Exercise Solutions for Math 20

Functions, Graphs, Symmetry

Nile Jocson <novoseiversia@gmail.com>

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1.1 Transform each of the following real-valued functions to a functional notation $f(x)$; find its domain and range; and compute $f(1)$.

1.1.a f assigns the number 10 to any one-digit integer.

$\Rightarrow f(x) = 10$	Final answer.
$\Rightarrow \text{dom}(f) = [-9, 9]$	The domain is the set of one digit integers.
$\Rightarrow \text{ran}(f) = 10$	The function can only equate to 10.
$\Rightarrow f(1) = 10$	
	■

1.1.b f maps a given real number to the real number that is 4 more than its square root.

$\Rightarrow f(x) = \sqrt{x} + 4$	Final answer. Assuming that only the principal root is needed.
$\Rightarrow \text{dom}(f) = [0, +\infty)$	The square root of a negative number cannot be real.
$\Rightarrow \text{ran}(f) = [4, +\infty]$	
$\Rightarrow f(1) \in \{3, 5\}$	
	■

1.1.c $\{(x, y) \mid y = \frac{1}{6-x}\}$

$\Rightarrow f(x) = \frac{1}{6-x}$	Final answer.
$\Rightarrow \text{dom}(f) = \mathbb{R} \setminus \{6\}$	$x = 6$ is an undefined point.
$\Rightarrow \text{ran}(f) = \mathbb{R}$	
$\Rightarrow f(1) = \frac{1}{5}$	
	■

1.1.d $\{(x, y) \mid y = \sqrt{x^2 - 5x + 6}\}$

$\Rightarrow (x-2)(x-3) \geq 0$	Find the domain by solving $x^2 - 5x + 6 \geq 0$; factor by grouping.
	Create a table of signs.

	2	3	
$x - 2$	$-$	$+$	$+$
$x - 3$	$-$	$-$	$+$
$(x-2)(x-3)$	$+$	$-$	$+$

$\Rightarrow x \in (-\infty, 2] \cup [3, +\infty)$	
$\Rightarrow f(x) = \sqrt{x^2 - 5x + 6}$	Final answer.

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$\Rightarrow \text{dom}(f) = (-\infty, 2] \cup [3, +\infty)$ $\Rightarrow \text{ran}(f) = [0, +\infty)$ $\Rightarrow f(1) = \sqrt{2}$ ■

1.1.e $\{(x, y) \mid y = \frac{1}{\sqrt[3]{x^2-1}}\}$

$\Rightarrow (x-1)(x+1) = 0$ $\Rightarrow x \in \{-1, 1\}$	Find undefined points by solving $x^2 - 1 = 0$; factor using difference of two squares.
$\Rightarrow x = \frac{1}{\sqrt[3]{y^2-1}}$ $\Rightarrow \frac{1}{x} = \sqrt[3]{y^2-1}$ $\Rightarrow \frac{1}{x^3} = y^2 - 1$ $\Rightarrow y^2 = \frac{1}{x^3} + 1$ $\Rightarrow y^2 = \frac{1}{x^3} + \frac{x^3}{x^3}$ $\Rightarrow y^2 = \frac{1+x^3}{x^3}$ $\Rightarrow f'(x) = \pm\sqrt{\frac{1+x^3}{x^3}}$	Find the inverse function.
$\Rightarrow \text{dom}(f') = (-\infty, -1] \cup (0, +\infty)$	Find the domain of the inverse function; 0 is also an undefined point.
$\Rightarrow f(x) = \frac{1}{\sqrt[3]{x^2-1}}$ $\Rightarrow \text{dom}(f) = \mathbb{R} \setminus \{-1, 1\}$ $\Rightarrow \text{ran}(f) = (-\infty, -1] \cup (0, +\infty)$ $\Rightarrow f(1)$ is undefined. ■	Final answer.