Exercise Solutions for Math 20

Radicals and Complex Numbers

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Contents

1 Simplify the following. Rationalize the denominators.

1.1 $\dfrac{24c^{-\frac{1}{2}}d^{\frac{2}{3}}}{18c^{-\frac{1}{7}}d^{-\frac{3}{5}}}$

$\Rightarrow \dfrac{4c^{-\frac{1}{2}}d^{\frac{2}{3}}}{3c^{-\frac{1}{7}}d^{-\frac{3}{5}}}$	Simplify the fraction to lowest terms.
$\Rightarrow \dfrac{4d^{\frac{2}{3}}c^{\frac{1}{7}}d^{\frac{3}{5}}}{3c^{\frac{1}{2}}}$	$a^{-\frac{b}{c}} = \frac{1}{a^{\frac{b}{c}}}$
$\Rightarrow \dfrac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{\frac{1}{7}-\frac{1}{2}}$	$\frac{a^m}{a^n} = a^{m-n}$
$\Rightarrow \dfrac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{\frac{2}{14}-\frac{7}{14}}$	LCM $= 14$
$\Rightarrow \dfrac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{-\frac{5}{14}}$	
$\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{2}{3}+\frac{3}{5}}$	$a^m a^n = a^{m+n}$
$\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{10}{15}+\frac{9}{15}}$	LCM $= 15$
$\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{19}{15}}$	
$\Rightarrow \dfrac{4d^{\frac{19}{15}}}{3c^{\frac{5}{14}}}$	$a^{-\frac{b}{c}} = \frac{1}{a^{\frac{b}{c}}}$
$\Rightarrow \dfrac{4\sqrt[15]{d^{19}}}{3\sqrt[14]{c^5}}$	
$\Rightarrow \dfrac{4\sqrt[15]{d^{19}}}{3\sqrt[14]{c^5}} \cdot \dfrac{\sqrt[14]{c^9}}{\sqrt[14]{c^9}}$	Rationalize.
$\Rightarrow \dfrac{4\sqrt[14]{c^9}\sqrt[15]{d^{19}}}{3c}$	
	■

1.2 $(u^{\frac{1}{3}} + (uv)^{\frac{1}{6}} + v^{\frac{1}{3}})(u^{\frac{1}{6}} - v^{\frac{1}{6}})$

$\Rightarrow (u^{\frac{1}{3}} + u^{\frac{1}{6}}v^{\frac{1}{6}} + v^{\frac{1}{3}})(u^{\frac{1}{6}} - v^{\frac{1}{6}})$	Distribute exponent.
$\Rightarrow u^{\frac{1}{2}} - v^{\frac{1}{2}}$	Use difference of two cubes.
$\Rightarrow \sqrt{u} - \sqrt{v}$	
	■

1.3 $\sqrt[3]{-8^4}$

$\Rightarrow -\sqrt[3]{8^4}$	$\sqrt[m]{-a} = -\sqrt[m]{a}$ for odd m
$\Rightarrow -\sqrt[3]{(2^3)^4}$	
$\Rightarrow -\sqrt[3]{(2^4)^3}$	$(a^m)^n = (a^n)^m$
$\Rightarrow -2^4$	
$\Rightarrow -16$	
	■

1.4 $\sqrt[4]{9x^8}$

$\Rightarrow \sqrt[4]{9}\sqrt[4]{x^8}$ $\qquad \sqrt[m]{ab} = \sqrt[m]{a}\,\sqrt[m]{b}$

$\Rightarrow \sqrt[4]{3^2}\sqrt[4]{x^8}$

$\Rightarrow x^2\sqrt{3}$

■

1.5 $\sqrt{\sqrt[3]{9a^4b^4}}$

$\Rightarrow \sqrt[6]{9a^4b^4}$ $\qquad \sqrt[m]{\sqrt[n]{a}} = \sqrt[m+n]{a}$

$\Rightarrow \sqrt[6]{3^2a^4b^4}$

$\Rightarrow \sqrt[3]{3a^2b^2}$

■

1.6 $\frac{2\sqrt{5}}{\sqrt{8}} + \frac{9}{\sqrt[3]{16}}$

$\Rightarrow \frac{2\sqrt{5}}{\sqrt{8}} \cdot \frac{\sqrt{2}}{\sqrt{2}} + \frac{9}{\sqrt[3]{16}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}}$ $\qquad$ Rationalize.

$\Rightarrow \frac{2\sqrt{5}\sqrt{2}}{\sqrt{16}} + \frac{9\,\sqrt[3]{4}}{\sqrt[3]{64}}$

$\Rightarrow \frac{2\sqrt{5}\sqrt{2}}{4} + \frac{9\,\sqrt[3]{4}}{4}$

$\Rightarrow \frac{2\sqrt{5}\sqrt{2}+9\,\sqrt[3]{4}}{4}$

$\Rightarrow \frac{2\sqrt{10}+9\,\sqrt[3]{4}}{4}$

■

1.7 $\frac{x^2-2x+1}{\sqrt{x}+1}$

$\Rightarrow \frac{x^2-2x+1}{\sqrt{x}+1} \cdot \frac{\sqrt{x}-1}{\sqrt{x}-1}$ $\qquad$ Rationalize using difference of two squares.

$\Rightarrow \frac{(x^2-2x+1)(\sqrt{x}-1)}{x-1}$

$\Rightarrow \frac{(x-1)^2(\sqrt{x}-1)}{x-1}$ $\qquad$ Factor by grouping.

