Exercise Solutions for Math 20

Sum, Difference, Cofunction, Double Measure Identities

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1 Evaluate the following without using a calculator.

1.1 $\sin\left(\frac{19\pi}{12}\right)$

$\Rightarrow \sin\left(\frac{10\pi}{12} + \frac{9\pi}{12}\right)$	
$\Rightarrow \sin\left(\frac{5\pi}{6} + \frac{3\pi}{4}\right)$	
$\Rightarrow \sin\left(\frac{5\pi}{6}\right)\cos\left(\frac{3\pi}{4}\right) + \cos\left(\frac{5\pi}{6}\right)\sin\left(\frac{3\pi}{4}\right)$	$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
$\Rightarrow (\frac{1}{2})(-\frac{\sqrt{2}}{2}) + (-\frac{\sqrt{3}}{2})(\frac{\sqrt{2}}{2})$	
$\Rightarrow -\frac{\sqrt{2}}{4} - \frac{\sqrt{3}\sqrt{2}}{4}$	
$\Rightarrow -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$	
$\Rightarrow -\frac{\sqrt{2}+\sqrt{6}}{4}$	Final answer.

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1.2 $\cos(33°)\cos(27°) - \sin(33°)\sin(27°)$

$\Rightarrow \cos(33° + 27°)$	$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
$\Rightarrow \cos(60°)$	
$\Rightarrow \frac{1}{2}$	Final answer.

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2 If $\cot(\theta) = -\frac{5}{12}$ and $\theta \in (-\frac{\pi}{2}, 0)$, find $\cos\big(\theta + \frac{\pi}{3}\big)$.

$\Rightarrow O = -12, A = 5$	$\cot(\theta) = \frac{A}{O}$, since $\theta \in (-\frac{\pi}{2}, 0)$, $O < 0$ and $A > 0$
$\Rightarrow H = \sqrt{(-12)^2 + 5^2}$	$H = \sqrt{O^2 + A^2}$
$\Rightarrow H = \sqrt{144 + 25}$	
$\Rightarrow H = \sqrt{169}$	
$\Rightarrow H = 13$	
$\Rightarrow \cos\big(\theta + \frac{\pi}{3}\big) = \cos(\theta)\cos\big(\frac{\pi}{3}\big) - \sin(\theta)\sin\big(\frac{\pi}{3}\big)$	$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
$\Rightarrow \cos\big(\theta + \frac{\pi}{3}\big) = \frac{5}{13}\cos\big(\frac{\pi}{3}\big) - \sin(\theta)\sin\big(\frac{\pi}{3}\big)$	$\cos(\theta) = \frac{A}{H}$
$\Rightarrow \cos\big(\theta + \frac{\pi}{3}\big) = \frac{5}{13}\cos\big(\frac{\pi}{3}\big) + \frac{12}{13}\sin\big(\frac{\pi}{3}\big)$	$\sin(\theta) = \frac{O}{H}$
$\Rightarrow \cos\big(\theta + \frac{\pi}{3}\big) = (\frac{5}{13})(\frac{1}{2}) + (\frac{12}{13})(\frac{\sqrt{3}}{2})$	
$\Rightarrow \cos\big(\theta + \frac{\pi}{3}\big) = \frac{5}{26} + \frac{12\sqrt{3}}{26}$	
$\Rightarrow \cos\big(\theta + \frac{\pi}{3}\big) = \frac{5+12\sqrt{3}}{26}$	
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