

Exercise Solutions for Math 20

Conics (Hyperbola), Systems of Linear Equations

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November 18, 2024

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1.1 Identify the following conic sections.

1.1.a $2x^2 - 3y^2 + 4x + 6y - 1 = 0$

$\Rightarrow 2x^2 + 4x - 3y^2 + 6y = 1$	Group terms.
$\Rightarrow 2(x^2 + 2x) - 3(y^2 - 2y) = 1$	
$\Rightarrow 2(x^2 + 2x + 1) - 3(y^2 - 2y) = 1 + 2(1)$	Complete the square.
$\Rightarrow 2(x^2 + 2x + 1) - 3(y^2 - 2y) = 3$	
$\Rightarrow 2(x + 1)^2 - 3(y^2 - 2y) = 3$	
$\Rightarrow 2(x + 1)^2 - 3(y^2 - 2y + 1) = 3 - 3(1)$	Complete the square.
$\Rightarrow 2(x + 1)^2 - 3(y^2 - 2y + 1) = 0$	
\Rightarrow Not a conic.	Final answer. Cannot divide both sides. ■

1.1.b $2x^2 + 3y^2 + 16x - 18y - 53 = 0$

$\Rightarrow 2x^2 + 16x + 3y^2 - 18y = 53$	Group terms.
$\Rightarrow 2(x^2 + 8x) + 3(y^2 - 6y) = 53$	
$\Rightarrow 2(x^2 + 8x + 16) + 3(y^2 - 6y) = 53 + 2(16)$	Complete the square.
$\Rightarrow 2(x^2 + 8x + 16) + 3(y^2 - 6y) = 85$	
$\Rightarrow 2(x + 4)^2 + 3(y^2 - 6y) = 85$	
$\Rightarrow 2(x + 4)^2 + 3(y^2 - 6y + 9) = 85 + 3(9)$	Complete the square.
$\Rightarrow 2(x + 4)^2 + 3(y^2 - 6y + 9) = 112$	
$\Rightarrow 2(x + 4)^2 + 3(y - 3)^2 = 112$	
\Rightarrow Ellipse.	Final answer. ■

1.1.c $9x + y^2 + 4y - 5 = 0$

$\Rightarrow y^2 + 4y = -9x + 5$	Group terms.
$\Rightarrow y^2 + 4y + 4 = -9x + 9$	Complete the square.
$\Rightarrow (y + 2)^2 = -9(x - 1)$	
\Rightarrow Parabola.	Final answer. ■

1.1.d $4x^2 - x = y^2 + 1$

$\Rightarrow 4x^2 - x - y^2 = 1$	Group terms.
$\Rightarrow 4(x^2 - \frac{1}{4}x) - y^2 = 1$	
$\Rightarrow 4(x^2 - \frac{1}{4}x + \frac{1}{64}) - y^2 = 1 + \frac{1}{16}$	Complete the square.

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$$\Rightarrow 4(x - \frac{1}{8})^2 - y^2 = \frac{17}{16}$$

\Rightarrow Hyperbola.

Final answer.



1.1.e $7y - y^2 - x = 0$

$$\Rightarrow -y^2 + 7y = x$$

Group terms.

$$\Rightarrow y^2 - 7y = -x$$

$$\Rightarrow y^2 - 7y + \frac{49}{4} = -x + \frac{49}{4}$$

Complete the square.

$$\Rightarrow (y^2 - \frac{7}{2})^2 = -1(x - \frac{49}{4})$$

\Rightarrow Parabola.

Final answer.



1.2 Sketch the graph of the following hyperbolas.

1.2.a $x^2 - 16y^2 = 40$

$$\Rightarrow \frac{x^2}{40} - \frac{16y^2}{40} = 1$$

Rewrite in standard form.

$$\Rightarrow \frac{x^2}{40} - \frac{y^2}{\frac{1}{16}(40)} = 1$$

$$\Rightarrow \frac{x^2}{40} - \frac{y^2}{\frac{5}{2}} = 1$$

$$\Rightarrow \frac{x^2}{\sqrt{40}^2} - \frac{y^2}{(\sqrt{\frac{5}{2}})^2} = 1$$

$$\Rightarrow \frac{x^2}{(\sqrt{4}\sqrt{10})^2} - \frac{y^2}{(\sqrt{\frac{5}{2}})^2} = 1$$

$$\Rightarrow \frac{x^2}{(2\sqrt{10})^2} - \frac{y^2}{(\sqrt{\frac{5}{2}})^2} = 1$$

\Rightarrow See Figure 1.

