

Exercise Solutions for Math 20

Equations in Quadratic Form and with Radicals and Absolute Values

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Contents

1	Find the solution set of the following inequalities.	3
1.1	$\frac{2x+1}{4} \leq \frac{2x}{3} + \frac{1}{6}$	3
1.2	$-2 < 5 + 3x < 20$	3
1.3	$\frac{x}{x-1} > -1$	3
1.4	$\frac{x}{x+1} \geq \frac{2}{x+3}$	3
1.5	$\left \frac{9-2x}{4x} \right \geq 1$	4
1.6	$\left \frac{x}{2x-3} \right \leq 1$	5
1.7	$0 < x - 5 < 2$	6
1.8	$\frac{2x-7}{x^2-6x+8} \leq 1$	6

1 Find the solution set of the following inequalities.

1.1 $\frac{2x+1}{4} \leq \frac{2x}{3} + \frac{1}{6}$

$\Rightarrow \frac{3(2x+1)}{12} \leq \frac{4(2x)}{12} + \frac{2}{12}$ $\Rightarrow \frac{6x+3}{12} \leq \frac{8x}{12} + \frac{2}{12}$ $\Rightarrow \frac{6x+3}{12} \leq \frac{8x+2}{12}$ $\Rightarrow 6x+3 \leq 8x+2$ $\Rightarrow 3-2 \leq 8x-6x$ $\Rightarrow 1 \leq 2x$ $\Rightarrow x \geq \frac{1}{2}$	LCM = 12
$\Rightarrow x \in [\frac{1}{2}, +\infty)$	Final answer. ■

1.2 $-2 < 5 + 3x < 20$

$\Rightarrow -7 < 3x < 15$ $\Rightarrow -\frac{7}{3} < x < 5$	Solve for x .
$\Rightarrow x \in (-\frac{7}{3}, 5)$	Final answer. ■

1.3 $\frac{x}{x-1} > -1$

$\Rightarrow \frac{x}{x-1} + 1 > 0$ $\Rightarrow \frac{x}{x-1} + \frac{x-1}{x-1} > 0$ $\Rightarrow \frac{x+x-1}{x-1} > 0$ $\Rightarrow \frac{2x-1}{x-1} > 0$	Solve for x .																
<div style="text-align: right;">$x = 1$ is an undefined point.</div> <div style="text-align: right;">Create a table of signs.</div> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td></td> <td style="text-align: center;">$\frac{1}{2}$</td> <td colspan="2" style="text-align: center;">1</td> </tr> <tr> <td style="text-align: center;">$2x - 1$</td> <td style="text-align: center;">-</td> <td style="text-align: center;">+</td> <td style="text-align: center;">+</td> </tr> <tr> <td style="text-align: center;">$x - 1$</td> <td style="text-align: center;">-</td> <td style="text-align: center;">-</td> <td style="text-align: center;">+</td> </tr> <tr> <td style="text-align: center;">$\frac{2x-1}{x-1}$</td> <td style="text-align: center;">+</td> <td style="text-align: center;">-</td> <td style="text-align: center;">+</td> </tr> </table>			$\frac{1}{2}$	1		$2x - 1$	-	+	+	$x - 1$	-	-	+	$\frac{2x-1}{x-1}$	+	-	+
	$\frac{1}{2}$	1															
$2x - 1$	-	+	+														
$x - 1$	-	-	+														
$\frac{2x-1}{x-1}$	+	-	+														
$\Rightarrow x \in (-\infty, \frac{1}{2}) \cup (1, +\infty)$	Final answer. ■																

1.4 $\frac{x}{x+1} \geq \frac{2}{x+3}$

$\Rightarrow \frac{x}{x+1} - \frac{2}{x+3} \geq 0$	Solve for x .
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$\Rightarrow \frac{x(x+3)}{(x+1)(x+3)} - \frac{2(x+1)}{(x+1)(x+3)} \geq 0$ LCM = $(x+1)(x+3)$
 $\Rightarrow \frac{x(x+3)-2(x+1)}{(x+1)(x+3)} \geq 0$
 $\Rightarrow \frac{x^2+3x-2x-2}{(x+1)(x+3)} \geq 0$
 $\Rightarrow \frac{x^2+x-2}{(x+1)(x+3)} \geq 0$
 $\Rightarrow \frac{(x-1)(x+2)}{(x+1)(x+3)} \geq 0$ Factor by grouping. $x \in \{-3, -1\}$ are undefined points.

