

Exercise Solutions for Math 20

Factoring Polynomials and Simplifying Rational Expressions

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1 Factor the following completely.

1.1 $16x^4 - 1$

$\Rightarrow (4x^2 - 1)(4x^2 + 1)$	Factor using difference of two squares.
$\Rightarrow (2x - 1)(2x + 1)(4x^2 + 1)$	Factor using difference of two squares.

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1.2 $8j^3 - 125k^6$

$\Rightarrow (2j - 5k^2)(4j^2 + 10jk^2 + 25k^4)$	Factor using difference of two cubes.
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1.3 $s^2 + 7s + 10$

$\Rightarrow (s + 2)(s + 5)$	Factor by grouping.
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1.4 $4n^2 - 12n + 9$

$\Rightarrow 4n^2 - 6n - 6n + 9$	Factor by grouping.
$\Rightarrow 2n(2n - 3) - 3(2n - 3)$	
$\Rightarrow (2n - 3)^2$	

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1.5 $x^3 - x^2 - x + 1$

$\Rightarrow x^2(x - 1) - 1(x - 1)$	Factor by grouping.
$\Rightarrow (x^2 - 1)(x - 1)$	
$\Rightarrow (x - 1)(x + 1)(x - 1)$	Factor using difference of two squares.
$\Rightarrow (x - 1)^2(x + 1)$	

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1.6 $48 - 13q - q^2$

$\Rightarrow -q^2 - 13q + 48$	Rewrite in standard form.
$\Rightarrow -(q^2 + 13q - 48)$	
$\Rightarrow -(q - 3)(q + 16)$	Factor by grouping.

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2 Reduce the following rational expressions to lowest terms.

2.1 $\frac{a^2-b^2}{a^3-b^3}$

$\Rightarrow \frac{(a-b)(a+b)}{a^3-b^3}$	Factor using difference of two squares.
$\Rightarrow \frac{(a-b)(a+b)}{(a-b)(a^2+ab+b^2)}$	Factor using difference of two cubes.
$\Rightarrow \frac{a+b}{a^2+ab+b^2}$	■