



## 2-3 TREE



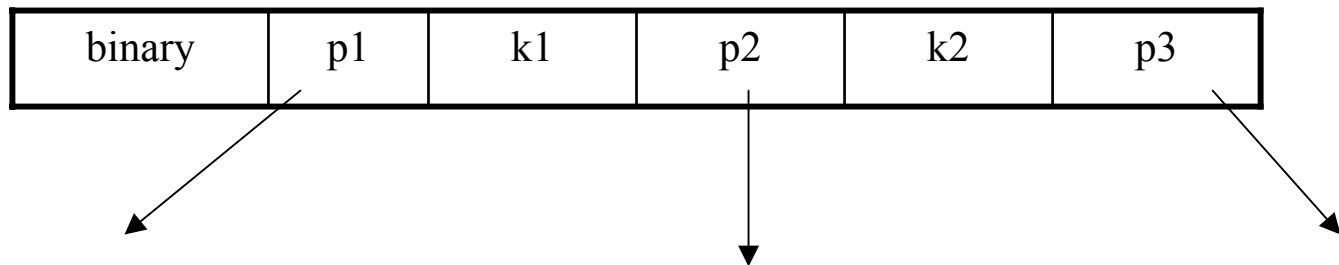
## 2-3 Tree Definition

- All leaf nodes are at the same level.
- All non leaf nodes are either binary or ternary.
- Binary nodes are like BST nodes with single key  $k$ .
- Ternary nodes have two keys  $k_1$  and  $k_2$  and three children pointers  $p_1$ ,  $p_2$ ,  $p_3$ .
- All keys  $< k_1$  reside in sub-tree pointed to by  $p_1$ . All keys  $> k_2$  reside in sub-tree pointed to by  $p_3$ . Others reside in the sub-tree pointed to by  $p_2$ . All keys are distinct .



## 2-3 Tree node

```
typedef struct nt {  
    T      k1, k2;  
    int    binary;  
    struct nt * p1, * p2, * p3;  
} 23treenode;
```





# Operations

➤ If  $h$  is the height of a 2-3 tree having  $n$  nodes,

$$\text{then } 2^h - 1 \leq n \leq 3^h - 1$$

$$\text{Thus, } \log_3(n+1) \leq h \leq \log_2(n+1)$$

Operations:

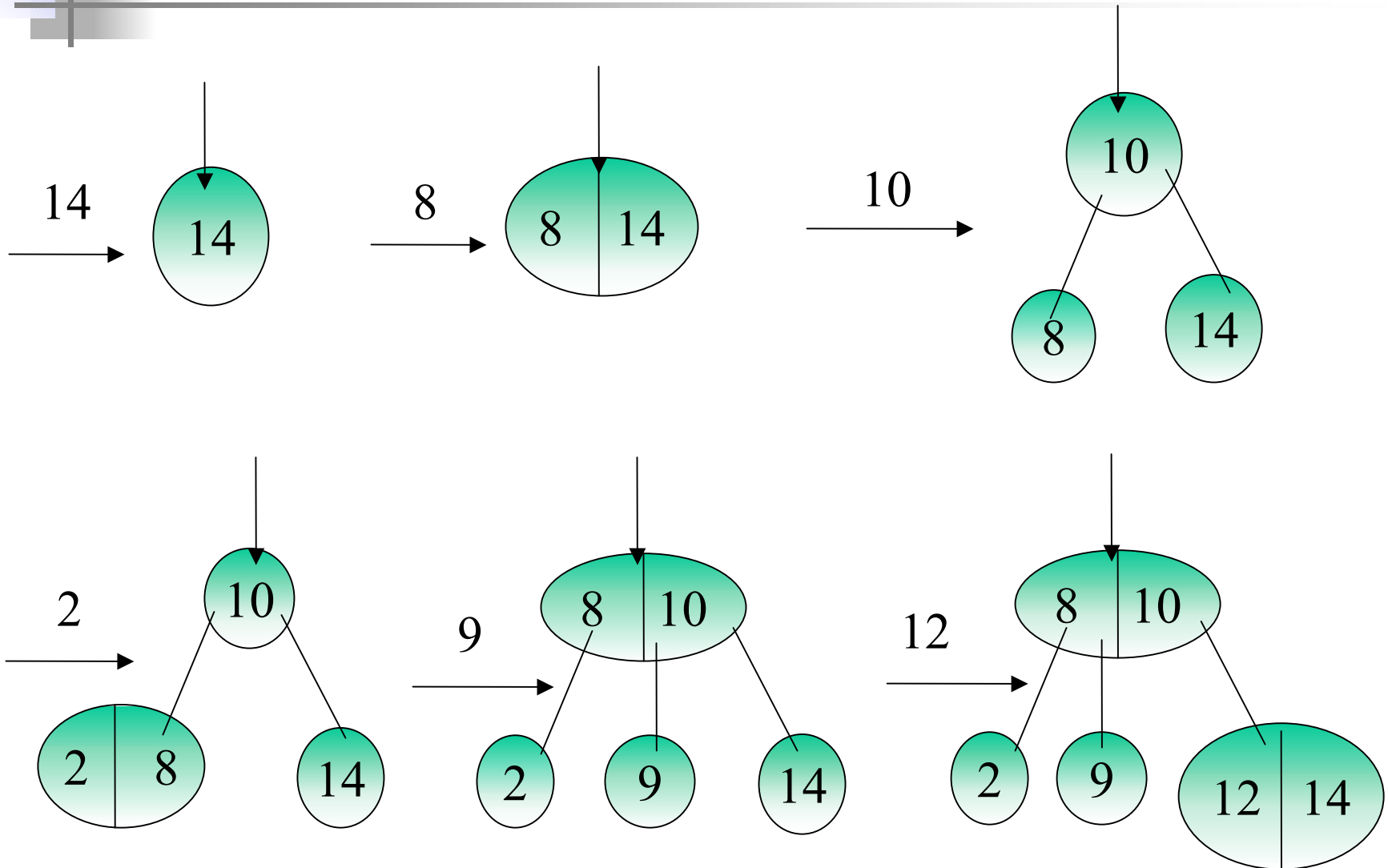
Insert node

Delete node

Search node

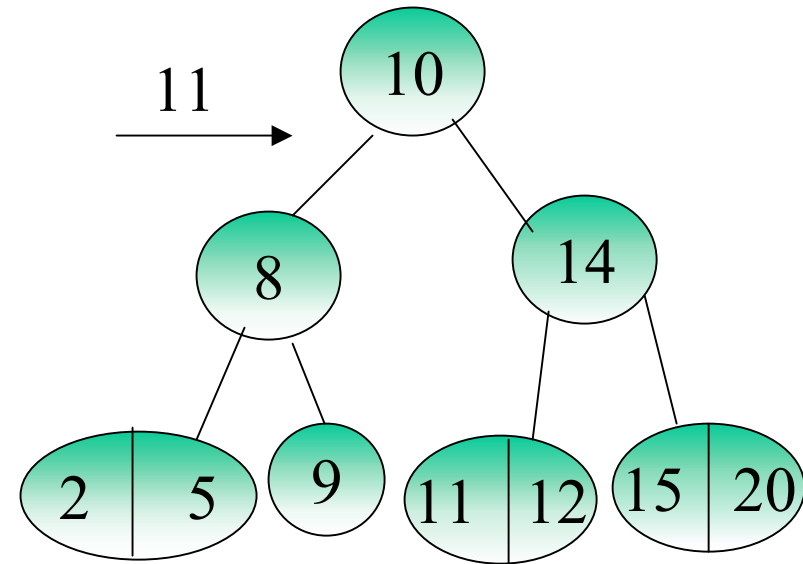
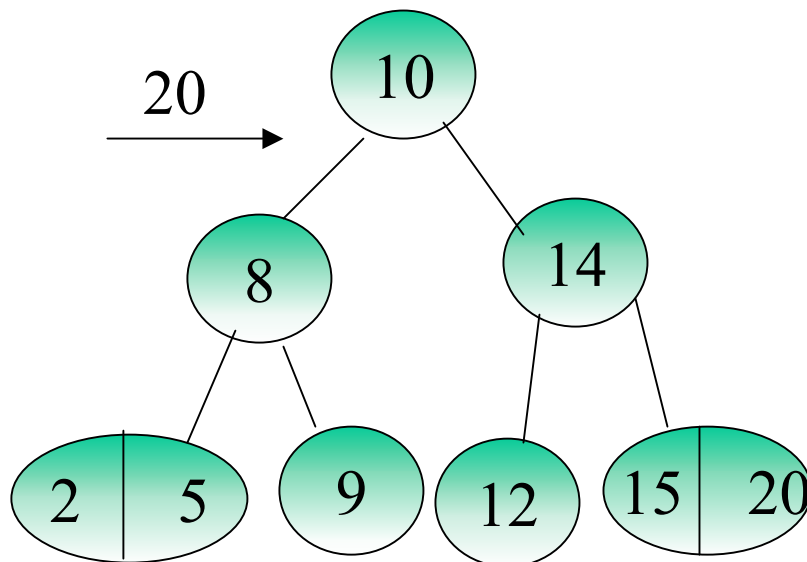
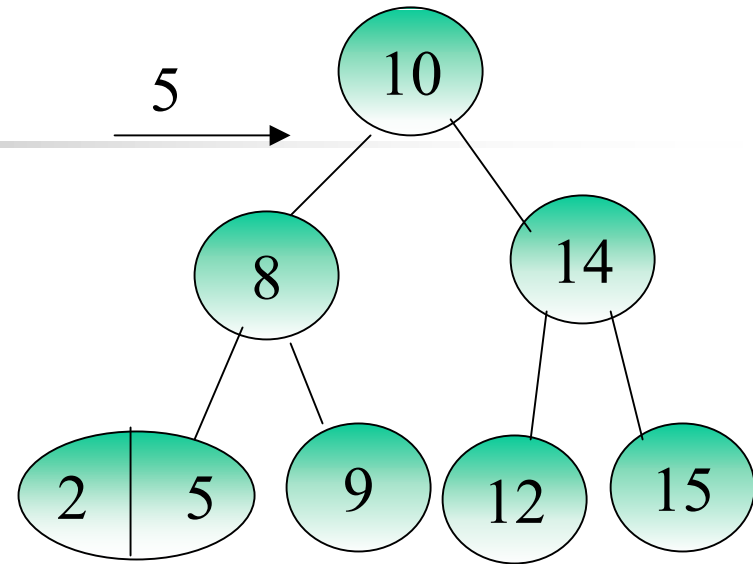
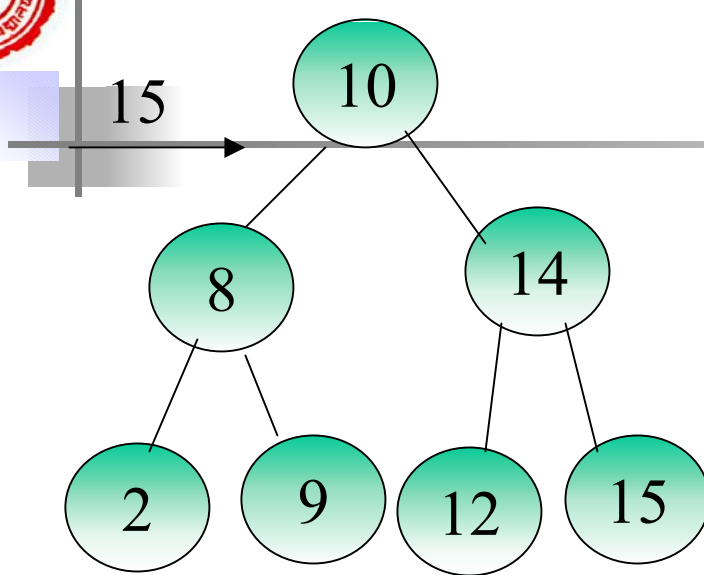