Create a table of signs.

	-3	-2	-1	1	
$x-1$	-	-	-	-	+
$x+2$	-	-	+	+	+
$x+1$	-	-	-	+	+
$x+3$	-	+	+	+	+
$\frac{(x-1)(x+2)}{(x+1)(x+3)}$	+	-	+	-	+

$\Rightarrow (-\infty, -3) \cup [-2, -1) \cup [1, +\infty)$ Final answer. Don't include undefined points. ■

1.5 $\left| \frac{9-2x}{4x} \right| \geq 1$

$\Rightarrow \frac{9-2x}{4x} \geq 1$ $|a| \geq b \Rightarrow a \geq b$ or $a \leq -b$. Solve for $a \geq b$.
 $\Rightarrow \frac{9-2x}{4x} - 1 \geq 0$
 $\Rightarrow \frac{9-2x}{4x} - \frac{4x}{4x} \geq 0$
 $\Rightarrow \frac{9-2x-4x}{4x} \geq 0$
 $\Rightarrow \frac{9-6x}{4x} \geq 0$
 $\Rightarrow \frac{9-6x}{x} \geq 0$
 $\Rightarrow \frac{-3(2x-3)}{x} \geq 0$
 $\Rightarrow \frac{2x-3}{x} \leq 0$ $x = 0$ is an undefined point.

Create a table of signs.

	0	$\frac{3}{2}$	
$2x-3$	-	-	+
x	-	+	+
$\frac{2x-3}{x}$	+	-	+

$\Rightarrow x \in (0, \frac{3}{2}]$

$\Rightarrow \frac{9-2x}{4x} \leq -1$ $|a| \geq b \Rightarrow a \geq b$ or $a \leq -b$. Solve for $a \leq -b$.
 $\Rightarrow \frac{9-2x}{4x} + 1 \leq 0$
 $\Rightarrow \frac{9-2x}{4x} + \frac{4x}{4x} \leq 0$
 $\Rightarrow \frac{9-2x+4x}{4x} \leq 0$
 $\Rightarrow \frac{2x+9}{4x} \leq 0$

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$\Rightarrow \frac{2x+9}{x} \leq 0$	$x = 0$ is an undefined point.
Create a table of signs.	
	$-\frac{9}{2} \quad 0$
$2x + 9$	$- \quad + \quad +$
x	$- \quad - \quad +$
$\frac{2x+9}{x}$	$+ \quad - \quad +$
$\Rightarrow x \in [-\frac{9}{2}, 0)$	
$\Rightarrow x \in [-\frac{9}{2}, 0) \cup (0, \frac{3}{2}]$	Final answer. Combine intervals. ■

1.6 $\left| \frac{x}{2x-3} \right| \leq 1$

$\Rightarrow \frac{x}{2x-3} \leq 1$	$ a \leq b \Rightarrow a \leq b \text{ and } a \geq -b. \text{ Solve for } a \leq b.$
$\Rightarrow \frac{x}{2x-3} - 1 \leq 0$	
$\Rightarrow \frac{x}{2x-3} - \frac{2x-3}{2x-3} \leq 0$	
$\Rightarrow \frac{x-(2x-3)}{2x-3} \leq 0$	
$\Rightarrow \frac{x-2x+3}{2x-3} \leq 0$	
$\Rightarrow \frac{-x+3}{2x-3} \leq 0$	
$\Rightarrow \frac{-(x-3)}{2x-3} \leq 0$	
$\Rightarrow \frac{x-3}{2x-3} \geq 0$	$x = \frac{3}{2}$ is an undefined point.
Create a table of signs.	
	$\frac{3}{2} \quad 3$
$x - 3$	$- \quad - \quad +$
$2x - 3$	$- \quad + \quad +$
$\frac{x-3}{2x-3}$	$+ \quad - \quad +$
$\Rightarrow x \in (-\infty, \frac{3}{2}) \cup [3, +\infty)$	
$\Rightarrow \frac{x}{2x-3} \geq -1$	$ a \leq b \Rightarrow a \leq b \text{ and } a \geq -b. \text{ Solve for } a \geq -b.$
$\Rightarrow \frac{x}{2x-3} + 1 \geq 0$	
$\Rightarrow \frac{x}{2x-3} + \frac{2x-3}{2x-3} \geq 0$	
$\Rightarrow \frac{x+2x-3}{2x-3} \geq 0$	
$\Rightarrow \frac{3x-3}{2x-3} \geq 0$	
$\Rightarrow \frac{3(x-1)}{2x-3} \geq 0$	
$\Rightarrow \frac{x-1}{2x-3} \geq 0$	$x = \frac{3}{2}$ is an undefined point.
Create a table of signs.	

