Exercise Solutions for Math 20

Conics (Parabola and Ellipse)

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1.1 Determine the vertex and orientation of the following parabolas.

1.1.a $4y^2 + 4y + x = 2$

$\Rightarrow 4y^2 + 4y = -x + 2$	Isolate y.
$\Rightarrow y^2 + y = -\frac{x}{4} + \frac{2}{4}$	
$\Rightarrow y^2 + y = -\frac{x}{4} + \frac{1}{2}$	
$\Rightarrow y^2 + y + \frac{1}{4} = -\frac{x}{4} + \frac{1}{2} + \frac{1}{4}$	Complete the square.
$\Rightarrow (y + \frac{1}{2})^2 = -\frac{x}{4} + \frac{3}{4}$	
$\Rightarrow (y + \frac{1}{2})^2 = -\frac{1}{4}(x - 3)$	
$\Rightarrow (y + \frac{1}{2})^2 = 4(-\frac{1}{16})(x - 3)$	
$\Rightarrow$ Opening leftward, $(h, k) = (3, -\frac{1}{2})$	Final answer.
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1.1.b $x^2 - 6x - 2y = 7$

$\Rightarrow x^2 - 6x = 2y + 7$	Isolate x.
$\Rightarrow x^2 - 6x + 9 = 2y + 7 + 9$	Complete the square.
$\Rightarrow (x - 3)^2 = 2y + 16$	
$\Rightarrow (x - 3)^2 = 2(y + 8)$	
$\Rightarrow (x - 3)^2 = 4(\frac{1}{2})(y + 8)$	
$\Rightarrow$ Opening upward, $(h, k) = (3, -8)$	Final answer.
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1.1.c $2y^2 - 6y - 9x = 0$

$\Rightarrow 2y^2 - 6y = 9x$	Isolate y.
$\Rightarrow y^2 - 3y = \frac{9}{2}x$	
$\Rightarrow y^2 - 3y + \frac{9}{4} = \frac{9}{2}x + \frac{9}{4}$	Complete the square.
$\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}x + \frac{9}{4}$	
$\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{9}{4} \cdot \frac{2}{9})$	
$\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{18}{36})$	
$\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{1}{2})$	
$\Rightarrow (y - \frac{3}{2})^2 = 4(\frac{9}{8})(x + \frac{1}{2})$	
$\Rightarrow$ Opening rightward, $(h, k) = (-\frac{1}{2}, \frac{3}{2})$	Final answer.
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1.2 Sketch the graph of the following parabolas.

1.2.a $3y^2 = 8x$

$\Rightarrow y^2 = 4(\frac{2}{3})x$	Rewrite in standard form.
$\Rightarrow$ See Figure 1.	Final answer. Graph the parabola. ■

