

Exercise Solutions for Math 20

Radicals and Complex Numbers

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1 Simplify the following. Rationalize the denominators.

1.1 $\frac{24c^{-\frac{1}{2}}d^{\frac{2}{3}}}{18c^{-\frac{1}{7}}d^{-\frac{3}{5}}}$

$$\begin{aligned} &\Rightarrow \frac{4c^{-\frac{1}{2}}d^{\frac{2}{3}}}{3c^{-\frac{1}{7}}d^{-\frac{3}{5}}} \\ &\Rightarrow \frac{4d^{\frac{2}{3}}c^{\frac{1}{7}}d^{\frac{3}{5}}}{3c^{\frac{1}{2}}} \\ &\Rightarrow \frac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{\frac{1}{7}-\frac{1}{2}} \\ &\Rightarrow \frac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{\frac{2}{14}-\frac{7}{14}} \\ &\Rightarrow \frac{4d^{\frac{2}{3}}d^{\frac{3}{5}}}{3}c^{-\frac{5}{14}} \\ &\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{2}{3}+\frac{3}{5}} \\ &\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{10}{15}+\frac{9}{15}} \\ &\Rightarrow \frac{4}{3}c^{-\frac{5}{14}}d^{\frac{19}{15}} \\ &\Rightarrow \frac{4d^{\frac{19}{15}}}{3c^{\frac{5}{14}}} \\ &\Rightarrow \frac{4}{3}\frac{\sqrt[15]{d^{19}}}{\sqrt[14]{c^5}} \\ &\Rightarrow \frac{4}{3}\frac{\sqrt[15]{d^{19}}}{\sqrt[14]{c^5}} \cdot \frac{\sqrt[14]{c^9}}{\sqrt[14]{c^9}} \\ &\Rightarrow \frac{4}{3}\frac{\sqrt[14]{c^9}\sqrt[15]{d^{19}}}{\sqrt[14]{c^5}\sqrt[14]{c^9}} \end{aligned}$$

Simplify the fraction to lowest terms.

$$a^{-\frac{b}{c}} = \frac{1}{a^{\frac{b}{c}}}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\text{LCM} = 14$$

$$a^m a^n = a^{m+n}$$

$$\text{LCM} = 15$$

$$a^{-\frac{b}{c}} = \frac{1}{a^{\frac{b}{c}}}$$

Rationalize.

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1.2 $(u^{\frac{1}{3}} + (uv)^{\frac{1}{6}} + v^{\frac{1}{3}})(u^{\frac{1}{6}} - v^{\frac{1}{6}})$

$$\begin{aligned} &\Rightarrow (u^{\frac{1}{3}} + u^{\frac{1}{6}}v^{\frac{1}{6}} + v^{\frac{1}{3}})(u^{\frac{1}{6}} - v^{\frac{1}{6}}) \\ &\Rightarrow u^{\frac{1}{2}} - v^{\frac{1}{2}} \\ &\Rightarrow \sqrt{u} - \sqrt{v} \end{aligned}$$

Distribute exponent.

Use difference of two cubes.

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1.3 $\sqrt[3]{-8^4}$

$$\begin{aligned} &\Rightarrow -\sqrt[3]{8^4} \\ &\Rightarrow -\sqrt[3]{(2^3)^4} \\ &\Rightarrow -\sqrt[3]{(2^4)^3} \\ &\Rightarrow -2^4 \\ &\Rightarrow -16 \end{aligned}$$

$$\sqrt[m]{-a} = -\sqrt[m]{a} \text{ for odd } m$$

$$(a^m)^n = (a^n)^m$$

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1.4 $\sqrt[4]{9x^8}$

$$\begin{aligned} &\Rightarrow \sqrt[4]{9}\sqrt[4]{x^8} & \sqrt[n]{ab} &= \sqrt[n]{a}\sqrt[n]{b} \\ &\Rightarrow \sqrt[4]{3^2}\sqrt[4]{x^8} \\ &\Rightarrow x^2\sqrt{3} \end{aligned}$$

