

Exercise Solutions for Math 20

Conics (Parabola and Ellipse)

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1.1 Determine the vertex and orientation of the following parabolas.

1.1.a $4y^2 + 4y + x = 2$

$\Rightarrow 4y^2 + 4y = -x + 2$ $\Rightarrow y^2 + y = -\frac{x}{4} + \frac{2}{4}$ $\Rightarrow y^2 + y = -\frac{x}{4} + \frac{1}{2}$	Isolate y .
$\Rightarrow y^2 + y + \frac{1}{4} = -\frac{x}{4} + \frac{1}{2} + \frac{1}{4}$ $\Rightarrow (y + \frac{1}{2})^2 = -\frac{x}{4} + \frac{3}{4}$ $\Rightarrow (y + \frac{1}{2})^2 = -\frac{1}{4}(x - 3)$ $\Rightarrow (y + \frac{1}{2})^2 = 4(-\frac{1}{16})(x - 3)$	Complete the square.
\Rightarrow Opening leftwards, $(h, k) = (3, -\frac{1}{2})$	Final answer. ■

1.1.b $x^2 - 6x - 2y = 7$

$\Rightarrow x^2 - 6x = 2y + 7$	Isolate x .
$\Rightarrow x^2 - 6x + 9 = 2y + 7 + 9$ $\Rightarrow (x - 3)^2 = 2y + 16$ $\Rightarrow (x - 3)^2 = 2(y + 8)$ $\Rightarrow (x - 3)^2 = 4(\frac{1}{2})(y + 8)$	Complete the square.
\Rightarrow Opening upwards, $(h, k) = (3, -8)$	Final answer. ■

1.1.c $2y^2 - 6y - 9x = 0$

$\Rightarrow 2y^2 - 6y = 9x$ $\Rightarrow y^2 - 3y = \frac{9}{2}x$	Isolate y .
$\Rightarrow y^2 - 3y + \frac{9}{4} = \frac{9}{2}x + \frac{9}{4}$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}x + \frac{9}{4}$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{9}{4} \cdot \frac{2}{9})$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{18}{36})$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{1}{2})$ $\Rightarrow (y - \frac{3}{2})^2 = 4(\frac{9}{8})(x + \frac{1}{2})$	Complete the square.
\Rightarrow Opening rightwards, $(h, k) = (-\frac{1}{2}, \frac{3}{2})$	Final answer. ■

1.2 Sketch the graph of the following parabolas.

1.2.a $3y^2 = 8x$

$\Rightarrow y^2 = 4(\frac{2}{3})x$	Rewrite in standard form.
\Rightarrow See Figure 1.	Final answer. Graph the parabola. ■