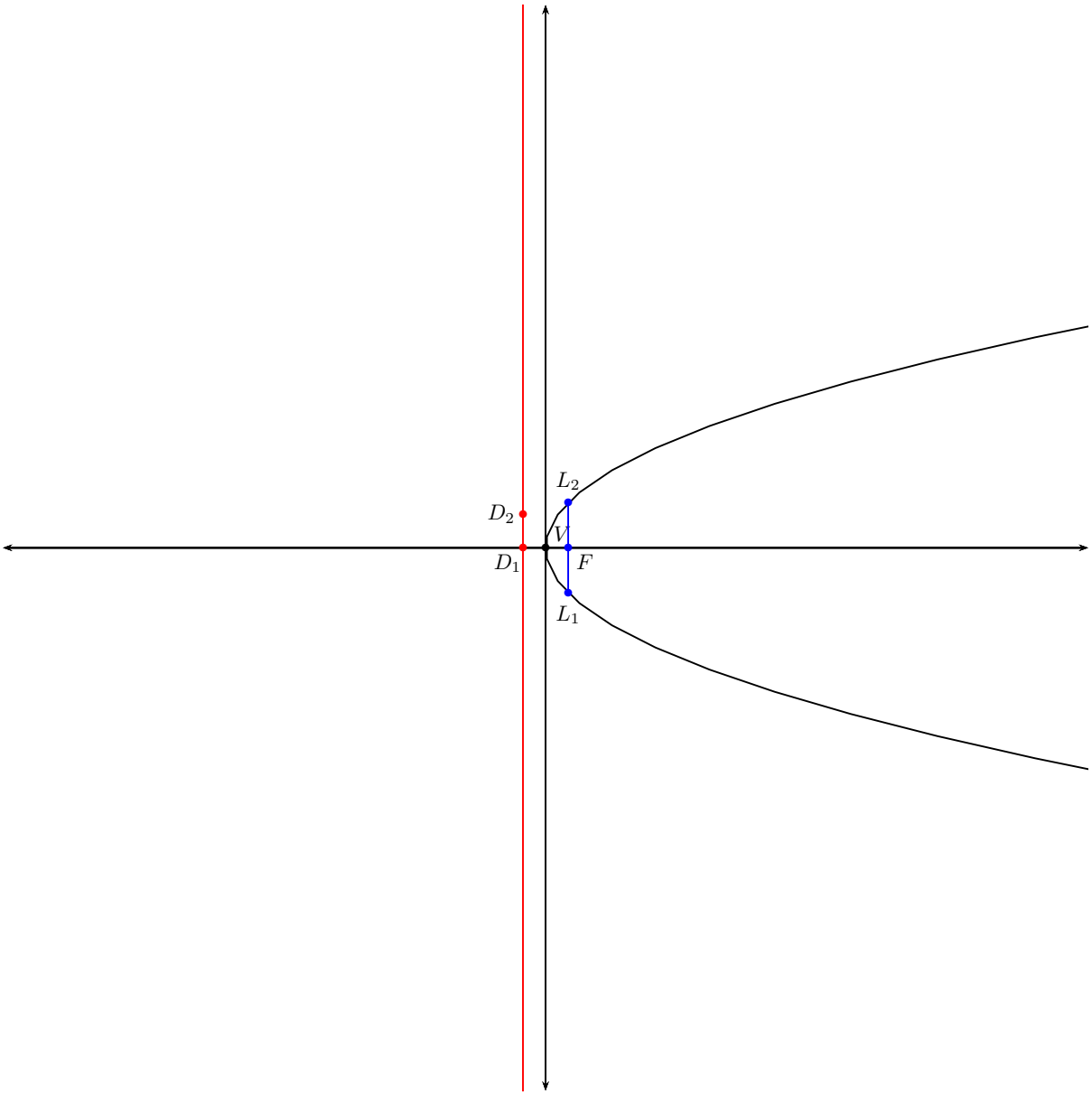

Figure 1. Graph of $y^2 = 4(\frac{2}{3})x$.

1.2.b $x^2 - 8x + 4y = -10$

$\Rightarrow x^2 - 8x = -4y - 10$	Rewrite in standard form.
$\Rightarrow x^2 - 8x + 16 = -4y - 10 + 16$	Complete the square.
$\Rightarrow (x-4)^2 = -4y + 6$	

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$\Rightarrow (x-4)^2 = 4(-1)(y-\frac{3}{2})$	
$\Rightarrow$ See Figure 2.	Final answer. Graph the parabola. ■

