

Exercise Solutions for Math 20

Radicals and Complex Numbers

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1 Simplify the following. Rationalize the denominators.

1.1 $\frac{24c^{-\frac{1}{2}}d^{\frac{2}{3}}}{18c^{-\frac{1}{7}}d^{-\frac{3}{5}}}$

| | |
|--|--|
| $\Rightarrow \frac{4c^{-\frac{1}{2}}d^{\frac{2}{3}}}{3c^{-\frac{1}{7}}d^{-\frac{3}{5}}}$ | Simplify the fraction to lowest terms. |
| $\Rightarrow \frac{4d^{\frac{2}{3}}c^{\frac{1}{7}}d^{\frac{3}{5}}}{3c^{\frac{1}{2}}}$ | $a^{-\frac{b}{c}} = \frac{1}{a^{\frac{b}{c}}}$ |
| $\Rightarrow \frac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{\frac{1}{7}-\frac{1}{2}}$ | $\frac{a^m}{a^n} = a^{m-n}$ |
| $\Rightarrow \frac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{\frac{2}{14}-\frac{7}{14}}$ | LCM = 14 |
| $\Rightarrow \frac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{-\frac{5}{14}}$ | |
| $\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{2}{3}+\frac{3}{5}}$ | $a^m a^n = a^{m+n}$ |
| $\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{10}{15}+\frac{9}{15}}$ | LCM = 15 |
| $\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{19}{15}}$ | |
| $\Rightarrow \frac{4d^{\frac{19}{15}}}{3c^{\frac{5}{14}}}$ | $a^{-\frac{b}{c}} = \frac{1}{a^{\frac{b}{c}}}$ |
| $\Rightarrow \frac{4}{3} \frac{\sqrt[15]{d^{19}}}{\sqrt[14]{c^5}}$ | |
| $\Rightarrow \frac{4}{3} \frac{\sqrt[15]{d^{19}}}{\sqrt[14]{c^5}} \cdot \frac{\sqrt[14]{c^9}}{\sqrt[14]{c^9}}$ | Rationalize. |
| $\Rightarrow \frac{4}{3c} \frac{\sqrt[14]{c^9} \sqrt[15]{d^{19}}}{\sqrt[14]{c^5}}$ | |

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1.2 $(u^{\frac{1}{3}} + (uv)^{\frac{1}{6}} + v^{\frac{1}{3}})(u^{\frac{1}{6}} - v^{\frac{1}{6}})$

| | |
|---|------------------------------|
| $\Rightarrow (u^{\frac{1}{3}} + u^{\frac{1}{6}}v^{\frac{1}{6}} + v^{\frac{1}{3}})(u^{\frac{1}{6}} - v^{\frac{1}{6}})$ | Distribute exponent. |
| $\Rightarrow u^{\frac{1}{2}} - v^{\frac{1}{2}}$ | Use difference of two cubes. |
| $\Rightarrow \sqrt{u} - \sqrt{v}$ | |

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1.3 $\sqrt[3]{-8^4}$

| | |
|----------------------------------|---|
| $\Rightarrow -\sqrt[3]{8^4}$ | $\sqrt[n]{-a} = -\sqrt[n]{a}$ for odd n |
| $\Rightarrow -\sqrt[3]{(2^3)^4}$ | |
| $\Rightarrow -\sqrt[3]{(2^4)^3}$ | $(a^m)^n = (a^n)^m$ |
| $\Rightarrow -2^4$ | |
| $\Rightarrow -16$ | |

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1.4 $\sqrt[4]{9x^8}$

$$\begin{aligned} &\Rightarrow \sqrt[4]{9}\sqrt[4]{x^8} & \sqrt[n]{ab} &= \sqrt[n]{a}\sqrt[n]{b} \\ &\Rightarrow \sqrt[4]{3^2}\sqrt[4]{x^8} \\ &\Rightarrow x^2\sqrt{3} \end{aligned}$$

■

1.5 $\sqrt{\sqrt[3]{9a^4b^4}}$

$$\begin{aligned} &\Rightarrow \sqrt[6]{9a^4b^4} & \sqrt[n]{\sqrt[n]{a}} &= \sqrt[n+n]{a} \\ &\Rightarrow \sqrt[6]{3^2a^4b^4} \\ &\Rightarrow \sqrt[3]{3a^2b^2} \end{aligned}$$