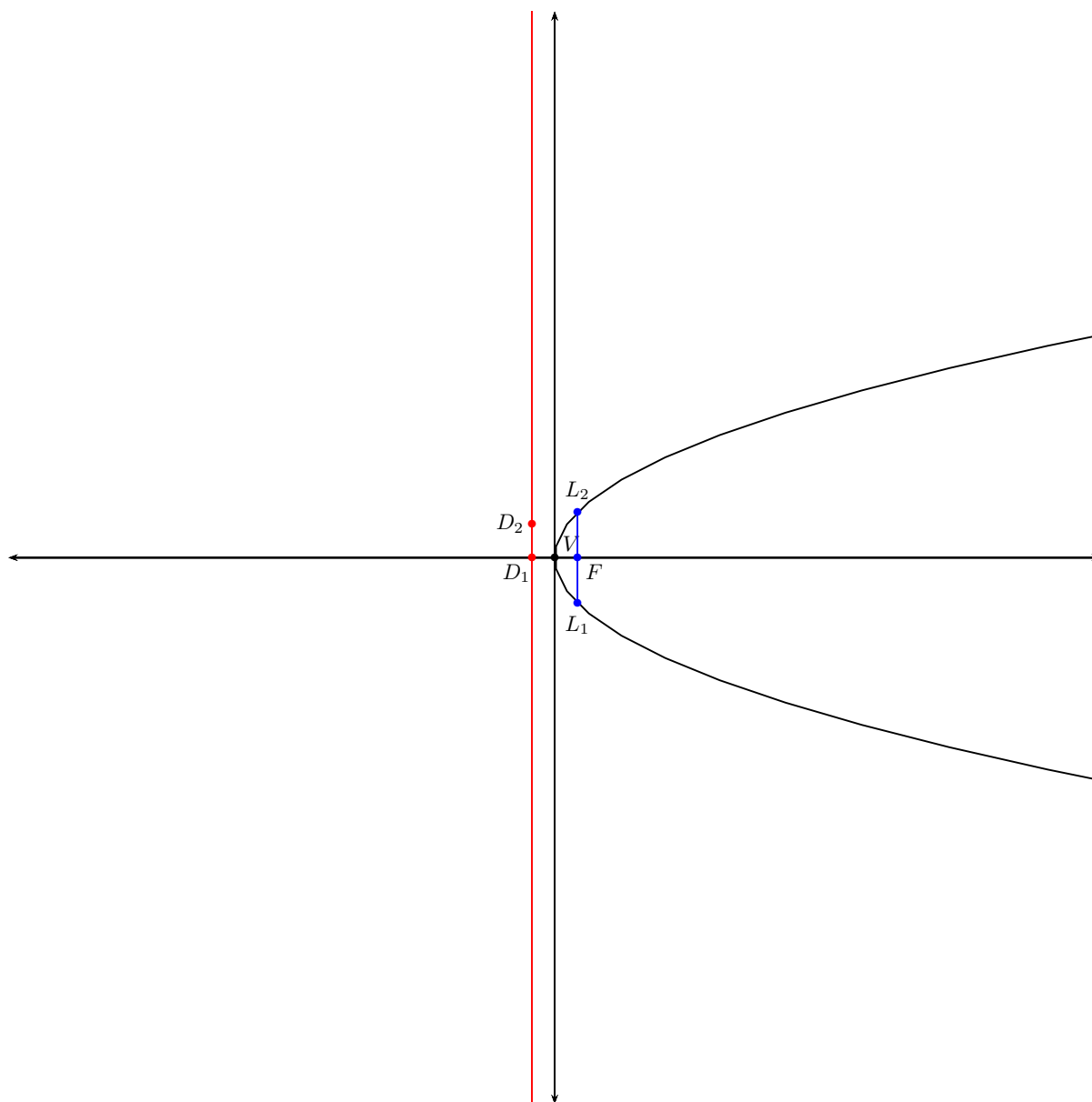


Figure 1. Graph of $y^2 = 4(\frac{2}{3})x$.

1.2.b $x^2 - 8x + 4y = -10$

$\Rightarrow x^2 - 8x = -4y - 10$	Rewrite in standard form.
$\Rightarrow x^2 - 8x + 16 = -4y - 10 + 16$	Complete the square.
$\Rightarrow (x - 4)^2 = -4y + 6$	

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$$\Rightarrow (x - 4)^2 = 4(-1)(y - \frac{3}{2})$$

\Rightarrow See Figure 2.

Final answer. Graph the parabola.

