Exercise Solutions for Math 20

Radicals and Complex Numbers

Nile Jocson <novoseiversia@gmail.com>

November 8, 2024

Contents

1 Simplify the following. Rationalize the denominators.

1.1 $\dfrac{24c^{-\frac{1}{2}}d^{\frac{2}{3}}}{18c^{-\frac{1}{7}}d^{-\frac{3}{5}}}$

$\Rightarrow \dfrac{4c^{-\frac{1}{2}}d^{\frac{2}{3}}}{3c^{-\frac{1}{7}}d^{-\frac{3}{5}}}$	Simplify the fraction to lowest terms.
$\Rightarrow \dfrac{4d^{\frac{2}{3}}c^{\frac{1}{7}}d^{\frac{3}{5}}}{3c^{\frac{1}{2}}}$	$a^{-\frac{b}{c}} = \dfrac{1}{a^{\frac{b}{c}}}$
$\Rightarrow \dfrac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{\frac{1}{7}-\frac{1}{2}}$	$\dfrac{a^m}{a^n} = a^{m-n}$
$\Rightarrow \dfrac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{\frac{2}{14}-\frac{7}{14}}$	LCM = 14
$\Rightarrow \dfrac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{-\frac{5}{14}}$	
$\Rightarrow \dfrac{4}{3}c^{-\frac{5}{14}}d^{\frac{2}{3}+\frac{3}{5}}$	$a^m a^n = a^{m+n}$
$\Rightarrow \dfrac{4}{3}c^{-\frac{5}{14}}d^{\frac{10}{15}+\frac{9}{15}}$	LCM = 15
$\Rightarrow \dfrac{4}{3}c^{-\frac{5}{14}}d^{\frac{19}{15}}$	
$\Rightarrow \dfrac{4d^{\frac{19}{15}}}{3c^{\frac{5}{14}}}$	$a^{-\frac{b}{c}} = \dfrac{1}{a^{\frac{b}{c}}}$
$\Rightarrow \dfrac{4\sqrt[15]{d^{19}}}{3\sqrt[14]{c^5}}$	
$\Rightarrow \dfrac{4\sqrt[15]{d^{19}}}{3\sqrt[14]{c^5}} \cdot \dfrac{\sqrt[14]{c^9}}{\sqrt[14]{c^9}}$	Rationalize.
$\Rightarrow \dfrac{4\sqrt[14]{c^9}\sqrt[15]{d^{19}}}{3c}$	
	$\blacksquare$

1.2 $\left(u^{\frac{1}{3}} + (uv)^{\frac{1}{6}} + v^{\frac{1}{3}}\right)\left(u^{\frac{1}{6}} - v^{\frac{1}{6}}\right)$

$\Rightarrow (u^{\frac{1}{3}} + u^{\frac{1}{6}}v^{\frac{1}{6}} + v^{\frac{1}{3}})(u^{\frac{1}{6}} - v^{\frac{1}{6}})$	Distribute exponent.
$\Rightarrow u^{\frac{1}{2}} - v^{\frac{1}{2}}$	Use difference of two cubes.
$\Rightarrow \sqrt{u} - \sqrt{v}$	
	$\blacksquare$