Final answer. Graph the hyperbola.



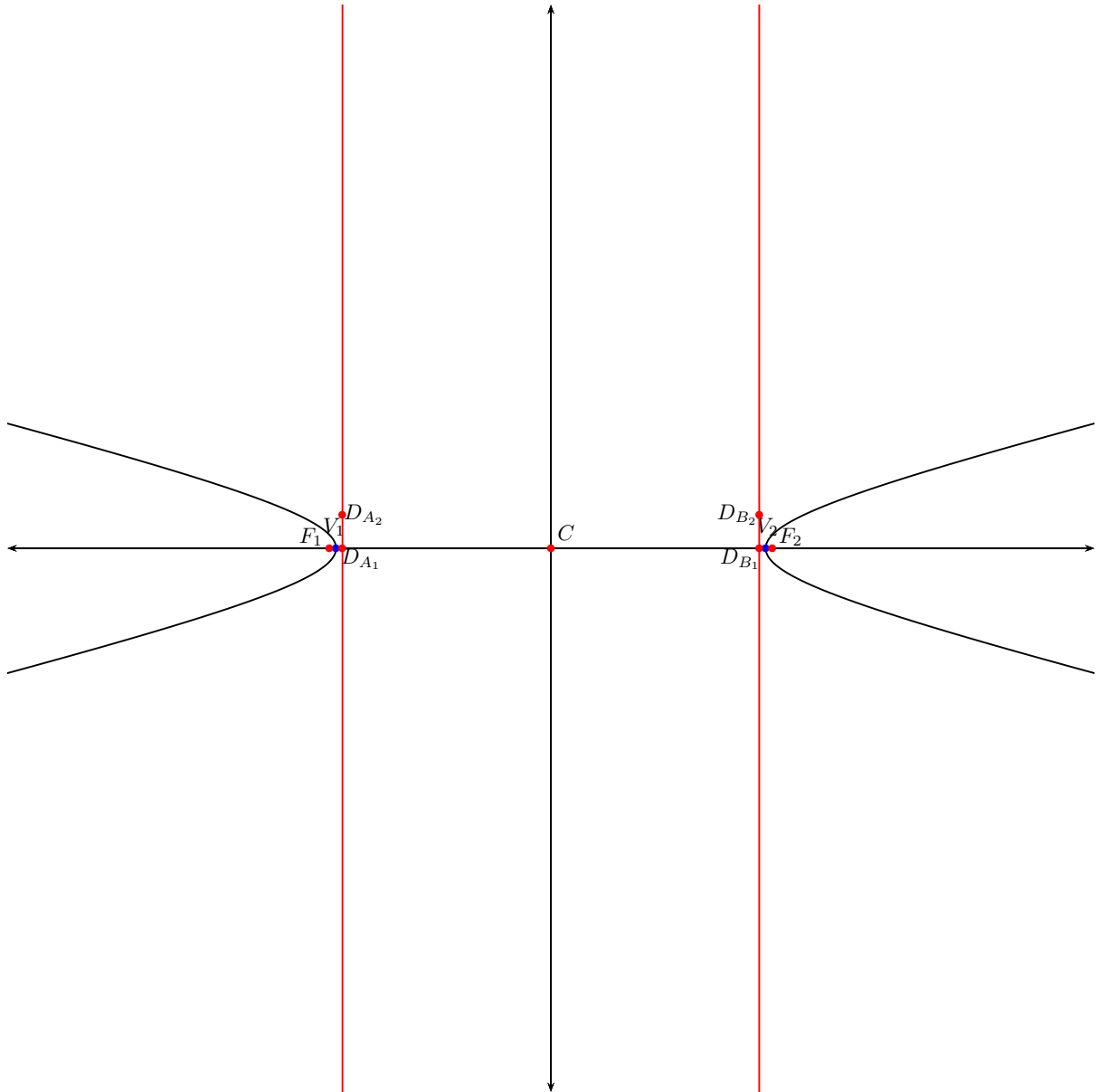


Figure 1. Graph of $\frac{x^2}{(2\sqrt{10})^2} - \frac{y^2}{(\sqrt{\frac{5}{2}})^2} = 1$.

