

# Exercise Solutions for Math 20

## The 2-Dimensional Coordinate System

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# 1

**1.1** Given  $A(4, 2)$ ,  $B(6, -4)$  and  $C(2, -7)$ , find the distance between C and the midpoint  $M$  of  $\overline{AB}$ .

$\Rightarrow M = (\frac{4+6}{2}, \frac{2-4}{2})$	Use the midpoint formula.
$\Rightarrow M = (\frac{10}{2}, \frac{-2}{2})$	
$\Rightarrow M = (5, -1)$	
$\Rightarrow d_{CM} = \sqrt{(5-2)^2 + (-7+1)^2}$	Use the distance formula.
$\Rightarrow d_{CM} = \sqrt{(3)^2 + (-6)^2}$	
$\Rightarrow d_{CM} = \sqrt{9+36}$	
$\Rightarrow d_{CM} = \sqrt{45}$	
$\Rightarrow d_{CM} = \sqrt{9}\sqrt{5}$	
$\Rightarrow d_{CM} = 3\sqrt{5}$	Final answer. <span style="float: right;">■</span>

**1.2** Solve algebraically for the x- and y-intercepts of the graphs of the following equations.

**1.2.a**  $y^2 = x - 2$

$\Rightarrow 0 = x - 2$	Find the x-intercepts.
$\Rightarrow x_i = 2$	
$\Rightarrow y^2 = 0 - 2$	Find the y-intercepts.
$\Rightarrow y^2 = -2$	
$\Rightarrow y_i \in \emptyset$	No y-intercepts. The square of a real number cannot be negative.
$\Rightarrow x_i = 2, y_i \in \emptyset$	Final answer. <span style="float: right;">■</span>

**1.2.b**  $y = \frac{4x^2+9}{x^2-9}$

$\Rightarrow \frac{4x^2+9}{x^2-9} = 0$	Find the x-intercepts.
$\Rightarrow \frac{4x^2+9}{(x-3)(x+3)} = 0$	Factor using difference of two squares.
$\Rightarrow 4x^2 + 9 = 0$	$x \in \{-3, 3\}$ are undefined points.
$\Rightarrow \frac{\pm\sqrt{-4(4)(9)}}{2(4)}$	Use the quadratic formula.
$\Rightarrow \frac{\pm\sqrt{-144}}{8}$	
$\Rightarrow x_i \in \emptyset$	No x-intercepts. The square root of a negative number is imaginary.
$\Rightarrow y = \frac{4(0)^2+9}{0^2-9}$	Find the y-intercepts.
$\Rightarrow y = \frac{9}{-9}$	
$\Rightarrow y_i = -1$	

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$$\Rightarrow x_i \in \emptyset, y_i = -1$$

Final answer.



**1.2.c**  $y = |x - 2| - 2$

$$\Rightarrow |x - 2| - 2 = 0$$

Find the x-intercepts.

$$\Rightarrow |x - 2| = 2$$

$$\Rightarrow x - 2 = 2$$

$|a| = b \Rightarrow a = \pm b$ . Solve for  $a = b$ .

$$\Rightarrow x_i = 4$$

$$\Rightarrow x - 2 = -2$$

$|a| = b \Rightarrow a = \pm b$ . Solve for  $a = -b$ .

$$\Rightarrow x_i = 0$$

$$\Rightarrow y = |0 - 2| - 2$$

Find the y-intercepts.

$$\Rightarrow y = |-2| - 2$$

$$\Rightarrow y = 2 - 2$$

$$\Rightarrow y_i = 0$$

$$\Rightarrow x_i \in \{0, 4\}, y_i = 0$$

Final answer.



**1.2.d**  $y = x^2 + 1$

$$\Rightarrow x^2 + 1 = 0$$

Find the x-intercepts.

$$\Rightarrow \frac{\pm\sqrt{-(-1)(1)}}{2(1)}$$

Use the quadratic equation.

$$\Rightarrow \frac{\pm\sqrt{-1}}{2}$$

$$\Rightarrow x_i \in \emptyset$$

No x-intercepts. The square root of a negative number is imaginary.

$$\Rightarrow y = 0^2 + 1$$

Find the y-intercepts.

$$\Rightarrow y_i = 1$$

$$\Rightarrow x_i \in \emptyset, y_i = 1$$

Final answer.



