Exercise Solutions for Math 20

Conics (Hyperbola), Systems of Linear Equations

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1.1 Identify the following conic sections.

1.1.a $2x^2 - 3y^2 + 4x + 6y - 1 = 0$

$\Rightarrow 2x^2 + 4x - 3y^2 + 6y = 1$	Group terms.
$\Rightarrow 2(x^2 + 2x) - 3(y^2 - 2y) = 1$	
$\Rightarrow 2(x^2 + 2x + 1) - 3(y^2 - 2y) = 1 + 2(1)$	Complete the square.
$\Rightarrow 2(x^2 + 2x + 1) - 3(y^2 - 2y) = 3$	
$\Rightarrow 2(x+1)^2 - 3(y^2 - 2y) = 3$	
$\Rightarrow 2(x+1)^2 - 3(y^2 - 2y + 1) = 3 - 3(1)$	Complete the square.
$\Rightarrow 2(x+1)^2 - 3(y^2 - 2y + 1) = 0$	
$\Rightarrow$ Not a conic.	Final answer. Cannot divide both sides.

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1.1.b $2x^2 + 3y^2 + 16x - 18y - 53 = 0$

$\Rightarrow 2x^2 + 16x + 3y^2 - 18y = 53$	Group terms.
$\Rightarrow 2(x^2 + 8x) + 3(y^2 - 6y) = 53$	
$\Rightarrow 2(x^2 + 8x + 16) + 3(y^2 - 6y) = 53 + 2(16)$	Complete the square.
$\Rightarrow 2(x^2 + 8x + 16) + 3(y^2 - 6y) = 85$	
$\Rightarrow 2(x+4)^2 + 3(y^2 - 6y) = 85$	
$\Rightarrow 2(x+4)^2 + 3(y^2 - 6y + 9) = 85 + 3(9)$	Complete the square.
$\Rightarrow 2(x+4)^2 + 3(y^2 - 6y + 9) = 112$	
$\Rightarrow 2(x+4)^2 + 3(y-3)^2 = 112$	
$\Rightarrow$ Ellipse.	Final answer.

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1.1.c $9x + y^2 + 4y - 5 = 0$

$\Rightarrow y^2 + 4y = -9x + 5$	Group terms.
$\Rightarrow y^2 + 4y + 4 = -9x + 9$	Complete the square.
$\Rightarrow (y+2)^2 = -9(x-1)$	
$\Rightarrow$ Parabola.	Final answer.

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1.1.d $4x^2 - x = y^2 + 1$

$\Rightarrow 4x^2 - x - y^2 = 1$	Group terms.
$\Rightarrow 4(x^2 - \frac{1}{4}x) - y^2 = 1$	
$\Rightarrow 4(x^2 - \frac{1}{4}x + \frac{1}{64}) - y^2 = 1 + \frac{1}{16}$	Complete the square.

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$\Rightarrow 4(x-\frac{1}{8})^2 - y^2 = \frac{17}{16}$	
$\Rightarrow$ Hyperbola.	Final answer.
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1.1.e $7y - y^2 - x = 0$

$\Rightarrow -y^2 + 7y = x$	Group terms.
$\Rightarrow y^2 - 7y = -x$	
$\Rightarrow y^2 - 7y + \frac{49}{4} = -x + \frac{49}{4}$	Complete the square.
$\Rightarrow (y^2 - \frac{7}{2})^2 = -1(x - \frac{49}{4})$	
$\Rightarrow$ Parabola.	Final answer.
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1.2 Sketch the graph of the following hyperbolas.

