Exercise Solutions for Math 20

Linear Inequalities System, Nonlinear Systems

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1.1 Solve algebraically for the solution sets of the following systems of equations.

1.1.a

$$\begin{cases} y = x^2 - 2x - 8 \\ 4x + 3y + 3 = 0 \end{cases}$$

$\Rightarrow 3y = -4x - 3$	Rewrite in terms of y.
$\Rightarrow y = -\frac{4}{3}x - 1$	
$\Rightarrow x^2 - 2x - 8 = -\frac{4}{3}x - 1$	Solve for x.
$\Rightarrow x^2 - 2x + \frac{4}{3}x - 8 + 1 = 0$	
$\Rightarrow x^2 - \frac{6}{3}x + \frac{4}{3}x - 7 = 0$	
$\Rightarrow x^2 - \frac{2}{3}x - 7 = 0$	
$\Rightarrow 3x^2 - 2x - 21 = 0$	
$\Rightarrow 3x^2 - 9x + 7x - 21 = 0$	Factor by grouping.
$\Rightarrow 3x(x-3) + 7(x-3) = 0$	
$\Rightarrow (3x+7)(x-3) = 0$	
$\Rightarrow x \in \{-\frac{7}{3}, 3\}$	
$\Rightarrow 4(-\frac{7}{3}) + 3y + 3 = 0$	Solve for $y, x = -\frac{7}{3}$
$\Rightarrow -\frac{28}{3} + 3y + 3 = 0$	
$\Rightarrow -\frac{28}{3} + 3y + \frac{9}{3} = 0$	
$\Rightarrow 3y - \frac{19}{3} = 0$	
$\Rightarrow 3y = \frac{19}{3}$	
$\Rightarrow y = \frac{19}{9}$	
$\Rightarrow 4(3) + 3y + 3 = 0$	Solve for $y, x = 3$
$\Rightarrow 12 + 3y + 3 = 0$	
$\Rightarrow 3y + 15 = 0$	
$\Rightarrow 3y = -15$	
$\Rightarrow y = -5$	
$\Rightarrow (x, y) \in \{(-\frac{7}{3}, \frac{19}{9}), (3, -5)\}$	Final answer.
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1.1.b

$$\begin{cases} 10x^2 - xy + 4y^2 = 28 \\ 2x^2 - 3xy - 2y^2 = 0 \end{cases}$$

$\Rightarrow 2x^2 - 3yx - 2y^2 = 0$	Rewrite in terms of x.
$\Rightarrow \frac{3y \pm \sqrt{(3y)^2 - 4(2)(-2y^2)}}{4}$	Use the quadratic formula.
$\Rightarrow \frac{3y \pm \sqrt{9y^2 + 16y^2}}{4}$	
$\Rightarrow \frac{3y \pm \sqrt{25y^2}}{4}$	

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$\Rightarrow \frac{3y \pm 5y}{4}$	
$\Rightarrow \frac{3y+5y}{4}, \frac{3y-5y}{4}$	
$\Rightarrow \frac{8y}{4}, \frac{-2y}{4}$	
$\Rightarrow x \in \{-\frac{1}{2}y, 2y\}$	
$\Rightarrow 10(-\frac{1}{2}y)^2 - (-\frac{1}{2}y)y + 4y^2 = 28$	Solve for $y, x = -\frac{1}{2}y$.
$\Rightarrow 10(\frac{1}{4}y^2) + \frac{1}{2}y^2 + 4y^2 = 28$	
$\Rightarrow \frac{5}{2}y^2 + \frac{1}{2}y^2 + 4y^2 = 28$	
$\Rightarrow \frac{5}{2}y^2 + \frac{1}{2}y^2 + \frac{8}{2}y^2 = 28$	
$\Rightarrow \frac{14}{2}y^2 = 28$	
$\Rightarrow 7y^2 = 28$	
$\Rightarrow y^2 = 4$	
$\Rightarrow y = \pm 2$	
$\Rightarrow x = -\frac{1}{2}(-2)$	Solve for $x, y = -2$
$\Rightarrow x = 1$	
$\Rightarrow x = -\frac{1}{2}(2)$	Solve for $x, y = 2$
$\Rightarrow x = -1$	
$\Rightarrow 10(2y)^2 - (2y)y + 4y^2 = 28$	Solve for $y, x = 2y$.
$\Rightarrow 10(4y^2) - 2y^2 + 4y^2 = 28$	
$\Rightarrow 40y^2 - 2y^2 + 4y^2 = 28$	
$\Rightarrow 42y^2 = 28$	
$\Rightarrow y^2 = \frac{28}{42}$	
$\Rightarrow y^2 = \frac{2}{3}$	
$\Rightarrow y = \pm\sqrt{\frac{2}{3}}$	
$\Rightarrow x = -2\sqrt{\frac{2}{3}}$	Solve for $x, y = -\sqrt{\frac{2}{3}}$
$\Rightarrow x = 2\sqrt{\frac{2}{3}}$	Solve for $x, y = \sqrt{\frac{2}{3}}$
$\Rightarrow (x, y) = \{(1, -2), (-1, 2), (-2\sqrt{\frac{2}{3}}, -\sqrt{\frac{2}{3}}), (2\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}})\}$	Final answer.
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1.2 Sketch the solution region for each of the given system of inequalities.

