

Exercise Solutions for Math 20

Some Types of Functions, Operations

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November 20, 2024


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1.1

Given

$$g(x) = \begin{cases} x + 4, & \text{if } x < -2 \\ |x|, & \text{if } -1 < x < 1 \\ 2, & \text{if } x > 3 \end{cases}$$

Sketch its graph and label its x- and y- intercepts.

$\Rightarrow 0 = x + 4$ $\Rightarrow x_i = -4$	Find the x-intercepts of $y = x + 4$
$\Rightarrow 0 = x $ $\Rightarrow x_i = 0$	Find the x-intercepts of $y = x $
$\Rightarrow y = 0 $ $\Rightarrow y_i = 0$	Find the y-intercepts of $y = x $
\Rightarrow See Figure 1.	Final answer. Graph the system. 

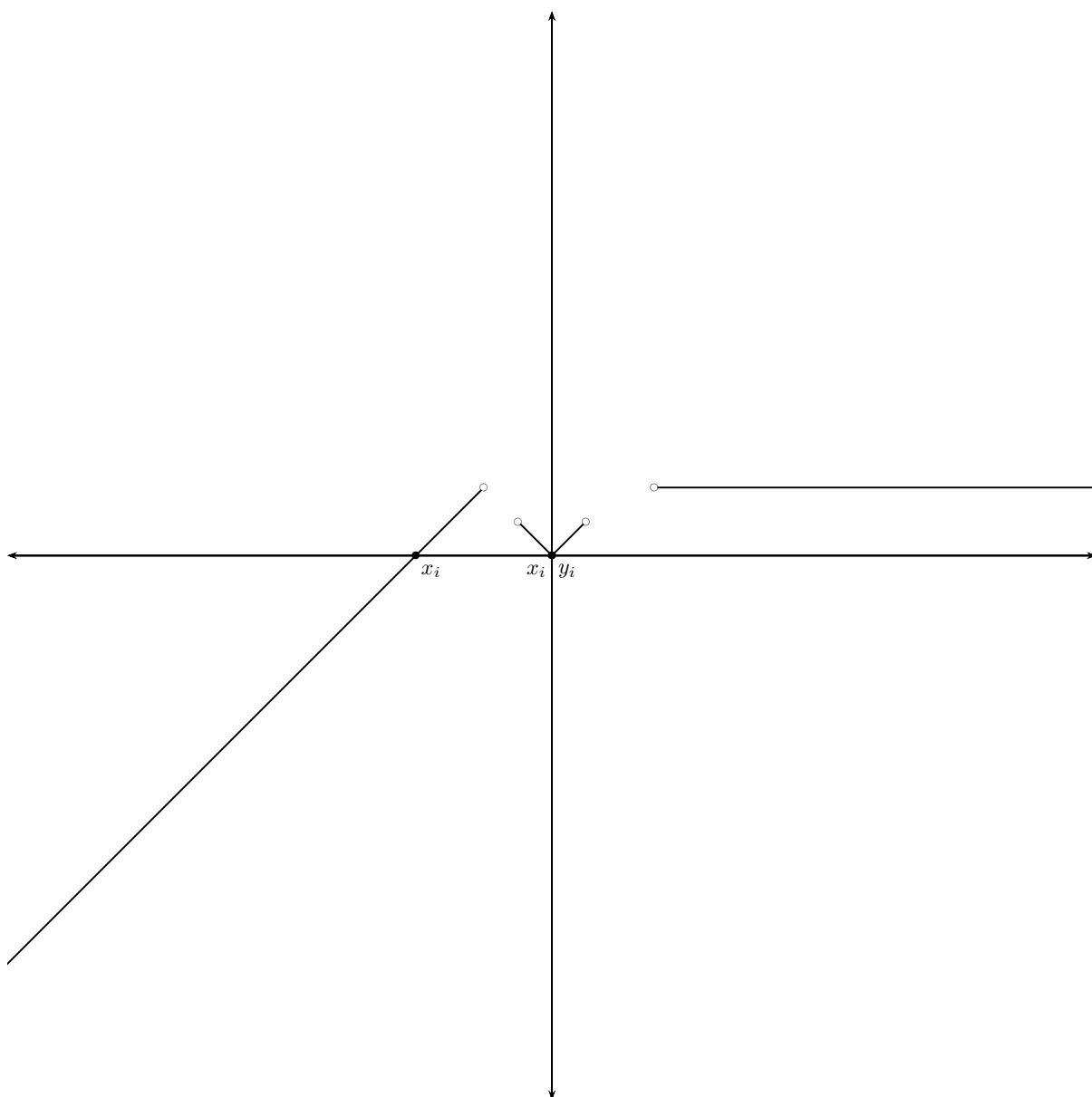


Figure 1.

