Exercise Solutions for Math 20

The 2-Dimensional Coordinate System

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1.1 Given $A(4,2)$, $B(6,-4)$ and $C(2,-7)$, find the distance between C and the midpoint M of $\overline{AB}$.

$\Rightarrow M = (\frac{4+6}{2}, \frac{2-4}{2})$	Use the midpoint formula.
$\Rightarrow M = (\frac{10}{2}, \frac{-2}{2})$	
$\Rightarrow M = (5, -1)$	
$\Rightarrow d_{CM} = \sqrt{(5-2)^2 + (-7+1)^2}$	Use the distance formula.
$\Rightarrow d_{CM} = \sqrt{(3)^2 + (-6)^2}$	
$\Rightarrow d_{CM} = \sqrt{9+36}$	
$\Rightarrow d_{CM} = \sqrt{45}$	
$\Rightarrow d_{CM} = \sqrt{9}\sqrt{5}$	
$\Rightarrow d_{CM} = 3\sqrt{5}$	Final answer.
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1.2 Solve algebraically for the x- and y-intercepts of the graphs of the following equations.

1.2.a $y^2 = x - 2$

$\Rightarrow 0 = x - 2$	Find the x-intercepts.
$\Rightarrow x_i = 2$	
$\Rightarrow y^2 = 0 - 2$	Find the y-intercepts.
$\Rightarrow y^2 = -2$	
$\Rightarrow y_i \in \emptyset$	No y-intercepts. The square of a real number cannot be negative.
$\Rightarrow x_i = 2, y_i \in \emptyset$	Final answer.
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1.2.b $y = \frac{4x^2+9}{x^2-9}$

$\Rightarrow \frac{4x^2+9}{x^2-9} = 0$	Find the x-intercepts.
$\Rightarrow \frac{4x^2+9}{(x-3)(x+3)} = 0$	Factor using difference of two squares.
$\Rightarrow 4x^2 + 9 = 0$	$x \in \{-3, 3\}$ are undefined points.
$\Rightarrow \frac{\pm\sqrt{-4(4)(9)}}{2(4)}$	Use the quadratic formula.
$\Rightarrow \frac{\pm\sqrt{-144}}{8}$	
$\Rightarrow x_i \in \emptyset$	No x-intercepts. The square root of a negative number is imaginary.
$\Rightarrow y = \frac{4(0)^2+9}{0^2-9}$	Find the y-intercepts.
$\Rightarrow y = \frac{9}{-9}$	
$\Rightarrow y_i = -1$	

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$\Rightarrow x_i \in \emptyset, y_i = -1$	Final answer.
	$\blacksquare$

1.2.c $y = |x-2| - 2$

$\Rightarrow |x-2| - 2 = 0$	Find the x-intercepts.
$\Rightarrow |x-2| = 2$	
$\Rightarrow x - 2 = 2$	$|a| = b \Rightarrow a = \pm b$. Solve for $a = b$.
$\Rightarrow x_i = 4$	
$\Rightarrow x - 2 = -2$	$|a| = b \Rightarrow a = \pm b$. Solve for $a = -b$.
$\Rightarrow x_i = 0$	
$\Rightarrow y = |0-2| - 2$	Find the y-intercepts.
$\Rightarrow y = |-2| - 2$	
$\Rightarrow y = 2 - 2$	
$\Rightarrow y_i = 0$	
$\Rightarrow x_i \in \{0, 4\}, y_i = 0$	Final answer.
	$\blacksquare$

1.2.d $y = x^2 + 1$

$\Rightarrow x^2 + 1 = 0$	Find the x-intercepts.
$\Rightarrow \frac{\pm\sqrt{-(1)(1)}}{2(1)}$	Use the quadratic equation.
$\Rightarrow \frac{\pm\sqrt{-1}}{2}$	
$\Rightarrow x_i \in \emptyset$	No x-intercepts. The square root of a negative number is imaginary.
$\Rightarrow y = 0^2 + 1$	Find the y-intercepts.
$\Rightarrow y_i = 1$	
$\Rightarrow x_i \in \emptyset, y_i = 1$	Final answer.
	$\blacksquare$

1.2.e $x^2 + y^2 = 25$

$\Rightarrow x^2 + 0^2 = 25$	Find the x-intercepts.
$\Rightarrow x^2 = 25$	
$\Rightarrow x = \pm 5$	
$\Rightarrow x_i \in \{-5, 5\}$	
$\Rightarrow 0^2 + y^2 = 25$	Find the y-intercepts.
$\Rightarrow y^2 = 25$	
$\Rightarrow y = \pm 5$	
$\Rightarrow y_i \in \{-5, 5\}$	
$\Rightarrow x_i \in \{-5, 5\}, y_i \in \{-5, 5\}$	Final answer.
	$\blacksquare$

1.2.f $x = \frac{y^2}{y^4-4}$

$\Rightarrow x = \frac{0^2}{0^2-4}$	Find the x-intercepts.
$\Rightarrow x_i = 0$	
$\Rightarrow \frac{y^2}{y^4-4} = 0$	Find the y-intercepts.
$\Rightarrow \frac{y^2}{(y^2-2)(y^2+2)} = 0$	
$\Rightarrow y^2 = 0$	$y \in \{-\sqrt{2}, \sqrt{2}\}$ are undefined points.
$\Rightarrow y_i = 0$	$0^2 = 0$
$\Rightarrow x_i = 0, y_i = 0$	Final answer.
	■

1.2.g $xy = 1$

$\Rightarrow x(0) = 1$	Find the x-intercepts.
$\Rightarrow 0 = 1$	
$\Rightarrow x_i \in \emptyset$	
$\Rightarrow 0y = 1$	Find the y-intercepts.
$\Rightarrow 0 = 1$	
$\Rightarrow y_i \in \emptyset$	
$\Rightarrow x_i \in \emptyset, y_i \in \emptyset$	Final answer.
	■

1.3 Determine the value(s) of k for which the graph of $y = 2x^2 + kx + 8$ does not intersect the x-axis. How about if the graph is to intersect the x-axis at exactly one point?

$\Rightarrow 2x^2 + kx + 8 = 0$	Set $y = 0$, and rewrite in standard form.
$\Rightarrow k^2 - 4(2)(8) < 0$	If the discriminant is negative, the quadratic equation has no real solutions, and as such, no x-intercepts.
$\Rightarrow k^2 - 64 < 0$	
$\Rightarrow (k-8)(k+8) < 0$	Factor using difference of two squares.
	Create a table of signs.

	-4	4	
$k - 8$	$-$	$-$	$+$
$k + 8$	$-$	$+$	$+$
$(k-8)(k+8)$	$+$	$-$	$+$

$\Rightarrow k \in (-8, 8)$

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$\Rightarrow k^2 - 4(2)(8) = 0$	If the disctiminant is zero, the quadratic equation has exactly one real solution, and as such, exactly one x-intercept.
$\Rightarrow k^2 - 16 = 0$	
$\Rightarrow (k-8)(k+8) = 0$	Factor using difference of two squares.
$\Rightarrow k \in \{-8, 8\}$	
$\Rightarrow k \in (-8, 8)$	Final answer. If the graph should have no x-intercept.
$\Rightarrow k \in \{-8, 8\}$	If the graph should have exactly one x-intercept.
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