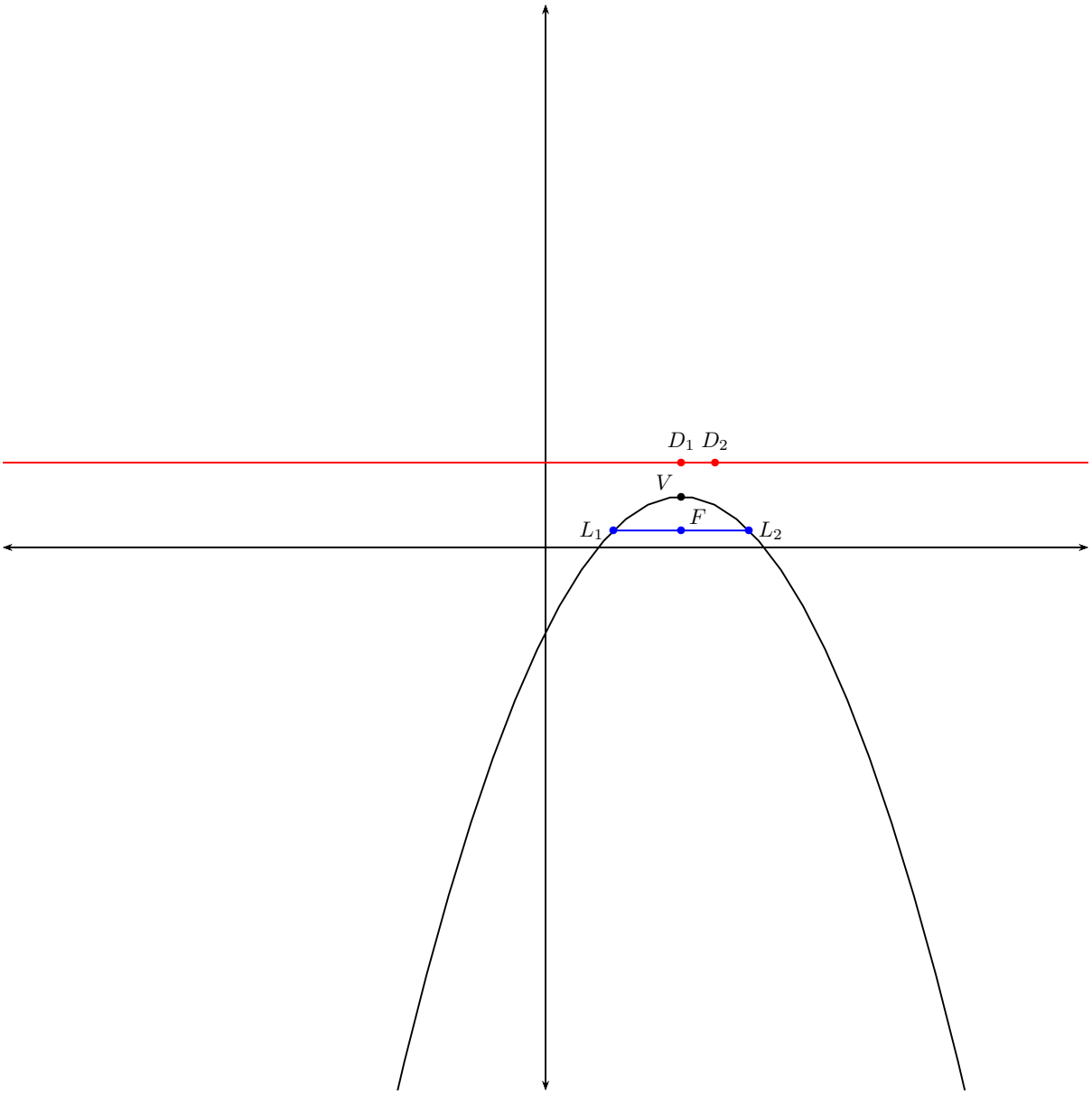

Figure 2. Graph of $(x-4)^2 = 4(-1)(y-\frac{3}{2})$.

1.3 Sketch the graph of $y = -x^2 + 6x - 8$. Label the vertex, x- and y-intercept(s).

$\Rightarrow -x^2 + 6x = y + 8$	Rewrite in standard form.
$\Rightarrow x^2 - 6x = -y - 8$	
$\Rightarrow x^2 - 6x + 9 = -y - 8 + 9$	Complete the square.
$\Rightarrow (x-3)^2 = -y + 1$	

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$\Rightarrow (x-3)^2 = 4(-\frac{1}{4})(y-1)$	
$\Rightarrow (x-3)^2 = 4(-\frac{1}{4})(-1)$ $\Rightarrow (x-3)^2 = 4(\frac{1}{4})$ $\Rightarrow (x-3)^2 = 1$ $\Rightarrow x = \pm 1 + 3$ $\Rightarrow x = 1+3, x = -1+3$ $\Rightarrow x_i \in \{2, 4\}$	Find the x-intercepts.
$\Rightarrow (0-3)^2 = 4(-\frac{1}{4})(y-1)$ $\Rightarrow (-3)^2 = 4(-\frac{1}{4})(y-1)$ $\Rightarrow 9 = -(y-1)$ $\Rightarrow 9 = -y+1$ $\Rightarrow y = 1-9$ $\Rightarrow y_i = -8$	Find the y-intercepts.
$\Rightarrow$ See Figure 3. ■	Final answer. Graph the parabola.

