Exercise Solutions for Math 20

Functions, Graphs, Symmetry

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1.1 Transform each of the following real-valued functions to a functional notation $f(x)$; find its domain and range; and compute $f(1)$.

1.1.a f assigns the number 10 to any one-digit integer.

$\Rightarrow f(x) = 10$ Final answer.

$\Rightarrow \mathrm{dom}(f) = [-9, 9]$ The domain is the set of one digit integers.

$\Rightarrow \mathrm{ran}(f) = 10$ The function can only equate to 10.

$\Rightarrow f(1) = 10$

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1.1.b f maps a given real number to the real number that is 4 more than its square root.

$\Rightarrow f(x) = \sqrt{x} + 4$ Final answer. Assuming that only the principal root is needed.

$\Rightarrow \mathrm{dom}(f) = [0, +\infty)$ The square root of a negative number cannot be real.

$\Rightarrow \mathrm{ran}(f) = [4, +\infty]$

$\Rightarrow f(1) = 5$

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1.1.c $\{(x, y) \mid y = \frac{1}{6-x}\}$

$\Rightarrow f(x) = \frac{1}{6-x}$ Final answer.

$\Rightarrow \mathrm{dom}(f) = \mathbb{R} \setminus \{6\}$ $x = 6$ is an undefined point.

$\Rightarrow \mathrm{ran}(f) = \mathbb{R}$

$\Rightarrow f(1) = \frac{1}{5}$

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1.1.d $\{(x, y) \mid y = \sqrt{x^2 - 5x + 6}\}$

$\Rightarrow (x-2)(x-3) \geq 0$ Find the domain by solving $x^2 - 5x + 6 \geq 0$; factor by grouping.

Create a table of signs.

		2		3	
$x-2$	$-$		$+$		$+$
$x-3$	$-$		$-$		$+$
$(x-2)(x-3)$	$+$		$-$		$+$

$\Rightarrow x \in (-\infty, 2] \cup [3, +\infty)$

$\Rightarrow f(x) = \sqrt{x^2 - 5x + 6}$ Final answer.

$\Rightarrow \operatorname{dom}(f) = (-\infty, 2] \cup [3, +\infty)$

$\Rightarrow \operatorname{ran}(f) = [0, +\infty)$

$\Rightarrow f(1) = \sqrt{2}$

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1.1.e $\left\{(x, y) \mid y = \frac{1}{\sqrt[3]{x^2 - 1}}\right\}$

$\Rightarrow (x - 1)(x + 1) = 0$	Find undefined points by solving $x^2 - 1 = 0$; factor using difference of two squares.
$\Rightarrow x \in \{-1, 1\}$	

$\Rightarrow x = \frac{1}{\sqrt[3]{y^2 - 1}}$ *Find the inverse function.*

$\Rightarrow \frac{1}{x} = \sqrt[3]{y^2 - 1}$

$\Rightarrow \frac{1}{x^3} = y^2 - 1$

$\Rightarrow y^2 = \frac{1}{x^3} + 1$

$\Rightarrow y^2 = \frac{1}{x^3} + \frac{x^3}{x^3}$

$\Rightarrow y^2 = \frac{1 + x^3}{x^3}$

$\Rightarrow f'(x) = \pm\sqrt{\frac{1 + x^3}{x^3}}$

$\Rightarrow \sqrt{\frac{1 + x^3}{x^3}} > 0$ Solve for the domain. Note that $x = 0$ is an undefined point.

Create a table of signs.

		-1		0	
$1 + x^3$		$-$	$+$		$+$
x^3		$-$	$-$		$+$
$\sqrt{\frac{1+x^3}{x^3}}$		$+$	$-$		$+$

$\Rightarrow \operatorname{dom}(f') = (-\infty, -1] \cup (0, +\infty)$

$\Rightarrow f(x) = \frac{1}{\sqrt[3]{x^2 - 1}}$ *Final answer.*

$\Rightarrow \operatorname{dom}(f) = \mathbb{R} \setminus \{-1, 1\}$

$\Rightarrow \operatorname{ran}(f) = (-\infty, -1] \cup (0, +\infty)$

$\Rightarrow f(1)$ is undefined.

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