1.2.a $x^2 - 16y^2 = 40$

$\Rightarrow \frac{x^2}{40} - \frac{16y^2}{40} = 1$	Rewrite in standard form.
$\Rightarrow \frac{x^2}{40} - \frac{y^2}{\frac{1}{16}(40)} = 1$	
$\Rightarrow \frac{x^2}{40} - \frac{y^2}{\frac{5}{2}} = 1$	
$\Rightarrow \frac{x^2}{\sqrt{40}^2} - \frac{y^2}{(\sqrt{\frac{5}{2}})^2} = 1$	
$\Rightarrow \frac{x^2}{(\sqrt{4}\sqrt{10})^2} - \frac{y^2}{(\sqrt{\frac{5}{2}})^2} = 1$	
$\Rightarrow \frac{x^2}{(2\sqrt{10})^2} - \frac{y^2}{(\sqrt{\frac{5}{2}})^2} = 1$	
$\Rightarrow$ See Figure 1.	Final answer. Graph the hyperbola.
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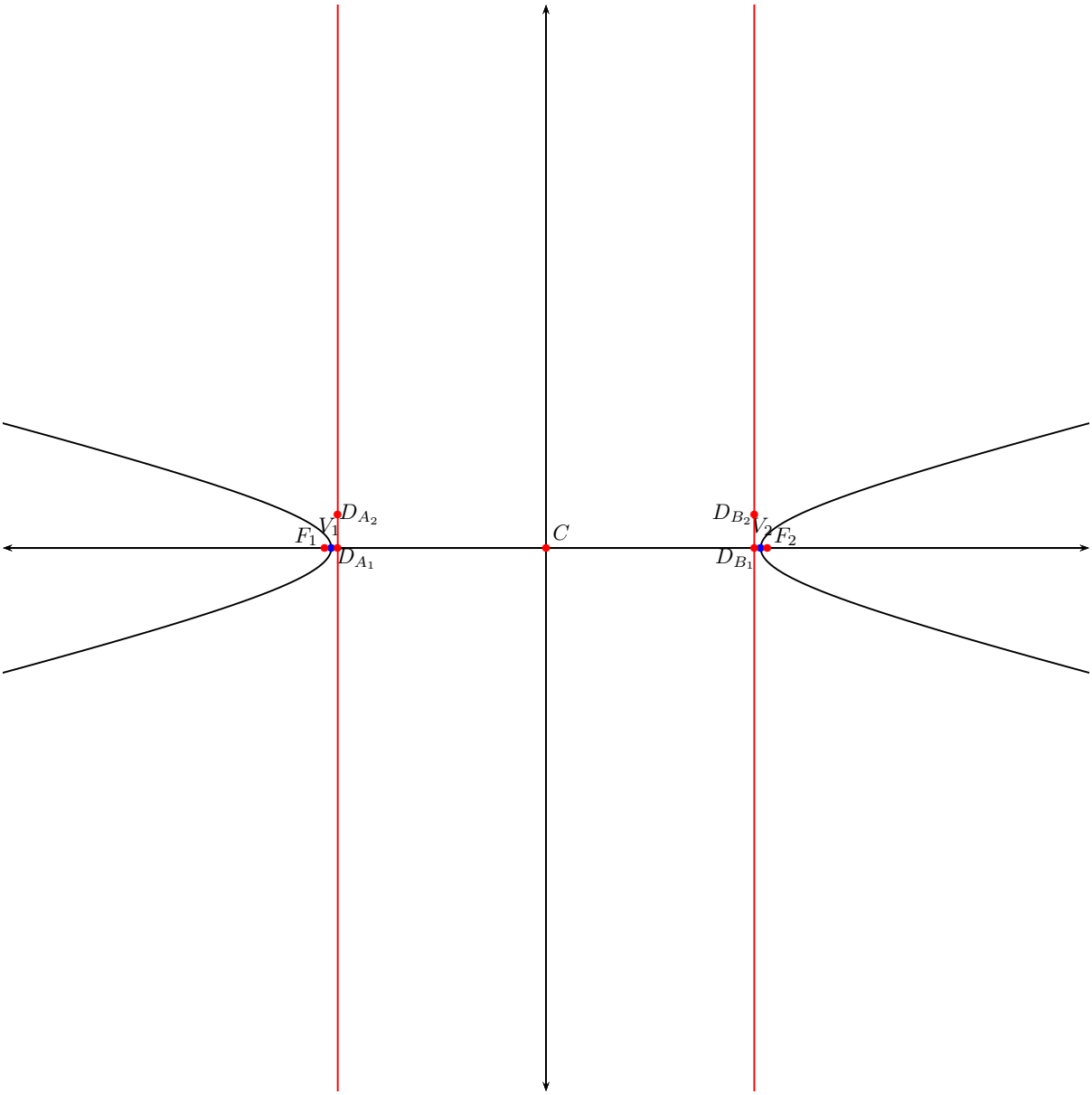

Figure 1. Graph of $\frac{x^2}{(2\sqrt{10})^2} - \frac{y^2}{(\sqrt{\frac{5}{2}})^2} = 1$.