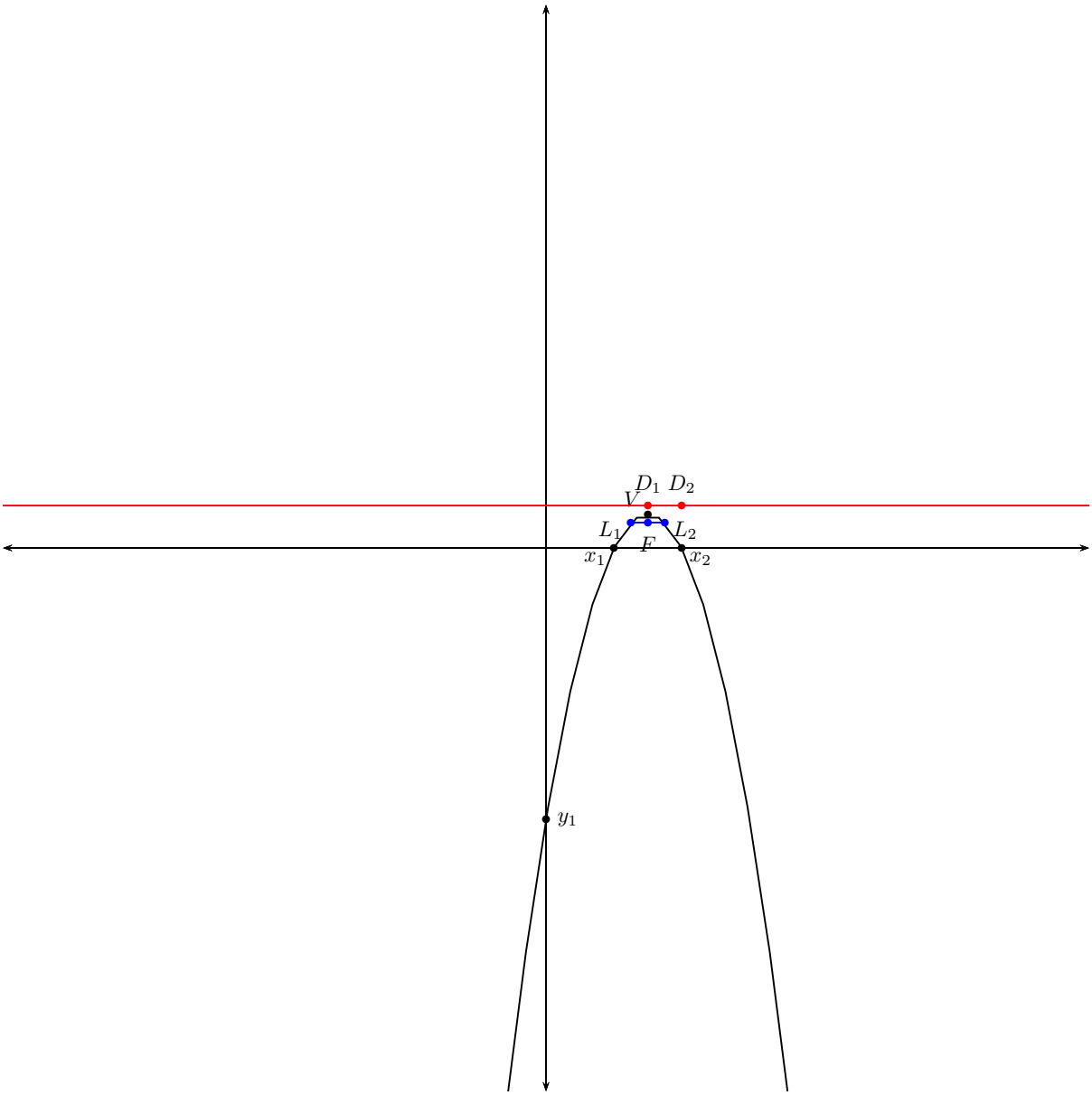

Figure 3. Graph of $(x-3)^2 = 4(-\frac{1}{4})(y-1)$ with x- and y-intercepts.

