# Typing rules

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$$(var) \quad \frac{x : \alpha \in \Gamma}{C^e \Gamma \vdash x : \alpha}$$

$$(app) \quad \frac{C^r \Gamma \vdash e_1 : C^s \alpha \to \beta \qquad C^t \Gamma \vdash e_2 : \alpha}{C^{r \vee (s \oplus t)} \Gamma \vdash e_1\, e_2 : \beta}$$

$$(sub) \quad \frac{C^s \Gamma \vdash e : \alpha}{C^r \Gamma \vdash e : \alpha} \; (s \leqslant r)$$

$$(abs) \quad \frac{C^{r \wedge s} (\Gamma, x : \alpha) \vdash e : \beta}{C^r \Gamma \vdash \lambda x.e : C^s \alpha \to \beta}$$

Figure 1: Type system for the flat coeffect language $\lambda_{fc}$

**TYPING RULES** $\quad C^r \Gamma \vdash e : \alpha$

$(var) \quad \dfrac{}{C^e(v : \alpha) \vdash v : \alpha}$

$(const) \quad \dfrac{c : \alpha \in \Phi}{C^1() \vdash c : \alpha}$

$(fun) \quad \dfrac{C^{r \times s}(\Gamma, v : \alpha) \vdash e : \beta \quad v \notin \Gamma}{C^r \Gamma \vdash \lambda v.e : C^s \alpha \to \beta}$

$(app) \quad \dfrac{C^s \Gamma_1 \vdash e_1 : C^r \alpha \to \beta \quad C^t \Gamma_2 \vdash e_2 : \alpha}{C^{s \times (r \oplus t)}(\Gamma_1, \Gamma_2) \vdash e_1 \, e_2 : \beta}$

$(let) \quad \dfrac{C^{r \times s}(\Gamma_1, v : \alpha) \vdash e_1 : \beta \quad C^t \Gamma_2 \vdash e_2 : \alpha}{C^{r \times (s \oplus t)}(\Gamma_1, \Gamma_2) \vdash \mathbf{let}\ x = e_2\ \mathbf{in}\ e_1 : \beta}$

$(ctx) \quad \dfrac{C^r \Gamma \vdash e : \alpha \quad C^{r'} \Gamma' \Rightarrow_c C^r \Gamma}{C^{r'} \Gamma' \vdash e : \alpha}$

**CONTEXT RULES** $\quad C^r \Gamma \Rightarrow_c C^r \Gamma$

$(nest) \quad \dfrac{C^{r'} \Gamma_1' \Rightarrow_c C^r \Gamma_1}{C^{r' \times s}(\Gamma_1', \Gamma_2) \Rightarrow_c C^{r \times s}(\Gamma_1, \Gamma_2)}$

$(nest) \quad C^{r \times s}(\Gamma_1, \Gamma_2) \Rightarrow_c C^{s \times r}(\Gamma_2, \Gamma_1)$

$(empty) \quad C^{r \times 1}(\Gamma, ()) \Rightarrow_c C^r \Gamma$

$(weak) \quad C^{r \times 0}(\Gamma, x : \alpha) \Rightarrow_c C^r \Gamma$

$(contr) \quad C^{r + s}(x : \alpha) \Rightarrow_c C^{r \times s}(x : \alpha, x : \alpha)$

$(assoc) \quad C^{r \times (s \times t)}(\Gamma_1, (\Gamma_2, \Gamma_3)) \Rightarrow_c C^{(r \times s) \times t}((\Gamma_1, \Gamma_2), \Gamma_3)$

$(sub) \quad C^r \Gamma \Rightarrow_c C^s \Gamma \quad (\text{when } s \leqslant r)$

Figure 2: Type system for the structural coeffect language $\lambda_{sc}$

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$$(var) \quad \frac{x : \alpha \in \Gamma}{\Gamma \vdash x : \alpha}$$

$$(app) \quad \frac{\Gamma \vdash e_1 : \alpha \to \beta \quad \Gamma \vdash e_2 : \alpha}{\Gamma \vdash e_1 \; e_2 : \beta}$$

$$(abs) \quad \frac{\Gamma, x : \alpha \vdash e : \beta}{\Gamma \vdash \lambda x.e : \alpha \to \beta}$$

$$(letbox) \quad \frac{\Gamma \vdash e_1 : C^{r \oplus s}\alpha \quad \Gamma, x : C^r\alpha \vdash e_2 : \beta}{\Gamma \vdash \mathbf{let\ box}\ x = e_1\ \mathbf{in}\ e_2 : C^s\beta}$$

$$(eval) \quad \frac{\Gamma \vdash e : C^e\alpha}{\Gamma \vdash !e : \alpha}$$

$$(sub) \quad \frac{\Gamma \vdash e : C^s\alpha}{\Gamma \vdash e : C^r\alpha} \quad (s \leqslant r)$$

Figure 3: Type system for the coeffect meta-language $\lambda_{\mathsf{cm}}$

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