Exercise Solutions for Math 20

The 2-Dimensional Coordinate System

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1.1 Given A(4,2), B(6,-4) and C(2,-7), find the distance between C and the midpoint M of \overline{AB} .

$$\Rightarrow M = (\frac{4+6}{2}, \frac{2-4}{2})$$
 Use the midpoint formula.
$$\Rightarrow M = (\frac{10}{2}, \frac{-2}{2})$$

$$\Rightarrow M = (5, -1)$$
 Use the distance formula.
$$\Rightarrow d_{CM} = \sqrt{(5-2)^2 + (-7+1)^2}$$
 Use the distance formula.
$$\Rightarrow d_{CM} = \sqrt{(3)^2 + (-6)^2}$$

$$\Rightarrow d_{CM} = \sqrt{9+36}$$

$$\Rightarrow d_{CM} = \sqrt{45}$$

$$\Rightarrow d_{CM} = \sqrt{9}\sqrt{5}$$
 Final answer.
$$\Rightarrow d_{CM} = 3\sqrt{5}$$

1.2 Solve algebraically for the x- and y-intercepts of the graphs of the following equations.

1.2.a $y^2 = x - 2$

$\Rightarrow 0 = x - 2$	Find the x-intercepts.
$\Rightarrow x_i = 2$	
$\Rightarrow y^2 = 0 - 2$	Find the y-intercepts.
$\Rightarrow y^2 = -2$	
$\Rightarrow y_i \in \emptyset$	No y-intercepts. The square of a real number cannot be negative.
$\Rightarrow x_i = 2, y_i \in \emptyset$	Final answer.

1.2.b $y = \frac{4x^2 + 9}{x^2 - 9}$

$$\Rightarrow \frac{4x^2+9}{x^2-9}=0 \qquad \qquad \text{Find the x-intercepts.}$$

$$\Rightarrow \frac{4x^2+9}{(x-3)(x+3)}=0 \qquad \qquad \text{Factor using difference of two squares.}$$

$$\Rightarrow 4x^2+9=0 \qquad \qquad x\in \{-3,3\} \text{ are undefined points.}$$

$$\Rightarrow \frac{\pm\sqrt{-4(4)(9)}}{2(4)} \qquad \qquad \text{Use the quadratic formula.}$$

$$\Rightarrow \frac{\pm\sqrt{-144}}{8}$$

$$\Rightarrow x_i \in \emptyset \qquad \qquad \text{No x-intercepts. The square root of a negative number is imaginary.}$$

$$\Rightarrow y = \frac{4(0)^2+9}{0^2-9} \qquad \qquad \text{Find the y-intercepts.}$$

$$\Rightarrow y = \frac{9}{-9}$$

$$\Rightarrow y_i = -1$$

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$$\Rightarrow x_i \in \emptyset, y_i = -1$$
 Final answer.

1.2.c y = |x - 2| - 2

$\Rightarrow x-2 - 2 = 0$	Find the x-intercepts.
$\Rightarrow x-2 = 2$	
$\Rightarrow x - 2 = 2$	$ a = b \Rightarrow a = \pm b$. Solve for $a = b$.
$\Rightarrow x_i = 4$	
$\Rightarrow x - 2 = -2$	$ a = b \Rightarrow a = \pm b$. Solve for $a = -b$.
$\Rightarrow x_i = 0$	
$\Rightarrow y = 0 - 2 - 2$	Find the y-intercepts.
$\Rightarrow y = -2 - 2$	
$\Rightarrow y = 2 - 2$	
$\Rightarrow y_i = 0$	
$\Rightarrow x_i \in \{0,4\}, y_i = 0$	Final answer.

1.2.d $y = x^2 + 1$

$\Rightarrow x^2 + 1 = 0$	Find the x-intercepts.
$\Rightarrow \frac{\pm\sqrt{-(1)(1)}}{2(1)}$	Use the quadratic equation.
$\Rightarrow \frac{\pm\sqrt{-1}}{2}$	
$\Rightarrow x_i \in \emptyset$	No x-intercepts. The square root of a negative number is imaginary.
$\Rightarrow y = 0^2 + 1$	Find the y-intercepts.
$\Rightarrow y_i = 1$	
$\Rightarrow x_i \in \emptyset, y_i = 1$	Final answer.

1.2.e $x^2 + y^2 = 25$

$\Rightarrow x^2 + 0^2 = 25$	Find the x-intercepts.
$\Rightarrow x^2 = 25$	
$\Rightarrow x = \pm 5$	
$\Rightarrow x_i \in \{-5, 5\}$	
$\Rightarrow 0^2 + y^2 = 25$	Find the y-intercepts.
$\Rightarrow y^2 = 25$	
$\Rightarrow y = \pm 5$	
$\Rightarrow y_i \in \{-5, 5\}$	
$\Rightarrow x_i \in \{-5, 5\}, y_i \in \{-5, 5\}$	Final answer.