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2.1 Sketch the graph of the following ellipses.

2.1.a $\frac{x^2}{4} + \frac{(y-1)^2}{9} = 1$

$\Rightarrow \frac{x^2}{2^2} + \frac{(y-1)^2}{3^2} = 1$	Rewrite in standard form.
$\Rightarrow$ See Figure 4.	Final answer. Graph the ellipse.
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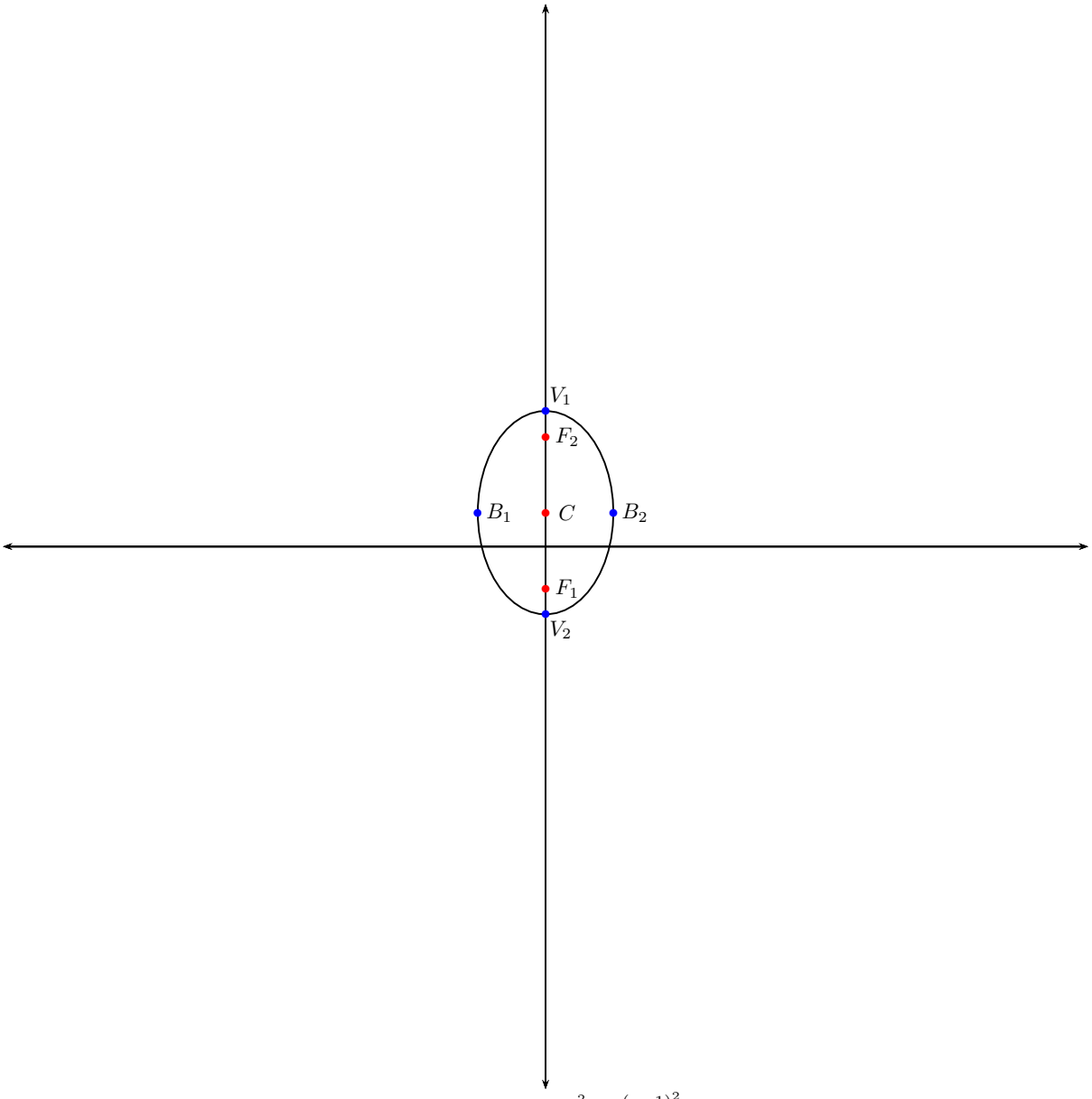

