Exercise Solutions for Math 20

Linear, Quadratic, and Rational Equations

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November 14, 2024

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1.1 Find the solution set of the following equations.

1.1.a $x + 9 = 5 - 3x$

$\Rightarrow x + 3x = 5 - 9$	Solve for x.
$\Rightarrow 4x = -4$	
$\Rightarrow x = -1$	Final answer.
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1.1.b $\frac{2x+3}{4} - \frac{x-1}{2} = -\frac{1}{3}$

$\Rightarrow \frac{3(2x+3)}{12} - \frac{6(x-1)}{12} = -\frac{4}{12}$	LCM = 12
$\Rightarrow \frac{6x+9}{12} - \frac{6x-6}{12} = -\frac{4}{12}$	
$\Rightarrow (6x+9) - (6x-6) = -4$	
$\Rightarrow 6x + 9 - 6x + 6 = -4$	
$\Rightarrow 15 = -4$	
$\Rightarrow x \in \emptyset$	Final answer.
	■

1.1.c $3x^2 - 2x + 1 = 0$

$\Rightarrow \frac{-(-2)\pm\sqrt{(-2)^2-4(3)(1)}}{2(3)}$	Use the quadratic formula.
$\Rightarrow \frac{2\pm\sqrt{-8}}{6}$	
$\Rightarrow \frac{2\pm\sqrt{4}\sqrt{-2}}{6}$	
$\Rightarrow \frac{2\pm2i\sqrt{2}}{6}$	
$\Rightarrow \frac{1\pm i\sqrt{2}}{3}$	
$\Rightarrow \frac{1}{3} \pm \frac{\sqrt{2}}{3}i$	
$\Rightarrow x \in \{\frac{1}{3} + \frac{\sqrt{2}}{3}i, \frac{1}{3} - \frac{\sqrt{2}}{3}i\}$	Final answer.
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1.1.d $4x^2 + 2x = 2$

$\Rightarrow 4x^2 + 2x - 2 = 0$	Rewrite in standard form.
$\Rightarrow 2x^2 + x - 1 = 0$	
$\Rightarrow 2x^2 + 2x - x - 1 = 0$	Factor by grouping.
$\Rightarrow 2x(x+1) - 1(x+1) = 0$	
$\Rightarrow (2x-1)(x+1) = 0$	
$\Rightarrow x \in \{-1, \frac{1}{2}\}$	Final answer.
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1.1.e $16x^2 + 9 = 24x$

$\Rightarrow 16x^2 - 24x + 9 = 0$	Rewrite in standard form.
$\Rightarrow 16x^2 - 12x - 12x + 9 = 0$	Factor by grouping.
$\Rightarrow 4x(4x-3) - 3(4x-3) = 0$	
$\Rightarrow (4x-3)^2 = 0$	
$\Rightarrow x = \frac{3}{4}$	Final answer.
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1.1.f $\frac{x}{x-1} + \frac{x-5}{x^2+2x-3} = \frac{1}{x+3}$

$\Rightarrow \frac{x}{x-1} + \frac{x-5}{(x-1)(x+3)} = \frac{1}{x+3}$	Factor by grouping.
$\Rightarrow \frac{x(x+3)}{(x-1)(x+3)} + \frac{x-5}{(x-1)(x+3)} = \frac{x-1}{(x-1)(x+3)}$	LCM $= (x-1)(x+3)$
$\Rightarrow \frac{x^2+3x}{(x-1)(x+3)} + \frac{x-5}{(x-1)(x+3)} = \frac{x-1}{(x-1)(x+3)}$	
$\Rightarrow x^2 + 3x + x - 5 = x - 1$	Eliminate denominator. $x \in \{-3, 1\}$ are undefined points.
$\Rightarrow x^2 + 3x + x - 5 - x + 1 = 0$	
$\Rightarrow x^2 + 3x - 4 = 0$	
$\Rightarrow (x+4)(x-1) = 0$	Factor by grouping.
$\Rightarrow x = -4$	Final answer. Discard undefined point $x = 1$.
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1.2 Find all real values of k such that the equation $x^2 + kx + k = x - 2$ has exactly one solution.

$\Rightarrow x^2 + kx + k - x + 2 = 0$	Rewrite in standard form.
$\Rightarrow x^2 + (k-1)x + (k+2) = 0$	
$\Rightarrow (k-1)^2 - 4(1)(k+2) = 0$	A quadratic equation has exactly one solution if the value of its discriminant is 0.
$\Rightarrow k^2 - 2k + 1 - 4k - 8 = 0$	
$\Rightarrow k^2 - 6k - 7 = 0$	
$\Rightarrow (k-7)(k+1) = 0$	Factor by grouping.
$\Rightarrow k \in \{-1, 7\}$	Final answer.
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