Exercise Solutions for Math 20

Some Types of Functions, Operations

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1.1

Given

$$g(x) = \begin{cases} x+4, & \text{if } x < -2 \\ |x|, & \text{if } -1 < x < 1 \\ 2, & \text{if } x > 3 \end{cases}$$

Sketch its graph and label its x- and y- intercepts.

$\Rightarrow 0 = x + 4$	Find the x-intercepts of $y = x + 4$
$\Rightarrow x_i = -4$	
$\Rightarrow 0 = \|x\|$	Find the x-intercepts of $y = \|x\|$
$\Rightarrow x_i = 0$	
$\Rightarrow y = \|0\|$	Find the y-intercepts of $y = \|x\|$
$\Rightarrow y_i = 0$	
$\Rightarrow$ See Figure 1.	Final answer. Graph the system. ■

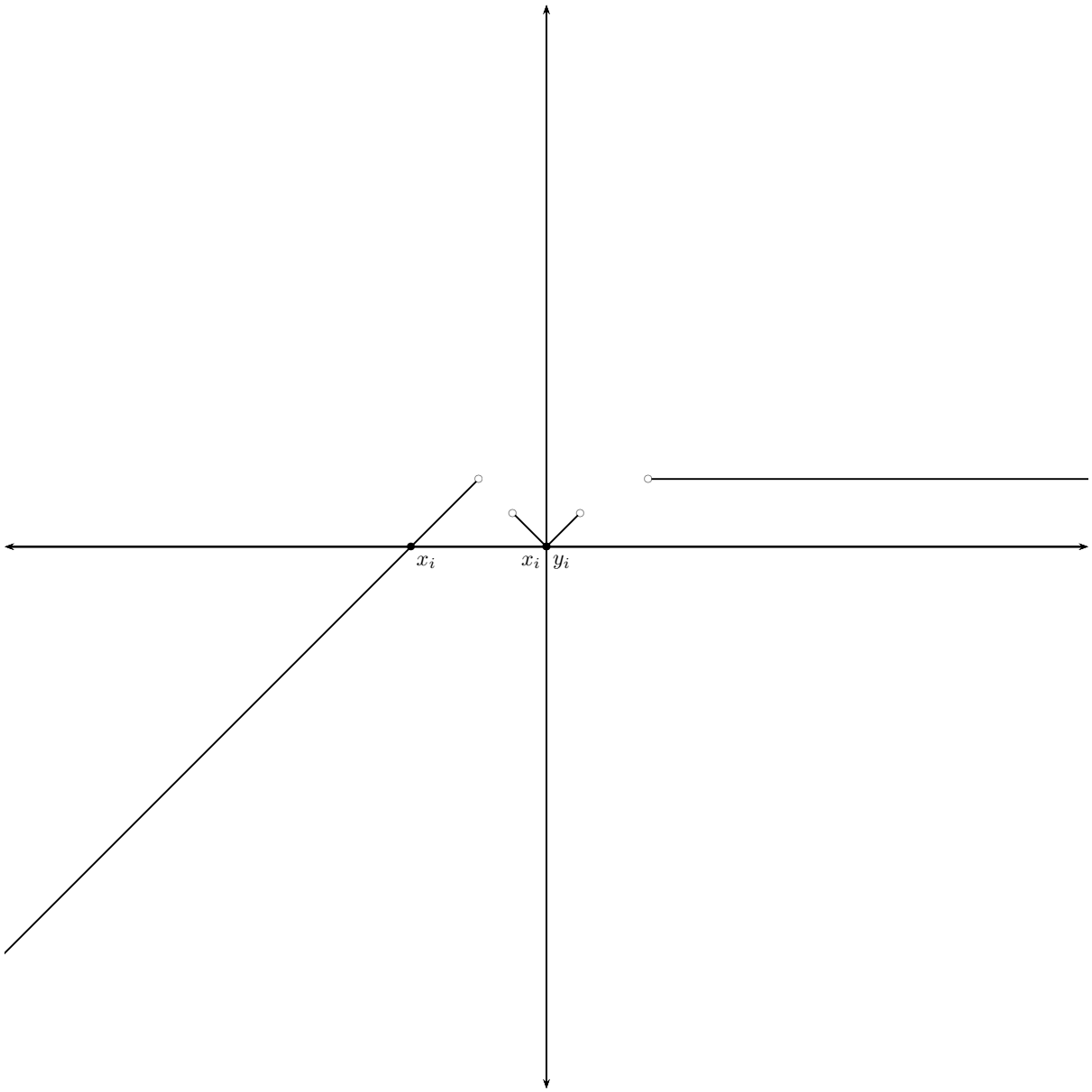

Figure 1.

1.2

Let f, g, and h be the functions defined from the expressions below.

$$f(x) = \frac{x+2}{x-3}$$

$$g(x) = \frac{x+3}{x+2}$$

$$h(x) = \sqrt{3x-1}$$

Determine $(f-g)(x)$, $(\frac{f}{g})(x)$, $(fg)(x)$, $(f\circ g)(x)$, $(h\circ f)(x)$, $(g\circ h)(x)$ and obtain their respective domains.

$\Rightarrow (f-g)(x) = \frac{x+2}{x-3} - \frac{x+3}{x+2}$ $\Rightarrow (f-g)(x) = \frac{(x+2)(x+2)}{(x-3)(x+2)} - \frac{(x-3)(x+3)}{(x-3)(x+2)}$ $\Rightarrow (f-g)(x) = \frac{(x+2)(x+2)-(x-3)(x+3)}{(x-3)(x+2)}$ $\Rightarrow (f-g)(x) = \frac{x^2+4x+4-(x^2-9)}{(x-3)(x+2)}$ $\Rightarrow (f-g)(x) = \frac{x^2+4x+4-x^2+9}{(x-3)(x+2)}$ $\Rightarrow (f-g)(x) = \frac{4x+13}{(x-3)(x+2)}$ $\Rightarrow \text{dom}(f-g) = \mathbb{R} \setminus \{-2, 3\}$	Find $(f-g)(x)$.