Figure 4. Graph of $\frac{x^2}{2^2} + \frac{(y-1)^2}{3^2} = 1$.

2.1.b $\frac{(x-3)^2}{25} + \frac{y^2+4y+4}{9} = 1$

$\Rightarrow \frac{(x-3)^2}{25} + \frac{(y+2)^2}{9} = 1$	Factor by grouping.
$\Rightarrow \frac{(x-3)^2}{5^2} + \frac{(y+2)^2}{3^2} = 1$	Rewrite in standard form.
$\Rightarrow$ See Figure 5.	Final answer. Graph the ellipse. ■

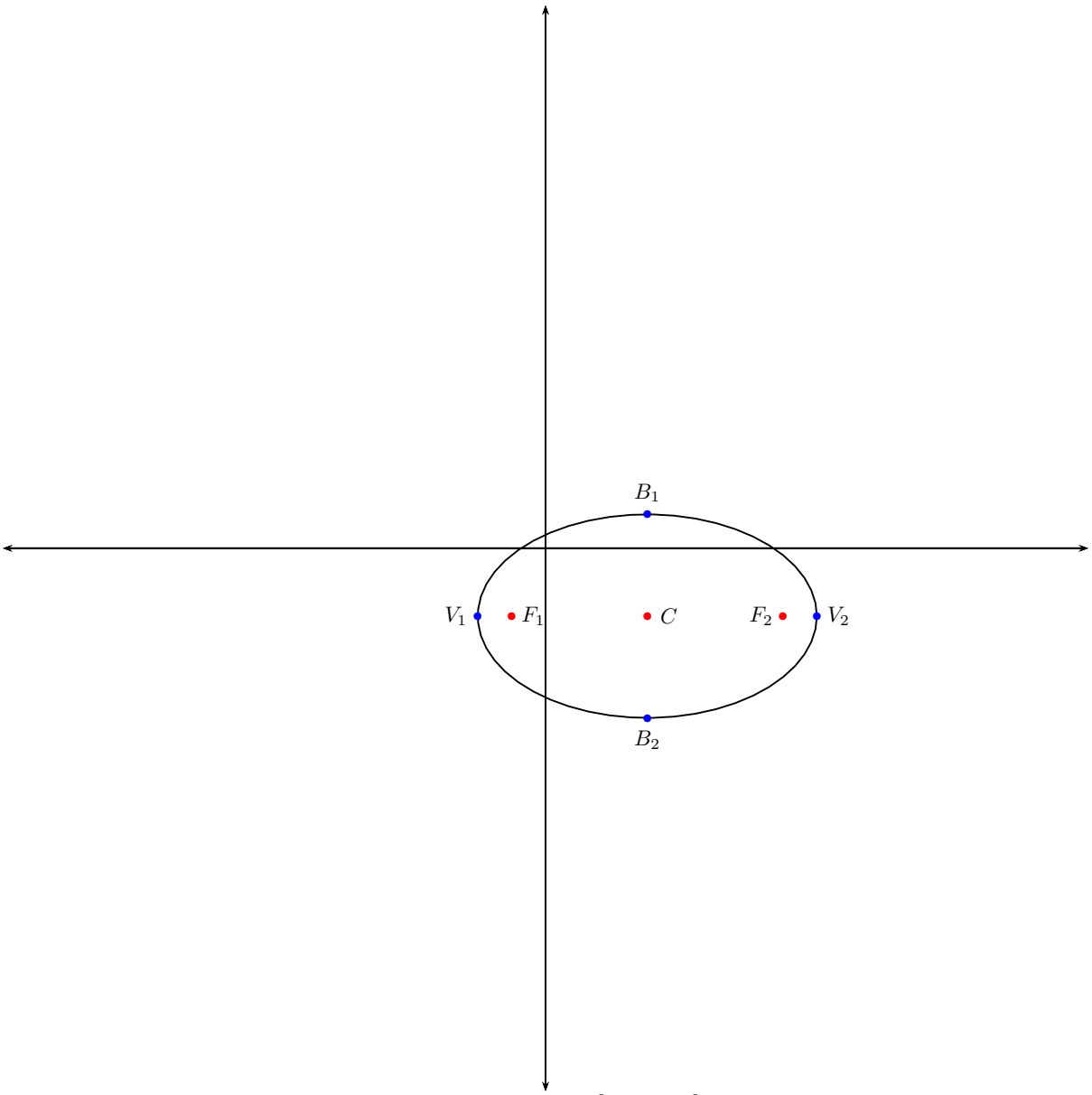

Figure 5. Graph of $\frac{(x-3)^2}{5^2} + \frac{(y+2)^2}{3^2} = 1$.

2.1.c $2x^2 + 3y^2 + 16x - 18y = 13$

$\Rightarrow 2x^2 + 16x + 3y^2 - 18y = 13$	Group terms.
$\Rightarrow 2(x^2 + 8x) + 3(y^2 - 6y) = 13$	

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$\Rightarrow 2(x^2+8x+16)+3(y^2-6y)=13+2(16)$	Complete the square.