1.2.a

$$\begin{cases} y \leq 2x + 1 \\ x < 5 \\ y < x + 2 \end{cases}$$

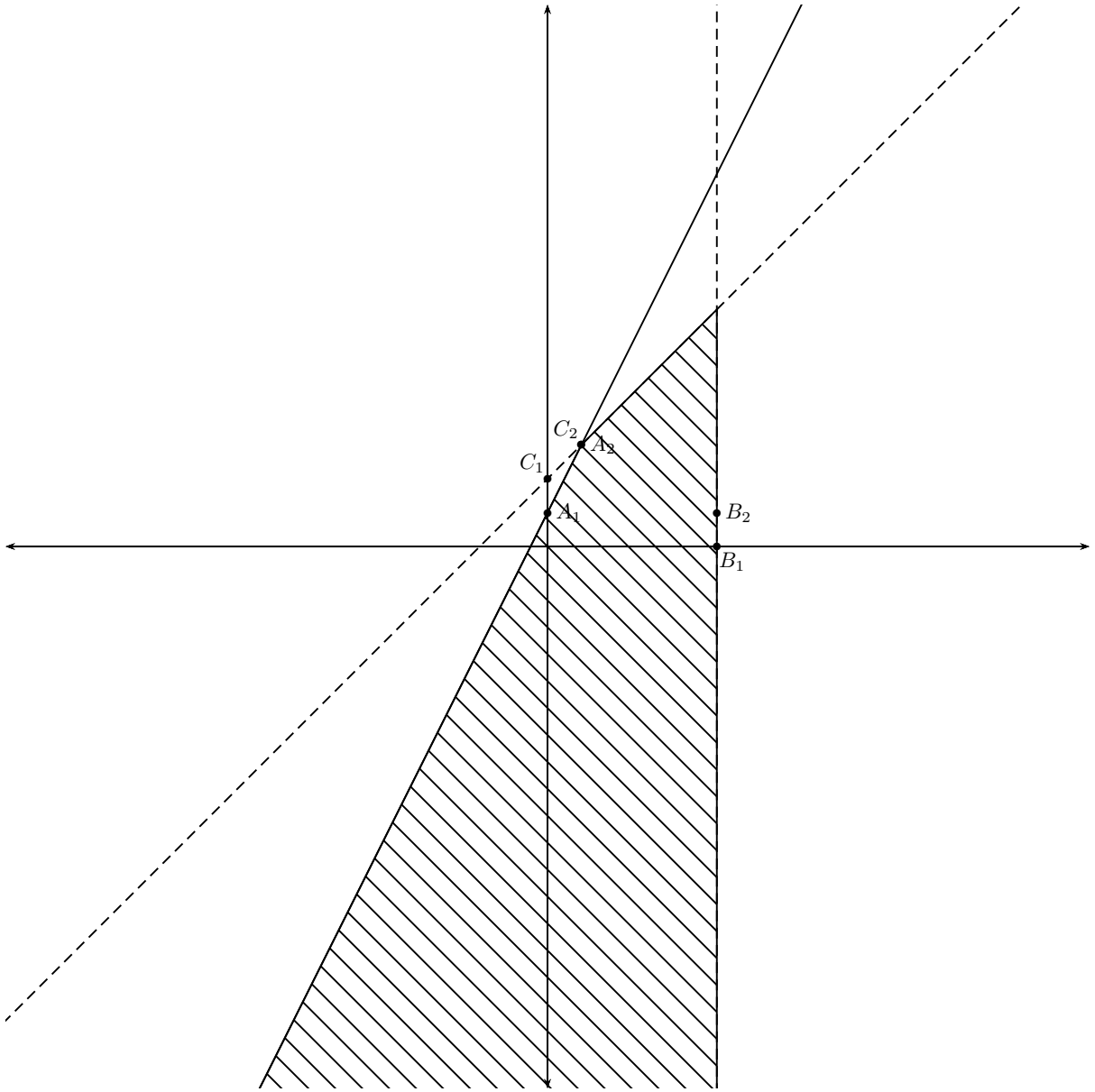

Figure 1.