# INSERTION EXAMPLES



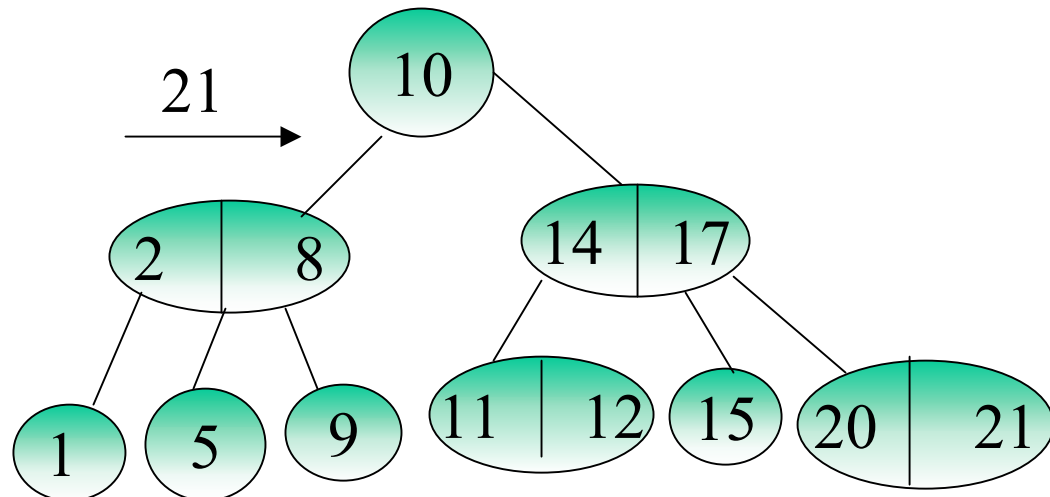
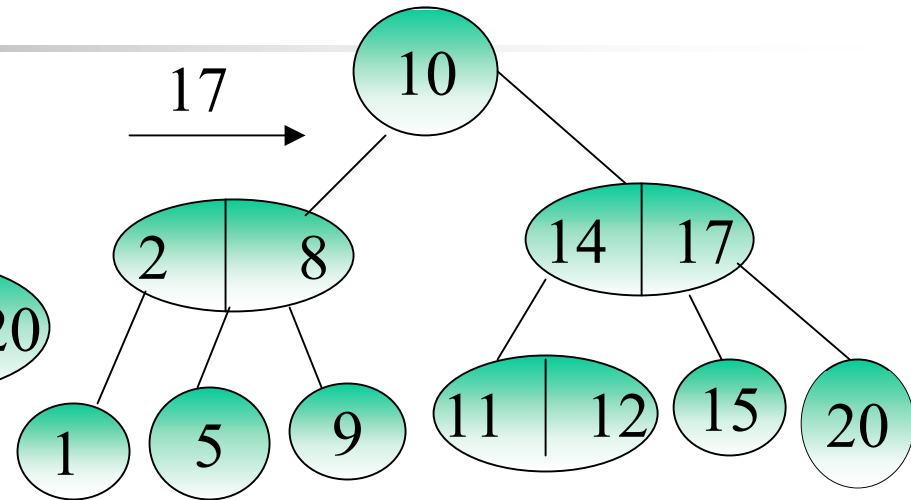
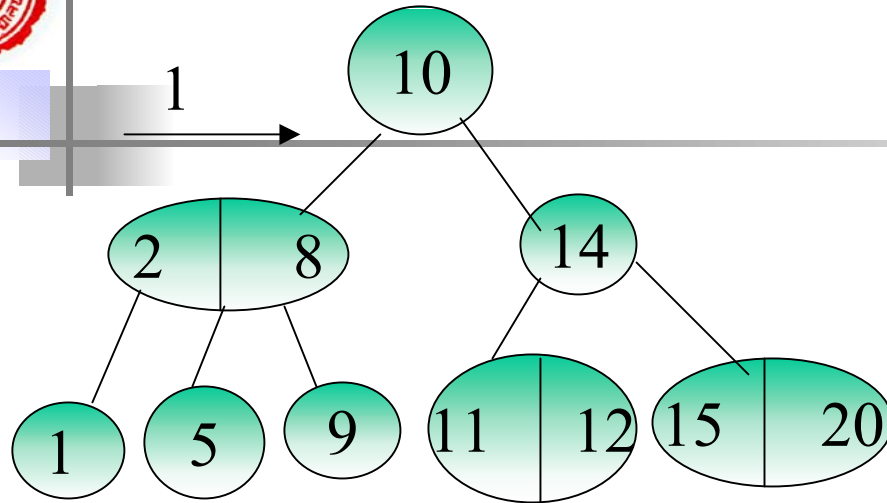


## INSERTION EXAMPLES ...



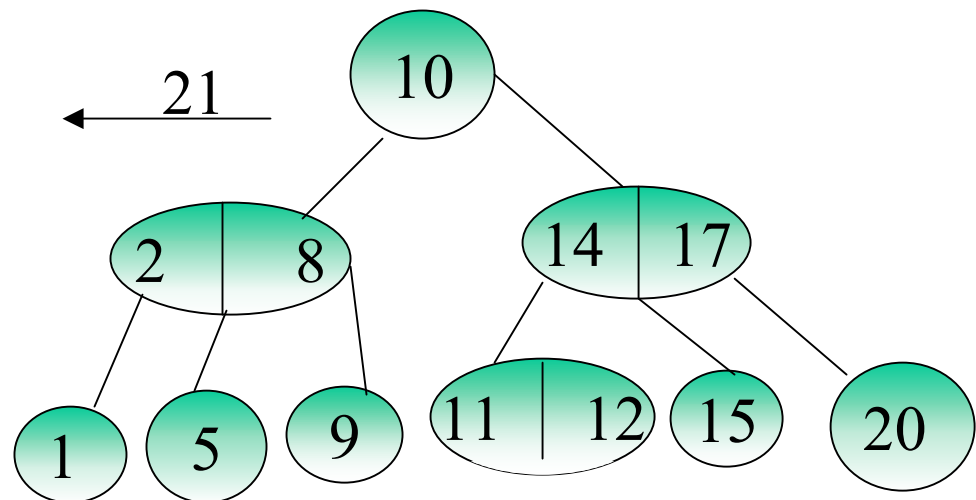
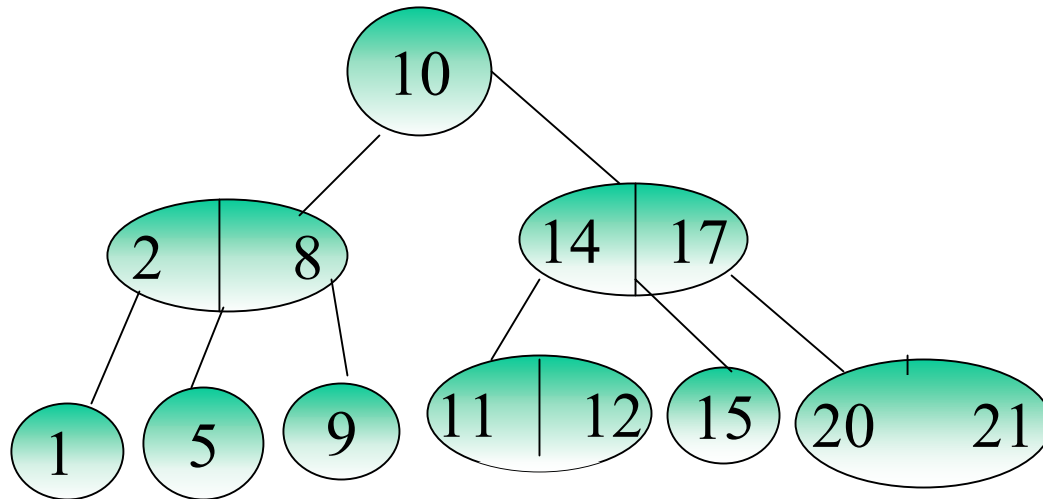