**1.2.e**  $x^2 + y^2 = 25$

$$\Rightarrow x^2 + 0^2 = 25$$

Find the x-intercepts.

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = \pm 5$$

$$\Rightarrow x_i \in \{-5, 5\}$$

$$\Rightarrow 0^2 + y^2 = 25$$

Find the y-intercepts.

$$\Rightarrow y^2 = 25$$

$$\Rightarrow y = \pm 5$$

$$\Rightarrow y_i \in \{-5, 5\}$$

$$\Rightarrow x_i \in \{-5, 5\}, y_i \in \{-5, 5\}$$

Final answer.



**1.2.f**  $x = \frac{y^2}{y^4-4}$

$\Rightarrow x = \frac{0^2}{0^2-4}$ $\Rightarrow x_i = 0$	Find the x-intercepts.
$\Rightarrow \frac{y^2}{y^4-4} = 0$ $\Rightarrow \frac{y^2}{(y^2-2)(y^2+2)} = 0$ $\Rightarrow y^2 = 0$ $\Rightarrow y_i = 0$	Find the y-intercepts.  $y \in \{-\sqrt{2}, \sqrt{2}\}$ are undefined points. $0^2 = 0$
$\Rightarrow x_i = 0, y_i = 0$	Final answer. ■

**1.2.g**  $xy = 1$

$\Rightarrow x(0) = 1$ $\Rightarrow 0 = 1$ $\Rightarrow x_i \in \emptyset$	Find the x-intercepts.
$\Rightarrow 0y = 1$ $\Rightarrow 0 = 1$ $\Rightarrow y_i \in \emptyset$	Find the y-intercepts.
$\Rightarrow x_i \in \emptyset, y_i \in \emptyset$	Final answer. ■

**1.3 Determine the value(s) of  $k$  for which the graph of  $y = 2x^2 + kx + 8$  does not intersect the x-axis. How about if the graph is to intersect the x-axis at exactly one point?**

$\Rightarrow 2x^2 + kx + 8 = 0$	Set $y = 0$ , and rewrite in standard form.															
$\Rightarrow k^2 - 4(2)(8) < 0$	If the discriminant is negative, the quadratic equation has no real solutions, and as such, no x-intercepts.															
$\Rightarrow k^2 - 16 < 0$																
$\Rightarrow (k - 4)(k + 4) < 0$	Factor using difference of two squares.															
Create a table of signs.																
<table><tr><td></td><td>-4</td><td>4</td></tr><tr><td><math>k - 4</math></td><td>-</td><td>-</td><td>+</td></tr><tr><td><math>k + 4</math></td><td>-</td><td>+</td><td>+</td></tr><tr><td><math>(k - 4)(k + 4)</math></td><td>+</td><td>-</td><td>+</td></tr></table>		-4	4	$k - 4$	-	-	+	$k + 4$	-	+	+	$(k - 4)(k + 4)$	+	-	+	
	-4	4														
$k - 4$	-	-	+													
$k + 4$	-	+	+													
$(k - 4)(k + 4)$	+	-	+													
$\Rightarrow k \in (-4, 4)$																

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$\Rightarrow k^2 - 4(2)(8) = 0$	If the discriminant is zero, the quadratic equation has exactly one real solution, and as such, exactly one x-intercept.
$\Rightarrow k^2 - 16 = 0$	
$\Rightarrow (k - 4)(k + 4) = 0$	Factor using difference of two squares.
$\Rightarrow k \in \{-4, 4\}$	
$\Rightarrow k \in (-4, 4)$	Final answer. If the graph should have no x-intercept.
$\Rightarrow k \in \{-4, 4\}$	Final answer. If the graph should have exactly one x-intercept. ■