1.2.b $4y^2 - (x+3)^2 = 16$

$$\Rightarrow \frac{4y^2}{16} - \frac{(x+3)^2}{16} = 1$$

Rewrite in standard form.

$$\Rightarrow \frac{y^2}{4} - \frac{(x+3)^2}{16} = 1$$

$$\Rightarrow \frac{y^2}{2^2} - \frac{(x+3)^2}{4^2} = 1$$

\Rightarrow See Figure 2.

Final answer. Graph the hyperbola. ■

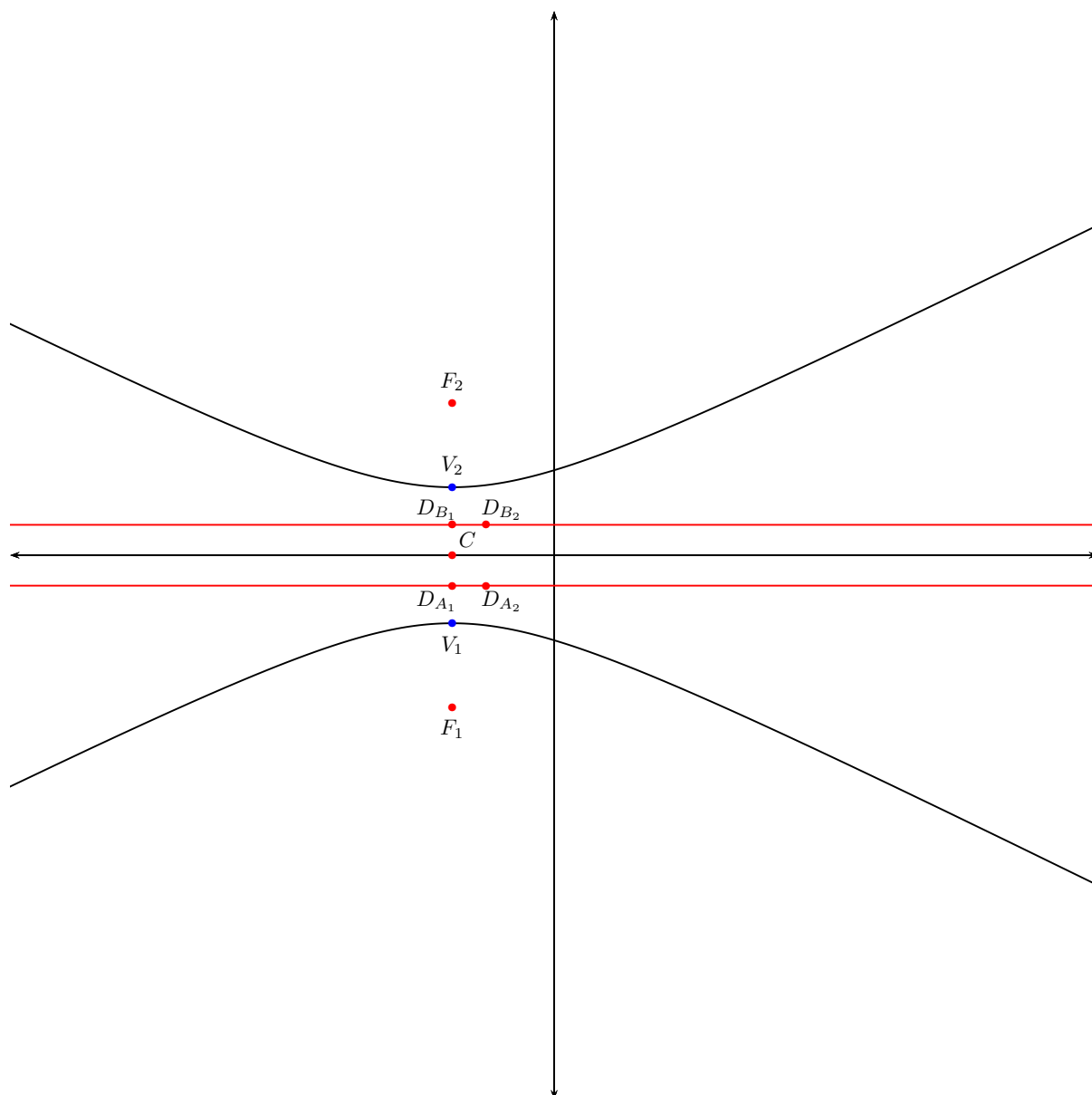


Figure 2. Graph of $\frac{y^2}{2^2} - \frac{(x+3)^2}{4^2} = 1$.

1.2.c $x^2 - y^2 + 6x - 4y = 4$

$$\Rightarrow x^2 + 6x - y^2 - 4y = 4$$

Group terms.

$$\Rightarrow x^2 + 6x + 9 - y^2 - 4y = 4 + 9$$

Complete the square.

$$\Rightarrow x^2 + 6x + 9 - y^2 - 4y = 13$$

$$\Rightarrow (x + 3)^2 - y^2 - 4y = 13$$

$$\Rightarrow (x + 3)^2 - (y^2 + 4y) = 13$$

$$\Rightarrow (x + 3)^2 - (y^2 + 4y + 4) = 13 - 4$$

Complete the square.

$$\Rightarrow (x + 3)^2 - (y^2 + 4y + 4) = 9$$

$$\Rightarrow (x + 3)^2 - (y + 2)^2 = 9$$

$$\Rightarrow \frac{(x+3)^2}{9} - \frac{(y+2)^2}{9} = 1$$

Rewrite in standard form.

$$\Rightarrow \frac{(x+3)^2}{3^2} - \frac{(y+2)^2}{3^2} = 1$$

\Rightarrow See Figure 3.

Final answer. Graph the hyperbola.