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1.5 $\sqrt{\sqrt[3]{9a^4b^4}}$

$$\begin{aligned} &\Rightarrow \sqrt[6]{9a^4b^4} & \sqrt[n]{\sqrt[n]{a}} &= \sqrt[n+n]{a} \\ &\Rightarrow \sqrt[6]{3^2a^4b^4} \\ &\Rightarrow \sqrt[3]{3a^2b^2} \end{aligned}$$

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1.6 $\frac{2\sqrt{5}}{\sqrt{8}} + \frac{9}{\sqrt[3]{16}}$

$$\begin{aligned} &\Rightarrow \frac{2\sqrt{5}}{\sqrt{8}} \cdot \frac{\sqrt{2}}{\sqrt{2}} + \frac{9}{\sqrt[3]{16}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} & \text{Rationalize.} \\ &\Rightarrow \frac{2\sqrt{5}\sqrt{2}}{\sqrt{16}} + \frac{9\sqrt[3]{4}}{\sqrt[3]{64}} \\ &\Rightarrow \frac{2\sqrt{5}\sqrt{2}}{4} + \frac{9\sqrt[3]{4}}{4} \\ &\Rightarrow \frac{2\sqrt{5}\sqrt{2}+9\sqrt[3]{4}}{4} \\ &\Rightarrow \frac{2\sqrt{10}+9\sqrt[3]{4}}{4} \end{aligned}$$

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1.7 $\frac{x^2-2x+1}{\sqrt{x+1}}$

$$\begin{aligned} &\Rightarrow \frac{x^2-2x+1}{\sqrt{x+1}} \cdot \frac{\sqrt{x}-1}{\sqrt{x}-1} & \text{Rationalize using difference of two squares.} \\ &\Rightarrow \frac{(x^2-2x+1)(\sqrt{x}-1)}{x-1} \\ &\Rightarrow \frac{(x-1)^2(\sqrt{x}-1)}{x-1} & \text{Factor by grouping.} \\ &\Rightarrow (x-1)(\sqrt{x}-1) \end{aligned}$$

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1.8 $\frac{1}{\sqrt[3]{4}-\sqrt[3]{-27}}$

$$\Rightarrow \frac{1}{\sqrt[3]{4}+\sqrt[3]{27}}$$

$$\Rightarrow \frac{1}{\sqrt[3]{4}+\sqrt[3]{27}} \cdot \frac{\sqrt[3]{4^2}-\sqrt[3]{4}\sqrt[3]{27}+\sqrt[3]{27^2}}{\sqrt[3]{4^2}-\sqrt[3]{4}\sqrt[3]{27}+\sqrt[3]{27^2}}$$

$$\Rightarrow \frac{\sqrt[3]{4^2}-\sqrt[3]{4}\sqrt[3]{27}+\sqrt[3]{27^2}}{4+27}$$

$$\Rightarrow \frac{\sqrt[3]{4^2}-3\sqrt[3]{4}+\sqrt[3]{27^2}}{4+27}$$

$$\Rightarrow \frac{\sqrt[3]{4^2}-3\sqrt[3]{4}+\sqrt[3]{27^2}}{31}$$

$$\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+\sqrt[3]{27^2}}{31}$$

$$\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+\sqrt[3]{(3^3)^2}}{31}$$

$$\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+\sqrt[3]{(3^2)^3}}{31}$$

$$\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+3^2}{31}$$

$$\Rightarrow \frac{\sqrt[3]{16}-3\sqrt[3]{4}+9}{31}$$

$$\Rightarrow \frac{\sqrt[3]{8}\sqrt[3]{2}-3\sqrt[3]{4}+9}{31}$$

$$\Rightarrow \frac{2\sqrt[3]{2}-3\sqrt[3]{4}+9}{31}$$

$$\sqrt[m]{-a} = -\sqrt[m]{a} \text{ for odd } m$$

Rationalize using difference of two cubes.

$$(a^m)^n = (a^n)^m$$

$$\sqrt[m]{ab} = \sqrt[m]{a}\sqrt[m]{b}$$

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2 Perform the following operations and simplify.

