A Bond Graph modeling perspective to price dynamics in the global oil-economic system

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***Abstract*—**

I. Introduction

The global energy system is undergoing a mayor shift. Carbon-based energy carriers, such as crude oil and natural gas, are being partly replaced by electrification alternatives such as electricity storage and hydrogen. [Global Energy and Climate Outlook 2019: Electrification for the low-carbon transition]. This shift to a hybrid and more complex energy system calls for new energy models that are capable of dealing with complexity, non-equilibrium conditions and uncertainty. [Emergence of New Economics Energy Transition Models: A Review].

Current modelling techniques to model oil-economic systems can be subdivided in three categories; structural, computational, and reduced form (econometric models) [Hillard G. Huntington. Oil markets and price movements: A survey of models. SSRN Electronic Journal, May 2003.]. Structural models are first-principle, agent-based systems models. Structural models lack the ability to yield quantitatively reliable results. Computational models are first-principle equilibrium models of economic systems. These models yield reliable quantitative results, but lack the ability to model non-equilibrium systems. Reduced form (econometric) models are regression models on time-series data. These models can accurately model short-term variability, but are not able to make predictions outside the variable space on which these models are trained. This inability is a crucial shortcoming of regression models given the shifting energy system.

In this paper, we introduce the use of dynamical systems theory to model the oil market. In particular, we use the bond graph modelling method to derive a first-principles model of the price and inventory dynamics in the oil market.

Bond graphs are graphical tools used to model dynamical systems. The bond graph method was introduced by Henry Paynter who noted that the dynamics of engineering systems from different domains, e.g. electrical, mechanical, or hydraulic engineering, were described by differential equations of the same form. In a bond graph, elements are interconnected by power bonds that represent the time rate of energy flow between the elements as the product of *flow* and *effort* variables. Furthermore, each power bond indicates the causality at each element port, i.e. whether an element is driven by a flow variable, or its dual effort variable. The assignment of causality throughout a system must follow structured rules, leaving no room for ambiguous causal relations. Bond graph theory has been extended to mechatronics, thermodynamics, hybrid and switching systems, and even social and economic systems.

The extension to economic systems was first introduced by Brewer. Brewer's analogy has been further used in a port-Hamiltonian formulation of macroeconomic systems in ?? and in a macroeconomic bond graph including fractional-order elements in machado. In Brewer's analogy, money is the economic analog of energy. As a result of this choice, power bonds represent the cash flow between elements where cash flow is the product of price, the effort variable, and commodity flow, the flow variable. This however entails that, besides inventory stock, the state variable is the time-integral of price, which Brewer refers to as the *economic impulse*. Using this analogy it is therefore not possible to derive a first-principles model of the price dynamics of an economic process.

In this paper we introduce an alternative economic bond graph analog in which cash flow, or income, is the economic analog of energy. Economic bond graph elements are then linked by bonds that represent the exchange of growth. Here, growth is the product of price movement and commodity flow, representing the economic effort and flow variables, respectively. In this analogy, bond graph models can be used to model the dynamics of inventory stocks and of prices.

II. Bond graph modeling of dynamical systems

Bond graphs describe systems as the interconnection of subsystems, or elements. Each element has a *port* through which it can be connected to its environment. The ports of elements are interconnected by bonds that represent the bi-directional flow of two power variables, effort and flow. The product of the effort and flow determines the rate of energy exchange between elements per unit of time, i.e. the power. Power and energy are domain neutral phenomena. Effort and flow are the generalized description of the signals that make up power in specific engineering domains. For example, in mechanical systems power is the product of force (effort) and velocity (flow) and in electrical systems power is the product of voltage (effort) and current (flow). For this reason, force and velocity are said to be the *analog* of voltage and current, respectively.[1]

The basic bond graph elements are divided into 1-port, 2-port, and multi-port elements. The 1-port elements are the inertia, the capacitor, the resistor, and the active effort source

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[1] In alternative, equally valid analogy voltage is the analog of velocity.[**?**].