$\Rightarrow (x-1)(\sqrt{x}-1)$

■

1.8 $\frac{1}{\sqrt[3]{4}-\sqrt[3]{-27}}$

$$\Rightarrow \frac{1}{\sqrt[3]{4}+\sqrt[3]{27}}$$

$\sqrt[m]{-a} = -\sqrt[m]{a}$ for odd m

$$\Rightarrow \frac{1}{\sqrt[3]{4}+\sqrt[3]{27}} \cdot \frac{\sqrt[3]{4^2}-\sqrt[3]{4}\sqrt[3]{27}+\sqrt[3]{27^2}}{\sqrt[3]{4^2}-\sqrt[3]{4}\sqrt[3]{27}+\sqrt[3]{27^2}}$$

Rationalize using difference of two cubes.

$$\Rightarrow \frac{\sqrt[3]{4^2}-\sqrt[3]{4}\sqrt[3]{27}+\sqrt[3]{27^2}}{4+27}$$

$$\Rightarrow \frac{\sqrt[3]{4^2}-3\sqrt[3]{4}+\sqrt[3]{27^2}}{4+27}$$

$$\Rightarrow \frac{\sqrt[3]{4^2}-3\sqrt[3]{4}+\sqrt[3]{27^2}}{31}$$

$$\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+\sqrt[3]{27^2}}{31}$$

$$\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+\sqrt[3]{(3^3)^2}}{31}$$

$$\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+\sqrt[3]{(3^2)^3}}{31}$$

$(a^m)^n = (a^n)^m$

$$\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+3^2}{31}$$

$$\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+9}{31}$$

$$\Rightarrow \frac{\sqrt[3]{8}\sqrt[3]{2}-3\sqrt[3]{4}+9}{31}$$

$\sqrt[m]{ab} = \sqrt[m]{a}\sqrt[m]{b}$

$$\Rightarrow \frac{2\sqrt[3]{2}-3\sqrt[3]{4}+9}{31}$$

$\blacksquare$

2 Perform the following operations and simplify.

2.1 $3i(i^2 - i^3 + 5i^5 - i^{-2})$

$\Rightarrow 3i(-1 - i^3 + 5i^5 - i^{-2})$	$i^2 = -1$
$\Rightarrow 3i(-1 + i + 5i^5 - i^{-2})$	$i^3 = -i$
$\Rightarrow 3i(-1 + i + 5i - i^{-2})$	$i^5 = i$
$\Rightarrow 3i(-1 + i + 5i + 1)$ $\Rightarrow 3i(6i)$ $\Rightarrow 18i^2$	$i^{-2} = -1$
$\Rightarrow -18$ ■	$i^2 = -1$

2.2 $(3 - 5i)(7 + 4i)$

$\Rightarrow 21 + 12i - 35i - 20i^2$	Expand.
$\Rightarrow 21 + 12i - 35i + 20$ $\Rightarrow 41 - 23i$ ■	$i^2 = -1$

2.3 $\frac{3i-2}{3i+2}$

$\Rightarrow \frac{-2+3i}{2+3i}$	Rewrite in standard form.
$\Rightarrow \frac{-2+3i}{2+3i} \cdot \frac{2-3i}{2-3i}$ $\Rightarrow \frac{(-2+3i)(2-3i)}{(2+3i)(2-3i)}$ $\Rightarrow \frac{-4+6i+6i-9i^2}{4-9i^2}$ $\Rightarrow \frac{-4+12i-9i^2}{4-9i^2}$	Multiply by conjugate to eliminate the complex denominator.
$\Rightarrow \frac{-4+12i+9}{4+9}$ $\Rightarrow \frac{5+12i}{13}$ $\Rightarrow \frac{5}{13} + \frac{12}{13}i$ ■	$i^2 = -1$

2.4 $\frac{7+i-4(3-i)}{6-5i^3}$

$\Rightarrow \frac{7+i-4(3-i)}{6+5i}$ $\quad i^3 = -i$

$\Rightarrow \frac{7+i-12+4i}{6+5i}$

$\Rightarrow \frac{-5+5i}{6+5i}$

$\Rightarrow \frac{-5+5i}{6+5i} \cdot \frac{6-5i}{6-5i}$ Multiply by conjugate to eliminate the complex denominator.

$\Rightarrow \frac{(-5+5i)(6-5i)}{(6+5i)(6-5i)}$

$\Rightarrow \frac{-30+25i+30i-25i^2}{36-25i^2}$

$\Rightarrow \frac{-30+25i+30i+25}{36+25}$ $\quad i^2 = -1$

$\Rightarrow \frac{-5+55i}{61}$

$\Rightarrow -\frac{5}{61} + \frac{55}{61}i$

■

2.5 $\frac{2-2(\overline{i+1})}{2-\sqrt{-4}}$

$\Rightarrow \frac{2-2(i-1)}{2-\sqrt{-4}}$ Line above a complex number denotes its conjugate.

$\Rightarrow \frac{2-2i+2}{2-\sqrt{-4}}$

$\Rightarrow \frac{4-2i}{2-\sqrt{-4}}$

$\Rightarrow \frac{4-2i}{2-2i}$ $\quad \sqrt{-a} = i\sqrt{a}$

$\Rightarrow \frac{4-2i}{2-2i} \cdot \frac{2+2i}{2+2i}$ Multiply by conjugate to eliminate the complex denominator.

$\Rightarrow \frac{(4-2i)(2+2i)}{(2-2i)(2+2i)}$

$\Rightarrow \frac{8+8i-4i-4i^2}{4-4i^2}$

$\Rightarrow \frac{8+4i-4i^2}{4-4i^2}$

$\Rightarrow \frac{8+4i+4}{4+4}$ $\quad i^2 = -1$

$\Rightarrow \frac{12+4i}{8}$

$\Rightarrow \frac{3+i}{2}$

$\Rightarrow \frac{3}{2} + \frac{1}{2}i$

■