Exercise Solutions for Math 20

Fundamental Identities

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1.1 Use the fundamental identities to find the other five circular function values of x given that $\tan(x) = \frac{4}{3}$ and $\cos(x) < 0$.

$\Rightarrow H = \sqrt{4^2 + 3^2}$	Find the hypotenuse using Pythagoras; the opposite and adjacent is given from the definition of $\tan(x) = \frac{O}{A}$.
$\Rightarrow H = \sqrt{16 + 9}$	
$\Rightarrow H = \sqrt{25}$	
$\Rightarrow H = 5$	
$\Rightarrow \cos(x) = -\frac{3}{5}$	Final answer. $\cos(x) = \frac{A}{H}$, $\cos(x) < 0$.
$\Rightarrow \sin(x) = -\frac{4}{5}$	$\sin(x) = \frac{O}{H}$, and since $\cos(x)$ is negative, $\sin(x)$ also has to be negative for $\tan(x)$ to be positive.
$\Rightarrow \cot(x) = \frac{3}{4}$	
$\Rightarrow \sec(x) = -\frac{5}{3}$	
$\Rightarrow \csc(x) = -\frac{5}{4}$	
	■

1.2 Prove the following identities.

1.2.a $\cos^2(\theta) = \frac{\cot^2(\theta)}{1+\cot^2(\theta)}$

$\Rightarrow \cos^2(\theta) = \frac{\cot^2(\theta)}{\csc^2(\theta)}$	$\csc^2(\theta) = 1 + \cot^2(\theta)$
$\Rightarrow \cos^2(\theta) = \frac{\frac{\cos^2(\theta)}{\sin^2(\theta)}}{\csc^2(\theta)}$	$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
$\Rightarrow \cos^2(\theta) = \frac{\frac{\cos^2(\theta)}{\sin^2(\theta)}}{\frac{1}{\sin^2(\theta)}}$	$\csc(\theta) = \frac{1}{\sin(\theta)}$
$\Rightarrow \cos^2(\theta) = \frac{\cos^2(\theta)\sin^2(\theta)}{\sin^2(\theta)}$	
$\Rightarrow \cos^2(\theta) = \cos^2(\theta)$	Final answer.
	■

1.2.b $\frac{1}{\sec(\theta)-\tan(\theta)} = \sec(\theta) + \tan(\theta)$

$\Rightarrow \sec(\theta) + \tan(\theta) = \frac{1}{\sec(\theta)-\tan(\theta)} \cdot \frac{\sec(\theta)+\tan(\theta)}{\sec(\theta)+\tan(\theta)}$	Use difference of two squares.
$\Rightarrow \sec(\theta) + \tan(\theta) = \frac{\sec(\theta)+\tan(\theta)}{\sec^2(\theta)-\tan^2(\theta)}$	
$\Rightarrow \sec(\theta) + \tan(\theta) = \frac{\sec(\theta)+\tan(\theta)}{1}$	$\sec^2(\theta) = 1 + \tan^2(\theta)$
$\Rightarrow \sec(\theta) + \tan(\theta) = \sec(\theta) + \tan(\theta)$	Final answer.
	■

1.2.c $\frac{\sin(\theta)}{1+\cos(\theta)} = \csc(\theta) - \cot(\theta)$

$\Rightarrow \csc(\theta) - \cot(\theta) = \frac{\sin(\theta)}{1+\cos(\theta)} \cdot \frac{1-\cos(\theta)}{1-\cos(\theta)}$	Use difference of two squares.
$\Rightarrow \csc(\theta) - \cot(\theta) = \frac{\sin(\theta)(1-\cos(\theta))}{1-\cos^2(\theta)}$	
$\Rightarrow \csc(\theta) - \cot(\theta) = \frac{\sin(\theta)(1-\cos(\theta))}{\sin^2(\theta)}$	$\sin^2(\theta) + \cos^2(\theta) = 1$
$\Rightarrow \csc(\theta) - \cot(\theta) = \frac{1-\cos(\theta)}{\sin(\theta)}$	
$\Rightarrow \csc(\theta) - \cot(\theta) = \frac{1}{\sin(\theta)} - \frac{\cos(\theta)}{\sin(\theta)}$	
$\Rightarrow \csc(\theta) - \cot(\theta) = \csc(\theta) - \frac{\cos(\theta)}{\sin(\theta)}$	$\csc(\theta) = \frac{1}{\sin(\theta)}$
$\Rightarrow \csc(\theta) - \cot(\theta) = \csc(\theta) - \cot(\theta)$	Final answer. $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
	$\blacksquare$