and flow source. Each 1-port element has a constitutive relation that relates the two power variables. In case of the inertia and the capacitor this relation includes a time integral that keeps track of a stored state variable. The constitutive relation of a inertia is $f = \phi_I^{-1}(p)$, where $p = \int e dt$ and a capacitor is $e = \phi_C(q)$, where $q = \int f dt$, The dynamics of the state variables are defined by the entire bond graph model. This time integral also implies a preferred causality, i.e. a preferred effort or flow input. The resistor dissipates energy and does not have a preferred causality; its causality depends on how it is interconnected to the rest of the system. A resistor may return an effort for a flow input, $e = \phi_R(f)$, or return a flow for an effort input, $e = \phi_R^{-1}(e)$. The 2-port elements are the transformer and gyrator. A transformer links one effort e_1 to another effort e_2 and one flow f_1 to another flow f_2 in a power-preserving manner, i.e. $e_1 f_1 = e_2 f_2$. The gyrator links one effort e_1 to a flow f_2 and vice versa, again in a power-preserving manner. The multi-port elements are the 0-junction and the 1-junction. The multi-port elements represent the interconnection structure between elements. The 0-junction is an interconnection structure in which the flow variables sum up to zero and the effort variable is common,$f_1 + \cdots + f_n = 0$, $e_1 = \cdots = e_n$, e.g. an electrical node. In the interconnection structure of the 1-junction the effort variables sum up to zero and the flow variable is common, $e_1 + \cdots + e_n = 0$, $f_1 = \cdots = f_n$, e.g. an electrical loop.

make tables.

A. Bond graphs for economics

Brewer extended the bond graph analogy in [?] by choosing the rate of orders as flow variable and the unit commodity price as effort variable. The motivation behind this choice is that the power conservation at junctions,

$$e_1 f_1 + e_2 f_2 + \ldots e_n f_n = 0,$$

represents Walras' law for proper cash accounting.

However, choosing price as an effort variable entails that the time-integral of price, which Brewer calls the *economic impulse*, is the economic generalized momentum.

This choice of variables, in particular the choice of price as an effort variable excludes the possibility of deriving price dynamics.

We propose to view price not as an effort variable, but as a generalized momentum. This implies that the effort becomes a signal that provokes a rate of price movement, analogous to how a force provokes a rate of change in momentum. In general, we refer to the economic effort variable as a *desirability*.

Using this analogy, the power conservation at junctions represents the conservation of growth in an economic system.

III. Oil-Market Dynamics

A. What drives crude oil prices

The oil-economic literature identifies multiple factors that affect the price of crude oil. The US Energy Information Administration (EIA) captures the four most important factors in a simple model, shown in Figure 2. The four factors consist of supply, demand, inventory balance, and the financial markets. An extensive analysis of the EIA model supported by a literature review is provided in [?]. In this section, we discuss how the four factors can be modeled with dynamical systems theory resulting in a dynamical systems model for oil prices.

B. Oil Price as dynamical state

We consider the oil price as a dynamical state. The oil price changes over time due to the effect of supply, demand, inventories, and financial markets. We model this by the following dynamical law

$$p(t) = \int F(t) dt, \tag{1}$$

where $p(t)$ is the oil price in \$/# and $F(t)$ is an "economic force" applied by on or more factors in \$/#$\cdot$ yr. We introduce the specific economic force of each factor in the rest of this section.

The notion of price as a dynamical state is analogous to the notion of inventory, or stock, as a dynamical state. Inventory dynamics follow the physical law

$$q(t) = \int Q(t) dt, \tag{2}$$

where $Q(t)$ is the net flow of assets in #/yr.

C. Competitive and OPEC Oil supply

Oil supply is identified as a driver of the long-term equilibrium price. Oil supply is typically divided in Organization of the Petroleum Exporting Countries (OPEC) and non-OPEC production. This division is made because the OPEC members produce with respect to a central coordination and are mostly in hands of national oil companies, whereas non-OPEC producers are mostly in hands of investor-owned companies that make independent decisions.

To make the same distinction between OPEC and non-OPEC members, we will model non-OPEC members as inductive elements and OPEC members as controlled current sources.

1) non-OPEC members: Non-OPEC oil producers make independent decisions. In addition, they are assumed to make decisions so as to make return on investments at the actual market price.

