Exercise Solutions for Math 20

Equations in Quadratic Form and with Radicals and Absolute Values

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Contents

1 Solve for x

1.1 $\sqrt{2x+3} - \sqrt{x-2} = \sqrt{x+1}$

$\Rightarrow \left(\sqrt{2x+3} - \sqrt{x-2}\right)^2 = x+1$ Square both sides.

$\Rightarrow 2x+3-2\sqrt{2x+3}\sqrt{x-2}+x-2 = x+1$

$\Rightarrow 2x+3+x-2-x-1 = 2\sqrt{2x+3}\sqrt{x-2}$

$\Rightarrow 2x = 2\sqrt{2x+3}\sqrt{x-2}$

$\Rightarrow x = \sqrt{2x+3}\sqrt{x-2}$

$\Rightarrow x^2 = (2x+3)(x-2)$ Square both sides.

$\Rightarrow x^2 = 2x^2 - 4x + 3x - 6$

$\Rightarrow x^2 = 2x^2 - x - 6$

$\Rightarrow 2x^2 - x^2 - x - 6 = 0$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x-3)(x+2) = 0$ Factor by grouping.

$\Rightarrow x \subseteq \{-2, 3\}$

$\Rightarrow \sqrt{2(-2)+3} - \sqrt{-2-2} = \sqrt{-2+1}$ Verify $x = -2$

$\Rightarrow \sqrt{-4+3} - \sqrt{-2-2} = \sqrt{-2+1}$

$\Rightarrow \sqrt{-1} - \sqrt{-4} = \sqrt{-1}$

$\Rightarrow i - 2i = i$

$\Rightarrow -i = i$

$\Rightarrow x \neq -2$

$\Rightarrow \sqrt{2(3)+3} - \sqrt{3-2} = \sqrt{3+1}$ Verify $x = 3$

$\Rightarrow \sqrt{6+3} - \sqrt{3-2} = \sqrt{3+1}$

$\Rightarrow \sqrt{9} - \sqrt{1} = \sqrt{4}$

$\Rightarrow 3 - 1 = 2$

$\Rightarrow 2 = 2$

$\Rightarrow x = 3$

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