

# Exercise Solutions for Math 20

## Radicals and Complex Numbers

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# 1 Simplify the following. Rationalize the denominators.

1.1  $\frac{24c^{-\frac{1}{2}}d^{\frac{2}{3}}}{18c^{-\frac{1}{7}}d^{-\frac{3}{5}}}$

$\Rightarrow \frac{4c^{-\frac{1}{2}}d^{\frac{2}{3}}}{3c^{-\frac{1}{7}}d^{-\frac{3}{5}}}$	Simplify the fraction to lowest terms.
$\Rightarrow \frac{4d^{\frac{2}{3}}c^{\frac{1}{7}}d^{\frac{3}{5}}}{3c^{\frac{1}{2}}}$	$a^{-\frac{b}{c}} = \frac{1}{a^{\frac{b}{c}}}$
$\Rightarrow \frac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{\frac{1}{7}-\frac{1}{2}}$	$\frac{a^m}{a^n} = a^{m-n}$
$\Rightarrow \frac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{\frac{2}{14}-\frac{7}{14}}$ $\Rightarrow \frac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{-\frac{5}{14}}$	LCM = 14
$\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{2}{3}+\frac{3}{5}}$	$a^m a^n = a^{m+n}$
$\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{10}{15}+\frac{9}{15}}$ $\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{19}{15}}$	LCM = 15
$\Rightarrow \frac{4d^{\frac{19}{15}}}{3c^{\frac{5}{14}}}$ $\Rightarrow \frac{4}{3} \frac{\sqrt[15]{d^{19}}}{\sqrt[14]{c^5}}$	$a^{-\frac{b}{c}} = \frac{1}{a^{\frac{b}{c}}}$
$\Rightarrow \frac{4}{3} \frac{\sqrt[15]{d^{19}}}{\sqrt[14]{c^5}} \cdot \frac{\sqrt[14]{c^9}}{\sqrt[14]{c^9}}$ $\Rightarrow \frac{4}{3c} \frac{\sqrt[14]{c^9} \sqrt[15]{d^{19}}}{\sqrt[14]{c^5}}$	Rationalize.
	■

1.2  $(u^{\frac{1}{3}} + (uv)^{\frac{1}{6}} + v^{\frac{1}{3}})(u^{\frac{1}{6}} - v^{\frac{1}{6}})$

$\Rightarrow (u^{\frac{1}{3}} + u^{\frac{1}{6}}v^{\frac{1}{6}} + v^{\frac{1}{3}})(u^{\frac{1}{6}} - v^{\frac{1}{6}})$	Distribute exponent.
$\Rightarrow u^{\frac{1}{2}} - v^{\frac{1}{2}}$	Use difference of two cubes.
$\Rightarrow \sqrt{u} - \sqrt{v}$	
	■

1.3  $\sqrt[3]{-8^4}$

$\Rightarrow -\sqrt[3]{8^4}$ $\Rightarrow -\sqrt[3]{(2^3)^4}$	$\sqrt[n]{-a} = -\sqrt[n]{a}$ for odd $n$
$\Rightarrow -\sqrt[3]{(2^4)^3}$ $\Rightarrow -2^4$ $\Rightarrow -16$	$(a^m)^n = (a^n)^m$
	■

#### 1.4 $\sqrt[4]{9x^8}$

$$\Rightarrow \sqrt[4]{9}\sqrt[4]{x^8}$$

$$\Rightarrow \sqrt[4]{3^2}\sqrt[4]{x^8}$$

$$\Rightarrow x^2\sqrt{3}$$

$$\sqrt[m]{ab} = \sqrt[m]{a} \sqrt[m]{b}$$