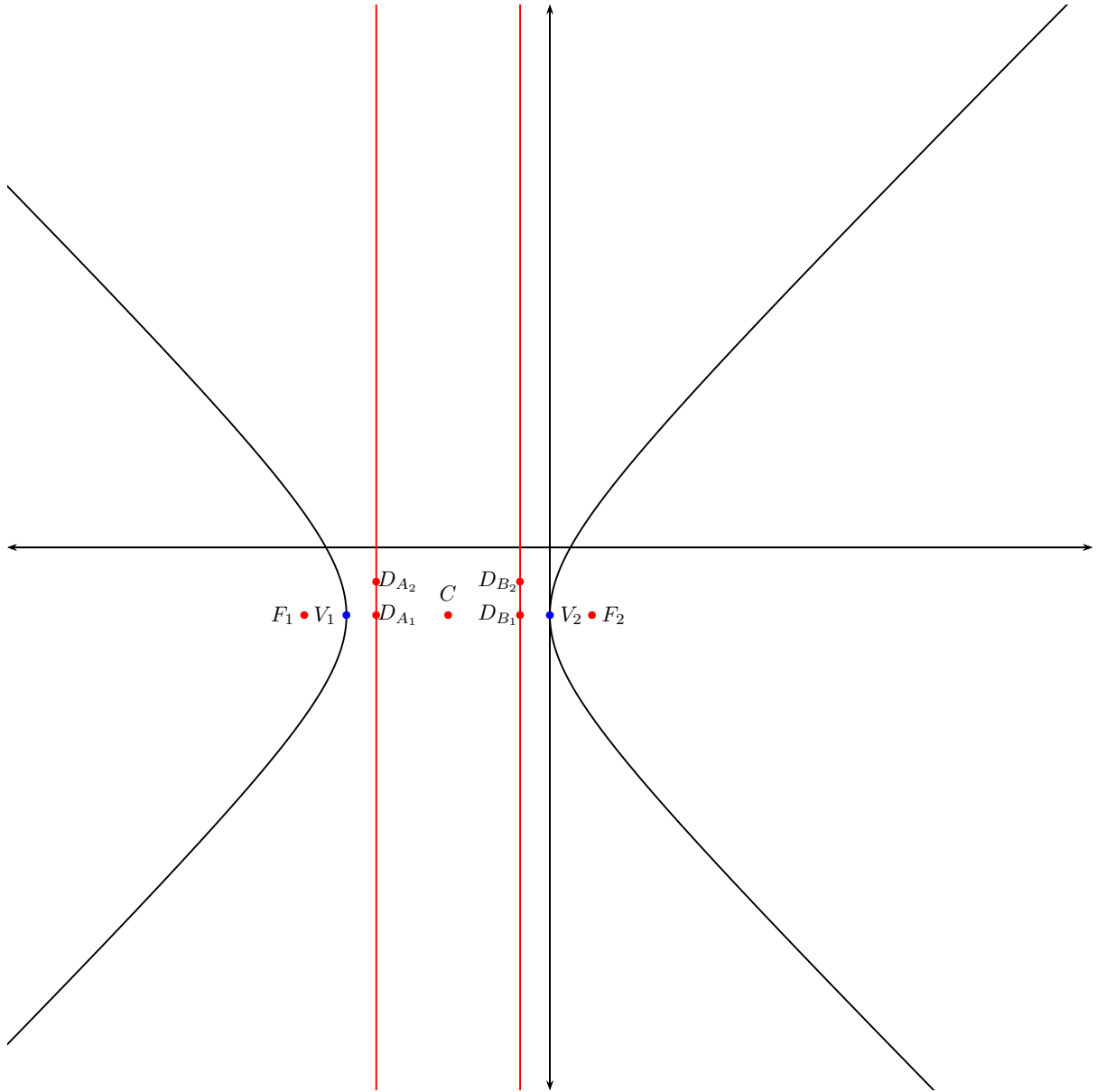


Figure 3. Graph of $\frac{(x+3)^2}{3^2} - \frac{(y+2)^2}{3^2} = 1$.

1.3 Find an equation of the hyperbola having $(2, 1)$ and $(-2, 1)$ as its foci and has a conjugate axis of length 3.

$\Rightarrow h = \frac{2+(-2)}{2}, k = 1$	This is a hyperbola with a horizontal transverse axis (from the changing x-coordinate of its foci). h can be found by averaging the x-coordinates of the foci, and k is simply equivalent to the y-coordinates of the foci.
$\Rightarrow h = 0, k = 1$	
$\Rightarrow 0 - c = 2$	Find c using $F_1(h - c, k)$.
$\Rightarrow c = -2$	
$\Rightarrow b = \frac{3}{2}$	By definition, b is half the length of the conjugate axis.
$\Rightarrow (-2)^2 = a^2 + (\frac{3}{2})^2$	Find a using $c^2 = a^2 + b^2$.
$\Rightarrow 4 = a^2 + \frac{9}{4}$	
$\Rightarrow a^2 = 4 - \frac{9}{4}$	
$\Rightarrow a^2 = \frac{16}{4} - \frac{9}{4}$	
$\Rightarrow a^2 = \frac{7}{4}$	
$\Rightarrow a = \sqrt{\frac{7}{4}}$	
$\Rightarrow \frac{x^2}{(\sqrt{\frac{7}{4}})^2} - \frac{(y-1)^2}{(\frac{3}{2})^2} = 1$	Final answer. Write the hyperbola equation using (h, k) , a , and b . ■

2 Solve the following systems of equations.

2.1

$$\begin{cases} 2x + 3y = 4 \\ x - 2y = -5 \end{cases}$$

$\Rightarrow 2x + 3y = 4, -2x + 4y = 10$	Solve for y
$\Rightarrow 7y = 14$	
$\Rightarrow y = 2$	
$\Rightarrow x - 2(2) = -5$	Solve for x .
$\Rightarrow x - 4 = -5$	
$\Rightarrow x = -1$	
$\Rightarrow x = -1, y = 2$	Final answer. ■

