^2+y^2}{(x-y)(x+y)} \cdot \frac{x(x-y)}{(x-y)(x+y)(x^2+y^2)} \div \frac{x}{x^2+2xy+y^2}$<br>$\Rightarrow \frac{x}{(x-y)(x+y)^2} \div \frac{x}{x^2+2xy+y^2}$ | Factor using difference of two squares. |
| $\Rightarrow \frac{x}{(x-y)(x+y)^2} \div \frac{x}{(x+y)^2}$ | Factor using perfect square trinomial. |
| $\Rightarrow \frac{x}{(x-y)(x+y)^2} \div \frac{(x+y)^2}{x}$<br>$\Rightarrow \frac{x}{(x-y)(x+y)^2} \cdot \frac{(x+y)^2}{x}$<br>$\Rightarrow \frac{1}{x-y}$<br>■ | $a \div b = a \cdot \frac{1}{b}$ |

**3.3** $\dfrac{\frac{3}{p+q}+\frac{1}{p-2q}}{1+\frac{p-q}{p-2q}}$

| | |
|---|---|
| $\Rightarrow \dfrac{\frac{3(p-2q)}{(p+q)(p-2q)}+\frac{p+q}{(p+q)(p-2q)}}{1+\frac{p-q}{p-2q}}$<br>$\Rightarrow \dfrac{\frac{3(p-2q)+p+q}{(p+q)(p-2q)}}{1+\frac{p-q}{p-2q}}$<br>$\Rightarrow \dfrac{\frac{4p-5q}{(p+q)(p-2q)}}{1+\frac{p-q}{p-2q}}$ | $\text{LCM} = (p+q)(p-2q)$ |
| $\Rightarrow \dfrac{\frac{4p-5q}{(p+q)(p-2q)}}{\frac{p-2q}{p-2q}+\frac{p-q}{p-2q}}$<br>$\Rightarrow \dfrac{\frac{4p-5q}{(p+q)(p-2q)}}{\frac{p-2q+p-q}{p-2q}}$<br>$\Rightarrow \dfrac{\frac{4p-5q}{(p+q)(p-2q)}}{\frac{2p-3q}{p-2q}}$ | $\frac{a}{a} = 1$ |
| $\Rightarrow \dfrac{4p-5q}{(p+q)(p-2q)} \cdot \dfrac{p-2q}{2p-3q}$<br>$\Rightarrow \dfrac{4p-5q}{(p+q)(2p-3q)}$<br>■ | $a \div b = a \cdot \frac{1}{b}$ |