

Exercise Solutions for Math 20

Linear, Quadratic, and Rational Equations

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November 8, 2024

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1.1 Find the solution set of the following equations.

1.1.a $x + 9 = 5 - 3x$

$$\Rightarrow x + 3x = 5 - 9$$

Solve for x .

$$\Rightarrow 4x = -4$$

$$\Rightarrow x = -1$$

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1.1.b $\frac{2x+3}{4} - \frac{x-1}{2} = -\frac{1}{3}$

$$\Rightarrow \frac{3(2x+3)}{12} - \frac{6(x-1)}{12} = -\frac{4}{12}$$

LCM = 12

$$\Rightarrow \frac{6x+9}{12} - \frac{6x-6}{12} = -\frac{4}{12}$$

$$\Rightarrow (6x + 9) - (6x - 6) = -4$$

$$\Rightarrow 6x + 9 - 6x + 6 = -4$$

$$\Rightarrow 15 = -4$$

$$\Rightarrow x \in \emptyset$$

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1.1.c $3x^2 - 2x + 1 = 0$

$$\Rightarrow \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(1)}}{2(3)}$$

Use the quadratic formula.

$$\Rightarrow \frac{2 \pm \sqrt{-8}}{6}$$

$$\Rightarrow \frac{2 \pm \sqrt{4}\sqrt{-2}}{6}$$

$$\Rightarrow \frac{2 \pm 2i\sqrt{2}}{6}$$

$$\Rightarrow \frac{1 \pm i\sqrt{2}}{3}$$

$$\Rightarrow \frac{1}{3} \pm \frac{\sqrt{2}}{3}i$$

$$\Rightarrow x \in \left\{ \frac{1}{3} + \frac{\sqrt{2}}{3}i, \frac{1}{3} - \frac{\sqrt{2}}{3}i \right\}$$

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1.1.d $4x^2 + 2x = 2$

$$\Rightarrow 4x^2 + 2x - 2 = 0$$

Rewrite in standard form.

$$\Rightarrow 2x^2 + x - 1 = 0$$

$$\Rightarrow 2x^2 + 2x - x - 1 = 0$$

Factor by grouping.

$$\Rightarrow 2x(x + 1) - 1(x + 1) = 0$$

$$\Rightarrow (2x - 1)(x + 1) = 0$$

$$\Rightarrow x \in \{-1, \frac{1}{2}\}$$

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1.1.e $16x^2 + 9 = 24x$

$$\Rightarrow 16x^2 - 24x + 9 = 0$$

Rewrite in standard form.

$$\Rightarrow 16x^2 - 12x - 12x + 9 = 0$$

Factor by grouping.

$$\Rightarrow 4x(4x - 3) - 3(4x - 3) = 0$$

$$\Rightarrow (4x - 3)^2 = 0$$

$$\Rightarrow x = \frac{3}{4}$$

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1.1.f $\frac{x}{x-1} + \frac{x-5}{x^2+2x-3} = \frac{1}{x+3}$

$$\Rightarrow \frac{x}{x-1} + \frac{x-5}{(x-1)(x+3)} = \frac{1}{x+3}$$

Factor by grouping.

$$\Rightarrow \frac{x(x+3)}{(x-1)(x+3)} + \frac{x-5}{(x-1)(x+3)} = \frac{x-1}{(x-1)(x+3)}$$

LCM = $(x - 1)(x + 3)$

$$\Rightarrow \frac{x^2+3x}{(x-1)(x+3)} + \frac{x-5}{(x-1)(x+3)} = \frac{x-1}{(x-1)(x+3)}$$

$$\Rightarrow x^2 + 3x + x - 5 = x - 1$$

Eliminate denominator. $x \in \{-3, 1\}$ are undefined points.

$$\Rightarrow x^2 + 3x + x - 5 - x + 1 = 0$$

$$\Rightarrow x^2 + 3x - 4 = 0$$

$$\Rightarrow (x + 4)(x - 1) = 0$$

Factor by grouping.

$$\Rightarrow x = -4$$

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