Exercise Solutions for Math 20

Equations in Quadratic Form and with Radicals and Absolute Values

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1 Find the solution set of the following inequalities.

1.1 $\frac{2x+1}{4} \leq \frac{2x}{3} + \frac{1}{6}$

$\Rightarrow \frac{3(2x+1)}{12} \leq \frac{4(2x)}{12} + \frac{2}{12}$ LCM = 12

$\Rightarrow \frac{6x+3}{12} \leq \frac{8x}{12} + \frac{2}{12}$

$\Rightarrow \frac{6x+3}{12} \leq \frac{8x+2}{12}$

$\Rightarrow 6x + 3 \leq 8x + 2$

$\Rightarrow 3 - 2 \leq 8x - 6x$

$\Rightarrow 1 \leq 2x$

$\Rightarrow x \geq \frac{1}{2}$

$\Rightarrow x \in [\frac{1}{2}, +\infty)$ Final answer.

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1.2 $-2 < 5 + 3x < 20$

$\Rightarrow -7 < 3x < 15$ Solve for x.

$\Rightarrow -\frac{7}{3} < x < 5$

$\Rightarrow x \in (-\frac{7}{3}, 5)$ Final answer.

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1.3 $\frac{x}{x-1} > -1$

$\Rightarrow \frac{x}{x-1} + 1 > 0$ Solve for x.

$\Rightarrow \frac{x}{x-1} + \frac{x-1}{x-1} > 0$

$\Rightarrow \frac{x+x-1}{x-1} > 0$

$\Rightarrow \frac{2x-1}{x-1} > 0$ $x = 1$ is an undefined point.

Create a table of signs.

		$\frac{1}{2}$	1	
$2x - 1$	$-$	$+$	$+$	
$x - 1$	$-$	$-$	$+$	
$\frac{2x-1}{x-1}$	$+$	$-$	$+$	

$\Rightarrow x \in (-\infty, \frac{1}{2}) \cup (1, +\infty)$ Final answer.

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1.4 $\frac{x}{x+1} \geq \frac{2}{x+3}$

$\Rightarrow \frac{x}{x+1} - \frac{2}{x+3} \geq 0$ Solve for x.

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$\Rightarrow \frac{x(x+3)}{(x+1)(x+3)} - \frac{2(x+1)}{(x+1)(x+3)} \geq 0$ LCM $= (x+1)(x+3)$

$\Rightarrow \frac{x(x+3)-2(x+1)}{(x+1)(x+3)} \geq 0$

$\Rightarrow \frac{x^2+3x-2x-2}{(x+1)(x+3)} \geq 0$

$\Rightarrow \frac{x^2+x-2}{(x+1)(x+3)} \geq 0$

$\Rightarrow \frac{(x-1)(x+2)}{(x+1)(x+3)} \geq 0$ Factor by grouping. $x \in \{-3, -1\}$ are undefined points.

Create a table of signs.

	-3		-2	-1	1
$x-1$	$-$	$-$	$-$	$-$	$+$
$x+2$	$-$	$-$	$+$	$+$	$+$
$x+1$	$-$	$-$	$-$	$+$	$+$
$x+3$	$-$	$+$	$+$	$+$	$+$
$\frac{(x-1)(x+2)}{(x+1)(x+3)}$	$+$	$-$	$+$	$-$	$+$

$\Rightarrow (-\infty, -3) \cup [-2, -1) \cup [1, +\infty)$ Final answer. Don't include undefined points.

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