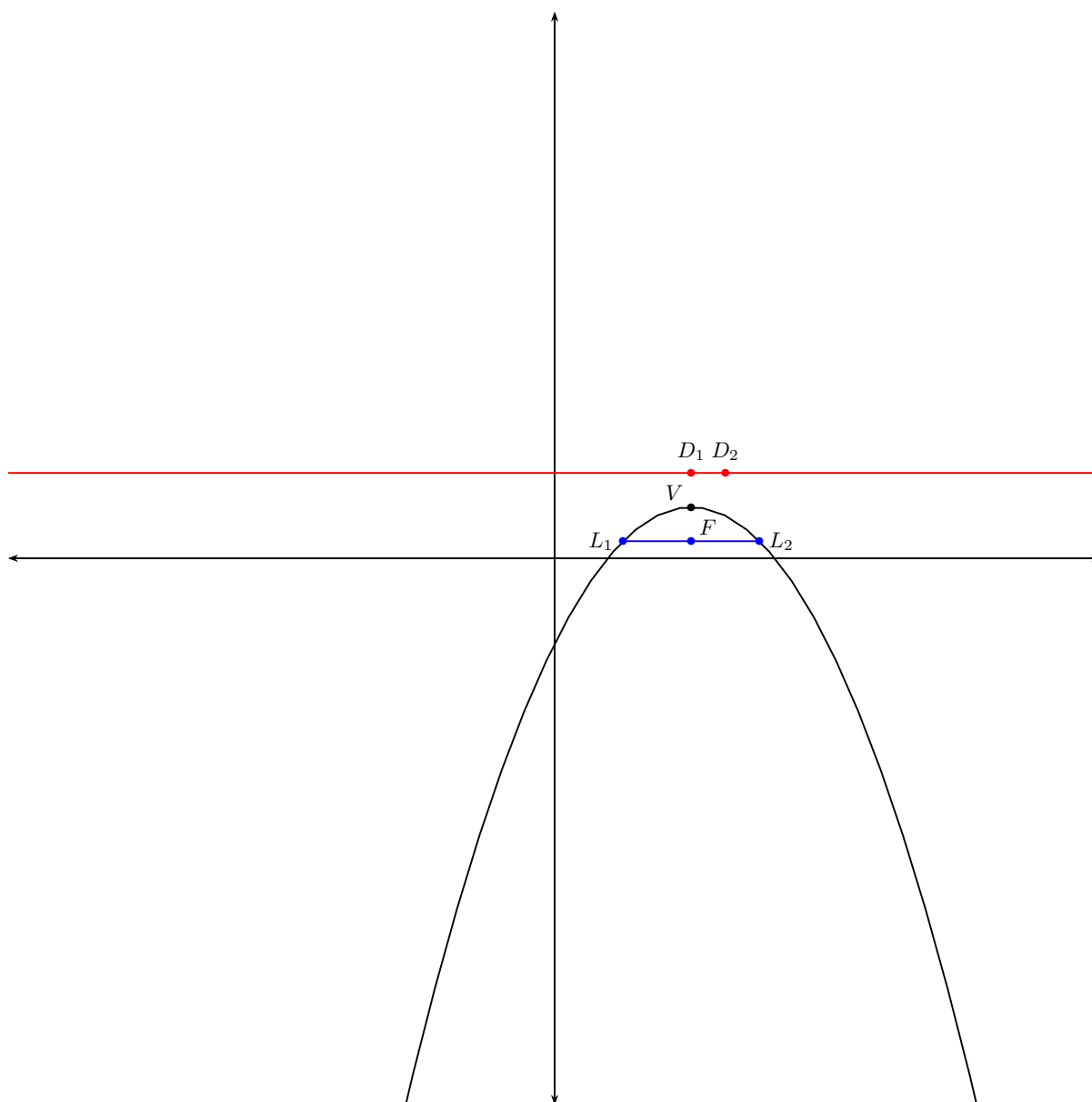


Figure 2. Graph of $(x - 4)^2 = 4(-1)(y - \frac{3}{2})$.

1.3 Sketch the graph of $y = -x^2 + 6x - 8$. Label the vertex, x- and y-intercept(s).

$$\Rightarrow -x^2 + 6x = y + 8$$

$$\Rightarrow x^2 - 6x = -y - 8$$

$$\Rightarrow x^2 - 6x + 9 = -y - 8 + 9$$

$$\Rightarrow (x - 3)^2 = -y + 1$$

Rewrite in standard form.

Complete the square.