2.2

$$\begin{cases} -\frac{1}{x} - \frac{3}{y} = 4 \\ \frac{2}{x} - \frac{1}{y} = 6 \end{cases}$$

$\Rightarrow -\frac{2}{x} - \frac{6}{y} = 8, \frac{2}{x} - \frac{1}{y} = 6$	Solve for y .
$\Rightarrow -\frac{7}{y} = 14$	
$\Rightarrow 14y = -7$	
$\Rightarrow y = -\frac{1}{2}$	
$\Rightarrow \frac{2}{x} - \frac{1}{-\frac{1}{2}} = 6$	Solve for x .
$\Rightarrow \frac{2}{x} + 2 = 6$	
$\Rightarrow \frac{2}{x} = 4$	
$\Rightarrow 4x = 2$	
$\Rightarrow x = \frac{1}{2}$	
$\Rightarrow x = \frac{1}{2}, y = -\frac{1}{2}$	Final answer. ■

2.3

$$\begin{cases} 3x - y + 2z = 5 \\ x + 3y - z = -5 \\ x + y + z = 1 \end{cases}$$

$\Rightarrow 3x - y + 2z = 5, -3x - 3y - 3z = -3$	Eliminate x .
$\Rightarrow -4y - z = 2$	
$\Rightarrow x + 3y - z = -5, -x - y - z = -1$	Eliminate x .
$\Rightarrow 2y - 2z = -6$	
$\Rightarrow -4y - z = 2, 4y - 4z = -12$	Solve for z .

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$\Rightarrow -5z = -10$ $\Rightarrow z = 2$	
$\Rightarrow -4y - 2 = 2$ $\Rightarrow -4y = 4$ $\Rightarrow y = -1$	Solve for y .
$\Rightarrow x - 1 + 2 = 1$ $\Rightarrow x = 1 + 1 - 2$ $\Rightarrow x = 0$	Solve for x .
$\Rightarrow x = 0, y = -1, z = 2$	Final answer. ■

2.4

$$\begin{cases} x + 2y - z = 4 \\ 3x - y + z = 5 \\ 2x + 3y + 2z = 7 \end{cases}$$

$\Rightarrow x + 2y - z = 4, 3x - y + z = 5$ $\Rightarrow 4x + y = 9$	Eliminate z .
$\Rightarrow 2x + 4y - 2z = 8, 2x + 3y + 2z = 7$ $\Rightarrow 4x + 7y = 15$	Eliminate z .
$\Rightarrow -4x - y = -9, 4x + 7y = 15$ $\Rightarrow 6y = 6$ $\Rightarrow y = 1$	Solve for y .
$\Rightarrow 4x + 1 = 9$ $\Rightarrow 4x = 8$ $\Rightarrow x = 2$	Solve for x .
$\Rightarrow 2 + 2(1) - z = 4$ $\Rightarrow 4 - z = 4$ $\Rightarrow z = 0$	Solve for z .
$\Rightarrow x = 2, y = 1, z = 0$	Final answer. ■

2.5

$$\begin{cases} 3x + y = 9 \\ -2x + z = -7 \\ 2y + 5z = -5 \end{cases}$$

$\Rightarrow 6x + 2y = 18, -6x + 3z = -21$ $\Rightarrow 2y + 3z = -3$	Eliminate x .
$\Rightarrow 2y + 5z = -5, -2y - 3z = 3$ $\Rightarrow 2z = -2$	Solve for z .

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$\Rightarrow z = -1$	
$\Rightarrow 2y + 5(-1) = -5$ $\Rightarrow 2y - 5 = -5$ $\Rightarrow 2y = 0$ $\Rightarrow y = 0$	Solve for y .
$\Rightarrow 3x + 0 = 9$ $\Rightarrow x = 3$	Solve for x .
$\Rightarrow x = 3, y = 0, z = -1$	Final answer. ■