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	1	$\frac{3}{2}$	
$x - 1$	−	+	+
$2x - 3$	−	−	+
$\frac{x-1}{2x-3}$	+	−	+

$\Rightarrow x \in (-\infty, 1] \cup (\frac{3}{2}, +\infty)$
 $\Rightarrow x \in ((-\infty, \frac{3}{2}) \cup [3, +\infty)) \cap ((-\infty, 1] \cup (\frac{3}{2}, +\infty))$ Combine intervals.
 $\Rightarrow x \in (-\infty, 1] \cup [3, +\infty)$ Final answer. ■

1.7 $0 < |x - 5| < 2$

$\Rightarrow x - 5 > 0, x - 5 < 2$	Split the inequality.
$\Rightarrow x - 5 > 0$ $\Rightarrow x > 5$ $\Rightarrow x \in (5, +\infty)$	$ a > b \Rightarrow a > b \text{ or } a < -b. \text{ Solve for } a > b.$
$\Rightarrow x - 5 < 0$ $\Rightarrow x < 5$ $\Rightarrow x \in (-\infty, 5)$	$ a > b \Rightarrow a > b \text{ or } a < -b. \text{ Solve for } a < -b.$
$\Rightarrow x - 5 < 2$ $\Rightarrow x < 7$ $\Rightarrow x \in (-\infty, 7)$	$ a < b \Rightarrow a < b \text{ and } a > -b. \text{ Solve for } a < b.$
$\Rightarrow x - 5 > -2$ $\Rightarrow x > 3$ $\Rightarrow x \in (3, +\infty)$	$ a < b \Rightarrow a < b \text{ and } a > -b. \text{ Solve for } a > -b.$
$\Rightarrow x \in ((-\infty, 5) \cup (5, +\infty)) \cap ((-\infty, 7) \cap (3, +\infty))$ $\Rightarrow x \in ((-\infty, 5) \cup (5, +\infty)) \cap (3, 7)$	Combine intervals.
$\Rightarrow x \in (3, 5) \cup (5, 7)$	Final answer. ■

1.8 $\frac{2x-7}{x^2-6x+8} \leq 1$

$\Rightarrow \frac{2x-7}{x^2-6x+8} - 1 \leq 0$ $\Rightarrow \frac{2x-7}{x^2-6x+8} - \frac{x^2-6x+8}{x^2-6x+8} \leq 0$ $\Rightarrow \frac{2x-7-(x^2-6x+8)}{x^2-6x+8} \leq 0$ $\Rightarrow \frac{2x-7-x^2+6x-8}{x^2-6x+8} \leq 0$ $\Rightarrow \frac{-x^2+8x-15}{x^2-6x+8} \leq 0$ $\Rightarrow \frac{-(x^2-8x+15)}{x^2-6x+8} \leq 0$	Solve for x.
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$$\Rightarrow \frac{x^2-8x+15}{x^2-6x+8} \geq 0$$

$$\Rightarrow \frac{(x-5)(x-3)}{(x-4)(x-2)} \geq 0$$

Factor by grouping. $x \in \{2, 4\}$ are undefined points.

Create a table of signs.

	2	3	4	5	
$x - 5$	-	-	-	-	+
$x - 3$	-	-	+	+	+
$x - 4$	-	-	-	+	+
$x - 2$	-	+	+	+	+
$\frac{(x-5)(x-3)}{(x-4)(x-2)}$	+	-	+	-	+

$$\Rightarrow x \in (-\infty, 2) \cup [3, 4) \cup [5, +\infty)$$

Final answer.