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$\Rightarrow (x - 3)^2 = 4(-\frac{1}{4})(y - 1)$	
$\Rightarrow (x - 3)^2 = 4(-\frac{1}{4})(-1)$	Find the x-intercepts.
$\Rightarrow (x - 3)^2 = 4(\frac{1}{4})$	
$\Rightarrow (x - 3)^2 = 1$	
$\Rightarrow x = \pm 1 + 3$	
$\Rightarrow x = 1 + 3, x = -1 + 3$	
$\Rightarrow x_i \in \{2, 4\}$	
$\Rightarrow (0 - 3)^2 = 4(-\frac{1}{4})(y - 1)$	Find the y-intercepts.
$\Rightarrow (-3)^2 = 4(-\frac{1}{4})(y - 1)$	
$\Rightarrow 9 = -(y - 1)$	
$\Rightarrow 9 = -y + 1$	
$\Rightarrow y = 1 - 9$	
$\Rightarrow y_i = -8$	
\Rightarrow See Figure 3.	Final answer. Graph the parabola. ■

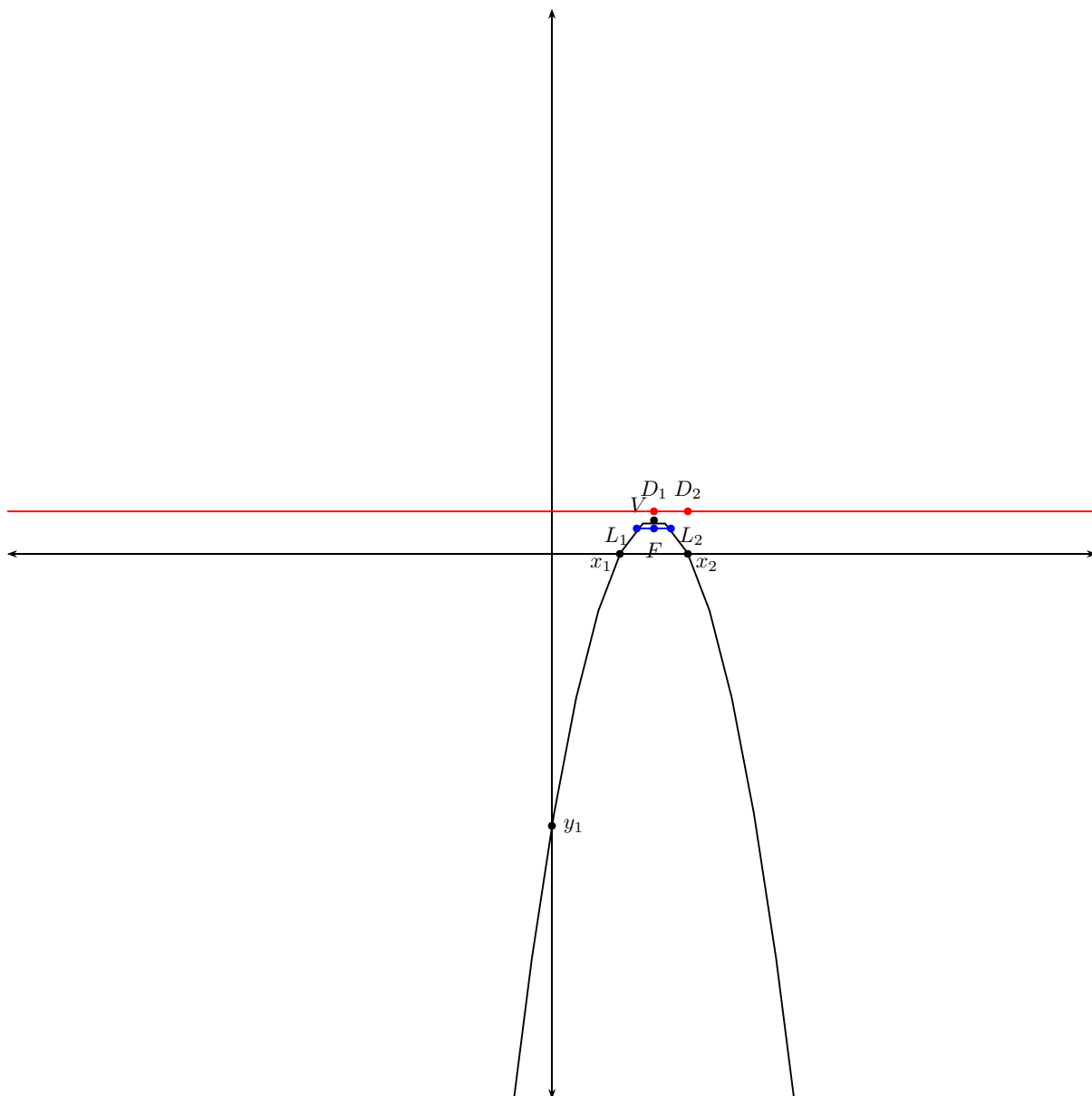


Figure 3. Graph of $(x - 3)^2 = 4(-\frac{1}{4})(y - 1)$ with x- and y-intercepts.

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2.1 Sketch the graph of the following ellipses.

2.1.a $\frac{x^2}{4} + \frac{(y-1)^2}{9} = 1$

$\Rightarrow \frac{x^2}{2^2} + \frac{(y-1)^2}{3^2} = 1$	Rewrite in standard form.
\Rightarrow See Figure 4.	Final answer. Graph the ellipse. ■