## INSERTION EXAMPLES ...





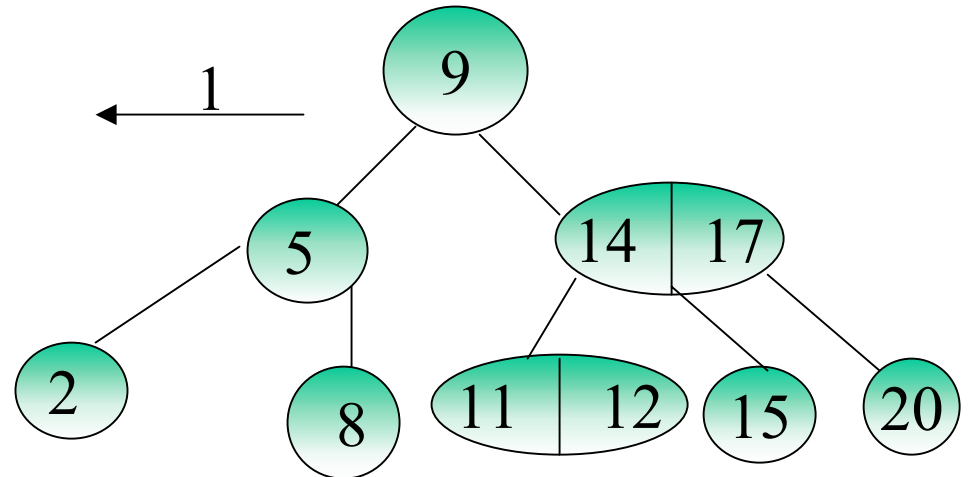
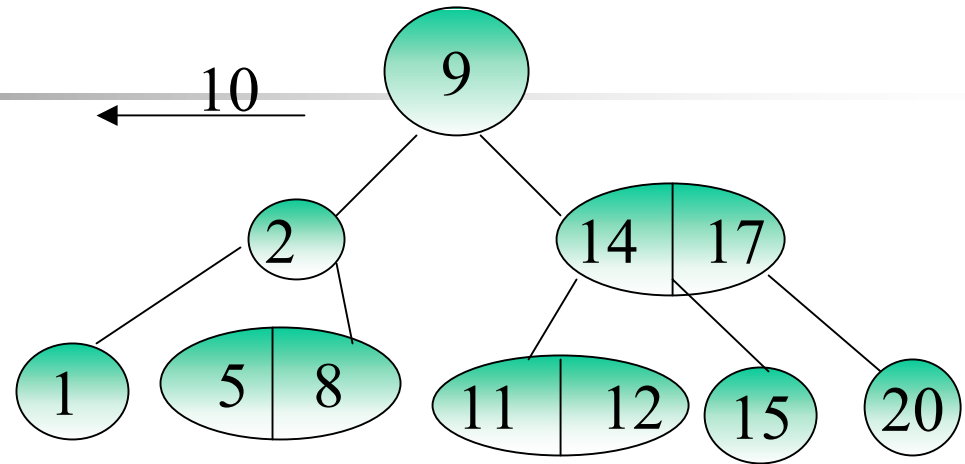
# DELETION EXAMPLES





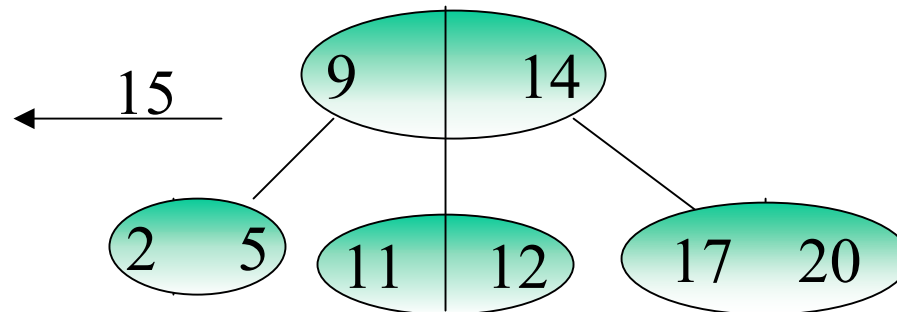
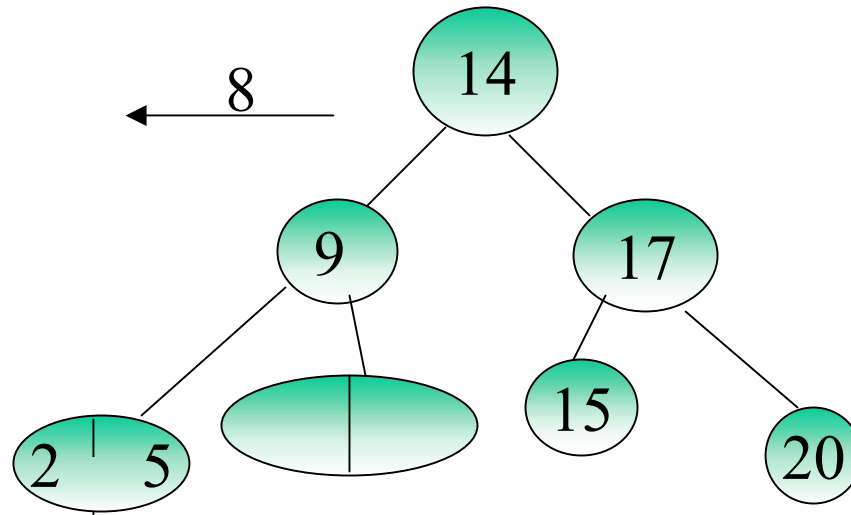


# DELETION EXAMPLES ...





## DELETION EXAMPLES ...





# B TREE



## B-Tree Definition

**A B-Tree of order  $d$  is a tree with following properties:**

- Each node ( except possibly the root node ) contains at most  $d$  records and at least  $\lfloor d/2 \rfloor$  records.
- The root node can have at most  $d$  records and as few as one record.
- An internal node containing  $k$  records ( $1 \leq k \leq d$ ) with key values  $k_1$  to  $k_k$ , have pointers to  $k+1$  subintervals of keys stored in sub-trees.
- A leaf node has empty sub-tree below it. All leaf nodes are at the same level.



