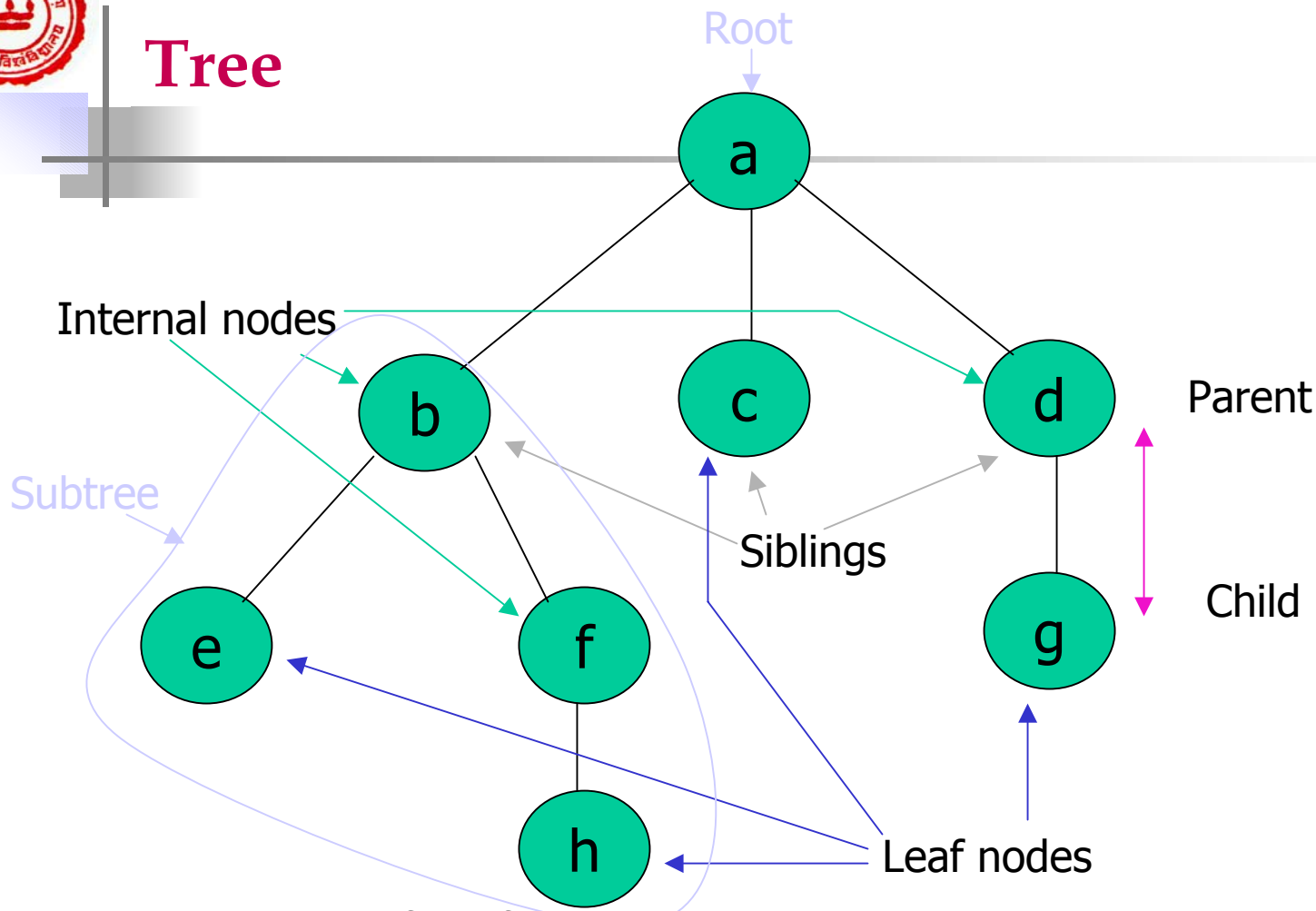
# TREE

# Tree




Edges – ab, ac, ad, be, bf, dg, fh

Path from a to e – abe

Path from a to h – abfh

Height = 4

**A tree with n nodes has n-1 edges**

# Binary Tree

- A tree in which every node has at most two children is a Binary Tree.
- One can distinguish between the left child and the right child.

- A Full Binary Tree of height n is one that contains maximum possible number of nodes up to level n.