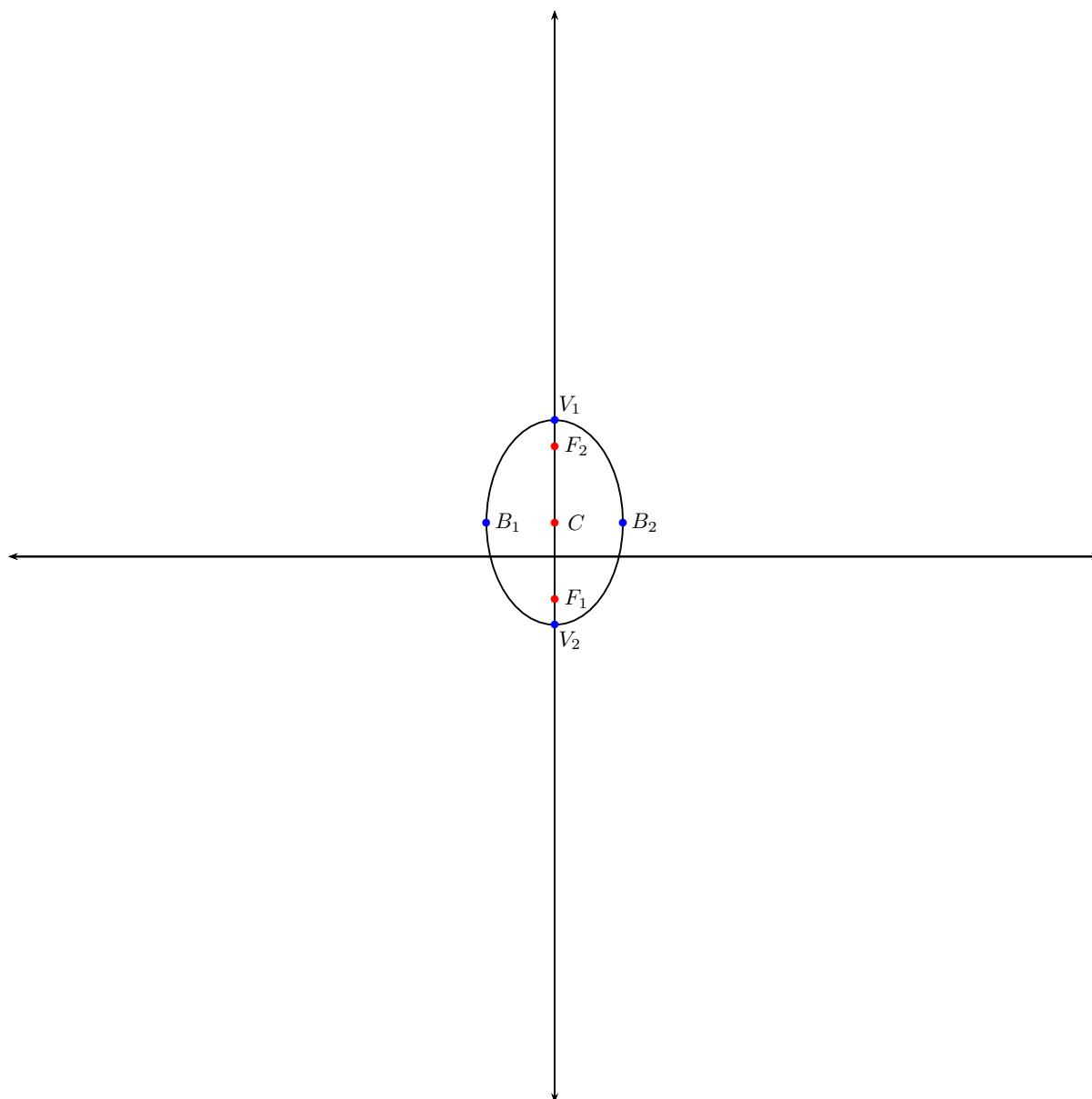


Figure 4. Graph of $\frac{x^2}{2^2} + \frac{(y-1)^2}{3^2} = 1$.

2.1.b $\frac{(x-3)^2}{25} + \frac{y^2+4y+4}{9} = 1$

$$\Rightarrow \frac{(x-3)^2}{25} + \frac{(y+2)^2}{9} = 1$$

Factor by grouping.

$$\Rightarrow \frac{(x-3)^2}{5^2} + \frac{(y+2)^2}{3^2} = 1$$

Rewrite in standard form.

\Rightarrow See Figure 5.