1.2.b

$$\begin{cases} x^2 + y^2 \leq 16 \\ y \leq x^2 - 3 \\ y < 0 \end{cases}$$

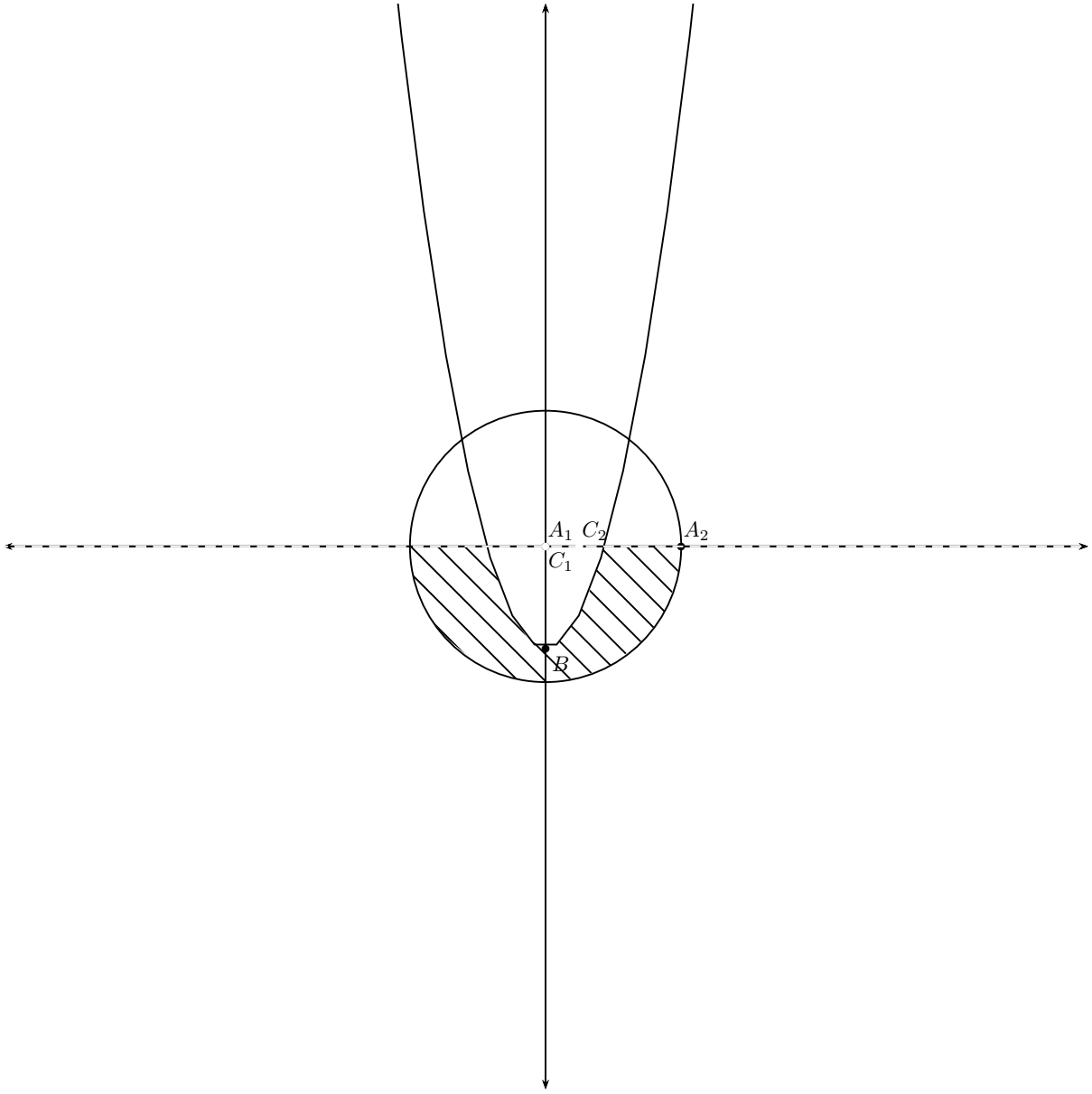

Figure 2.