# Exercise Solutions for Math 20

Conics (Parabola and Ellipse)

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# Contents

# 1

## 1.1 Determine the vertex and orientation of the following parabolas.

**1.1.a** $4y^2 + 4y + x = 2$

| | |
|---|---|
| $\Rightarrow 4y^2 + 4y = -x + 2$ | Isolate $y$. |
| $\Rightarrow y^2 + y = -\frac{x}{4} + \frac{2}{4}$ | |
| $\Rightarrow y^2 + y = -\frac{x}{4} + \frac{1}{2}$ | |
| $\Rightarrow y^2 + y + \frac{1}{4} = -\frac{x}{4} + \frac{1}{2} + \frac{1}{4}$ | Complete the square. |
| $\Rightarrow (y + \frac{1}{2})^2 = -\frac{x}{4} + \frac{3}{4}$ | |
| $\Rightarrow (y + \frac{1}{2})^2 = -\frac{1}{4}(x - 3)$ | |
| $\Rightarrow (y + \frac{1}{2})^2 = 4(-\frac{1}{16})(x - 3)$ | |
| $\Rightarrow$ Opening leftwards, $(h, k) = (3, -\frac{1}{2})$ | Final answer. |
| | ■ |

**1.1.b** $x^2 - 6x - 2y = 7$

| | |
|---|---|
| $\Rightarrow x^2 - 6x = 2y + 7$ | Isolate $x$. |
| $\Rightarrow x^2 - 6x + 9 = 2y + 7 + 9$ | Complete the square. |
| $\Rightarrow (x - 3)^2 = 2y + 16$ | |
| $\Rightarrow (x - 3)^2 = 2(y + 8)$ | |
| $\Rightarrow (x - 3)^2 = 4(\frac{1}{2})(y + 8)$ | |
| $\Rightarrow$ Opening upwards, $(h, k) = (3, -8)$ | Final answer. |
| | ■ |

**1.1.c** $2y^2 - 6y - 9x = 0$

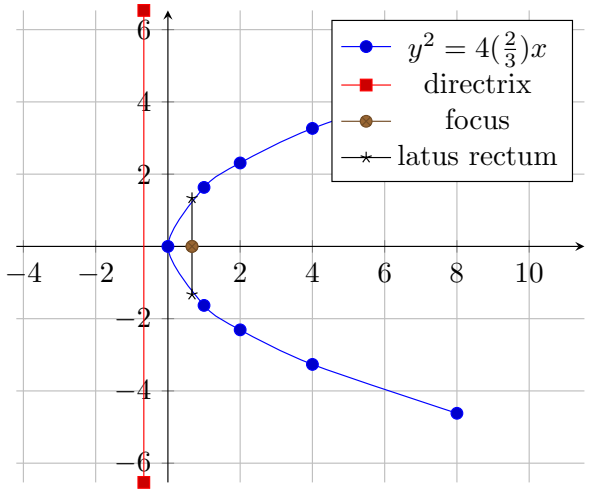
| | |
|---|---|
| $\Rightarrow 2y^2 - 6y = 9x$ | Isolate $y$. |
| $\Rightarrow y^2 - 3y = \frac{9}{2}x$ | |
| $\Rightarrow y^2 - 3y + \frac{9}{4} = \frac{9}{2}x + \frac{9}{4}$ | Complete the square. |
| $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}x + \frac{9}{4}$ | |
| $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{9}{4} \cdot \frac{2}{9})$ | |
| $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{18}{36})$ | |
| $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{1}{2})$ | |
| $\Rightarrow (y - \frac{3}{2})^2 = 4(\frac{9}{8})(x + \frac{1}{2})$ | |
| $\Rightarrow$ Opening rightwards, $(h, k) = (-\frac{1}{2}, \frac{3}{2})$ | Final answer. |
| | ■ |

## 1.2 Sketch the graph of the following parabolas.

**1.2.a** $3y^2 = 8x$

$\Rightarrow y^2 = 4(\frac{2}{3})x$ Rewrite in standard form.

Graph the equation.

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