Exercise Solutions for Math 20

Equations in Quadratic Form and with Radicals and Absolute Values

Nile Jocson <novoseiversia@gmail.com>

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1 Find the solution set of the following inequalities.

1.1 $\frac{2x+1}{4} \leq \frac{2x}{3} + \frac{1}{6}$

$\Rightarrow \frac{3(2x+1)}{12} \leq \frac{4(2x)}{12} + \frac{2}{12}$ LCM $= 12$

$\Rightarrow \frac{6x+3}{12} \leq \frac{8x}{12} + \frac{2}{12}$

$\Rightarrow \frac{6x+3}{12} \leq \frac{8x+2}{12}$

$\Rightarrow 6x + 3 \leq 8x + 2$

$\Rightarrow 3 - 2 \leq 8x - 6x$

$\Rightarrow 1 \leq 2x$

$\Rightarrow x \geq \frac{1}{2}$

$\Rightarrow x \in [\frac{1}{2}, +\infty)$ Final answer.

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1.2 $-2 < 5 + 3x < 20$

$\Rightarrow -7 < 3x < 15$ Solve for x.

$\Rightarrow -\frac{7}{3} < x < 5$

$\Rightarrow x \in (-\frac{7}{3}, 5)$ Final answer.

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1.3 $\frac{x}{x-1} > -1$

$\Rightarrow \frac{x}{x-1} + 1 > 0$ Solve for x.

$\Rightarrow \frac{x}{x-1} + \frac{x-1}{x-1} > 0$

$\Rightarrow \frac{x+x-1}{x-1} > 0$

$\Rightarrow \frac{2x-1}{x-1} > 0$ $x = 1$ is an undefined point.

Create a table of signs.

		$\frac{1}{2}$	1
$2x - 1$	$-$	$+$	$+$
$x - 1$	$-$	$-$	$+$
$\frac{2x-1}{x-1}$	$+$	$-$	$+$

$\Rightarrow x \in (-\infty, \frac{1}{2}) \cup (1, +\infty)$ Final answer.

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1.4 $\frac{x}{x+1} \geq \frac{2}{x+3}$

$\Rightarrow \frac{x}{x+1} - \frac{2}{x+3} \geq 0$ Solve for x.

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$\Rightarrow \frac{x(x+3)}{(x+1)(x+3)} - \frac{2(x+1)}{(x+1)(x+3)} \geq 0$ $\Rightarrow \frac{x(x+3)-2(x+1)}{(x+1)(x+3)} \geq 0$ $\Rightarrow \frac{x^2+3x-2x-2}{(x+1)(x+3)} \geq 0$ $\Rightarrow \frac{x^2+x-2}{(x+1)(x+3)} \geq 0$ $\Rightarrow \frac{(x-1)(x+2)}{(x+1)(x+3)} \geq 0$	LCM $= (x+1)(x+3)$ Factor by grouping. $x \in \{-3, -1\}$ are undefined points.

Create a table of signs.

	-3	-2	-1	1	
$x-1$	$-$	$-$	$-$	$-$	$+$
$x+2$	$-$	$-$	$+$	$+$	$+$
$x+1$	$-$	$-$	$-$	$+$	$+$
$x+3$	$-$	$+$	$+$	$+$	$+$
$\frac{(x-1)(x+2)}{(x+1)(x+3)}$	$+$	$-$	$+$	$-$	$+$

$\Rightarrow (-\infty, -3) \cup [-2, -1) \cup [1, +\infty)$ — Final answer. Don't include undefined points.

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1.5 $\left|\frac{9-2x}{4x}\right| \geq 1$

$\Rightarrow \frac{9-2x}{4x} \geq 1$ — $|a| \geq b \Rightarrow a \geq b$ or $a \leq -b$. Solve for $a \geq b$.

$\Rightarrow \frac{9-2x}{4x} - 1 \geq 0$

$\Rightarrow \frac{9-2x}{4x} - \frac{4x}{4x} \geq 0$

$\Rightarrow \frac{9-2x-4x}{4x} \geq 0$

$\Rightarrow \frac{9-6x}{4x} \geq 0$

$\Rightarrow \frac{9-6x}{x} \geq 0$

$\Rightarrow \frac{-3(2x-3)}{x} \geq 0$

$\Rightarrow \frac{2x-3}{x} \leq 0$ — $x = 0$ is an undefined point.

Create a table of signs.

	0	$\frac{3}{2}$	
$2x-3$	$-$	$-$	$+$
x	$-$	$+$	$+$
$\frac{2x-3}{x}$	$+$	$-$	$+$

$\Rightarrow x \in (0, \frac{3}{2}]$

$\Rightarrow \frac{9-2x}{4x} \leq -1$ — $|a| \geq b \Rightarrow a \geq b$ or $a \leq -b$. Solve for $a \leq -b$.

$\Rightarrow \frac{9-2x}{4x} + 1 \leq 0$

$\Rightarrow \frac{9-2x}{4x} + \frac{4x}{4x} \leq 0$

$\Rightarrow \frac{9-2x+4x}{4x} \leq 0$

$\Rightarrow \frac{2x+9}{4x} \leq 0$

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$\Rightarrow \frac{2x+9}{x} \leq 0$ $x = 0$ is an undefined point.

Create a table of signs.

	$-\frac{9}{2}$	0	
$2x+9$	$-$	$+$	$+$
x	$-$	$-$	$+$
$\frac{2x+9}{x}$	$+$	$-$	$+$

$\Rightarrow x \in [-\frac{9}{2}, 0)$

$\Rightarrow x \in [-\frac{9}{2}, 0) \cup (0, \frac{3}{2}]$ Final answer. Combine intervals.