$\Rightarrow (\frac{f}{g})(x) = \frac{\frac{x+2}{x-3}}{\frac{x+3}{x+2}}$ $\Rightarrow (\frac{f}{g})(x) = \frac{x+2}{x-3} \cdot \frac{x+2}{x+3}$ $\Rightarrow (\frac{f}{g})(x) = \frac{(x+2)^2}{(x-3)(x+3)}$ $\Rightarrow \text{dom}(\frac{f}{g}) = \mathbb{R} \setminus \{-3, -2, 3\}$	Find $(\frac{f}{g})(x)$.
$\Rightarrow (fg)(x) = \frac{x+2}{x-3} \cdot \frac{x+3}{x+2}$ $\Rightarrow (fg)(x) = \frac{(x+2)(x+3)}{(x-3)(x+2)}$ $\Rightarrow (fg)(x) = \frac{x+3}{x-3}$ $\Rightarrow \text{dom}(fg) = \mathbb{R} \setminus \{-2, 3\}$	Find $(fg)(x)$.
$\Rightarrow (f\circ g)(x) = \frac{\frac{x+3}{x+2}+2}{\frac{x+3}{x+2}-3}$ $\Rightarrow (f\circ g)(x) = \frac{\frac{x+3}{x+2}+\frac{2x+4}{x+2}}{\frac{x+3}{x+2}-\frac{3x+6}{x+2}}$ $\Rightarrow (f\circ g)(x) = \frac{\frac{x+3+(2x+4)}{x+2}}{\frac{x+3-(3x+6)}{x+2}}$ $\Rightarrow (f\circ g)(x) = \frac{\frac{x+3+2x+4}{x+2}}{\frac{x+3-3x-6}{x+2}}$ $\Rightarrow (f\circ g)(x) = \frac{\frac{3x+7}{x+2}}{\frac{-2x-3}{x+2}}$ $\Rightarrow (f\circ g)(x) = \frac{3x+7}{x+2} \cdot \frac{x+2}{-2x-3}$ $\Rightarrow (f\circ g)(x) = \frac{3x+7}{-2x-3}$ $\Rightarrow \text{dom}(f\circ g) = \mathbb{R} \setminus \{-2, -\frac{3}{2}\}$	Find $(f\circ g)(x)$.
$\Rightarrow (h\circ f)(x) = \sqrt{3(\frac{x+2}{x-3}) - 1}$ $\Rightarrow (h\circ f)(x) = \sqrt{\frac{3x+6}{x-3} - 1}$ $\Rightarrow (h\circ f)(x) = \sqrt{\frac{3x+6}{x-3} - \frac{x-3}{x-3}}$	Find $(h\circ f)(x)$.