1.2.b $4y^2 - (x+3)^2 = 16$

$\Rightarrow \frac{4y^2}{16} - \frac{(x+3)^2}{16} = 1$ $\Rightarrow \frac{y^2}{4} - \frac{(x+3)^2}{16} = 1$ $\Rightarrow \frac{y^2}{2^2} - \frac{(x+3)^2}{4^2} = 1$	Rewrite in standard form.
$\Rightarrow$ See Figure 2.	Final answer. Graph the hyperbola. ■

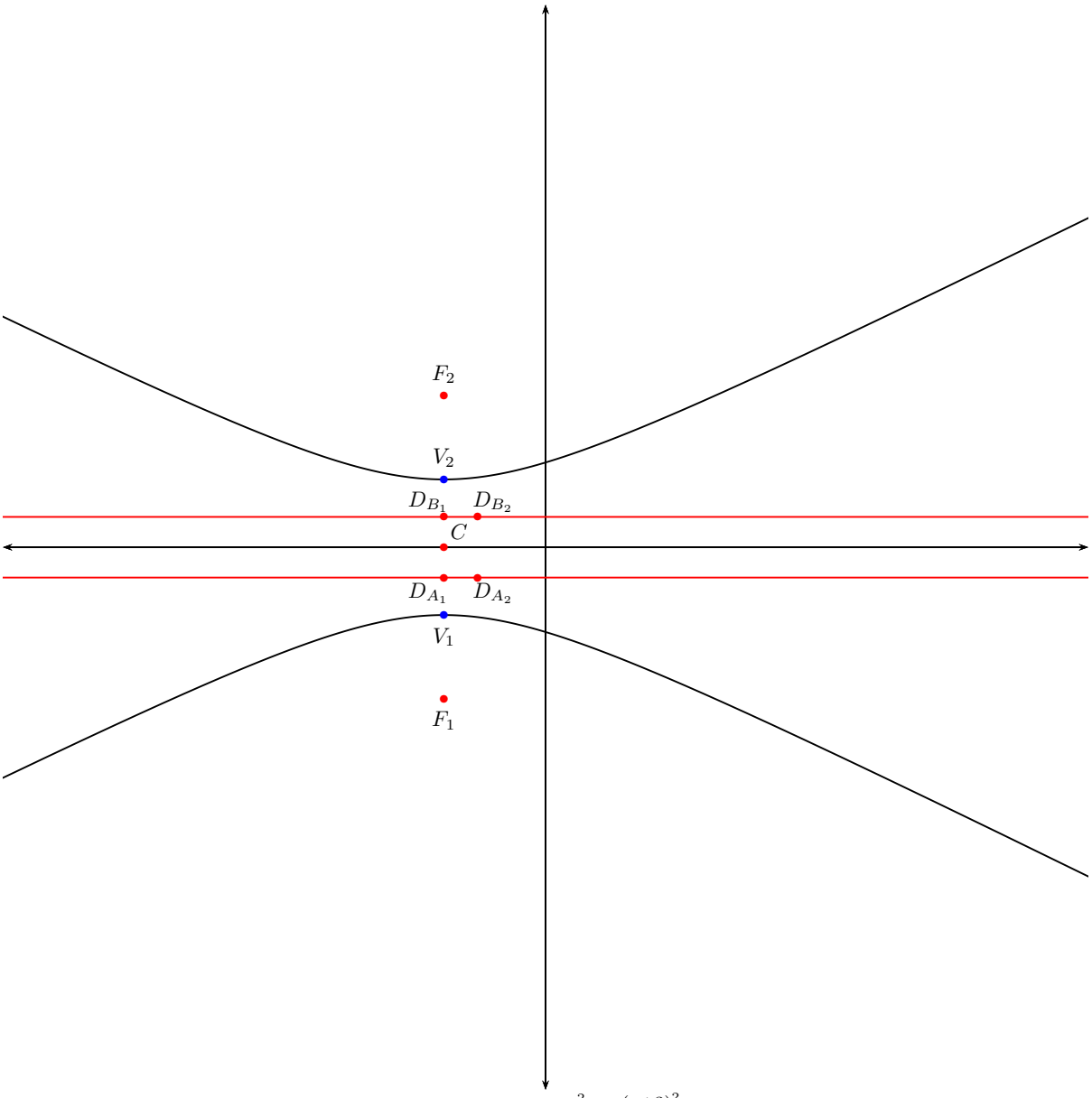

Figure 2. Graph of $\frac{y^2}{2^2} - \frac{(x+3)^2}{4^2} = 1$.

