

Exercise Solutions for Math 20

Linear Inequalities System, Nonlinear Systems

Nile Jocson <novoseiversia@gmail.com>

November 20, 2024

1.1 Solve algebraically for the solution sets of the following systems of equations.

$$\begin{cases} y = x^2 - 2x - 8 \\ 4x + 3y + 3 = 0 \end{cases}$$

$$\begin{cases} 10x^2 - xy + 4y^2 = 28 \\ 2x^2 - 3xy - 2y^2 = 0 \end{cases}$$

Use the quadratic formula.

2

$\Rightarrow \frac{3y \pm 5y}{4}$ $\Rightarrow \frac{3y+5y}{4}, \frac{3y-5y}{4}$ $\Rightarrow \frac{8y}{4}, \frac{-2y}{4}$ $\Rightarrow x \in \{-\frac{1}{2}y, 2y\}$	
$\Rightarrow 10(-\frac{1}{2}y)^2 - (-\frac{1}{2}y)y + 4y^2 = 28$ $\Rightarrow 10(\frac{1}{4}y^2) + \frac{1}{2}y^2 + 4y^2 = 28$ $\Rightarrow \frac{5}{2}y^2 + \frac{1}{2}y^2 + 4y^2 = 28$ $\Rightarrow \frac{5}{2}y^2 + \frac{1}{2}y^2 + \frac{8}{2}y^2 = 28$ $\Rightarrow \frac{14}{2}y^2 = 28$ $\Rightarrow 7y^2 = 28$ $\Rightarrow y^2 = 4$ $\Rightarrow y = \pm 2$	Solve for $y, x = -\frac{1}{2}y$.
$\Rightarrow x = -\frac{1}{2}(-2)$ $\Rightarrow x = 1$ $\Rightarrow x = -\frac{1}{2}(2)$ $\Rightarrow x = -1$	Solve for $x, y = -2$ Solve for $x, y = 2$
$\Rightarrow 10(2y)^2 - (2y)y + 4y^2 = 28$ $\Rightarrow 10(4y^2) - 2y^2 + 4y^2 = 28$ $\Rightarrow 40y^2 - 2y^2 + 4y^2 = 28$ $\Rightarrow 42y^2 = 28$ $\Rightarrow y^2 = \frac{28}{42}$ $\Rightarrow y^2 = \frac{2}{3}$ $\Rightarrow y = \pm \sqrt{\frac{2}{3}}$	Solve for $y, x = 2y$.
$\Rightarrow x = -2\sqrt{\frac{2}{3}}$ $\Rightarrow x = 2\sqrt{\frac{2}{3}}$	Solve for $x, y = -\sqrt{\frac{2}{3}}$ Solve for $x, y = \sqrt{\frac{2}{3}}$
$\Rightarrow (x, y) = \{(1, -2), (-1, 2), (-2\sqrt{\frac{2}{3}}, -\sqrt{\frac{2}{3}}), (2\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}})\}$	Final answer. ■

