Exercise Solutions for Math 20

Factoring Polynomials and Simplifying Rational Expressions

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Contents

1 Factor the following completely.

1.1 $16x^4 - 1$

$\Rightarrow (4x^2 - 1)(4x^2 + 1)$	Factor using difference of two squares.
$\Rightarrow (2x - 1)(2x + 1)(4x^2 + 1)$	Factor using difference of two squares. ■

1.2 $8j^3 - 125k^6$

$\Rightarrow (2j - 5k^2)(4j^2 + 10jk^2 + 25k^4)$	Factor using difference of two cubes. ■

1.3 $s^2 + 7s + 10$

$\Rightarrow (s + 2)(s + 5)$	Factor by grouping. ■

1.4 $4n^2 - 12n + 9$

$\Rightarrow 4n^2 - 6n - 6n + 9$ $\Rightarrow 2n(2n - 3) - 3(2n - 3)$ $\Rightarrow (2n - 3)^2$	Factor by grouping. ■

1.5 $x^3 - x^2 - x + 1$

$\Rightarrow x^2(x - 1) - 1(x - 1)$ $\Rightarrow (x^2 - 1)(x - 1)$	Factor by grouping.
$\Rightarrow (x - 1)(x + 1)(x - 1)$ $\Rightarrow (x - 1)^2(x + 1)$	Factor using difference of two squares. ■

1.6 $48 - 13q - q^2$

$\Rightarrow -q^2 - 13q + 48$ $\Rightarrow -(q^2 + 13q - 48)$	Rewrite in standard form.
$\Rightarrow -(q - 3)(q + 16)$	Factor by grouping. ■

2 Reduce the following rational expressions to lowest terms.

2.1 $\frac{a^2-b^2}{a^3-b^3}$

$\Rightarrow \frac{(a-b)(a+b)}{a^3-b^3}$	Factor using difference of two squares.
$\Rightarrow \frac{(a-b)(a+b)}{(a-b)(a^2+ab+b^2)}$ $\Rightarrow \frac{a+b}{a^2+ab+b^2}$	Factor using difference of two cubes. ■

2.2 $\frac{x^3-x^2y+xy^2-y^3}{x^6+y^6}$

$\Rightarrow \frac{x^2(x-y)+y^2(x-y)}{x^6+y^6}$ $\Rightarrow \frac{(x^2+y^2)(x-y)}{x^6+y^6}$	Factor by grouping.
$\Rightarrow \frac{(x^2+y^2)(x-y)}{(x^2+y^2)(x^4-x^2y^2+y^4)}$ $\Rightarrow \frac{x-y}{x^4-x^2y^2+y^4}$	Factor using difference of two cubes. ■