

Exercise Solutions for Math 20

Equations in Quadratic Form and with Radicals and Absolute Values

Nile Jocson <novoseiversia@gmail.com>

November 9, 2024

Contents

| | | |
|----------|--|----------|
| 1 | Solve for x. | 3 |
| 1.1 | $\sqrt{2x+3} - \sqrt{x-2} = \sqrt{x+1}$ | 3 |
| 1.2 | $1 = x + \sqrt{2x-3}$ | 3 |
| 1.3 | $\left \frac{3x-4}{2x+3} \right = 1$ | 4 |
| 1.4 | $-7\left(\frac{1}{x} - 1\right) = 4 - 2\left(\frac{1}{x} - 1\right)^2$ | 4 |
| 1.5 | $x^2(x^2 - 1) - 9(x^2 - 1) = 0$ | 5 |
| 1.6 | $2(x^2 + x + 1) + \sqrt{x^2 + x + 1} - 3 = 0$ | 5 |

1 Solve for x .

1.1 $\sqrt{2x+3} - \sqrt{x-2} = \sqrt{x+1}$

| | |
|--|---------------------|
| $\Rightarrow (\sqrt{2x+3} - \sqrt{x-2})^2 = x+1$ | Square both sides. |
| $\Rightarrow 2x+3 - 2\sqrt{2x+3}\sqrt{x-2} + x-2 = x+1$ | |
| $\Rightarrow 2x+3 + x-2 - x-1 = 2\sqrt{2x+3}\sqrt{x-2}$ | |
| $\Rightarrow 2x = 2\sqrt{2x+3}\sqrt{x-2}$ | |
| $\Rightarrow x = \sqrt{2x+3}\sqrt{x-2}$ | |
| $\Rightarrow x^2 = (2x+3)(x-2)$ | Square both sides. |
| $\Rightarrow x^2 = 2x^2 - 4x + 3x - 6$ | |
| $\Rightarrow x^2 = 2x^2 - x - 6$ | |
| $\Rightarrow 2x^2 - x^2 - x - 6 = 0$ | |
| $\Rightarrow x^2 - x - 6 = 0$ | |
| $\Rightarrow (x-3)(x+2) = 0$ | Factor by grouping. |
| $\Rightarrow x \subseteq \{-2, 3\}$ | |
| $\Rightarrow \sqrt{2(-2)+3} - \sqrt{-2-2} = \sqrt{-2+1}$ | Verify $x = -2$. |
| $\Rightarrow \sqrt{-4+3} - \sqrt{-2-2} = \sqrt{-2+1}$ | |
| $\Rightarrow \sqrt{-1} - \sqrt{-4} = \sqrt{-1}$ | |
| $\Rightarrow i - 2i = i$ | |
| $\Rightarrow -i = i$ | |
| $\Rightarrow x \neq -2$ | |
| $\Rightarrow \sqrt{2(3)+3} - \sqrt{3-2} = \sqrt{3+1}$ | Verify $x = 3$. |
| $\Rightarrow \sqrt{6+3} - \sqrt{3-2} = \sqrt{3+1}$ | |
| $\Rightarrow \sqrt{9} - \sqrt{1} = \sqrt{4}$ | |
| $\Rightarrow 3 - 1 = 2$ | |
| $\Rightarrow 2 = 2$ | |
| $\Rightarrow x = 3$ | |

■

1.2 $1 = x + \sqrt{2x-3}$

| | |
|---|---------------------|
| $\Rightarrow 1 - x = \sqrt{2x-3}$ | Isolate the root. |
| $\Rightarrow (1-x)^2 = 2x-3$ | Square both sides. |
| $\Rightarrow 1 - 2x + x^2 = 2x-3$ | |
| $\Rightarrow 1 - 2x + x^2 - 2x + 3 = 0$ | |
| $\Rightarrow x^2 - 4x + 4 = 0$ | |
| $\Rightarrow (x-2)^2$ | Factor by grouping. |
| $\Rightarrow x = 2$ | |
| $\Rightarrow 1 = 2 + \sqrt{2(2)-3}$ | Verify $x = 2$ |
| $\Rightarrow 1 = 2 + \sqrt{4-3}$ | |

