Exercise Solutions for Math 20

Angles and Their Measure, Trigonometric Functions of Angles

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1.1 Complete the following table.

rev	deg	rad
$\frac{1}{20}$	$18°$	$\frac{\pi}{10}$
$-\frac{2}{3}$	$-240°$	$-\frac{4\pi}{3}$

1.2 Marian eats one slice of a circular pie that is cut into six congruent slices. If the arclength of the side she ate is 24π inches, what is the radius of the pie?

$\Rightarrow \theta = \frac{2\pi}{6}$ $\Rightarrow \theta = \frac{\pi}{3}$	The pie is sliced into 6 pieces.
$\Rightarrow 24\pi = r(\frac{\pi}{3})$ $\Rightarrow r = \frac{24\pi}{\frac{\pi}{3}}$ $\Rightarrow r = \frac{24\pi(3)}{\pi}$	$s = r\theta$
$\Rightarrow r = 72$ inches	Final answer. ■

1.3 If the terminal side of an angle $\theta > 0$ contains the point $(1, -4\sqrt{3})$, find the six trigonometric functions of θ.

$\Rightarrow r = \sqrt{1^2 + (-4\sqrt{3})^2}$ $\Rightarrow r = \sqrt{1 + 16(3)}$ $\Rightarrow r = \sqrt{1 + 48}$ $\Rightarrow r = \sqrt{49}$ $\Rightarrow r = 7$	Use the Pythagorean Theorem to find r.
$\Rightarrow \cos(\theta) = \frac{1}{7}$ $\Rightarrow \sin(\theta) = -\frac{4\sqrt{3}}{7}$ $\Rightarrow \tan(\theta) = -4\sqrt{3}$ $\Rightarrow \cot(\theta) = -\frac{1}{4\sqrt{3}} = -\frac{\sqrt{3}}{12}$ $\Rightarrow \sec(\theta) = 7$ $\Rightarrow \csc(\theta) = -\frac{7}{4\sqrt{3}} = -\frac{7\sqrt{3}}{12}$	Final answer. $\cos(\theta) = \frac{x}{r}$ $\sin(\theta) = \frac{y}{r}$ $\sin(\theta) = \frac{y}{x}$ $\sin(\theta) = \frac{x}{y}$ $\sec(\theta) = \frac{r}{x}$ $\csc(\theta) = \frac{r}{y}$ ■

1.4 Find the six trigonometric functions of α if $\cos(\alpha) = -\frac{5}{13}$ and α is in Quadrant III.

$\Rightarrow x = -5, r = 13$	$\cos(\theta) = \frac{x}{r}$

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$\Rightarrow y = -\sqrt{13^2 - (-5)^2}$	Use the Pythagorean Theorem to find y. Since we're in Quadrant III, we want the negative case.
$\Rightarrow y = -\sqrt{169 - 25}$	
$\Rightarrow y = -\sqrt{144}$	
$\Rightarrow y = -12$	
$\Rightarrow \sin(\theta) = -\frac{12}{13}$	Final answer. $\sin(\theta) = \frac{y}{r}$
$\Rightarrow \tan(\theta) = \frac{-\frac{12}{13}}{-\frac{5}{13}} = \frac{12}{5}$	$\tan(\theta) = \frac{y}{x}$
$\Rightarrow \cot(\theta) = \frac{5}{12}$	$\cot(\theta) = \frac{1}{\tan(\theta)}$
$\Rightarrow \sec(\theta) = -\frac{13}{5}$	$\sec(\theta) = \frac{1}{\cos(\theta)}$
$\Rightarrow \csc(\theta) = -\frac{13}{12}$	$\csc(\theta) = \frac{1}{\sin(\theta)}$
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1.5 Evaluate the following.

1.5.a csc(315°)

$\Rightarrow \overline{\theta} = 360° - 315°$	Find the reference angle.
$\Rightarrow \overline{\theta} = 45°$	
$\Rightarrow \sin(315°) = -\sin(45°) = -\frac{\sqrt{2}}{2}$	Since 315° is in QIV, the result of sin will be negative.
$\Rightarrow \csc(315°) = -\sqrt{2}$	Final answer. $\csc(\theta) = \frac{1}{\sin(\theta)}$
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1.5.b cot(420°)

$\Rightarrow \theta = 60°$	Since $420° > 360°$, find 420° mod 360°.
$\Rightarrow \cos(60°) = \frac{1}{2}, \sin(60°) = \frac{\sqrt{3}}{2}$	Find $\cos(\theta)$ and $\sin(\theta)$. Both will be positive since 60° is in QI.
$\Rightarrow \cot(\theta) = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$	$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
$\Rightarrow \cot(\theta) = \frac{2}{2\sqrt{3}}$	
$\Rightarrow \cot(\theta) = \frac{2\sqrt{3}}{2(3)}$	Rationalize.
$\Rightarrow \cot(\theta) = \frac{\sqrt{3}}{3}$	Final answer.
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1.5.c tan($\frac{5}{8}$ rev)cos(660°)

$\Rightarrow \theta_1 = \frac{5}{8}(2\pi)$	Convert from revolutions to radians.
$\Rightarrow \theta_1 = \frac{10\pi}{8}$	
$\Rightarrow \theta_1 = \frac{5\pi}{4}$	
$\Rightarrow \tan(\frac{5\pi}{4}) = 1$	Since θ_1 is in QIII, tan will be positive.
$\Rightarrow \theta_2 = 300$	Find 660° mod 360°.
$\Rightarrow \overline{\theta_2} = 360 - 300$	Find the reference angle.

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$\Rightarrow \overline{\theta_2} = 60$	
$\Rightarrow \cos(660) = \cos(60) = \frac{1}{2}$	Since θ_2 is in QIV, cos will be positive.
$\Rightarrow \tan(\frac{5}{8}\text{ rev})\cos(660°) = \frac{1}{2}$	Final answer.
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