1.2.d $\frac{\cos(\theta)}{\sec(\theta)-\tan(\theta)} = 1 + \sin(\theta)$

$\Rightarrow 1 + \sin(\theta) = \frac{\cos(\theta)}{\sec(\theta)-\tan(\theta)} \cdot \frac{\sec(\theta)+\tan(\theta)}{\sec(\theta)+\tan(\theta)}$	Use difference of two squares.
$\Rightarrow 1 + \sin(\theta) = \frac{\cos(\theta)(\sec(\theta)+\tan(\theta))}{\sec^2(\theta)-\tan^2(\theta)}$	
$\Rightarrow 1 + \sin(\theta) = \frac{\cos(\theta)(\sec(\theta)+\tan(\theta))}{1}$	$\sec(\theta) = 1 + \tan(\theta)$
$\Rightarrow 1 + \sin(\theta) = \cos(\theta)(\sec(\theta) + \tan(\theta))$	
$\Rightarrow 1 + \sin(\theta) = \cos(\theta)(\frac{1}{\cos(\theta)} + \tan(\theta))$	$\sec(\theta) = \frac{1}{\cos(\theta)}$
$\Rightarrow 1 + \sin(\theta) = \cos(\theta)(\frac{1}{\cos(\theta)} + \frac{\sin(\theta)}{\cos(\theta)})$	$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$
$\Rightarrow 1 + \sin(\theta) = 1 + \sin(\theta)$	Final answer.
	$\blacksquare$

1.2.e $\frac{\sec(\theta)-\sin(\theta)\tan(\theta)}{\cot(\theta)} = \sin(\theta)$

$\Rightarrow \sin(\theta) = \frac{\sec(\theta)-\sin(\theta)\frac{\sin(\theta)}{\cos(\theta)}}{\cot(\theta)}$	$\tan(\theta) = \frac{\tan(\theta)}{\cos(\theta)}$
$\Rightarrow \sin(\theta) = \frac{\sec(\theta)-\frac{\sin^2(\theta)}{\cos(\theta)}}{\cot(\theta)}$	
$\Rightarrow \sin(\theta) = \frac{\frac{1}{\cos(\theta)}-\frac{\sin^2(\theta)}{\cos(\theta)}}{\cot(\theta)}$	$\sec(\theta) = \frac{1}{\cos(\theta)}$
$\Rightarrow \sin(\theta) = \frac{\frac{1-\sin^2(\theta)}{\cos(\theta)}}{\cot(\theta)}$	
$\Rightarrow \sin(\theta) = \frac{\frac{\cos^2(\theta)}{\cos(\theta)}}{\cot(\theta)}$	$\sin^2(\theta) + \cos^2(\theta) = 1$
$\Rightarrow \sin(\theta) = \frac{\cos(\theta)}{\cot(\theta)}$	
$\Rightarrow \sin(\theta) = \frac{\cos(\theta)}{\frac{\cos(\theta)}{\sin(\theta)}}$	$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
$\Rightarrow \sin(\theta) = \sin(\theta)$	Final answer.
	$\blacksquare$

1.2.f $\cos^4(\theta) - \sin^4(\theta) = 2\cos^2(\theta) - 1$

$\Rightarrow 2\cos^2(\theta) - 1 = (\cos^2(\theta) - \sin^2(\theta))(\cos^2(\theta) + \sin^2(\theta))$	Factor using difference of two squares.
$\Rightarrow 2\cos^2(\theta) - 1 = (\cos^2(\theta) - \sin^2(\theta))(1)$	$\sin^2(\theta) + \cos^2(\theta) = 1$
$\Rightarrow 2\cos^2(\theta) - 1 = \cos^2(\theta) - \sin^2(\theta)$	
$\Rightarrow 2\cos^2(\theta) - 1 = 2\cos^2(\theta) - 1$	Final answer. $\cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1$ ■