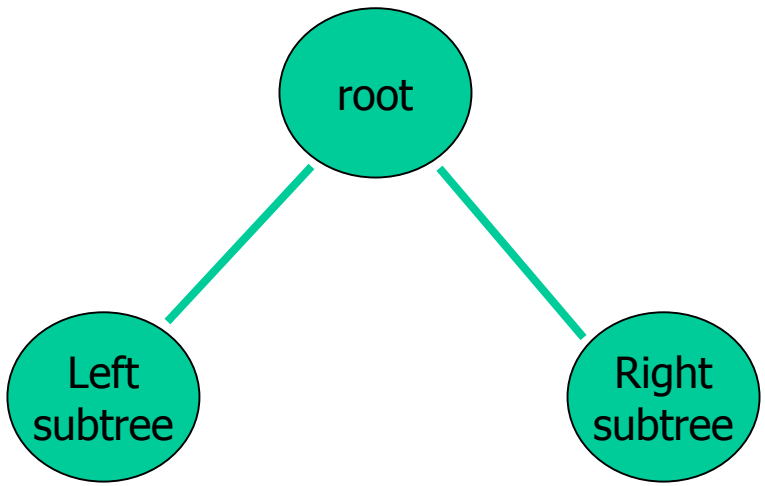
- A complete binary tree is one where

1. All the leaves are on level n or n-1.
2. On level 1 through n-2, all nodes have exactly two children.
3. On level n, the leaves are as far to the left as possible.

- Full and complete binary trees can be easily represented by arrays.

# Binary Tree ...

A binary tree can be viewed as composed of a root node, a left sub-tree and a right sub-tree. Both the sub-trees are binary.




Binary tree can be used to represent general trees.

# Properties of Binary Trees

1. The maximum no. of nodes on level i of a binary tree is $2^{i-1}$, i >= 1.

2. The maximum no. of nodes in a binary tree of depth (height) k is $2^k$-1, k>=1.

3. For any non-empty binary tree, if $n_0$ is the no. of terminal (leaf) nodes, and $n_2$ is the no. of nodes of degree 2 (having two children), then $n_0 = n_2 + 1$.

# ADT Binary Tree

- **Objects:** A binary tree is a structure of nodes of the same type. Each node is the root of a sub-tree, having a left and right child which are binary trees. Each node has a key field that identifies the entry.
- **Operations:**

  **init_t(t)** – initialize an empty tree.

  **empty_t(t)** – boolean function to return true if the tree t is empty.

  **height(t)** – function to return the height of tree t.

  **make_root(n)** – procedure to make node n the root of a tree.

  **addleft(t, n)** – add node n as a left child of tree t.

  **addright(t, n)** – add node n as a right child of tree t.

  **deleteleaf(n)** – delete the leaf node n.

# Binary Tree Traversals

Preorder Traverse
Inorder Traverse
Postorder Traverse

Perform a traversal of the nodes of the tree in the specified order and visit each node.

# A few problems

1. Delete a tree
2. Boolean function ***isleaf*** (t)
3. Function ***sizetree*** to count the no. of nodes in a tree
4. Function ***count leaf***
5. ***Equaltree*** ($t_1$ , $t_2$)
6. Function ***maxintree***
7. ***Copytree***(s,t)
8. ***Isintree***(t,n)
9. ***Printtree*** - LTR, TLR, LRT, RTL, TRL, RLT
10. Find
    a) In-order-predecessor
    b) In-order-successor

# Tree Implementation using C Pointers

```
typedef struct nt {
                T    info;
                struct nt *left, *right;
              } bintree;
bintree *t, *cur;

bintree *init_t() {       return    NULL; }

int empty_t(bintree *t) { return (t == NULL) ;}

bintree *makeroot(T  n) {
                        bintree *t;
                        if((t = malloc(sizeof(bintree))) == NULL)
                                perror("malloc error");
                        else {
                                t -> info = n;
                                t -> left =NULL;
                                t -> right = NULL;
                        }
                        return t;
                }
```

# Tree Implementation using C Pointers …

```
void addleft ( bintree *t , T  n ) {
                        t -> left = makeroot ( n );
                    }
 void addright ( bintree *t, T  n ) {
                        t -> right = makeroot ( n );
                     }
 int height ( bintree *t ) {
                if ( empty_t (t) )
                        return 0;
                else
                        return(1 + max(height(t -> left), height ( t -> right)));
            }

void preorder_traverse ( bintree *t ) {
                                    if ( !(empty_t (t)))
                                    {
                                                visit (t);
                                                preorder_traverse (t -> left);
                                                preorder_traverse (t ->right);
                                    }
                                }
```

# Threaded Binary Tree

- Frequent inorder-traversals of long binary trees using recursive routines involve lot of space and time.

- Use of right pointers in a node having no right child to point to the inorder-successor can avoid recursion. Such pointers are called Threads.

```
typedef struct nt {  T  info ;
                     struct nt *left, *right ;
                     int thread ;
                   } threaded_bintree ;
```

# Threaded Binary Tree ...

```
void th_inorder_traverse ( threaded_bintree *t) {
    threaded_bintree *cur ;
    cur = t ;
    while ( !empty_t (cur)) {
          while ( !(empty_t(cur -> left)))
                cur = cur -> left ;
          visit (cur) ;
          while((cur -> thread) & (!(empty_t(cur ->right)))) {
                cur = cur -> right ;
                visit (cur);
          }
          cur = cur -> right ;
    }
}
```

- Write other ADT functions for Threaded Binary Tree

# Application of Trees / Binary Trees

- Set representation
- Decision Trees
- Game Trees
- Searching