1.2 Sketch the solution region for each of the given system of inequalities.

1.2.a

$$\begin{cases} y \leq 2x + 1 \\ x < 5 \\ y < x + 2 \end{cases}$$

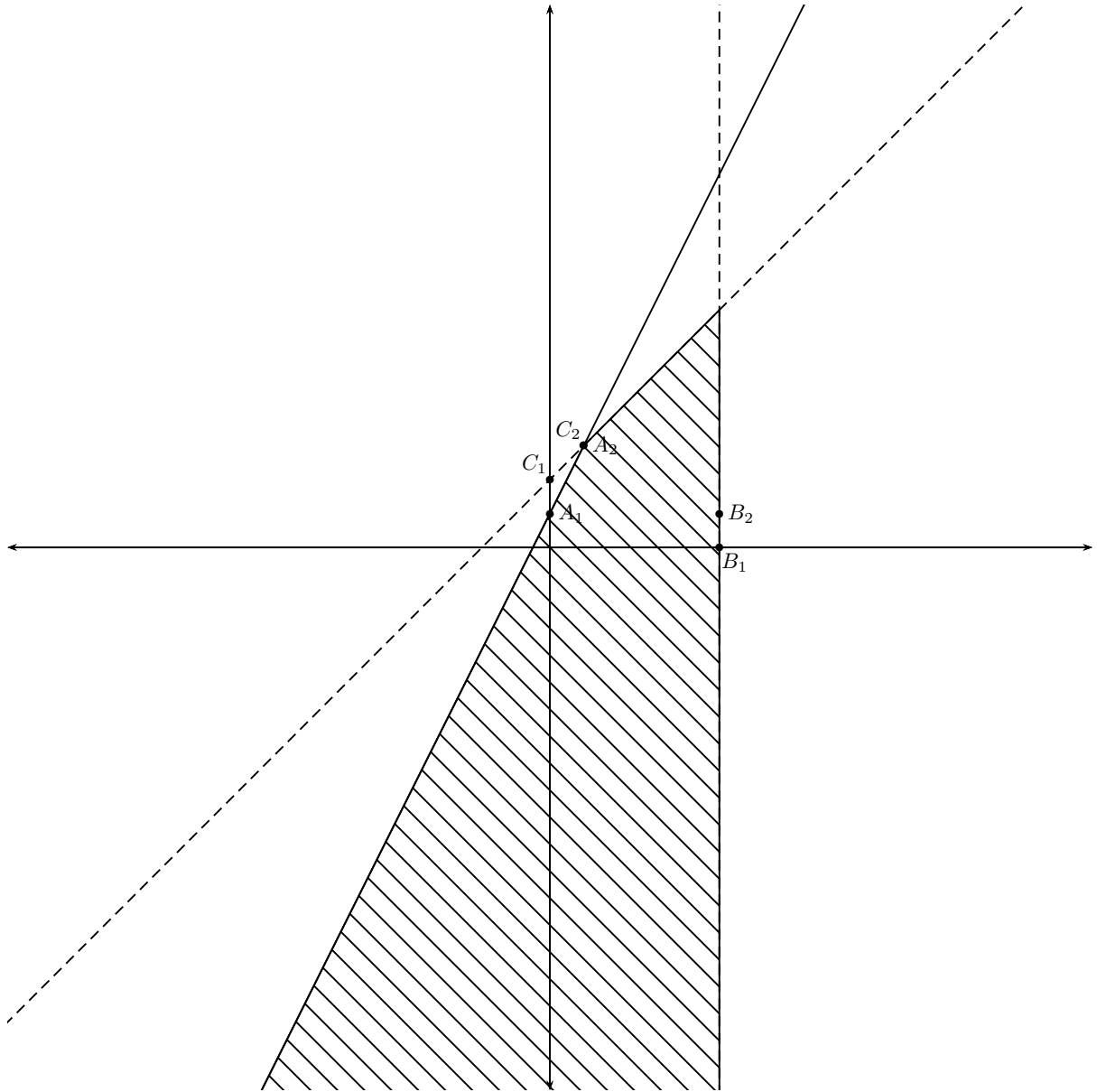


Figure 1.

1.2.b

$$\begin{cases} x^2 + y^2 \leq 16 \\ y \leq x^2 - 3 \\ y < 0 \end{cases}$$

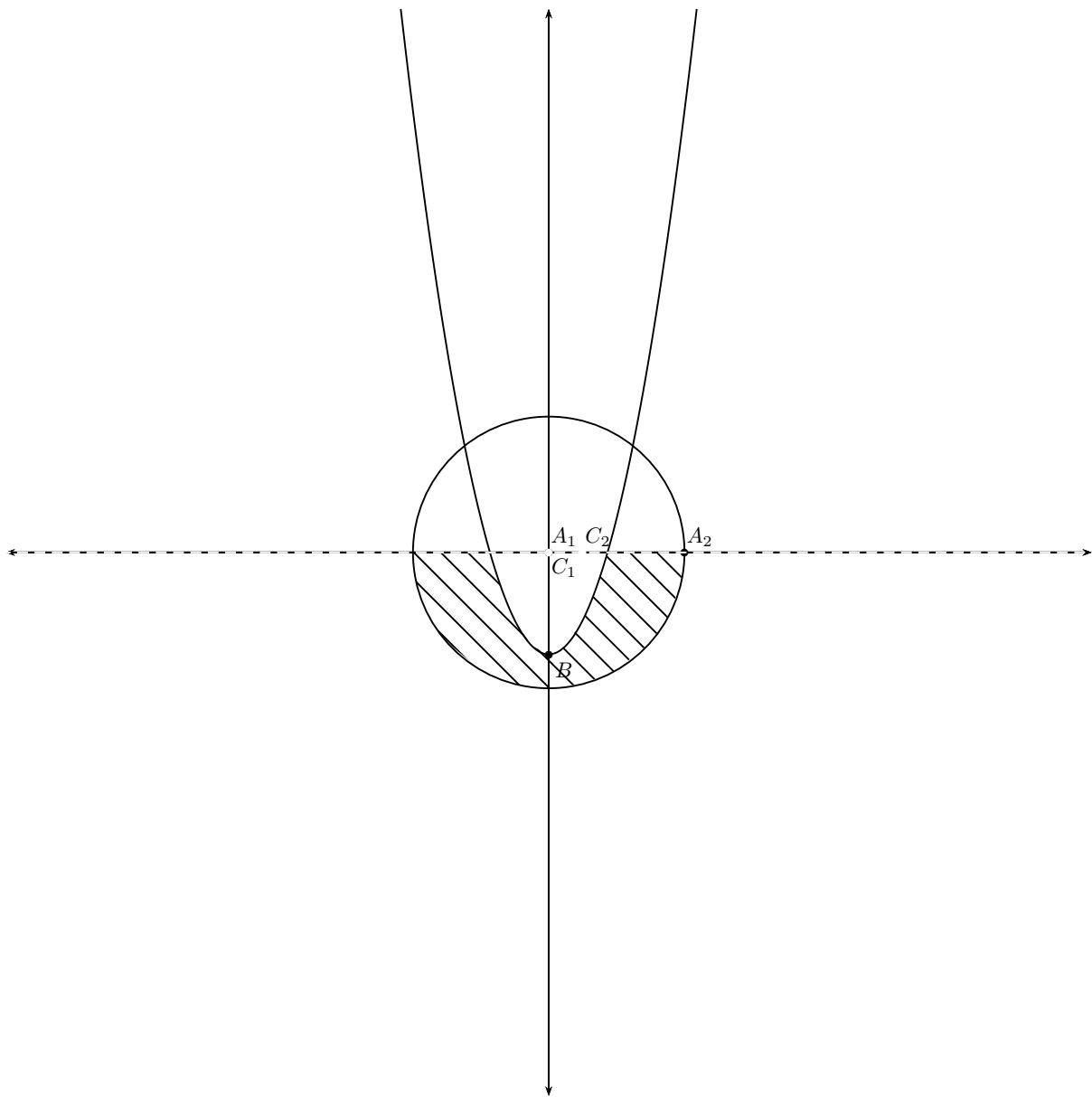


Figure 2.