We model each non-OPEC producer as a cost-minimizing agent, i.e. ignoring investments. Each producer $i \in I$ controls a production path γ_i for the upcoming period T_p. The i^{th} producer's problem is

$$\begin{aligned} \min_{\gamma_i} \int_0^{T_p} L_i(y_i)\, \mathrm{d}\tau \\ \text{s.t. } y_i \in Y_i \end{aligned} \tag{3}$$

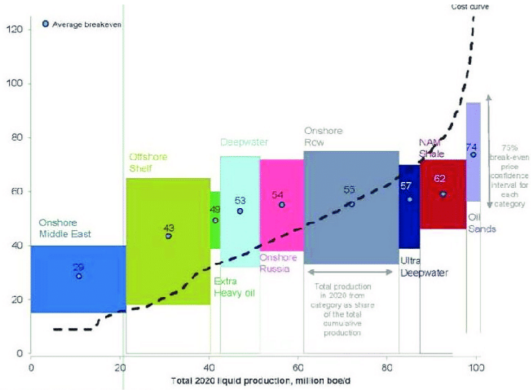

Fig. 1. Marginal cost curve of crude oil as piecewise-affine function

Here, Y_i is the set of achievable production levels of producer i. We assume that the running cost L_i takes the following form

$$L_i = \underbrace{\frac{1}{2} y_i(t)^T E_i^{-1} y_i(t)}_{\text{variable cost}} + \underbrace{p_{f,i}(t)^T y_i(t)}_{\text{fixed cost}} - \underbrace{\lambda(t)^T y_i(t)}_{\text{revenue}}, \quad (4)$$

where $y_i \in \mathbb{R}^1$ is the production, E_i is the price elasticity of supply of the i^{th} producer, $p_{f,i}$ are the fixed costs, and λ is the market price.

After applying the calculus of variations the optimization problem (3) results in the affine supply function

$$y_i(t) = \begin{cases} E_i(\lambda(t) - p_{f,i}) & \text{if } \lambda \geq p_{f,i} \text{ and } y_i(t) \in Y_i \\ 0 & \text{if} \lambda \leq p_{f,i} \end{cases} \quad (5)$$

2) OPEC: OPEC members do not produce according to a free market principle, i.e. basing production levels on the market price. Instead, the organization sets predetermined production targets for its member countries. The cost-minimization model used for non-OPEC members thus does not apply to OPEC members.

In this paper, we consider the supply from OPEC members as an external disturbance.

$$Q(t) = d(t) \quad (6)$$

In the discussion section we propose objective functions that may predict the oil supply from OPEC members.

D. Oil demand

On the demand side, the EIA model makes a distinction between member states of the Organization of Economic Cooperation and Development (OECD) and non-OECD countries. Most developed countries, e.g. the US and countries in the European Union, are members of the OECD. The oil consumption in OECD countries is less dependent on economic growth than for developing countries. This is because the GDP of developed countries is more stable, tax policies slow down oil consumption, and developed countries have larger service industries relative to manufacturing industries. As a result, the oil consumption in OECD countries is mostly dependent on the oil price.

We model each OECD consumer similar to how we model non-OPEC producers. This entails that each consumer $j \in J$ controls its consumption path .....

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Many non-OECD members are developing countries, among which China, India, and Saudi Arabia are the biggest consumers. In non-OECD counries, oil consumption is stronger related to economic growth rather than to oil prices. This is a result of on the one hand changing conditions in developing economies, such as increases in population and purchasing power, and on the other hand the higher share of manufacturing industry and use of oil as energy source and feedstock. Therefore, non-OECD consumption cannot be modeled like OECD consumption. Instead, we propose to model non-OECD consumption as an external disturbance that varies with the economic growth of these countries.

Some non-OECD countries control domestic oil prices, preventing consumers to react to global oil price changes. In the discussion section we propose a model of these countries as disturbance-rejecting controller.

E. Inventories

The difference between supply and demand is stored in inventory. The inventory level is a state variable q. The inventory level changes over time by the kinematic relation

$$\dot{q} = Q_S - Q_D, \quad (7)$$

where Q_S is the oil supply, and Q_D is the oil demand. In case $Q_S > Q_D$, inventory levels rise, and in case $Q_D > Q_S$ inventory levels decrease. A positive-valued inventory level $q(t)$ indicates a long position of oil that is physically stored, whereas a negative $q(t)$ indicates a short position registered in order books.

We model the convenience of having an inventory as an economic force that is proportional to the inventory level:

$$F_C(t) = -k(q(t) - q^*), \quad (8)$$

where F_C is the convenience "force", k is a constant indicating the elasticity of the storage, and q^* is a desired inventory level. The convenience equation implies that there is a positive (negative) force that drives the price of oil up (down) when the inventory level is below (above) the desired level.

