

Exercise Solutions for Math 20

Conics (Parabola and Ellipse)

Nile Jocson <novoseiversia@gmail.com>

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1.1 Determine the vertex and orientation of the following parabolas.

1.1.a $4y^2 + 4y + x = 2$

$\Rightarrow 4y^2 + 4y = -x + 2$ $\Rightarrow y^2 + y = -\frac{x}{4} + \frac{2}{4}$ $\Rightarrow y^2 + y = -\frac{x}{4} + \frac{1}{2}$	Isolate y .
$\Rightarrow y^2 + y + \frac{1}{4} = -\frac{x}{4} + \frac{1}{2} + \frac{1}{4}$ $\Rightarrow (y + \frac{1}{2})^2 = -\frac{x}{4} + \frac{3}{4}$ $\Rightarrow (y + \frac{1}{2})^2 = -\frac{1}{4}(x - 3)$ $\Rightarrow (y + \frac{1}{2})^2 = 4(-\frac{1}{16})(x - 3)$	Complete the square.
\Rightarrow Opening leftwards, $(h, k) = (3, -\frac{1}{2})$	Final answer. ■

1.1.b $x^2 - 6x - 2y = 7$

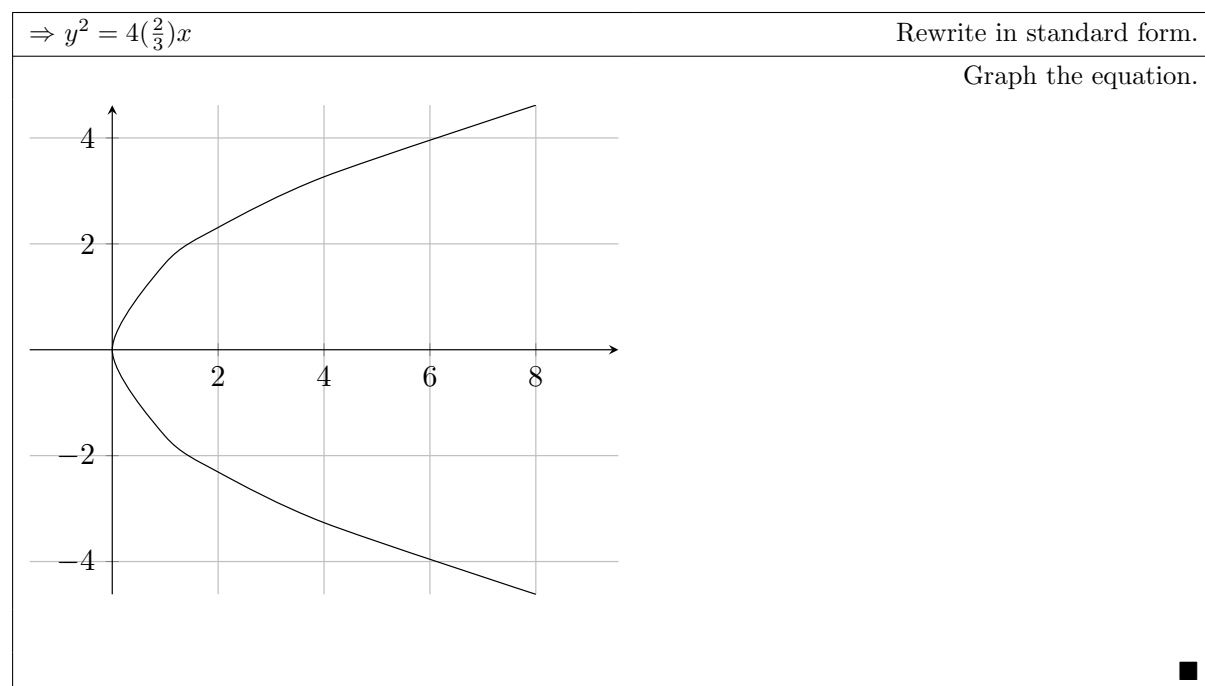
$\Rightarrow x^2 - 6x = 2y + 7$	Isolate x .
$\Rightarrow x^2 - 6x + 9 = 2y + 7 + 9$ $\Rightarrow (x - 3)^2 = 2y + 16$ $\Rightarrow (x - 3)^2 = 2(y + 8)$ $\Rightarrow (x - 3)^2 = 4(\frac{1}{2})(y + 8)$	Complete the square.
\Rightarrow Opening upwards, $(h, k) = (3, -8)$	Final answer. ■