## B-TREE Node

```
typedef struct btnode {  
    int      k;  
    T        datalist[maxkeyno];  
    struct btnode * ptr[maxkeynode + 1];  
}
```

**Height of a B-Tree of order  $d$  containing  $n$  nodes is given by**

$$\lfloor \log_{\lfloor d/2 \rfloor + 1} (n+1) \rfloor \geq h \geq \lceil \log_{d+1} (n+1) \rceil$$

**2-3 tree is a B-Tree of order 2**

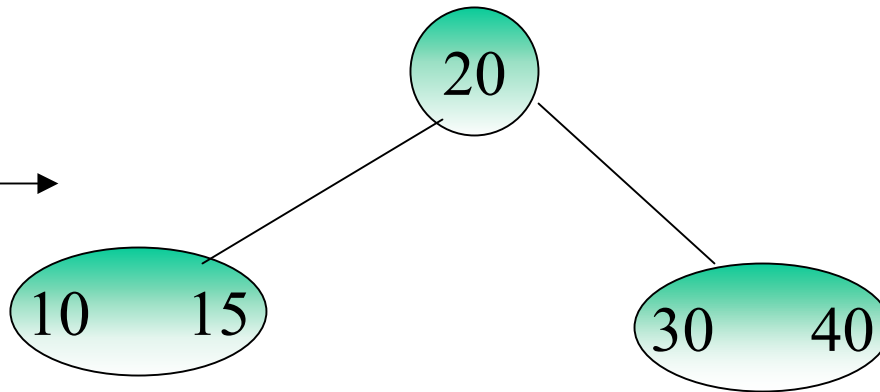


## Insertion Examples on a B-Tree of order 4

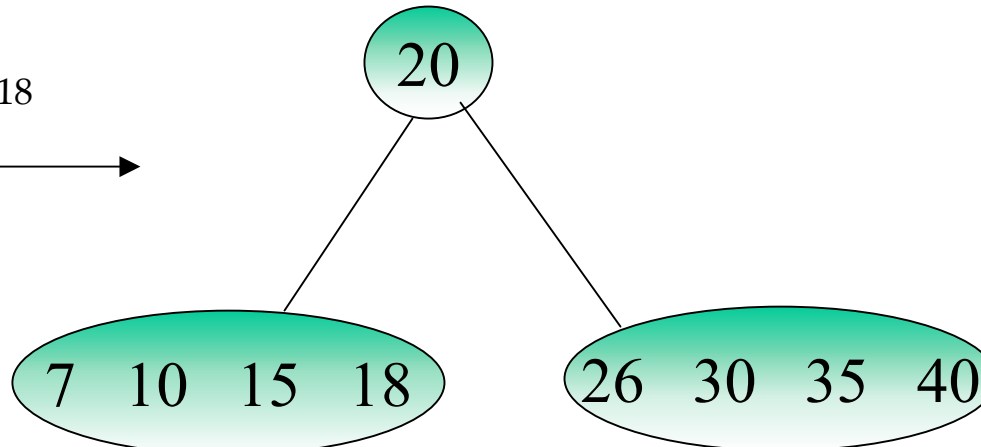
20, 40, 10, 30

10 20 30 40

15

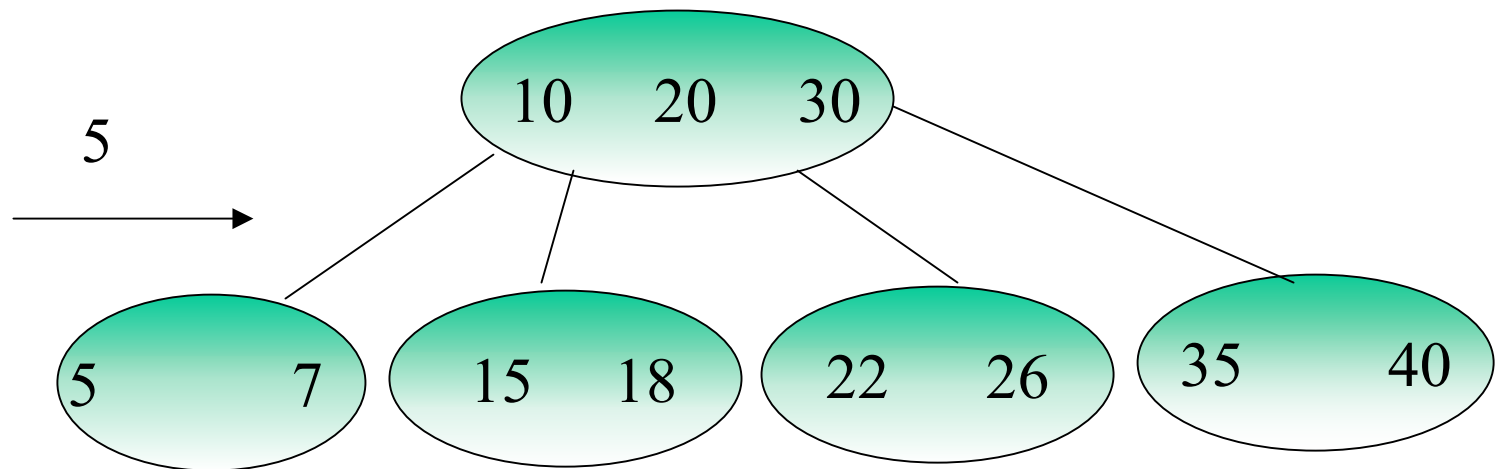
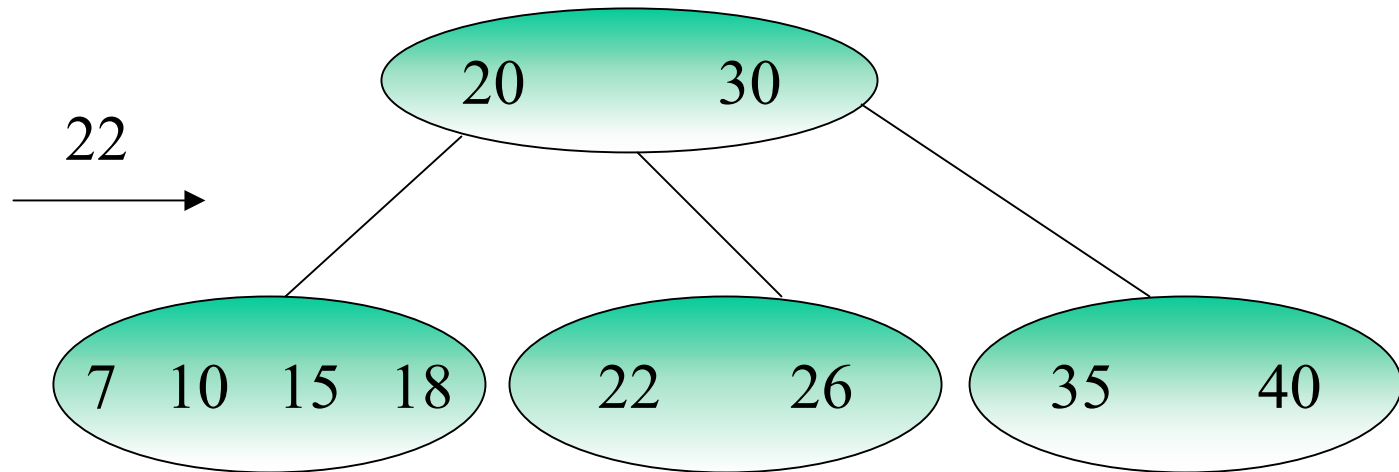