$\Rightarrow 2(x^2+8x+16)+3(y^2-6y)=13+32$	
$\Rightarrow 2(x^2+8x+16)+3(y^2-6y)=45$	
$\Rightarrow 2(x+4)^2+3(y^2-6y)=45$	
$\Rightarrow 2(x+4)^2+3(y^2-6y+9)=45+3(9)$	Complete the square.
$\Rightarrow 2(x+4)^2+3(y^2-6y+9)=45+27$	
$\Rightarrow 2(x+4)^2+3(y^2-6y+9)=72$	
$\Rightarrow 2(x+4)^2+3(y-3)^2=72$	
$\Rightarrow \frac{2(x+4)^2}{72}+\frac{3(y-3)^2}{72}=1$	
$\Rightarrow \frac{(x+4)^2}{36}+\frac{(y-3)^2}{24}=1$	
$\Rightarrow \frac{(x+4)^2}{6^2}+\frac{(y-3)^2}{(\sqrt{24})^2}=1$	Rewrite in standard form.
$\Rightarrow \frac{(x+4)^2}{6^2}+\frac{(y-3)^2}{(\sqrt{4}\sqrt{6})^2}=1$	
$\Rightarrow \frac{(x+4)^2}{6^2}+\frac{(y-3)^2}{(2\sqrt{6})^2}=1$	
$\Rightarrow$ See Figure 6.	Final answer. Graph the ellipse.