2.1 $3i(i^2 - i^3 + 5i^5 - i^{-2})$

$$\begin{aligned} &\Rightarrow 3i(-1 - i^3 + 5i^5 - i^{-2}) & i^2 &= -1 \\ &\Rightarrow 3i(-1 + i + 5i^5 - i^{-2}) & i^3 &= -i \\ &\Rightarrow 3i(-1 + i + 5i - i^{-2}) & i^5 &= i \\ &\Rightarrow 3i(-1 + i + 5i + 1) & i^{-2} &= -1 \\ &\Rightarrow 3i(6i) \\ &\Rightarrow 18i^2 \\ &\Rightarrow -18 & i^2 &= -1 \end{aligned}$$

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2.2 $(3 - 5i)(7 + 4i)$

$$\begin{aligned} &\Rightarrow 21 + 12i - 35i - 20i^2 & \text{Expand.} \\ &\Rightarrow 21 + 12i - 35i + 20 & i^2 &= -1 \\ &\Rightarrow 41 - 23i \end{aligned}$$

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2.3 $\frac{3i-2}{3i+2}$

$$\begin{aligned} &\Rightarrow \frac{-2+3i}{2+3i} & \text{Rewrite in standard form.} \\ &\Rightarrow \frac{-2+3i}{2+3i} \cdot \frac{2-3i}{2-3i} & \text{Multiply by conjugate to eliminate the complex denominator.} \\ &\Rightarrow \frac{(-2+3i)(2-3i)}{(2+3i)(2-3i)} \\ &\Rightarrow \frac{-4+6i+6i-9i^2}{4-9i^2} \\ &\Rightarrow \frac{-4+12i-9i^2}{4-9i^2} \\ &\Rightarrow \frac{-4+12i+9}{4+9} & i^2 &= -1 \\ &\Rightarrow \frac{5+12i}{13} \\ &\Rightarrow \frac{5}{13} + \frac{12}{13}i \end{aligned}$$

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2.4 $\frac{7+i-4(3-i)}{6-5i^3}$

$$\begin{aligned} \Rightarrow & \frac{7+i-4(3-i)}{6+5i} & i^3 = -i \\ \Rightarrow & \frac{7+i-12+4i}{6+5i} \\ \Rightarrow & \frac{-5+5i}{6+5i} \\ \Rightarrow & \frac{-5+5i}{6+5i} \cdot \frac{6-5i}{6-5i} & \text{Multiply by conjugate to eliminate the complex denominator.} \\ \Rightarrow & \frac{(-5+5i)(6-5i)}{(6+5i)(6-5i)} \\ \Rightarrow & \frac{-30+25i+30i-25i^2}{36-25i^2} \\ \Rightarrow & \frac{-30+25i+30i+25}{36+25} & i^2 = -1 \\ \Rightarrow & \frac{-5+55i}{61} \\ \Rightarrow & -\frac{5}{61} + \frac{55}{61}i \end{aligned}$$

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2.5 $\frac{2-2(\overline{i+1})}{2-\sqrt{-4}}$

$$\begin{aligned} \Rightarrow & \frac{2-2(\overline{i-1})}{2-\sqrt{-4}} & \text{Line above a complex number denotes its conjugate.} \\ \Rightarrow & \frac{2-2i+2}{2-\sqrt{-4}} \\ \Rightarrow & \frac{4-2i}{2-\sqrt{-4}} \\ \Rightarrow & \frac{4-2i}{2-2i} & \sqrt{-a} = i\sqrt{a} \\ \Rightarrow & \frac{4-2i}{2-2i} \cdot \frac{2+2i}{2+2i} & \text{Multiply by conjugate to eliminate the complex denominator.} \\ \Rightarrow & \frac{(4-2i)(2+2i)}{(2-2i)(2+2i)} \\ \Rightarrow & \frac{8+8i-4i-4i^2}{4-4i^2} \\ \Rightarrow & \frac{8+4i-4i^2}{4-4i^2} \\ \Rightarrow & \frac{8+4i+4}{4+4} & i^2 = -1 \\ \Rightarrow & \frac{12+4i}{8} \\ \Rightarrow & \frac{3+i}{2} \\ \Rightarrow & \frac{3}{2} + \frac{1}{2}i \end{aligned}$$

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