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1.6 $\left|\frac{x}{2x-3}\right| \leq 1$

$\Rightarrow \frac{x}{2x-3} \leq 1$ $|a| \leq b \Rightarrow a \leq b$ and $a \geq -b$. Solve for $a \leq b$.

$\Rightarrow \frac{x}{2x-3} - 1 \leq 0$

$\Rightarrow \frac{x}{2x-3} - \frac{2x-3}{2x-3} \leq 0$

$\Rightarrow \frac{x-(2x-3)}{2x-3} \leq 0$

$\Rightarrow \frac{x-2x+3}{2x-3} \leq 0$

$\Rightarrow \frac{-x+3}{2x-3} \leq 0$

$\Rightarrow \frac{-(x-3)}{2x-3} \leq 0$

$\Rightarrow \frac{x-3}{2x-3} \geq 0$ $x = \frac{3}{2}$ is an undefined point.

Create a table of signs.

	$\frac{3}{2}$	3	
$x-3$	$-$	$-$	$+$
$2x-3$	$-$	$+$	$+$
$\frac{x-3}{2x-3}$	$+$	$-$	$+$

$\Rightarrow x \in (-\infty, \frac{3}{2}) \cup [3, +\infty)$

$\Rightarrow \frac{x}{2x-3} \geq -1$ $|a| \leq b \Rightarrow a \leq b$ and $a \geq -b$. Solve for $a \geq -b$.

$\Rightarrow \frac{x}{2x-3} + 1 \geq 0$

$\Rightarrow \frac{x}{2x-3} + \frac{2x-3}{2x-3} \geq 0$

$\Rightarrow \frac{x+2x-3}{2x-3} \geq 0$

$\Rightarrow \frac{3x-3}{2x-3} \geq 0$

$\Rightarrow \frac{3(x-1)}{2x-3} \geq 0$

$\Rightarrow \frac{x-1}{2x-3} \geq 0$ $x = \frac{3}{2}$ is an undefined point.

Create a table of signs.

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	1	$\frac{3}{2}$	
$x-1$	$-$	$+$	$+$
$2x-3$	$-$	$-$	$+$
$\frac{x-1}{2x-3}$	$+$	$-$	$+$

$\Rightarrow x \in (-\infty, 1] \cup (\frac{3}{2}, +\infty)$

$\Rightarrow x \in ((-\infty, \frac{3}{2}) \cup [3, +\infty)) \cap ((-\infty, 1] \cup (\frac{3}{2}, +\infty))$	Combine intervals.
$\Rightarrow x \in (-\infty, 1] \cup [3, +\infty)$	Final answer.

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1.7 $0 < |x-5| < 2$

$\Rightarrow \|x-5\| > 0, \|x-5\| < 2$	Split the inequality.
$\Rightarrow x-5 > 0$ $\Rightarrow x > 5$ $\Rightarrow x \in (5, +\infty)$	$\|a\| > b \Rightarrow a > b$ or $a < -b$. Solve for $a > b$.
$\Rightarrow x-5 < 0$ $\Rightarrow x < 5$ $\Rightarrow x \in (-\infty, 5)$	$\|a\| > b \Rightarrow a > b$ or $a < -b$. Solve for $a < -b$.
$\Rightarrow x-5 < 2$ $\Rightarrow x < 7$ $\Rightarrow x \in (-\infty, 7)$	$\|a\| < b \Rightarrow a < b$ and $a > -b$. Solve for $a < b$.
$\Rightarrow x-5 > -2$ $\Rightarrow x > 3$ $\Rightarrow x \in (3, +\infty)$	$\|a\| < b \Rightarrow a < b$ and $a > -b$. Solve for $a > -b$.
$\Rightarrow x \in ((-\infty, 5) \cup (5, +\infty)) \cap ((-\infty, 7) \cap (3, +\infty))$ $\Rightarrow x \in ((-\infty, 5) \cup (5, +\infty)) \cap (3, 7)$	Combine intervals.
$\Rightarrow x \in (3, 5) \cup (5, 7)$	Final answer.

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1.8 $\frac{2x-7}{x^2-6x+8} \leq 1$

$\Rightarrow \frac{2x-7}{x^2-6x+8} - 1 \leq 0$ $\Rightarrow \frac{2x-7}{x^2-6x+8} - \frac{x^2-6x+8}{x^2-6x+8} \leq 0$ $\Rightarrow \frac{2x-7-(x^2-6x+8)}{x^2-6x+8} \leq 0$ $\Rightarrow \frac{2x-7-x^2+6x-8}{x^2-6x+8} \leq 0$ $\Rightarrow \frac{-x^2+8x-15}{x^2-6x+8} \leq 0$ $\Rightarrow \frac{-(x^2-8x+15)}{x^2-6x+8} \leq 0$	Solve for x.

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$\Rightarrow \frac{x^2-8x+15}{x^2-6x+8} \geq 0$

$\Rightarrow \frac{(x-5)(x-3)}{(x-4)(x-2)} \geq 0$ Factor by grouping. $x \in \{2, 4\}$ are undefined points.

Create a table of signs.

		2	3	4	5
$x-5$	$-$	$-$	$-$	$-$	$+$
$x-3$	$-$	$-$	$+$	$+$	$+$
$x-4$	$-$	$-$	$-$	$+$	$+$
$x-2$	$-$	$+$	$+$	$+$	$+$
$\frac{(x-5)(x-3)}{(x-4)(x-2)}$	$+$	$-$	$+$	$-$	$+$

$\Rightarrow x \in (-\infty, 2) \cup [3, 4) \cup [5, +\infty)$ Final answer.

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