35, 7, 26, 18



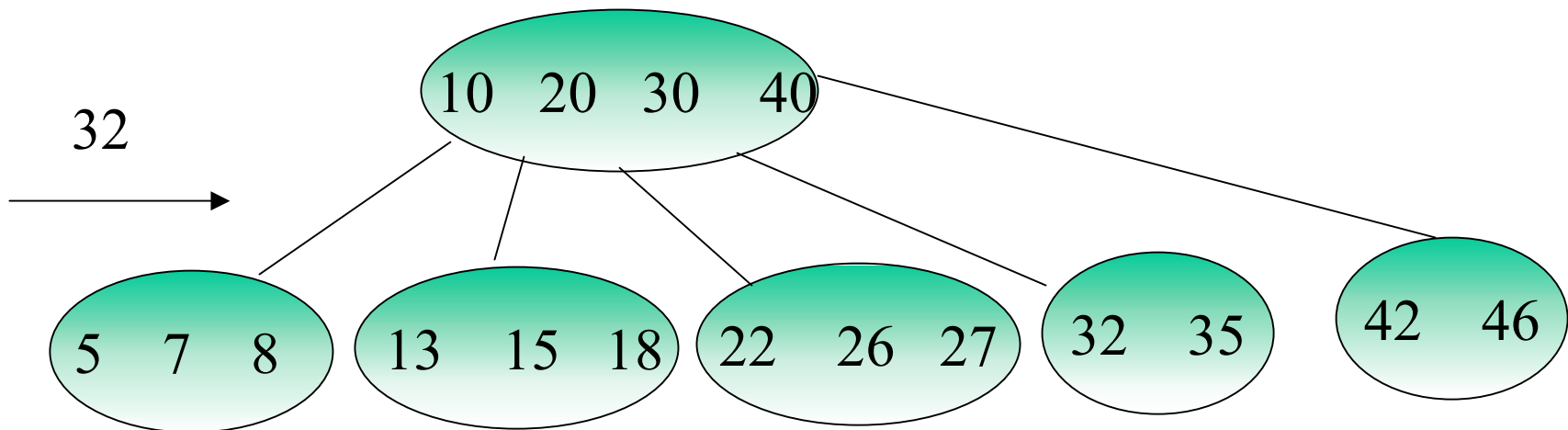
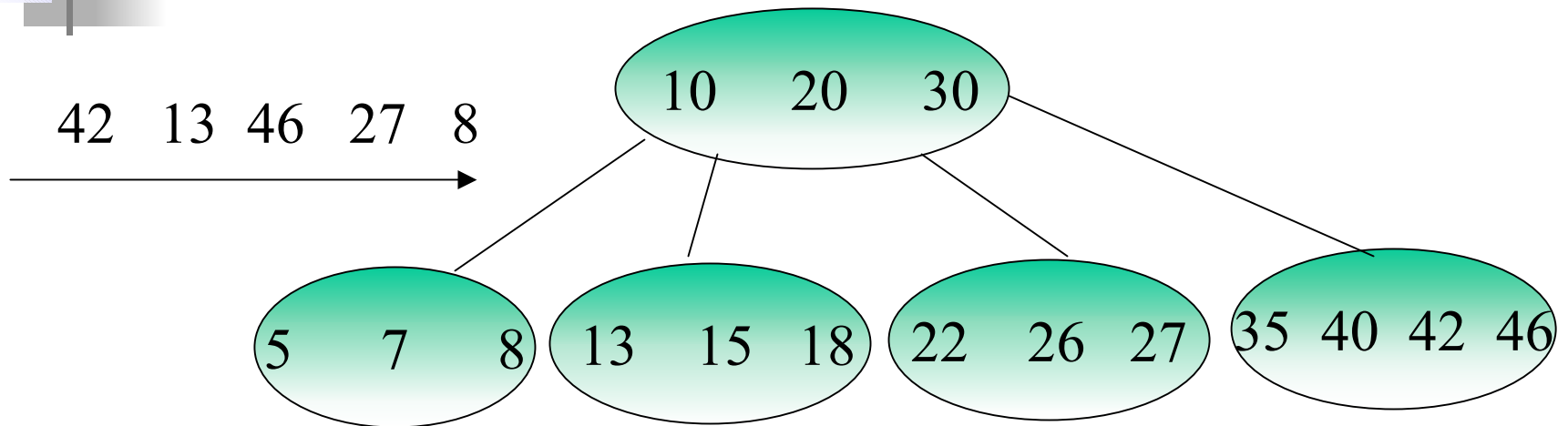