Final answer. Graph the ellipse. ■

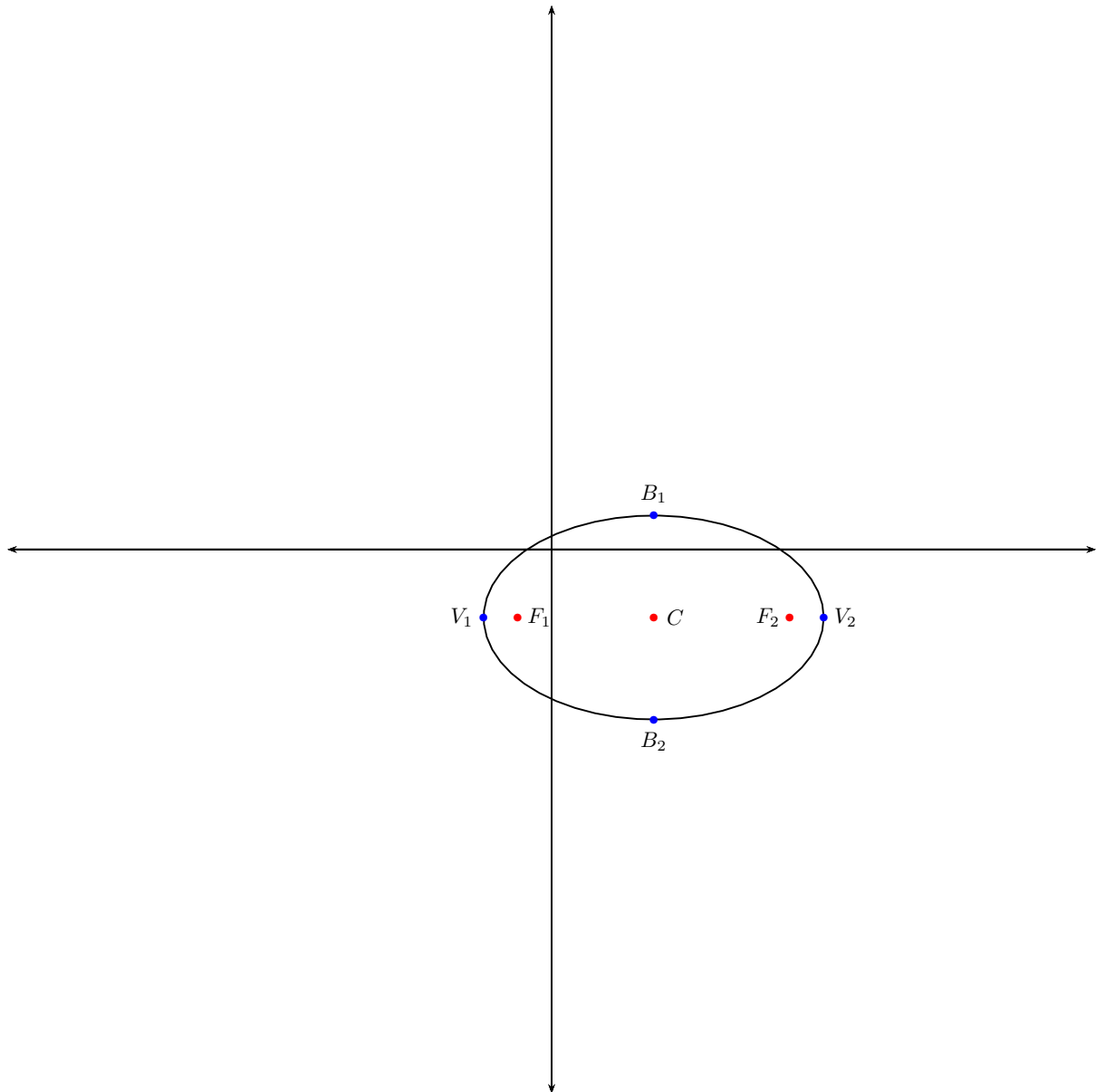


Figure 5. Graph of $\frac{(x-3)^2}{5^2} + \frac{(y+2)^2}{3^2} = 1$.

2.1.c $2x^2 + 3y^2 + 16x - 18y = 13$

$$\Rightarrow 2x^2 + 16x + 3y^2 - 18y = 13$$

Group terms.

$$\Rightarrow 2(x^2 + 8x) + 3(y^2 - 6y) = 13$$

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$\Rightarrow 2(x^2 + 8x + 16) + 3(y^2 - 6y) = 13 + 2(16)$	Complete the square.
$\Rightarrow 2(x^2 + 8x + 16) + 3(y^2 - 6y) = 13 + 32$	
$\Rightarrow 2(x^2 + 8x + 16) + 3(y^2 - 6y) = 45$	
$\Rightarrow 2(x + 4)^2 + 3(y^2 - 6y) = 45$	
$\Rightarrow 2(x + 4)^2 + 3(y^2 - 6y + 9) = 45 + 3(9)$	Complete the square.
$\Rightarrow 2(x + 4)^2 + 3(y^2 - 6y + 9) = 45 + 27$	
$\Rightarrow 2(x + 4)^2 + 3(y^2 - 6y + 9) = 72$	
$\Rightarrow 2(x + 4)^2 + 3(y - 3)^2 = 72$	
$\Rightarrow \frac{2(x+4)^2}{72} + \frac{3(y-3)^2}{72} = 1$	
$\Rightarrow \frac{(x+4)^2}{36} + \frac{(y-3)^2}{24} = 1$	
$\Rightarrow \frac{(x+4)^2}{6^2} + \frac{(y-3)^2}{(\sqrt{24})^2} = 1$	Rewrite in standard form.
$\Rightarrow \frac{(x+4)^2}{6^2} + \frac{(y-3)^2}{(\sqrt{4}\sqrt{6})^2} = 1$	
$\Rightarrow \frac{(x+4)^2}{6^2} + \frac{(y-3)^2}{(2\sqrt{6})^2} = 1$	
$\Rightarrow \text{See Figure 6.}$	Final answer. Graph the ellipse. ■

