Exercise Solutions for Math 20

Conics (Parabola and Ellipse)

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1.1 Determine the vertex and orientation of the following parabolas.

1.1.a $4y^2 + 4y + x = 2$

$\Rightarrow 4y^2 + 4y = -x + 2$	Isolate y.
$\Rightarrow y^2 + y = -\frac{x}{4} + \frac{2}{4}$	
$\Rightarrow y^2 + y = -\frac{x}{4} + \frac{1}{2}$	
$\Rightarrow y^2 + y + \frac{1}{4} = -\frac{x}{4} + \frac{1}{2} + \frac{1}{4}$	Complete the square.
$\Rightarrow \left(y + \frac{1}{2}\right)^2 = -\frac{x}{4} + \frac{3}{4}$	
$\Rightarrow \left(y + \frac{1}{2}\right)^2 = -\frac{1}{4}(x - 3)$	
$\Rightarrow \left(y + \frac{1}{2}\right)^2 = 4\left(-\frac{1}{16}\right)(x - 3)$	
$\Rightarrow$ Opening leftwards, $(h, k) = \left(3, -\frac{1}{2}\right)$	Final answer.
	∎

1.1.b $x^2 - 6x - 2y = 7$

$\Rightarrow x^2 - 6x = 2y + 7$	Isolate x.
$\Rightarrow x^2 - 6x + 9 = 2y + 7 + 9$	Complete the square.
$\Rightarrow (x - 3)^2 = 2y + 16$	
$\Rightarrow (x - 3)^2 = 2(y + 8)$	
$\Rightarrow (x - 3)^2 = 4\left(\frac{1}{2}\right)(y + 8)$	
$\Rightarrow$ Opening upwards, $(h, k) = (3, -8)$	Final answer.
	∎

1.1.c $2y^2 - 6y - 9x = 0$

$\Rightarrow 2y^2 - 6y = 9x$	Isolate y.
$\Rightarrow y^2 - 3y = \frac{9}{2}x$	
$\Rightarrow y^2 - 3y + \frac{9}{4} = \frac{9}{2}x + \frac{9}{4}$	Complete the square.
$\Rightarrow \left(y - \frac{3}{2}\right)^2 = \frac{9}{2}x + \frac{9}{4}$	
$\Rightarrow \left(y - \frac{3}{2}\right)^2 = \frac{9}{2}\left(x + \frac{9}{4} \cdot \frac{2}{9}\right)$	
$\Rightarrow \left(y - \frac{3}{2}\right)^2 = \frac{9}{2}\left(x + \frac{18}{36}\right)$	
$\Rightarrow \left(y - \frac{3}{2}\right)^2 = \frac{9}{2}\left(x + \frac{1}{2}\right)$	
$\Rightarrow \left(y - \frac{3}{2}\right)^2 = 4\left(\frac{9}{8}\right)\left(x + \frac{1}{2}\right)$	
$\Rightarrow$ Opening rightwards, $(h, k) = \left(-\frac{1}{2}, \frac{3}{2}\right)$	Final answer.
	∎

1.2   Sketch the graph of the following parabolas.

1.2.a $3y^2 = 8x$

$\Rightarrow y^2 = 4(\frac{2}{3})x$	Rewrite in standard form.

Graph the equation.

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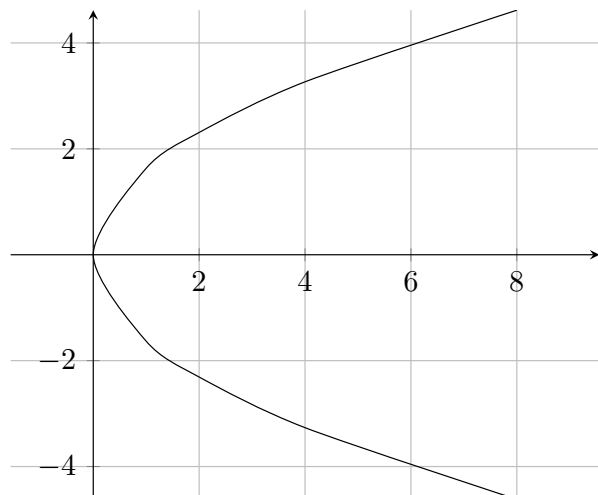
1.2.b $x^2 - 8x + 4y = -10$

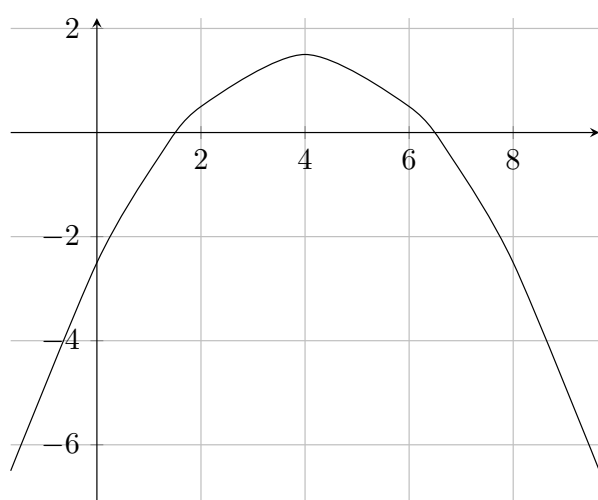|	
$\Rightarrow x^2 - 8x = -4y - 10$	Rewrite in standard form.
$\Rightarrow x^2 - 8x + 16 = -4y - 10 + 16$	Complete the square.
$\Rightarrow (x-4)^2 = -4y + 6$	
$\Rightarrow (x-4)^2 = 4(-1)(y - \frac{3}{2})$	

Graph the equation.

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1.3 **Sketch the graph of** $y = -x^2 + 6x - 8$. **Label the vertex, x- and y-intercept(s).**

$\Rightarrow -x^2 + 6x = y + 8$	Rewrite in standard form.
$\Rightarrow x^2 - 6x = -y - 8$	
$\Rightarrow x^2 - 6x + 9 = -y - 8 + 9$	Complete the square.
$\Rightarrow (x-3)^2 = -y + 1$	
$\Rightarrow (x-3)^2 = 4(-\frac{1}{4})(y-1)$	
$\Rightarrow (x-3)^2 = 4(-\frac{1}{4})(-1)$	Find the x-intercepts.
$\Rightarrow (x-3)^2 = 4(\frac{1}{4})$	
$\Rightarrow (x-3)^2 = 1$	
$\Rightarrow x = \pm 1 + 3$	
$\Rightarrow x = 1 + 3, x = -1 + 3$	
$\Rightarrow x_i \in \{2, 4\}$	
$\Rightarrow (0-3)^2 = 4(-\frac{1}{4})(y-1)$	Find the y-intercepts.
$\Rightarrow (-3)^2 = 4(-\frac{1}{4})(y-1)$	
$\Rightarrow 9 = -(y-1)$	
$\Rightarrow 9 = -y + 1$	
$\Rightarrow y = 1 - 9$	
$\Rightarrow y_i = -8$	
	Graph the equation.

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