1.2.c $x^2 - y^2 + 6x - 4y = 4$

$\Rightarrow x^2 + 6x - y^2 - 4y = 4$	Group terms.
$\Rightarrow x^2 + 6x + 9 - y^2 - 4y = 4 + 9$	Complete the square.
$\Rightarrow x^2 + 6x + 9 - y^2 - 4y = 13$	
$\Rightarrow (x+3)^2 - y^2 - 4y = 13$	
$\Rightarrow (x+3)^2 - (y^2 + 4y) = 13$	
$\Rightarrow (x+3)^2 - (y^2 + 4y + 4) = 13 - 4$	Complete the square.
$\Rightarrow (x+3)^2 - (y^2 + 4y + 4) = 9$	
$\Rightarrow (x+3)^2 - (y+2)^2 = 9$	
$\Rightarrow \frac{(x+3)^2}{9} - \frac{(y+2)^2}{9} = 1$	Rewrite in standard form.
$\Rightarrow \frac{(x+3)^2}{3^2} - \frac{(y+2)^2}{3^2} = 1$	
$\Rightarrow$ See Figure 3.	Final answer. Graph the hyperbola.

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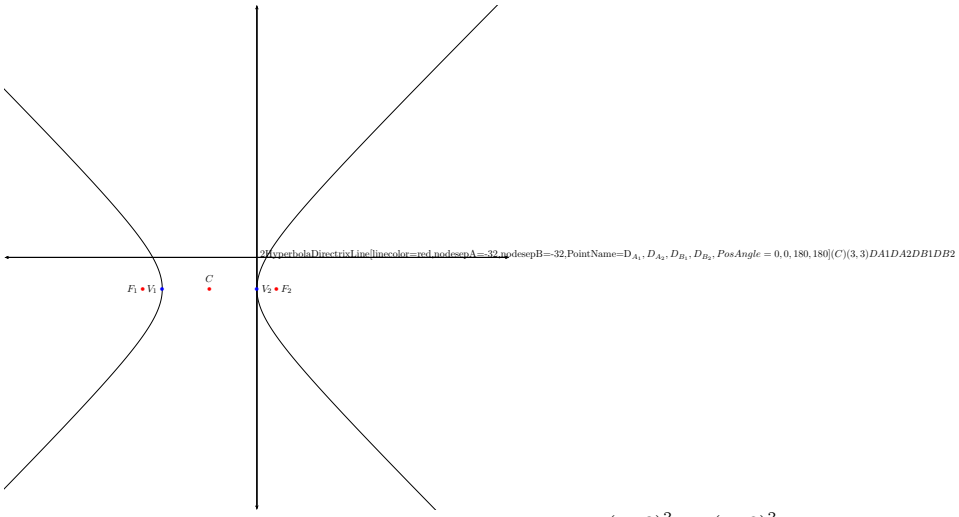

Figure 3. Graph of $\frac{(x+3)^2}{3^2} - \frac{(y+2)^2}{3^2} = 1$.

1.3 Find an equation of the hyperbola having $(2, 1)$ and $(-2, 1)$ as its foci and has a conjugate axis of length 3.

$\Rightarrow h = \frac{2+-2}{2}, k = 1$	This is a hyperbola with a horizontal transverse axis (from the changing x-coordinate of its foci). h can be found by averaging the x-coordinates of the foci, and k is simply equivalent to the y-coordinates of the foci.
$\Rightarrow h = 0, k = 1$	
$\Rightarrow 0 - c = 2$	Find c using $F_1(h - c, k)$.
$\Rightarrow c = -2$	
$\Rightarrow b = \frac{3}{2}$	By definition, b is half the length of the conjugate axis.
$\Rightarrow (-2)^2 = a^2 + (\frac{3}{2})^2$	Find a using $c^2 = a^2 + b^2$.
$\Rightarrow 4 = a^2 + \frac{9}{4}$	
$\Rightarrow a^2 = 4 - \frac{9}{4}$	
$\Rightarrow a^2 = \frac{16}{4} - \frac{9}{4}$	
$\Rightarrow a^2 = \frac{7}{4}$	
$\Rightarrow a = \sqrt{\frac{7}{4}}$	
$\Rightarrow \frac{x^2}{(\sqrt{\frac{7}{4}})^2} - \frac{(y-1)^2}{(\frac{3}{2})^2} = 1$	Final answer. Write the hyperbola equation using (h, k), a, and b.
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