## Insertion Examples ...





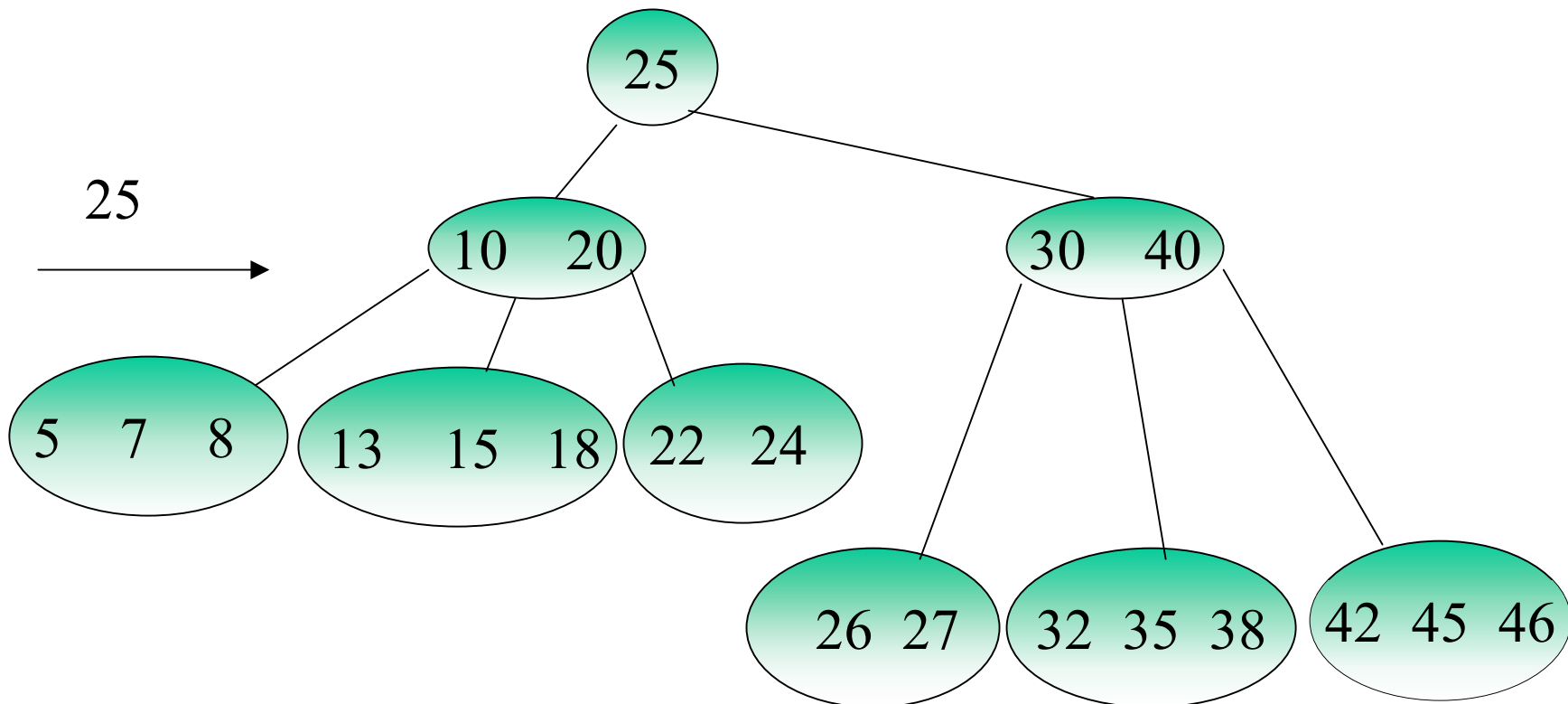
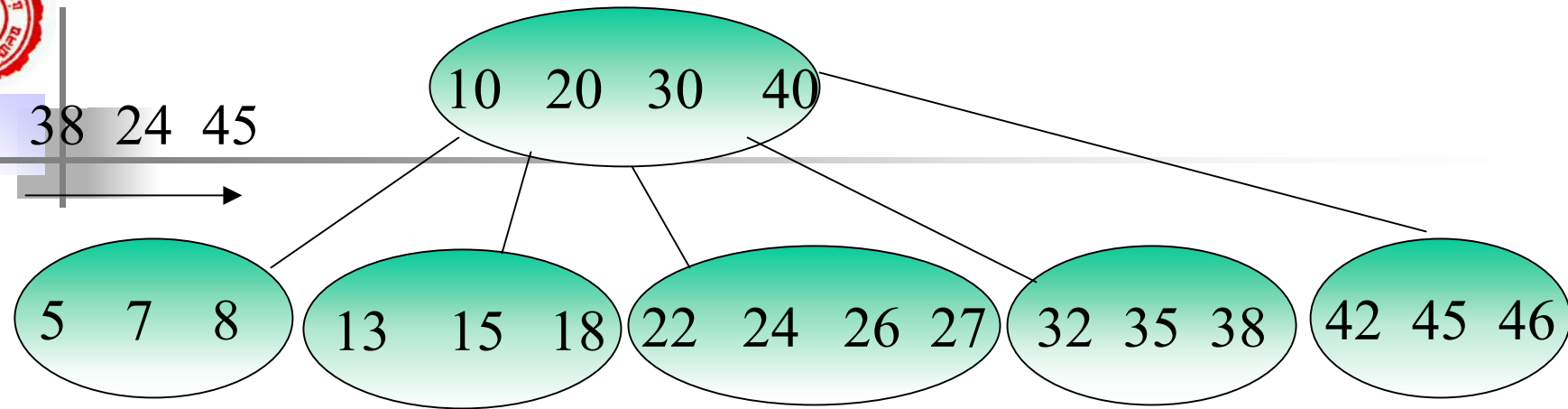
## Insertion Examples ...