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$\Rightarrow (h \circ f)(x) = \sqrt{\frac{3x+6-(x-3)}{x-3}}$

$\Rightarrow (h \circ f)(x) = \sqrt{\frac{3x+6-x+3}{x-3}}$

$\Rightarrow (h \circ f)(x) = \sqrt{\frac{2x+9}{x-3}}$

$\Rightarrow \frac{2x+9}{x-3} \geq 0$ Solve for the domain. Note that $x = 3$ is undefined.

Create a table of signs.

	$-\frac{9}{2}$		3
$2x+9$	$-$	$+$	$+$
$x-3$	$-$	$-$	$+$
$\frac{2x+9}{x-3}$	$+$	$-$	$+$

$\Rightarrow \text{dom}(h \circ f) = (-\infty, -\frac{9}{2}] \cup (3, +\infty)$

$\Rightarrow (g \circ h)(x) = \frac{\sqrt{3x-1}+3}{\sqrt{3x-1}+2}$ Find $(g \circ h)(x)$.

$\Rightarrow (g \circ h)(x) = \frac{\sqrt{3x-1}+3}{\sqrt{3x-1}+2} \cdot \frac{\sqrt{3x-1}-2}{\sqrt{3x-1}-2}$ Rationalize.

$\Rightarrow (g \circ h)(x) = \frac{(\sqrt{3x-1}+3)(\sqrt{3x-1}-2)}{3x-1-4}$

$\Rightarrow (g \circ h)(x) = \frac{(\sqrt{3x-1}+3)(\sqrt{3x-1}-2)}{3x-5}$

$\Rightarrow 3x - 1 \geq 0$ Solve for the domain. Note that $x = \frac{5}{3}$ is undefined.

$\Rightarrow 3x \geq 1$

$\Rightarrow x \geq \frac{1}{3}$

$\Rightarrow \text{dom}(g \circ h) = [\frac{1}{3}, +\infty) \setminus \{\frac{5}{3}\}$

$\Rightarrow (f - g)(x) = \frac{4x+13}{(x-3)(x+2)}$ Final answer.

$\Rightarrow \text{dom}(f - g) = \mathbb{R} \setminus \{-2, 3\}$

$\Rightarrow (\frac{f}{g})(x) = \frac{(x+2)^2}{(x-3)(x+3)}$

$\Rightarrow \text{dom}(\frac{f}{g}) = \mathbb{R} \setminus \{-3, -2, 3\}$

$\Rightarrow (fg)(x) = \frac{x+3}{x-3}$

$\Rightarrow \text{dom}(fg) = \mathbb{R} \setminus \{-2, 3\}$

$\Rightarrow (f \circ g)(x) = \frac{3x+7}{-2x-3}$

$\Rightarrow \text{dom}(f \circ g) = \mathbb{R} \setminus \{-2, -\frac{3}{2}\}$

$\Rightarrow (h \circ f)(x) = \sqrt{\frac{2x+9}{x-3}}$

$\Rightarrow \text{dom}(h \circ f) = (-\infty, -\frac{9}{2}] \cup (3, +\infty)$

$\Rightarrow (g \circ h)(x) = \frac{(\sqrt{3x-1}+3)(\sqrt{3x-1}-2)}{3x-5}$

$\Rightarrow \text{dom}(g \circ h) = [\frac{1}{3}, +\infty) \setminus \{\frac{5}{3}\}$

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