

# Exercise Solutions for Math 20

Conics (Parabola and Ellipse)

Nile Jocson <novoseiversia@gmail.com>

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# Contents

<b>1</b>		<b>3</b>
1.1	Determine the vertex and orientation of the following parabolas. . . . .	3
1.1.a	$4y^2 + 4y + x = 2$ . . . . .	3
1.1.b	$x^2 - 6x - 2y = 7$ . . . . .	3
1.1.c	$2y^2 - 6y - 9x = 0$ . . . . .	3
1.2	Sketch the graph of the following parabolas. . . . .	4
1.2.a	$3y^2 = 8x$ . . . . .	4
1.2.b	$x^2 - 8x + 4y = -10$ . . . . .	4

# 1

## 1.1 Determine the vertex and orientation of the following parabolas.

**1.1.a**  $4y^2 + 4y + x = 2$

$\Rightarrow 4y^2 + 4y = -x + 2$ $\Rightarrow y^2 + y = -\frac{x}{4} + \frac{2}{4}$ $\Rightarrow y^2 + y = -\frac{x}{4} + \frac{1}{2}$	Isolate $y$ .
$\Rightarrow y^2 + y + \frac{1}{4} = -\frac{x}{4} + \frac{1}{2} + \frac{1}{4}$ $\Rightarrow (y + \frac{1}{2})^2 = -\frac{x}{4} + \frac{3}{4}$ $\Rightarrow (y + \frac{1}{2})^2 = -\frac{1}{4}(x - 3)$ $\Rightarrow (y + \frac{1}{2})^2 = 4(-\frac{1}{16})(x - 3)$	Complete the square.
$\Rightarrow$ Opening leftwards, $(h, k) = (3, -\frac{1}{2})$	Final answer. <span style="float: right;">■</span>

**1.1.b**  $x^2 - 6x - 2y = 7$

$\Rightarrow x^2 - 6x = 2y + 7$	Isolate $x$ .
$\Rightarrow x^2 - 6x + 9 = 2y + 7 + 9$ $\Rightarrow (x - 3)^2 = 2y + 16$ $\Rightarrow (x - 3)^2 = 2(y + 8)$ $\Rightarrow (x - 3)^2 = 4(\frac{1}{2})(y + 8)$	Complete the square.
$\Rightarrow$ Opening upwards, $(h, k) = (3, -8)$	Final answer. <span style="float: right;">■</span>

**1.1.c**  $2y^2 - 6y - 9x = 0$

$\Rightarrow 2y^2 - 6y = 9x$ $\Rightarrow y^2 - 3y = \frac{9}{2}x$	Isolate $y$ .
$\Rightarrow y^2 - 3y + \frac{9}{4} = \frac{9}{2}x + \frac{9}{4}$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}x + \frac{9}{4}$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{9}{4} \cdot \frac{2}{9})$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{18}{36})$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{1}{2})$ $\Rightarrow (y - \frac{3}{2})^2 = 4(\frac{9}{8})(x + \frac{1}{2})$	Complete the square.
$\Rightarrow$ Opening rightwards, $(h, k) = (-\frac{1}{2}, \frac{3}{2})$	Final answer. <span style="float: right;">■</span>

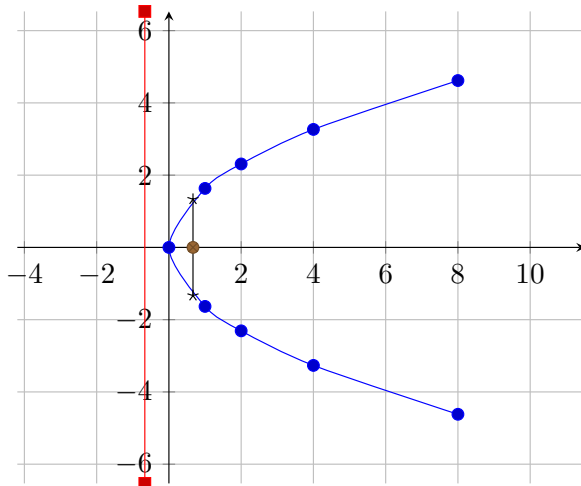
## 1.2 Sketch the graph of the following parabolas.

1.2.a  $3y^2 = 8x$

$$\Rightarrow y^2 = 4\left(\frac{2}{3}\right)x$$

Rewrite in standard form.

Graph the equation.



1.2.b  $x^2 - 8x + 4y = -10$

$$\Rightarrow x^2 - 8x = -4y - 10$$

Rewrite in standard form.

$$\Rightarrow x^2 - 8x + 16 = -4y - 10 + 16$$

Complete the square.

$$\Rightarrow (x - 4)^2 = -4y + 6$$

$$\Rightarrow (x - 4)^2 = 4(-1)\left(y - \frac{3}{2}\right)$$

Graph the equation.

