

# Exercise Solutions for Math 20

Equations in Quadratic Form and with Radicals and Absolute Values

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# 1 Solve for $x$

1.1  $\sqrt{2x+3} - \sqrt{x-2} = \sqrt{x+1}$

$$\Rightarrow (\sqrt{2x+3} - \sqrt{x-2})^2 = x+1 \quad \text{Square both sides.}$$

$$\Rightarrow 2x+3 - 2\sqrt{2x+3}\sqrt{x-2} + x-2 = x+1$$

$$\Rightarrow 2x+3 + x-2 - x-1 = 2\sqrt{2x+3}\sqrt{x-2}$$

$$\Rightarrow 2x = 2\sqrt{2x+3}\sqrt{x-2}$$

$$\Rightarrow x = \sqrt{2x+3}\sqrt{x-2}$$

$$\Rightarrow x^2 = (2x+3)(x-2) \quad \text{Square both sides.}$$

$$\Rightarrow x^2 = 2x^2 - 4x + 3x - 6$$

$$\Rightarrow x^2 = 2x^2 - x - 6$$

$$\Rightarrow 2x^2 - x^2 - x - 6 = 0$$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x-3)(x+2) = 0 \quad \text{Factor by grouping.}$$

$$\Rightarrow x \subseteq \{-2, 3\}$$

$$\Rightarrow \sqrt{2(-2)+3} - \sqrt{-2-2} = \sqrt{-2+1} \quad \text{Verify } x = -2$$

$$\Rightarrow \sqrt{-4+3} - \sqrt{-2-2} = \sqrt{-2+1}$$

$$\Rightarrow \sqrt{-1} - \sqrt{-4} = \sqrt{-1}$$

$$\Rightarrow i - 2i = i$$

$$\Rightarrow -i = i$$

$$\Rightarrow x \neq -2$$

$$\Rightarrow \sqrt{2(3)+3} - \sqrt{3-2} = \sqrt{3+1} \quad \text{Verify } x = 3$$

$$\Rightarrow \sqrt{6+3} - \sqrt{3-2} = \sqrt{3+1}$$

$$\Rightarrow \sqrt{9} - \sqrt{1} = \sqrt{4}$$

$$\Rightarrow 3 - 1 = 2$$

$$\Rightarrow 2 = 2$$

$$\Rightarrow x = 3$$

■

**1.2**  $1 = x + \sqrt{2x - 3}$

$$\Rightarrow 1 - x = \sqrt{2x - 3} \quad \text{Isolate the root.}$$

$$\Rightarrow (1 - x)^2 = 2x - 3 \quad \text{Square both sides.}$$

$$\Rightarrow 1 - 2x + x^2 = 2x - 3$$

$$\Rightarrow 1 - 2x + x^2 - 2x + 3 = 0$$

$$\Rightarrow x^2 - 4x + 4 = 0$$

$$\Rightarrow (x - 2)^2 \quad \text{Factor by grouping.}$$

$$\Rightarrow x = 2$$

$$\Rightarrow 1 = 2 + \sqrt{2(2) - 3} \quad \text{Verify } x = 2$$

$$\Rightarrow 1 = 2 + \sqrt{4 - 3}$$

$$\Rightarrow 1 = 2 + \sqrt{1}$$

$$\Rightarrow 1 = 2 + 1$$

$$\Rightarrow 1 = 3$$

$$\Rightarrow x \neq 2$$

$$\Rightarrow x \in \emptyset \quad \text{Final answer.}$$



**1.3**     $\left| \frac{3x-4}{2x+3} \right| = 1$

$\Rightarrow \frac{3x-4}{2x+3} = -1$	$ a  = b \Rightarrow a = \pm b$ . Solve for $a = -b$
$\Rightarrow \frac{3x-4}{2x+3} = -\frac{2x+3}{2x+3}$	
$\Rightarrow 3x - 4 = -(2x + 3)$	Eliminate denominator. $x = -\frac{3}{2}$ is an undefined point.
$\Rightarrow 3x - 4 = -2x - 3$	
$\Rightarrow 3x + 2x = -3 + 4$	
$\Rightarrow 5x = 1$	
$\Rightarrow x = \frac{1}{5}$	

$\Rightarrow \frac{3x-4}{2x+3} = 1$	$ a  = b \Rightarrow a = \pm b$ . Solve for $a = +b$
$\Rightarrow \frac{3x-4}{2x+3} = \frac{2x+3}{2x+3}$	
$\Rightarrow 3x - 4 = 2x + 3$	Eliminate denominator. $x = -\frac{3}{2}$ is an undefined point.
$\Rightarrow 3x - 2x = 3 + 4$	
$\Rightarrow x = 7$	

$\Rightarrow x \in \{\frac{1}{5}, 7\}$	Final answer.
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**1.4**  $-7\left(\frac{1}{x} - 1\right) = 4 - 2\left(\frac{1}{x} - 1\right)^2$

$\Rightarrow -7t = 4 - 2t^2$	$t = \left(\frac{1}{x} - 1\right)$ . $x = 0$ is an undefined point.
$\Rightarrow 2t^2 - 7t - 4 = 0$	
$\Rightarrow 2t^2 - 8t + t - 4 = 0$	Factor by grouping.
$\Rightarrow 2t(t - 4) + 1(t - 4) = 0$	
$\Rightarrow (2t + 1)(t - 4) = 0$	
$\Rightarrow (2t + 1)(t - 4) = 0$	
$\Rightarrow t \in \left\{-\frac{1}{2}, 4\right\}$	
$\Rightarrow \frac{1}{x} - 1 = -\frac{1}{2}$	Solve for $x$ using $t = -\frac{1}{2}$ .
$\Rightarrow \frac{1}{x} = -\frac{1}{2} + 1$	
$\Rightarrow \frac{1}{x} = \frac{1}{2}$	
$\Rightarrow x = 2$	
$\Rightarrow \frac{1}{x} - 1 = 4$	Solve for $x$ using $t = 4$ .
$\Rightarrow \frac{1}{x} = 4 + 1$	
$\Rightarrow \frac{1}{x} = 5$	
$\Rightarrow x = \frac{1}{5}$	
$\Rightarrow x \in \left\{\frac{1}{5}, 2\right\}$	Final answer. <span style="float: right;">■</span>

**1.5**  $x^2(x^2 - 1) - 9(x^2 - 1) = 0$

$\Rightarrow (x^2 - 9)(x^2 - 1) = 0$	Factor by grouping.
$\Rightarrow x^2 - 9 = 0$	Solve for x.
$\Rightarrow (x - 3)(x + 3) = 0$	Factor using difference of two squares.
$\Rightarrow x \in \{-3, 3\}$	
$\Rightarrow x^2 - 1 = 0$	Solve for x.
$\Rightarrow (x - 1)(x + 1) = 0$	Factor using difference of two squares.
$\Rightarrow x \in \{-1, 1\}$	
$\Rightarrow x \in \{-3, -1, 1, 3\}$	Final answer. <span style="float: right;">■</span>