1.2

Let f , g , and h be the functions defined from the expressions below.

$$f(x) = \frac{x+2}{x-3}$$

$$g(x) = \frac{x+3}{x+2}$$

$$h(x) = \sqrt{3x-1}$$

Determine $(f-g)(x)$, $(\frac{f}{g})(x)$, $(fg)(x)$, $(f \circ g)(x)$, $(h \circ f)(x)$, $(g \circ h)(x)$ and obtain their respective domains.

$\Rightarrow (f-g)(x) = \frac{x+2}{x-3} - \frac{x+3}{x+2}$ $\Rightarrow (f-g)(x) = \frac{(x+2)(x+2)}{(x-3)(x+2)} - \frac{(x-3)(x+3)}{(x-3)(x+2)}$ $\Rightarrow (f-g)(x) = \frac{(x+2)(x+2) - (x-3)(x+3)}{(x-3)(x+2)}$ $\Rightarrow (f-g)(x) = \frac{x^2+4x+4 - (x^2-9)}{(x-3)(x+2)}$ $\Rightarrow (f-g)(x) = \frac{x^2+4x+4-x^2+9}{(x-3)(x+2)}$ $\Rightarrow (f-g)(x) = \frac{4x+13}{(x-3)(x+2)}$ $\Rightarrow \text{dom}(f-g) = \mathbb{R} \setminus \{-2, 3\}$	Find $(f-g)(x)$.
$\Rightarrow (\frac{f}{g})(x) = \frac{\frac{x+2}{x-3}}{\frac{x+3}{x+2}}$ $\Rightarrow (\frac{f}{g})(x) = \frac{x+2}{x-3} \cdot \frac{x+2}{x+3}$ $\Rightarrow (\frac{f}{g})(x) = \frac{(x+2)^2}{(x-3)(x+3)}$ $\Rightarrow \text{dom}(\frac{f}{g}) = \mathbb{R} \setminus \{-3, -2, 3\}$	Find $(\frac{f}{g})(x)$.
$\Rightarrow (fg)(x) = \frac{x+2}{x-3} \cdot \frac{x+3}{x+2}$ $\Rightarrow (fg)(x) = \frac{(x+2)(x+3)}{(x-3)(x+2)}$ $\Rightarrow (fg)(x) = \frac{x+3}{x-3}$ $\Rightarrow \text{dom}(fg) = \mathbb{R} \setminus \{-2, 3\}$	Find $(fg)(x)$.
$\Rightarrow (f \circ g)(x) = \frac{\frac{x+3}{x+2} + 2}{\frac{x+3}{x+2} - 3}$ $\Rightarrow (f \circ g)(x) = \frac{\frac{x+3}{x+2} + \frac{2x+4}{x+2}}{\frac{x+3}{x+2} - \frac{3x+6}{x+2}}$ $\Rightarrow (f \circ g)(x) = \frac{\frac{x+3+(2x+4)}{x+2}}{\frac{x+3-(3x+6)}{x+2}}$ $\Rightarrow (f \circ g)(x) = \frac{\frac{x+3+2x+4}{x+2}}{\frac{x+3-3x-6}{x+2}}$ $\Rightarrow (f \circ g)(x) = \frac{\frac{3x+7}{x+2}}{\frac{-2x-3}{x+2}}$ $\Rightarrow (f \circ g)(x) = \frac{3x+7}{x+2} \cdot \frac{x+2}{-2x-3}$ $\Rightarrow (f \circ g)(x) = \frac{3x+7}{-2x-3}$ $\Rightarrow \text{dom}(f \circ g) = \mathbb{R} \setminus \{-2, -\frac{3}{2}\}$	Find $(f \circ g)(x)$.
$\Rightarrow (h \circ f)(x) = \sqrt{3(\frac{x+2}{x-3}) - 1}$ $\Rightarrow (h \circ f)(x) = \sqrt{\frac{3x+6}{x-3} - 1}$ $\Rightarrow (h \circ f)(x) = \sqrt{\frac{3x+6}{x-3} - \frac{x-3}{x-3}}$	Find $(h \circ f)(x)$.

