

Exercise Solutions for Math 20

The 2-Dimensional Coordinate System

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1.1 Given $A(4, 2)$, $B(6, -4)$ and $C(2, -7)$, find the distance between C and the midpoint M of \overline{AB} .

$\Rightarrow M = (\frac{4+6}{2}, \frac{2-4}{2})$	Use the midpoint formula.
$\Rightarrow M = (\frac{10}{2}, \frac{-2}{2})$	
$\Rightarrow M = (5, -1)$	
$\Rightarrow d_{CM} = \sqrt{(5-2)^2 + (-7+1)^2}$	Use the distance formula.
$\Rightarrow d_{CM} = \sqrt{(3)^2 + (-6)^2}$	
$\Rightarrow d_{CM} = \sqrt{9+36}$	
$\Rightarrow d_{CM} = \sqrt{45}$	
$\Rightarrow d_{CM} = \sqrt{9}\sqrt{5}$	
$\Rightarrow d_{CM} = 3\sqrt{5}$	Final answer. ■

1.2 Solve algebraically for the x- and y-intercepts of the graphs of the following equations.

1.2.a $y^2 = x - 2$

$\Rightarrow 0 = x - 2$	Find the x-intercepts.
$\Rightarrow x_i = 2$	
$\Rightarrow y^2 = 0 - 2$	Find the y-intercepts.
$\Rightarrow y^2 = -2$	
$\Rightarrow y_i \in \emptyset$	No y-intercepts. The square of a real number cannot be negative.
$\Rightarrow x_i = 2, y_i \in \emptyset$	Final answer. ■

1.2.b $y = \frac{4x^2+9}{x^2-9}$

$\Rightarrow \frac{4x^2+9}{x^2-9} = 0$	Find the x-intercepts.
$\Rightarrow \frac{4x^2+9}{(x-3)(x+3)} = 0$	Factor using difference of two squares.
$\Rightarrow 4x^2 + 9 = 0$	$x \in \{-3, 3\}$ are undefined points.
$\Rightarrow \frac{\pm\sqrt{-4(4)(9)}}{2(4)}$	Use the quadratic formula.
$\Rightarrow \frac{\pm\sqrt{-144}}{8}$	
$\Rightarrow x_i \in \emptyset$	No x-intercepts. The square root of a negative number is imaginary.
$\Rightarrow y = \frac{4(0)^2+9}{0^2-9}$	Find the y-intercepts.
$\Rightarrow y = \frac{9}{-9}$	
$\Rightarrow y_i = -1$	

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$$\Rightarrow x_i \in \emptyset, y_i = -1$$

Final answer.



1.2.c $y = |x - 2| - 2$

$$\Rightarrow |x - 2| - 2 = 0$$

Find the x-intercepts.

$$\Rightarrow |x - 2| = 2$$

$$\Rightarrow x - 2 = 2$$

$|a| = b \Rightarrow a = \pm b$. Solve for $a = b$.

$$\Rightarrow x_i = 4$$

$$\Rightarrow x - 2 = -2$$

$|a| = b \Rightarrow a = \pm b$. Solve for $a = -b$.

$$\Rightarrow x_i = 0$$

$$\Rightarrow y = |0 - 2| - 2$$

Find the y-intercepts.

$$\Rightarrow y = |-2| - 2$$

$$\Rightarrow y = 2 - 2$$

$$\Rightarrow y_i = 0$$

$$\Rightarrow x_i \in \{0, 4\}, y_i = 0$$

Final answer.



1.2.d $y = x^2 + 1$

$$\Rightarrow x^2 + 1 = 0$$

Find the x-intercepts.

$$\Rightarrow \frac{\pm\sqrt{-(-1)(1)}}{2(1)}$$

Use the quadratic equation.

$$\Rightarrow \frac{\pm\sqrt{-1}}{2}$$

$$\Rightarrow x_i \in \emptyset$$

No x-intercepts. The square root of a negative number is imaginary.

$$\Rightarrow y = 0^2 + 1$$

Find the y-intercepts.

$$\Rightarrow y_i = 1$$

$$\Rightarrow x_i \in \emptyset, y_i = 1$$

Final answer.



1.2.e $x^2 + y^2 = 25$

$$\Rightarrow x^2 + 0^2 = 25$$

Find the x-intercepts.

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = \pm 5$$

$$\Rightarrow x_i \in \{-5, 5\}$$

$$\Rightarrow 0^2 + y^2 = 25$$

Find the y-intercepts.

$$\Rightarrow y^2 = 25$$

$$\Rightarrow y = \pm 5$$

$$\Rightarrow y_i \in \{-5, 5\}$$

$$\Rightarrow x_i \in \{-5, 5\}, y_i \in \{-5, 5\}$$

Final answer.



1.2.f $x = \frac{y^2}{y^4-4}$

$\Rightarrow x = \frac{0^2}{0^2-4}$ $\Rightarrow x_i = 0$	Find the x-intercepts.
$\Rightarrow \frac{y^2}{y^4-4} = 0$ $\Rightarrow \frac{y^2}{(y^2-2)(y^2+2)} = 0$ $\Rightarrow y^2 = 0$ $\Rightarrow y_i = 0$	Find the y-intercepts. $y \in \{-\sqrt{2}, \sqrt{2}\}$ are undefined points. $0^2 = 0$
$\Rightarrow x_i = 0, y_i = 0$	Final answer. ■