Continued on next page

| | |
|--|--|
| $\Rightarrow 1 = 2 + \sqrt{1}$ $\Rightarrow 1 = 2 + 1$ $\Rightarrow 1 = 3$ $\Rightarrow x \neq 2$ | |
| $\Rightarrow x \in \emptyset$ | Final answer. ■ |

1.3 $\left| \frac{3x-4}{2x+3} \right| = 1$

| | |
|---|--|
| $\Rightarrow \frac{3x-4}{2x+3} = -1$ $\Rightarrow \frac{3x-4}{2x+3} = -\frac{2x+3}{2x+3}$ $\Rightarrow 3x - 4 = -(2x + 3)$ $\Rightarrow 3x - 4 = -2x - 3$ $\Rightarrow 3x + 2x = -3 + 4$ $\Rightarrow 5x = 1$ $\Rightarrow x = \frac{1}{5}$ | $ a = b \Rightarrow a = \pm b$. Solve for $a = -b$. Eliminate denominator. $x = -\frac{3}{2}$ is an undefined point. |
| $\Rightarrow \frac{3x-4}{2x+3} = 1$ $\Rightarrow \frac{3x-4}{2x+3} = \frac{2x+3}{2x+3}$ $\Rightarrow 3x - 4 = 2x + 3$ $\Rightarrow 3x - 2x = 3 + 4$ $\Rightarrow x = 7$ | $ a = b \Rightarrow a = \pm b$. Solve for $a = +b$. Eliminate denominator. $x = -\frac{3}{2}$ is an undefined point. |
| $\Rightarrow x \in \{\frac{1}{5}, 7\}$ | Final answer. ■ |

1.4 $-7(\frac{1}{x} - 1) = 4 - 2(\frac{1}{x} - 1)^2$

| | |
|---|---|
| $\Rightarrow -7t = 4 - 2t^2$ $\Rightarrow 2t^2 - 7t - 4 = 0$ $\Rightarrow 2t^2 - 8t + t - 4 = 0$ $\Rightarrow 2t(t - 4) + 1(t - 4) = 0$ $\Rightarrow (2t + 1)(t - 4) = 0$ $\Rightarrow (2t + 1)(t - 4) = 0$ $\Rightarrow t \in \{-\frac{1}{2}, 4\}$ | $t = (\frac{1}{x} - 1)$. $x = 0$ is an undefined point. Factor by grouping. |
| $\Rightarrow \frac{1}{x} - 1 = -\frac{1}{2}$ $\Rightarrow \frac{1}{x} = -\frac{1}{2} + 1$ $\Rightarrow \frac{1}{x} = \frac{1}{2}$ $\Rightarrow x = 2$ | Solve for x using $t = -\frac{1}{2}$. |
| $\Rightarrow \frac{1}{x} - 1 = 4$ $\Rightarrow \frac{1}{x} = 4 + 1$ $\Rightarrow \frac{1}{x} = 5$ | Solve for x using $t = 4$. |

Continued on next page

| | |
|--|--|
| $\Rightarrow x = \frac{1}{5}$ | |
| $\Rightarrow x \in \{\frac{1}{5}, 2\}$ | Final answer. ■ |

1.5 $x^2(x^2 - 1) - 9(x^2 - 1) = 0$

| | |
|--------------------------------------|--|
| $\Rightarrow (x^2 - 9)(x^2 - 1) = 0$ | Factor by grouping. |
| $\Rightarrow x^2 - 9 = 0$ | Solve for x. |
| $\Rightarrow (x - 3)(x + 3) = 0$ | Factor using difference of two squares. |
| $\Rightarrow x \in \{-3, 3\}$ | |
| $\Rightarrow x^2 - 1 = 0$ | Solve for x. |
| $\Rightarrow (x - 1)(x + 1) = 0$ | Factor using difference of two squares. |
| $\Rightarrow x \in \{-1, 1\}$ | |
| $\Rightarrow x \in \{-3, -1, 1, 3\}$ | Final answer. ■ |