Continued on next page

$\Rightarrow (h \circ f)(x) = \sqrt{\frac{3x+6-(x-3)}{x-3}}$ $\Rightarrow (h \circ f)(x) = \sqrt{\frac{3x+6-x+3}{x-3}}$ $\Rightarrow (h \circ f)(x) = \sqrt{\frac{2x+9}{x-3}}$ $\Rightarrow \frac{2x+9}{x-3} \geq 0$	Solve for the domain. Note that $x = 3$ is undefined.															
Create a table of signs.																
<table><tr><td></td><td>$-\frac{9}{2}$</td><td>3</td></tr><tr><td>$2x + 9$</td><td>$-$</td><td>$+$</td><td>$+$</td></tr><tr><td>$x - 3$</td><td>$-$</td><td>$-$</td><td>$+$</td></tr><tr><td>$\frac{2x+9}{x-3}$</td><td>$+$</td><td>$-$</td><td>$+$</td></tr></table>		$-\frac{9}{2}$	3	$2x + 9$	$-$	$+$	$+$	$x - 3$	$-$	$-$	$+$	$\frac{2x+9}{x-3}$	$+$	$-$	$+$	
	$-\frac{9}{2}$	3														
$2x + 9$	$-$	$+$	$+$													
$x - 3$	$-$	$-$	$+$													
$\frac{2x+9}{x-3}$	$+$	$-$	$+$													
$\Rightarrow \text{dom}(h \circ f) = (-\infty, -\frac{9}{2}] \cup (3, +\infty)$																
$\Rightarrow (g \circ h)(x) = \frac{\sqrt{3x-1}+3}{\sqrt{3x-1}+2}$ $\Rightarrow (g \circ h)(x) = \frac{\sqrt{3x-1}+3}{\sqrt{3x-1}+2} \cdot \frac{\sqrt{3x-1}-2}{\sqrt{3x-1}-2}$ $\Rightarrow (g \circ h)(x) = \frac{(\sqrt{3x-1}+3)(\sqrt{3x-1}-2)}{3x-1-4}$ $\Rightarrow (g \circ h)(x) = \frac{(\sqrt{3x-1}+3)(\sqrt{3x-1}-2)}{3x-5}$ $\Rightarrow 3x - 1 \geq 0$ $\Rightarrow 3x \geq 1$ $\Rightarrow x \geq \frac{1}{3}$ $\Rightarrow \text{dom}(g \circ h) = [\frac{1}{3}, +\infty) \setminus \{\frac{5}{3}\}$	Find $(g \circ h)(x)$. Rationalize.															
$\Rightarrow (f - g)(x) = \frac{4x+13}{(x-3)(x+2)}$ $\Rightarrow \text{dom}(f - g) = \mathbb{R} \setminus \{-2, 3\}$ $\Rightarrow (\frac{f}{g})(x) = \frac{(x+2)^2}{(x-3)(x+3)}$ $\Rightarrow \text{dom}(\frac{f}{g}) = \mathbb{R} \setminus \{-3, -2, 3\}$ $\Rightarrow (fg)(x) = \frac{x+3}{x-3}$ $\Rightarrow \text{dom}(fg) = \mathbb{R} \setminus \{-2, 3\}$ $\Rightarrow (f \circ g)(x) = \frac{3x+7}{-2x-3}$ $\Rightarrow \text{dom}(f \circ g) = \mathbb{R} \setminus \{-2, -\frac{3}{2}\}$ $\Rightarrow (h \circ f)(x) = \sqrt{\frac{2x+9}{x-3}}$ $\Rightarrow \text{dom}(h \circ f) = (-\infty, -\frac{9}{2}] \cup (3, +\infty)$ $\Rightarrow (g \circ h)(x) = \frac{(\sqrt{3x-1}+3)(\sqrt{3x-1}-2)}{3x-5}$ $\Rightarrow \text{dom}(g \circ h) = [\frac{1}{3}, +\infty) \setminus \{\frac{5}{3}\}$	Solve for the domain. Note that $x = \frac{5}{3}$ is undefined.															
	Final answer.															

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1.3

A ball is thrown upward from the roof of a building that is 30 meters high. If it is known that the position of the ball with respect to the ground after x seconds is $h(x) = -5x^2 + 20x + 30$ meters, determine the maximum height reached by the ball, and how long it takes before the ball hits the ground.

— Let:	
$\square M$	Maximum height.
$\square t$	Time to hit the ground.
$\Rightarrow -5x^2 + 20x + 30 = y$ $\Rightarrow -5x^2 + 20x = y - 30$ $\Rightarrow -5(x^2 - 4x) = y - 30$ $\Rightarrow -5(x^2 - 4x + 4) = y - 30 - 5(4)$ $\Rightarrow -5(x^2 - 4x + 4) = y - 30 - 20$ $\Rightarrow -5(x^2 - 4x + 4) = y - 50$ $\Rightarrow -5(x - 2)^2 = y - 50$ $\Rightarrow (x - 2)^2 = (-\frac{1}{5})y - 50$ $\Rightarrow (x - 2)^2 = 4(-\frac{1}{20})y - 50$	<p>The trajectory of a ball is most likely a parabola. Rewrite $h(x)$ in standard form and in terms of y.</p>
$\Rightarrow -5x^2 + 20x + 30 = 0$ $\Rightarrow x = \frac{-20 \pm \sqrt{(-20)^2 - 4(-5)(30)}}{2(-5)}$ $\Rightarrow x = \frac{-20 \pm \sqrt{400 + 600}}{-10}$ $\Rightarrow x = \frac{-20 \pm \sqrt{1000}}{-10}$ $\Rightarrow x = \frac{-20 \pm \sqrt{100}\sqrt{10}}{-10}$ $\Rightarrow x = \frac{-20 \pm 10\sqrt{10}}{-10}$ $\Rightarrow x = 2 \pm \sqrt{10}$ $\Rightarrow x = 2 + \sqrt{10}$	<p>The x-intercept of the function is the time it takes for the ball to hit the ground.</p> <p>Use the quadratic formula.</p> <p>We only care about the cases where $x \geq 0$ since we can't reverse time.</p>
$\Rightarrow M = 10 \text{ meters}$ $\Rightarrow t = 2 + \sqrt{10} \text{ seconds}$ $\Rightarrow \text{See Figure 2 for a visualization.}$	<p>Final answer. The maximum height is the y-coordinate of the vertex of the parabola.</p>