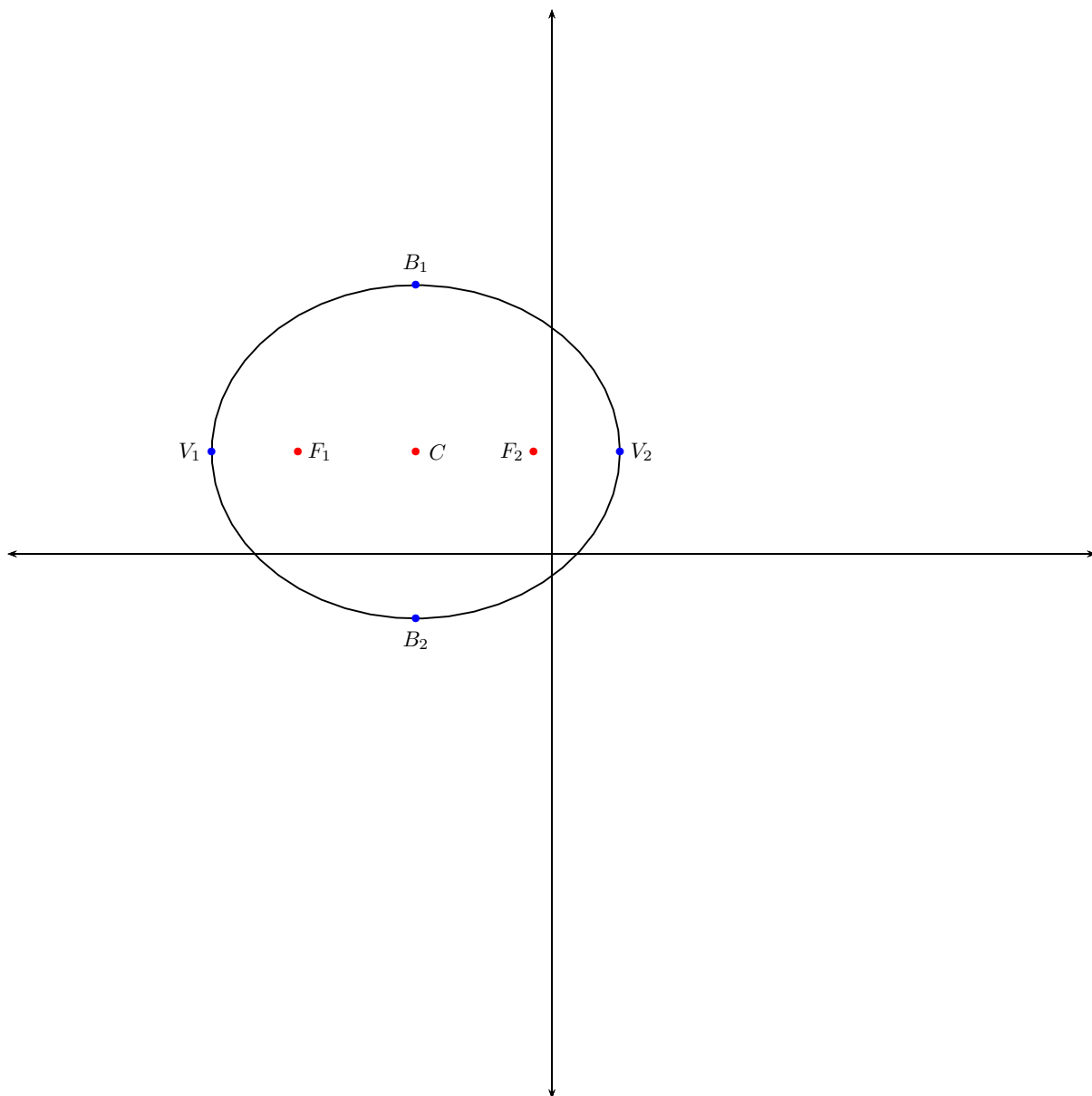


Figure 6. Graph of $\frac{(x+4)^2}{6^2} + \frac{(y-3)^2}{(2\sqrt{6})^2} = 1$