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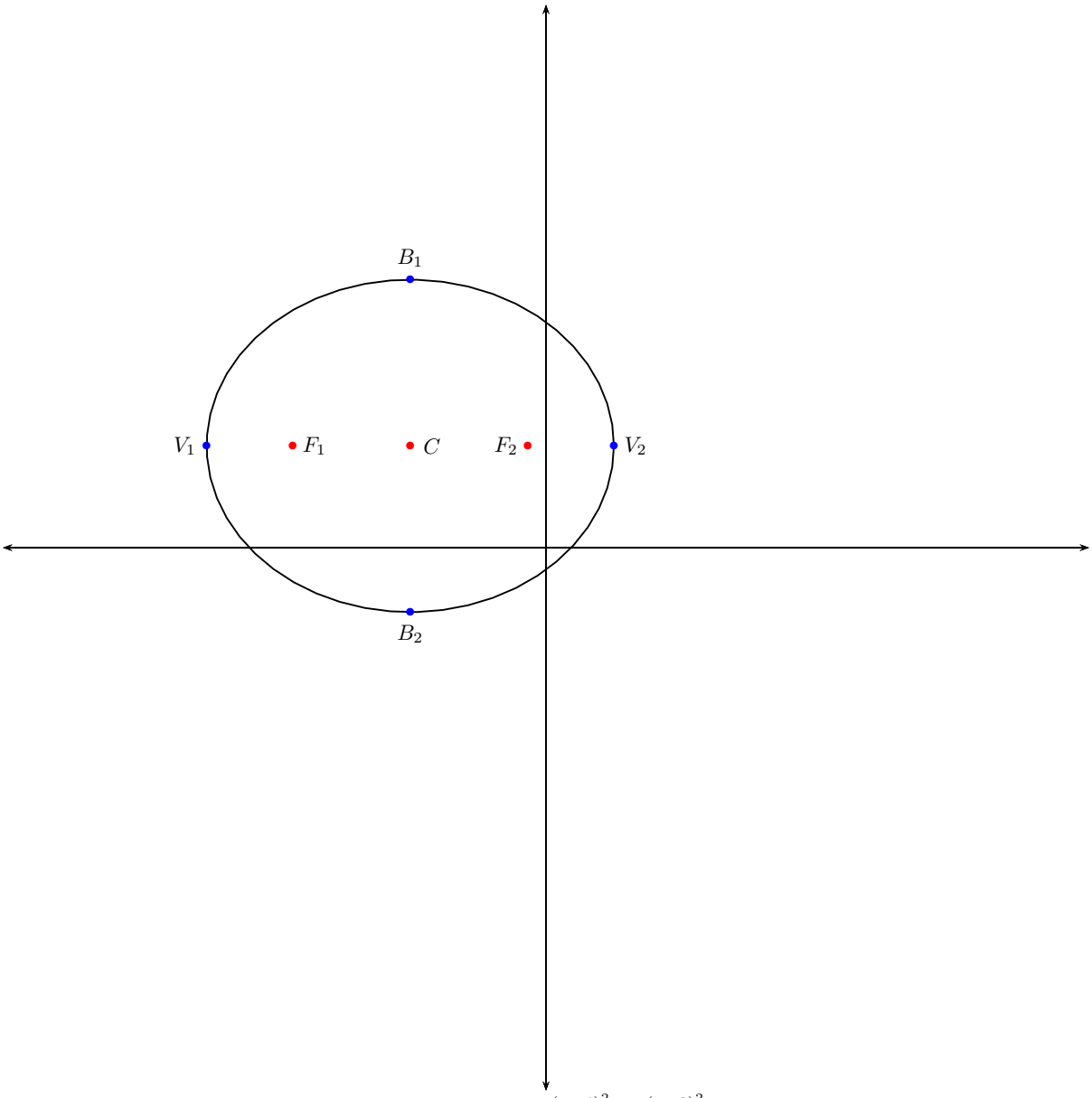

Figure 6. Graph of $\frac{(x+4)^2}{6^2} + \frac{(y-3)^2}{(2\sqrt{6})^2} = 1$

2.2 Find an equation of the parabola that opens downward and whose vertex and focus are the vertices of the ellipse $4(x-2)^2 + (y+1)^2 = 1$

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$\Rightarrow \frac{(x-2)^2}{\frac{1}{4}} + \frac{(y+1)^2}{1} = 1$	Rewrite in standard form.
$\Rightarrow \frac{(x-2)^2}{\frac{1}{2}^2} + \frac{(y+1)^2}{1^2} = 1$	Since $a < b$, this is an ellipse with a vertical major axis.
$\Rightarrow V_1 = (2, -2)$	$V_1 = (h, k-b)$ for ellipses with a vertical major axis.
$\Rightarrow V_2 = (2, 0)$	$V_2 = (h, k+b)$ for ellipses with a vertical major axis.
$\Rightarrow V = (2, 0), F = (2, -2)$	Derive the vertex and focus. Since this is a parabola opening downward, the lower point is the focus.
$\Rightarrow 0 + p = -2$	Derive p; $F = (h, k+p)$ for parabolas opening downward.
$\Rightarrow p = -2$	

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$\Rightarrow (x-2)^2 = 4(-2)y$	Final answer. Write the parabola equation using (h,k) and p. ■