1.1.c $2y^2 - 6y - 9x = 0$

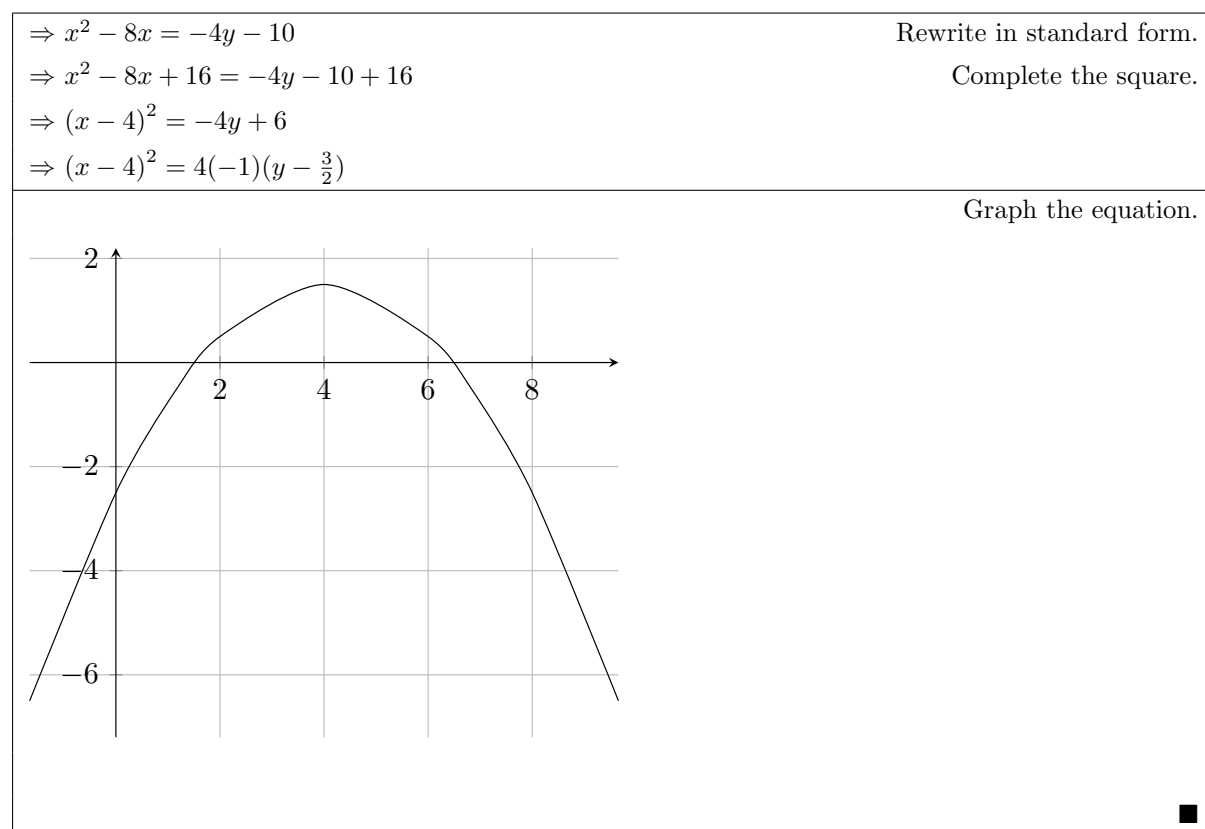
$\Rightarrow 2y^2 - 6y = 9x$ $\Rightarrow y^2 - 3y = \frac{9}{2}x$	Isolate y .
$\Rightarrow y^2 - 3y + \frac{9}{4} = \frac{9}{2}x + \frac{9}{4}$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}x + \frac{9}{4}$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{9}{4} \cdot \frac{2}{9})$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{18}{36})$ $\Rightarrow (y - \frac{3}{2})^2 = \frac{9}{2}(x + \frac{1}{2})$ $\Rightarrow (y - \frac{3}{2})^2 = 4(\frac{9}{8})(x + \frac{1}{2})$	Complete the square.
\Rightarrow Opening rightwards, $(h, k) = (-\frac{1}{2}, \frac{3}{2})$	Final answer. ■

1.2 Sketch the graph of the following parabolas.

1.2.a $3y^2 = 8x$



1.2.b $x^2 - 8x + 4y = -10$



1.3 Sketch the graph of $y = -x^2 + 6x - 8$. Label the vertex, x- and y-intercept(s).

$$\Rightarrow -x^2 + 6x = y + 8$$

Rewrite in standard form.

$$\Rightarrow x^2 - 6x = -y - 8$$

$$\Rightarrow x^2 - 6x + 9 = -y - 8 + 9$$

Complete the square.

$$\Rightarrow (x - 3)^2 = -y + 1$$

$$\Rightarrow (x - 3)^2 = 4(-\frac{1}{4})(y - 1)$$

$$\Rightarrow (x - 3)^2 = 4(-\frac{1}{4})(-1)$$

Find the x-intercepts.

$$\Rightarrow (x - 3)^2 = 4(\frac{1}{4})$$

$$\Rightarrow (x - 3)^2 = 1$$

$$\Rightarrow x = \pm 1 + 3$$

$$\Rightarrow x = 1 + 3, x = -1 + 3$$

$$\Rightarrow x_i \in \{2, 4\}$$

$$\Rightarrow (0 - 3)^2 = 4(-\frac{1}{4})(y - 1)$$

Find the y-intercepts.

$$\Rightarrow (-3)^2 = 4(-\frac{1}{4})(y - 1)$$

$$\Rightarrow 9 = -(y - 1)$$

$$\Rightarrow 9 = -y + 1$$

$$\Rightarrow y = 1 - 9$$

$$\Rightarrow y_i = -8$$

Graph the equation.

