Ref. No.: Ex/MATH/CSE/T/211/2016

B.CSE. 2ND YR 1ST SEMS EXAM, 2016

Mathematics

(Paper-IV)

Full Marks 100 | Time Three Hours

Answer Question number 1 and any six from the rest

1. Find a particular integral of the differential equation (4)

$$\frac{d^2y}{dx^2} - 9y = e^{3x}\cos x$$

2. (a) Find the series for $log(1+x)$ by integration and use Abel's Theorem to prove that (6)

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots = log2$$

(b) Find a power series solution of the initial value problem (10)

$$(x^2-1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + xy = 0, \qquad y(0) = 4, \qquad y'(0) = 6$$

Write atleast first five terms of the series

3. (a) Find Frobenius series solution about the regular singular point of the following differential equation (10)

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 8(x^2-1)y = 0$$

Write atleast first three terms of each series.

(b) State the orthogonality property of Chebyshev ploynomials of first kind. Use that property to find the expansion of $f(x) = x^3 + x, -1 \le x \le 1$ in terms of the Chebyshev polynomials of first kind (6)

4. (a) Prove that (10)

$$\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & , m \neq n \\ \frac{2}{2n+1} & , m = n \end{cases}$$

where $P_n(x)$ is the Legendra polynomial of degree n.

(b) Write generating function of Legendre ploynomials. Use that function to prove (6)

i. $P_n(1) = 1$

ii. $P_{2n}(0) = (-1)^n \frac{1.3.5.\ldots(2n-1)}{2^n n!}$

5. (a) Use the method of variation of parameters to find general solution of the equation (8)

$$\frac{d^2y}{dx^2} + y = \tan x$$

(b) Solve (8)

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = 2x \ln x$$

6. (a) If $f(z) = e^z$, describe the image under $f(z)$ of horizontal and vertical lines i.e find the sets $f(a + it)$ and $f(t + ib)$, where a,b are constants and t runs through all real numbers. (5)

(b) If the function $\frac{\bar{z}}{z}$ analytic in its domain of definition? (3)

(c) Suppose $f(z) = az^2 + bz\bar{z} + c\bar{z}^2$, where a, b, c are fixed complex numbers. By differentiating $f(z)$, show that $f(z)$ is complex differentiable at z iff $bz + 2c\bar{z} = 0$. (4)

(d) Derive the polar form of the Cauchy-Riemann equations for u and v: $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$, $\frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$ (4)

7. (a) Use Liouville's theorem to prove that every polynomial in z of degree $n(\geq 1)$ has a zero. (6)

(b) Find harmonic conjugate of $xy + 3x^2y - y^3$. (4)

(c) Define $u(z) = Im(\frac{1}{z^2})$ for $z \neq 0$ and set $u(0) = 0$, then show that (6)

i. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

ii. u is not harmonic on C.

iii. $\frac{\partial^2 u}{\partial x \partial y}$ does not exists at $(0, 0)$.

8. (a) Find (6)

$$\int_{\nu} f(z)dz$$

where $\nu = 3e^{it}$ for $t \in [0, 2\pi]$ and $f(z) = \bar{z}$.

(b) Show that if z_0 is an isolated singularity of $f(z)$ that is not removable, then z_0 is an essential singularity of $e^{f(z)}$. (4)

(c) By estimating the coefficient of the Laurent series, prove that if z_0 is an isolated singularity of f , and if $(z - z_0)f(z) \to 0$ as $z \to z_0$, then z_0 is removable. (6)

9. (a) Define Fourier series of a function $f(x)$. Find the Fourier series generated by a periodic function $f(x) = x^2$ in $-\pi \leq x \leq \pi$ and deduce that (8)

i. $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$.

ii. $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.

(b) Find the Fourier series for $f(x) = |x|$, $\quad -\pi < x < \pi$ (8)