■

1.6 $\frac{2\sqrt{5}}{\sqrt{8}} + \frac{9}{\sqrt[3]{16}}$

$$\begin{aligned} &\Rightarrow \frac{2\sqrt{5}}{\sqrt{8}} \cdot \frac{\sqrt{2}}{\sqrt{2}} + \frac{9}{\sqrt[3]{16}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} & \text{Rationalize.} \\ &\Rightarrow \frac{2\sqrt{5}\sqrt{2}}{\sqrt{16}} + \frac{9\sqrt[3]{4}}{\sqrt[3]{64}} \\ &\Rightarrow \frac{2\sqrt{5}\sqrt{2}}{4} + \frac{9\sqrt[3]{4}}{4} \\ &\Rightarrow \frac{2\sqrt{5}\sqrt{2}+9\sqrt[3]{4}}{4} \\ &\Rightarrow \frac{2\sqrt{10}+9\sqrt[3]{4}}{4} \end{aligned}$$

■

1.7 $\frac{x^2-2x+1}{\sqrt{x+1}}$

$$\begin{aligned} &\Rightarrow \frac{x^2-2x+1}{\sqrt{x+1}} \cdot \frac{\sqrt{x}-1}{\sqrt{x}-1} & \text{Rationalize using difference of two squares.} \\ &\Rightarrow \frac{(x^2-2x+1)(\sqrt{x}-1)}{x-1} \\ &\Rightarrow \frac{(x-1)^2(\sqrt{x}-1)}{x-1} & \text{Factor by grouping.} \\ &\Rightarrow (x-1)(\sqrt{x}-1) \end{aligned}$$

■

1.8 $\frac{1}{\sqrt[3]{4}-\sqrt[3]{-27}}$

| | |
|--|--|
| $\Rightarrow \frac{1}{\sqrt[3]{4}+\sqrt[3]{27}}$ | $\sqrt[m]{-a} = -\sqrt[m]{a}$ for odd m |
| $\Rightarrow \frac{1}{\sqrt[3]{4}+\sqrt[3]{27}} \cdot \frac{\sqrt[3]{4^2}-\sqrt[3]{4}\sqrt[3]{27}+\sqrt[3]{27^2}}{\sqrt[3]{4^2}-\sqrt[3]{4}\sqrt[3]{27}+\sqrt[3]{27^2}}$ $\Rightarrow \frac{\sqrt[3]{4^2}-\sqrt[3]{4}\sqrt[3]{27}+\sqrt[3]{27^2}}{4+27}$ $\Rightarrow \frac{\sqrt[3]{4^2}-3\sqrt[3]{4}+\sqrt[3]{27^2}}{4+27}$ $\Rightarrow \frac{\sqrt[3]{4^2}-3\sqrt[3]{4}+\sqrt[3]{27^2}}{31}$ $\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+\sqrt[3]{27^2}}{31}$ $\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+\sqrt[3]{(3^3)^2}}{31}$ | Rationalize using difference of two cubes. |
| $\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+\sqrt[3]{(3^2)^3}}{31}$ $\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+3^2}{31}$ $\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+9}{31}$ | $(a^m)^n = (a^n)^m$ |
| $\Rightarrow \frac{\sqrt[3]{8}\sqrt[3]{2}-3\sqrt[3]{4}+9}{31}$ $\Rightarrow \frac{2\sqrt[3]{2}-3\sqrt[3]{4}+9}{31}$ | $\sqrt[m]{ab} = \sqrt[m]{a}\sqrt[m]{b}$ |
| ■ | |

2 Perform the following operations and simplify.

2.1 $3i(i^2 - i^3 + 5i^5 - i^{-2})$

| | |
|--|-----------------|
| $\Rightarrow 3i(-1 - i^3 + 5i^5 - i^{-2})$ | $i^2 = -1$ |
| $\Rightarrow 3i(-1 + i + 5i^5 - i^{-2})$ | $i^3 = -i$ |
| $\Rightarrow 3i(-1 + i + 5i - i^{-2})$ | $i^5 = i$ |
| $\Rightarrow 3i(-1 + i + 5i + 1)$ $\Rightarrow 3i(6i)$ $\Rightarrow 18i^2$ | $i^{-2} = -1$ |
| $\Rightarrow -18$ | $i^2 = -1$ ■ |