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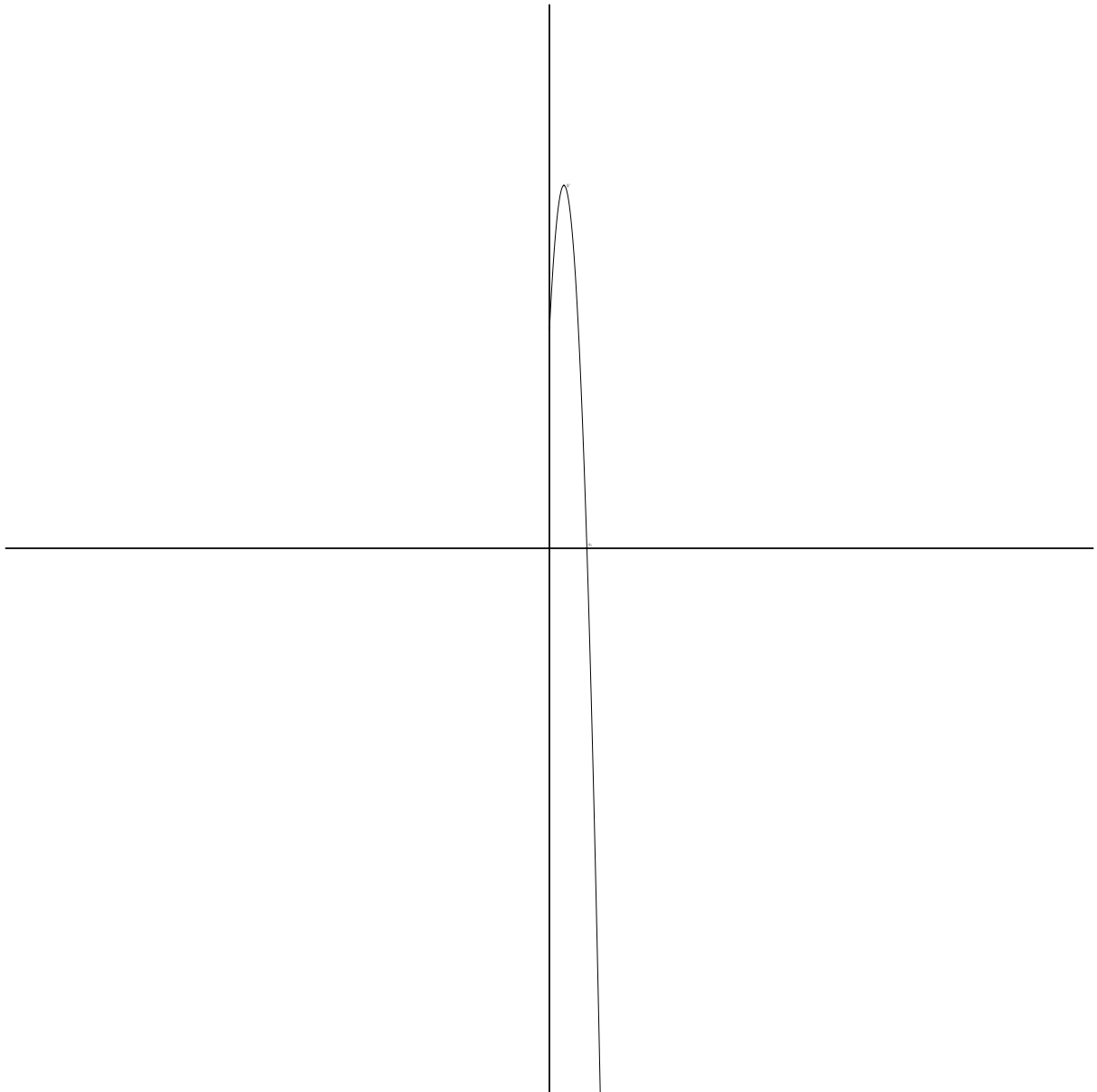


Figure 2. Zoom in to see labels.