Exercise Solutions for Math 20

Lines and Circles

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1.1 Find the value of k such that the lines with equations $3x + 2y - 4 = 0$ and $kx - 3y + 8$ are:

1.1.a Parallel.

$\Rightarrow 2y = -3x + 4$	Rewrite the first equation in slope-intercept form.
$\Rightarrow y = -\frac{3}{2}x + 4$	
$\Rightarrow -3y = -kx - 8$	Rewrite the second equation in slope-intercept form.
$\Rightarrow 3y = kx + 8$	
$\Rightarrow y = \frac{k}{3}x + \frac{8}{3}$	
$\Rightarrow \frac{k}{3} = -\frac{3}{2}$	Parallel slopes are equal.
$\Rightarrow k = -\frac{9}{2}$	Final answer.
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1.1.b Perpendicular.

$\Rightarrow \frac{k}{3} = -\frac{1}{-\frac{3}{2}}$	Perpendicular slopes are the negative reciprocal of each other.
$\Rightarrow \frac{k}{3} = \frac{2}{3}$	
$\Rightarrow k = 2$	Final answer.
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1.2 Line l is perpendicular to the line segment with endpoints $P(-4, 7)$ and $Q(2, -3)$. If l passes through the midpoint of the line segment $\overline{PQ}$, find an equation for l in slope-intercept form.

$\Rightarrow m = \frac{-3-7}{2+4}$	Find the slope of $\overline{PQ}$.
$\Rightarrow m = \frac{-10}{6}$	
$\Rightarrow m = -\frac{5}{3}$	
$\Rightarrow M = (\frac{-4+2}{2}, \frac{7-3}{2})$	Find the midpoint of $\overline{PQ}$.
$\Rightarrow M = (\frac{-2}{2}, \frac{4}{2})$	
$\Rightarrow M = (-1, 2)$	
$\Rightarrow y - 2 = -\frac{5}{3}(x + 1)$	Use the point-slope formula.
$\Rightarrow y - 2 = -\frac{5}{3}x - \frac{5}{3}$	
$\Rightarrow y = -\frac{5}{3}x - \frac{5}{3} + 2$	
$\Rightarrow y = -\frac{5}{3}x - \frac{5}{3} + \frac{6}{3}$	
$\Rightarrow y = -\frac{5}{3}x - \frac{1}{3}$	Final answer.
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1.3 Find a general equation of the line that is parallel to the line with equation $3x - y + 1 = 0$ and whose x-intercept is also the x-intercept of the line with equation $2x - 3y + 6 = 0$

$\Rightarrow -y = -3x - 1$	Find the slope of the first equation.
$\Rightarrow y = 3x + 1$	
$\Rightarrow m = 3$	
$\Rightarrow 2x - 3(0) + 6 = 0$	Find the x-intercept of the second equation.
$\Rightarrow 2x + 6 = 0$	
$\Rightarrow 2x = -6$	
$\Rightarrow x = -3$	
$\Rightarrow y - 0 = 3(x + 3)$	Use the point-slope formula.
$\Rightarrow y = 3x + 9$	Final answer.
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1.4 From the following equations, determine the center and radius of the circle if it exists.

1.4.a $(x+1)^2 + (y+3)^2 = 5$

$\Rightarrow (h,k) = (-1,-3), r = \sqrt{5}$	Final answer. Derive the center and radius.
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