1.6 $2(x^2 + x + 1) + \sqrt{x^2 + x + 1} - 3 = 0$

| | |
|---|---------------------------------------|
| $\Rightarrow 2t + \sqrt{t} - 3 = 0$ | $t = x^2 + x + 1$. |
| $\Rightarrow 2t - 3 = \sqrt{t}$ | Isolate the root. |
| $\Rightarrow (2t - 3)^2 = t$ | Square both sides. |
| $\Rightarrow 4t^2 - 12t + 9 = t$ | |
| $\Rightarrow 4t^2 - 13t + 9 = 0$ | |
| $\Rightarrow 4t^2 - 4t - 9t + 9 = 0$ | Factor by grouping. |
| $\Rightarrow 4t(t - 1) - 9(t - 1) = 0$ | |
| $\Rightarrow (4t - 9)(t - 1) = 0$ | |
| $\Rightarrow t \in \{1, \frac{9}{4}\}$ | |
| $\Rightarrow x^2 + x + 1 = 1$ | Solve for x using $t = 1$. |
| $\Rightarrow x^2 + x = 0$ | |
| $\Rightarrow x(x + 1) = 0$ | |
| $\Rightarrow x \in \{-1, 0\}$ | |
| $\Rightarrow x^2 + x + 1 = \frac{9}{4}$ | Solve for x using $t = \frac{9}{4}$. |
| $\Rightarrow x^2 + x + 1 - \frac{9}{4} = 0$ | |
| $\Rightarrow x^2 + x + \frac{4}{4} - \frac{9}{4} = 0$ | |
| $\Rightarrow x^2 + x - \frac{5}{4} = 0$ | |
| $\Rightarrow 4x^2 + 4x - 5 = 0$ | |
| $\Rightarrow \frac{-4 \pm \sqrt{4^2 - 4(4)(-5)}}{2(4)}$ | Use the quadratic formula. |
| $\Rightarrow \frac{-4 \pm \sqrt{16 - 4(4)(-5)}}{2(4)}$ | |
| $\Rightarrow \frac{-4 \pm \sqrt{16 + 80}}{8}$ | |
| $\Rightarrow \frac{-4 \pm \sqrt{96}}{8}$ | |

Continued on next page

| | |
|--|---|
| $\Rightarrow \frac{-4 \pm \sqrt{16} \sqrt{6}}{8}$ $\Rightarrow \frac{-4 \pm 4 \sqrt{6}}{8}$ $\Rightarrow \frac{-1 \pm \sqrt{6}}{2}$ $\Rightarrow x \in \left\{ \frac{-1 + \sqrt{6}}{2}, \frac{-1 - \sqrt{6}}{2} \right\}$ | |
| $\Rightarrow 2((-1)^2 - 1 + 1) + \sqrt{(-1)^2 - 1 + 1} - 3 = 0$ $\Rightarrow 2(1 - 1 + 1) + \sqrt{1 - 1 + 1} - 3 = 0$ $\Rightarrow 2(1) + \sqrt{1} - 3 = 0$ $\Rightarrow 2 + 1 - 3 = 0$ $\Rightarrow 0 = 0$ $\Rightarrow x = -1$ | Verify $x = -1$. |
| $\Rightarrow 2(0^2 - 0 + 1) + \sqrt{0^2 - 0 + 1} - 3 = 0$ $\Rightarrow 2(1) + \sqrt{1} - 3 = 0$ $\Rightarrow 2 + 1 - 3 = 0$ $\Rightarrow 0 = 0$ $\Rightarrow x = 0$ | Verify $x = 0$. |
| $\Rightarrow x \in \{-1, 0\}$ | <p>Final answer. A quadratic equation can have at most two solutions.</p> <p style="text-align: right;">■</p> |