Exercise Solutions for Math 20

Sum, Difference, Cofunction, Double Measure Identities

Nile Jocson <novoseiversia@gmail.com>

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1 Evaluate the following without using a calculator.

1.1 $\sin\left(\frac{19\pi}{12}\right)$

$\Rightarrow \sin\left(\frac{10\pi}{12} + \frac{9\pi}{12}\right)$	
$\Rightarrow \sin\left(\frac{5\pi}{6} + \frac{3\pi}{4}\right)$	
$\Rightarrow \sin\left(\frac{5\pi}{6}\right)\cos\left(\frac{3\pi}{4}\right) + \cos\left(\frac{5\pi}{6}\right)\sin\left(\frac{3\pi}{4}\right)$	$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
$\Rightarrow (\frac{1}{2})(-\frac{\sqrt{2}}{2}) + (-\frac{\sqrt{3}}{2})(\frac{\sqrt{2}}{2})$	
$\Rightarrow -\frac{\sqrt{2}}{4} - \frac{\sqrt{3}\sqrt{2}}{4}$	
$\Rightarrow -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$	
$\Rightarrow -\frac{\sqrt{2}+\sqrt{6}}{4}$	Final answer. ■

1.2 $\cos(33°)\cos(27°) - \sin(33°)\sin(27°)$

$\Rightarrow \cos(33° + 27°)$	$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
$\Rightarrow \cos(60°)$	
$\Rightarrow \frac{1}{2}$	Final answer. ■

2 If $\cot(\theta) = -\frac{5}{12}$ and $\theta \in (-\frac{\pi}{2}, 0)$, find $\cos\left(\theta + \frac{\pi}{3}\right)$.

$\Rightarrow O = -12, A = 5$	$\cot(\theta) = \frac{A}{O}$, and since $\theta \in (-\frac{\pi}{2}, 0)$, we are in QIV. Therefore, $O < 0$ and $A > 0$.
$\Rightarrow H = \sqrt{(-12)^2 + 5^2}$	$H = \sqrt{O^2 + A^2}$
$\Rightarrow H = \sqrt{144 + 25}$	
$\Rightarrow H = \sqrt{169}$	
$\Rightarrow H = 13$	
$\Rightarrow \cos\left(\theta + \frac{\pi}{3}\right) = \cos(\theta)\cos\left(\frac{\pi}{3}\right) - \sin(\theta)\sin\left(\frac{\pi}{3}\right)$	$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
$\Rightarrow \cos\left(\theta + \frac{\pi}{3}\right) = \frac{5}{13}\cos\left(\frac{\pi}{3}\right) - \sin(\theta)\sin\left(\frac{\pi}{3}\right)$	$\cos(\theta) = \frac{A}{H}$
$\Rightarrow \cos\left(\theta + \frac{\pi}{3}\right) = \frac{5}{13}\cos\left(\frac{\pi}{3}\right) + \frac{12}{13}\sin\left(\frac{\pi}{3}\right)$	$\sin(\theta) = \frac{O}{H}$
$\Rightarrow \cos\left(\theta + \frac{\pi}{3}\right) = (\frac{5}{13})(\frac{1}{2}) + (\frac{12}{13})(\frac{\sqrt{3}}{2})$	
$\Rightarrow \cos\left(\theta + \frac{\pi}{3}\right) = \frac{5}{26} + \frac{12\sqrt{3}}{26}$	
$\Rightarrow \cos\left(\theta + \frac{\pi}{3}\right) = \frac{5+12\sqrt{3}}{26}$	
	$\blacksquare$

3 Given $\cos(\alpha) = \frac{3}{5}$ where α lies in the interval $(\frac{\pi}{2}, 2\pi)$, find the following.

3.1 $\sin(2\alpha)$

$\Rightarrow A = 3, H = 5$	$\cos(\theta) = \frac{A}{H}$, and since $\cos(\alpha) > 0$ and $\alpha \in (\frac{\pi}{2}, 2\pi)$, we are in QIV. Therefore, $O < 0$ and $A > 0$.
$\Rightarrow O = -\sqrt{5^2 - 3^2}$	From $H = \sqrt{A^2 + O^2}$, we can derive $O = \pm\sqrt{H^2 - A^2}$. Remember that in this case, $A > 0$.
$\Rightarrow O = -\sqrt{25 - 9}$	
$\Rightarrow O = -\sqrt{16}$	
$\Rightarrow O = -4$	
$\Rightarrow \sin(\alpha) = -\frac{4}{5}$	$\sin(\theta) = \frac{O}{H}$
$\Rightarrow \sin(2\alpha) = 2(-\frac{4}{5})(\frac{3}{5})$	$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
$\Rightarrow \sin(2\alpha) = 2(-\frac{12}{25})$	
$\Rightarrow \sin(2\alpha) = -\frac{24}{25}$	Final answer.
	■

3.2 $\sin(3\alpha)$

$\Rightarrow \cos(2\alpha) = (\frac{3}{5})^2 - (-\frac{4}{5})^2$	$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
$\Rightarrow \cos(2\alpha) = \frac{9}{25} - \frac{16}{25}$	
$\Rightarrow \cos(2\alpha) = -\frac{7}{25}$	
$\Rightarrow \sin(3\alpha) = \sin(\alpha + 2\alpha)$	
$\Rightarrow \sin(\alpha + 2\alpha) = (-\frac{4}{5})(-\frac{7}{25}) + (\frac{3}{5})(-\frac{24}{25})$	$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
$\Rightarrow \sin(\alpha + 2\alpha) = \frac{28}{125} - \frac{72}{125}$	
$\Rightarrow \sin(\alpha + 2\alpha) = \frac{44}{125}$	
$\Rightarrow \sin(3\alpha) = \frac{44}{125}$	Final answer.
	■

4 Establish the following identities.

4.1