The oil inventories act as buffer between supply and demand. When supply exceeds demand, oil can be stored in inventory, and when demand exceeds supply oil may be extracted from the inventories. We recognize this principle as the analog of a capacitor in an electric circuit that may charge or discharge in case of alternating currents, or as the analog of a spring that can integrate the difference between two velocities. As the capacitor or spring, oil inventories do not only play a role in the kinematics of the system, i.e. describing the current, velocity, and flow of oil, respectively, but also in the dynamics describing the voltages, forces, and economic forces, respectively.

Fig. 2. Schematic view of what drives oil prices [eia].

$non-OPEC$ **I** $\mathbf{S}_f$ $non-OECD$

0 $ut](OECD);(OPEC)edge[e_in](0junc);(nOPEC)edge[e_in](0junc);(1junc)edge[e_out](0junc);(C)edge[e_out]($

$OPEC$ $\mathbf{S}_f$ **I** $OECD$

$Balance$ **C** **1** **R** $Fin.\ markets$

e

Fig. 3. Bondgraph model of Oil market based on the EIA model.

F. Financial markets

Multiple activities on the financial markets influence the price of oil. Oil future contracts, commodity exchange contracts, others. Some are complex.

In the model presented in this paper, we only consider the effect of oil traders.

$$F_T = c(Q_D - Q_S), \tag{9}$$

where c is a discounting constant.

In the discussion section, we propose how futures contracts may be modeled in the economic-engineering framework.

We omit the

IV. Dynamical Systems Model of the Oil Economy

A. Model Assumptions

- Inelastic, unknown OPEC Supply
- Elastic non-OPEC Supply
- Ignore Lead Time
- All oil is fungible
- Linear aggregation producers
- Linear aggregation consumers
- market price equals competitive production cost
- Ignore Futures Market
- Ignore Economic Growth (pre filtered)
- Ignore capacity constraints

Figure 3 shows an engineering bondgraph model that is analogous to the EIA oil model. Each bond represents a transmission of oil and of economic force between the connected element and its environment. The direction of the flow of oil is opposite to the direction of the economic force. The so-called causal stroke at the end of a bond indicates on which element the economic force acts. The half-arrow indicates the positive direction of the economic growth.

B. State-space model

The derived structural model of the oil market in state-space form is

$$\begin{bmatrix} \dot{q} \\ \dot{p}_S \\ \dot{p}_D \end{bmatrix} = \begin{bmatrix} 0 & I_S & -I_D \\ C & RI_S & -RI_D \\ -C & -RI_S & RI_D \end{bmatrix} \begin{bmatrix} q \\ p_S \\ p_D \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} d_{OPEC} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} d_{nOECD} \tag{10}$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q \\ p_S \\ p_D \end{bmatrix} \tag{11}$$

- q Inventories
- p_S Supply reservation price
- p_D Demand reservation price
- C Convenience
- I_S Elasticity of supply
- I_D Elasticity of demand
- R Brokerage

C. Supply and Demand

At the 0−junction, the sum of all oil flows adds to zero. The economic force is equal on all connected elements. From the left side, the produced oil from OPEC and non-OPEC sources is added to the market. The consumption from OECD and non-OECD countries extracts oil from the market.

In the model, OPEC supply and non-OECD demand are represented by a flow input and sink, respectively.

Non-Opec supply, and OECD consumption on the other hand are represented by inertias.

D. Balance

The remaining oil, positive or negative valued, is sent to the balancing and financial market. The balancing market (inventories) and financial markets are connected to a 1 junction. At the 1−junction, the sum of all economic forces adds to zero and the connected elements share the same oil flow, i.e. the uncleared oil flow. The Balancing market accumulates the uncleared oil in inventories and returns an economic force representing the inventory convenience.

E. Financial markets

The financial markets apply an economic force based on the size of uncleared oil flow. During an excess demand, the financial markets increase the oil price by asking a premium. During excess supply, the financial markets decrease the oil price by providing discounts to sell uncleared oil. The financial markets withdraw economic surplus from the oil market until the market is in equilibrium.

V. Simulation Studies

A. Venezuelan oil crisis

1) Model Identification:

- benchmark
- in sample
- out of sample

B. Demand Shocks

VI. Conclusions