Exercise Solutions for Math 20

Functions, Graphs, Symmetry

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1.1 Transform each of the following real-valued functions to a functional notation $f(x)$; find its domain and range; and compute $f(1)$.

1.1.a f assigns the number 10 to any one-digit integer.

$\Rightarrow f(x) = 10$	Final answer.
$\Rightarrow \operatorname{dom}(f) = [-9, 9]$	The domain is the set of one digit integers.
$\Rightarrow \operatorname{ran}(f) = 10$	The function can only equate to 10.
$\Rightarrow f(1) = 10$	
	$\blacksquare$

1.1.b f maps a given real number to the real number that is 4 more than its square root.

$\Rightarrow f(x) = \sqrt{x} + 4$	Final answer. Assuming that only the principal root is needed.
$\Rightarrow \operatorname{dom}(f) = [0, +\infty)$	The square root of a negative number cannot be real.
$\Rightarrow \operatorname{ran}(f) = [4, +\infty]$	
$\Rightarrow f(1) = 5$	
	$\blacksquare$

1.1.c $\{(x, y) \mid y = \frac{1}{6-x}\}$

$\Rightarrow f(x) = \frac{1}{6-x}$	Final answer.
$\Rightarrow \operatorname{dom}(f) = \mathbb{R} \setminus \{6\}$	$x = 6$ is an undefined point.
$\Rightarrow \operatorname{ran}(f) = \mathbb{R}$	
$\Rightarrow f(1) = \frac{1}{5}$	
	$\blacksquare$

1.1.d $\{(x, y) \mid y = \sqrt{x^2 - 5x + 6}\}$

$\Rightarrow (x-2)(x-3) \geq 0$	Find the domain by solving $x^2 - 5x + 6 \geq 0$; factor by grouping.
	Create a table of signs.

		2	3
$x - 2$	$-$	$+$	$+$
$x - 3$	$-$	$-$	$+$
$(x-2)(x-3)$	$+$	$-$	$+$

$\Rightarrow x \in (-\infty, 2] \cup [3, +\infty)$

$\Rightarrow f(x) = \sqrt{x^2 - 5x + 6}$	Final answer.

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$\Rightarrow \text{dom}(f) = (-\infty, 2] \cup [3, +\infty)$

$\Rightarrow \text{ran}(f) = [0, +\infty)$

$\Rightarrow f(1) = \sqrt{2}$

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1.1.e $\{(x, y) \mid y = \frac{1}{\sqrt[3]{x^2-1}}\}$

$\Rightarrow (x-1)(x+1) = 0$

Find undefined points by solving $x^2 - 1 = 0$; factor using difference of two squares.

$\Rightarrow x \in \{-1, 1\}$

$\Rightarrow x = \frac{1}{\sqrt[3]{y^2-1}}$

Find the inverse function.

$\Rightarrow \frac{1}{x} = \sqrt[3]{y^2 - 1}$

$\Rightarrow \frac{1}{x^3} = y^2 - 1$

$\Rightarrow y^2 = \frac{1}{x^3} + 1$

$\Rightarrow y^2 = \frac{1}{x^3} + \frac{x^3}{x^3}$

$\Rightarrow y^2 = \frac{1+x^3}{x^3}$

$\Rightarrow f'(x) = \pm\sqrt{\frac{1+x^3}{x^3}}$

$\Rightarrow \sqrt{\frac{1+x^3}{x^3}} > 0$

Solve for the domain. Note that $x = 0$ is an undefined point.

Create a table of signs.

	-1	0	
$1 + x^3$	$-$	$+$	$+$
x^3	$-$	$-$	$+$
$\sqrt{\frac{1+x^3}{x^3}}$	$+$	$-$	$+$

$\Rightarrow \text{dom}(f') = (-\infty, -1] \cup (0, +\infty)$

$\Rightarrow f(x) = \frac{1}{\sqrt[3]{x^2-1}}$

Final answer.

$\Rightarrow \text{dom}(f) = \mathbb{R} \setminus \{-1, 1\}$

$\Rightarrow \text{ran}(f) = (-\infty, -1] \cup (0, +\infty)$

$\Rightarrow f(1)$ is undefined.

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1.2 Given $f(x) = \frac{4-x^2}{x-2}$

1.2.a Find dom(f) and solve for its zeroes.

$\Rightarrow \frac{(2-x)(2+x)}{x-2} = 0$

Factor using difference of two squares.

$\Rightarrow \frac{-(x-2)(2+x)}{x-2} = 0$

$\Rightarrow -(2 + x) = 0$

$x = 2$ is an undefined point.

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⇒ $-2 - x = 0$	
⇒ $x = -2$	
⇒ $\mathrm{dom}(f) = \mathbb{R} \setminus \{2\}$	Final answer.
⇒ $x = -2$	
	■

1.2.b Is the function odd or even? Justify your answer algebraically.

⇒ $f(x) = -2 - x, f(-x) = -2 + x$	Check if the function is even. Note the simplified function, $f(x) = -2 - x$.
⇒ $f(-x) \neq f(x)$	
⇒ Not even.	
⇒ $-f(x) = -(-2 - x)$	Find $-f(x)$.
⇒ $-f(x) = 2 + x$	
⇒ $f(-x) \neq -f(x)$	
⇒ Not odd.	
⇒ Neither even nor odd.	Final answer.
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1.2.c Determine its x- and y- intercepts.

⇒ $x_i = -2$	The zeroes of a function are its x-intercepts.
⇒ $y = -2 - 0$	Find the y-intercepts. Use the simplified function.
⇒ $y_i = -2$	
⇒ $x_i = -2, y_i = -2$	Final answer.
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