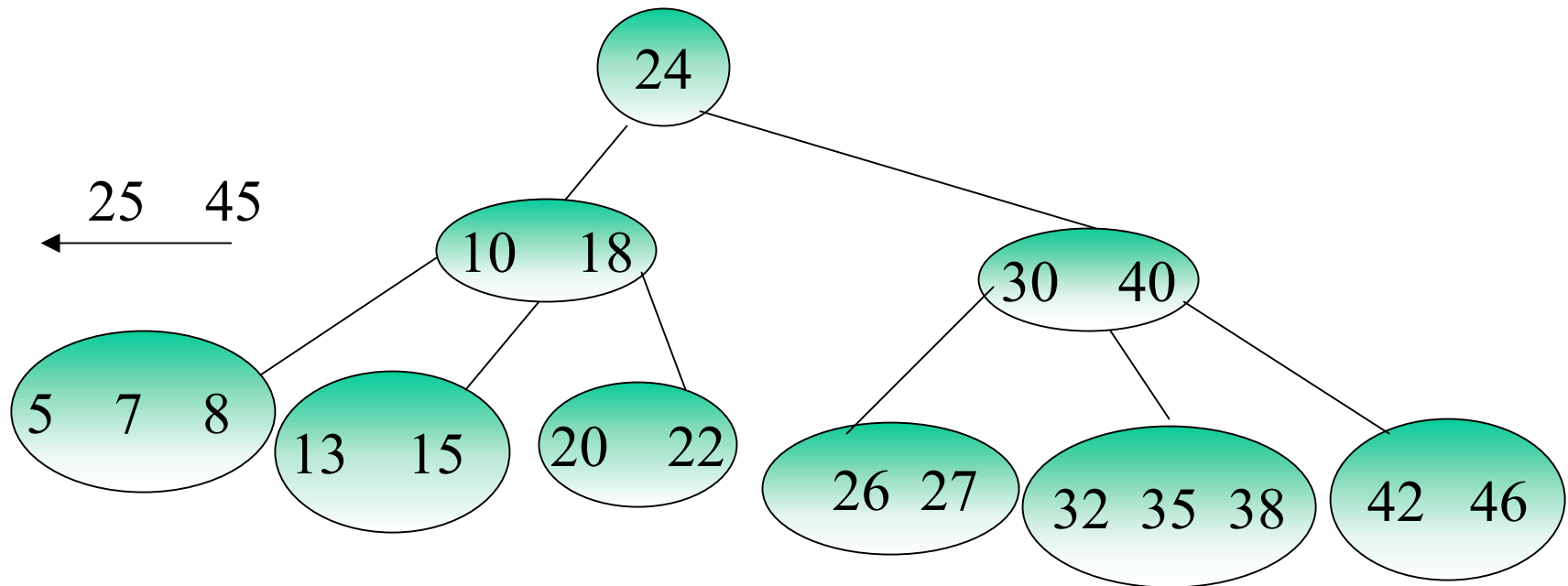


## Insertion Examples ...



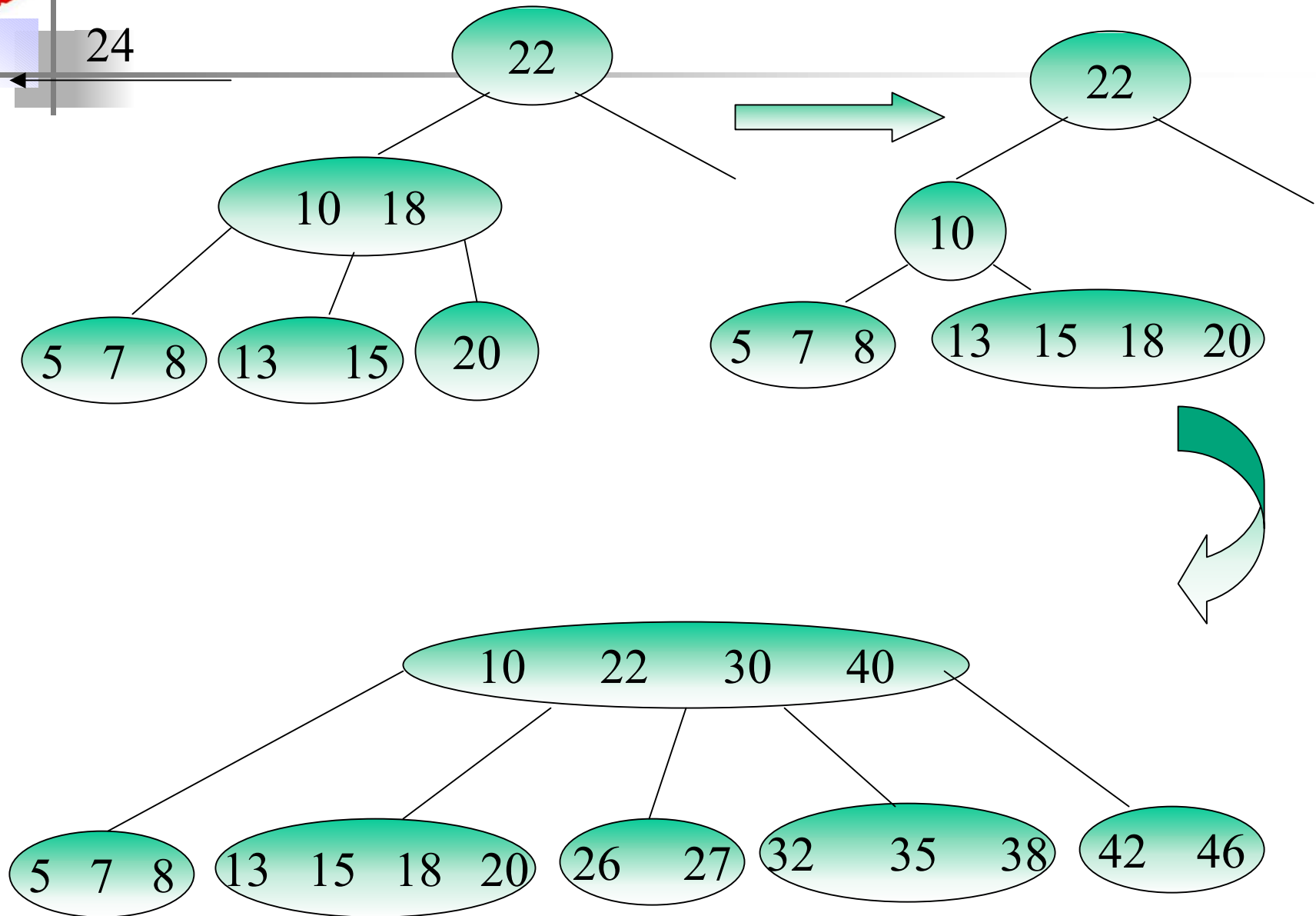


## Deletion examples



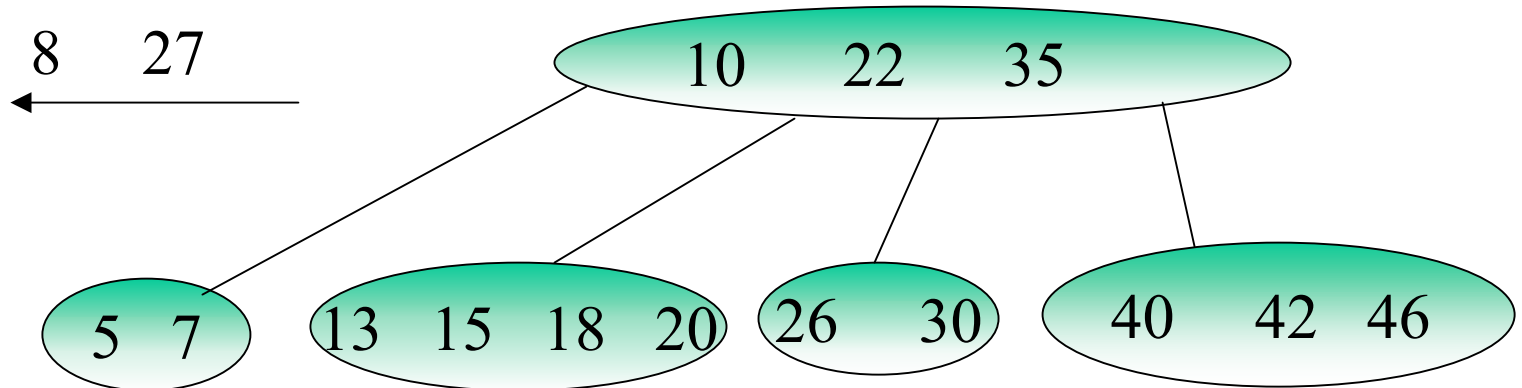
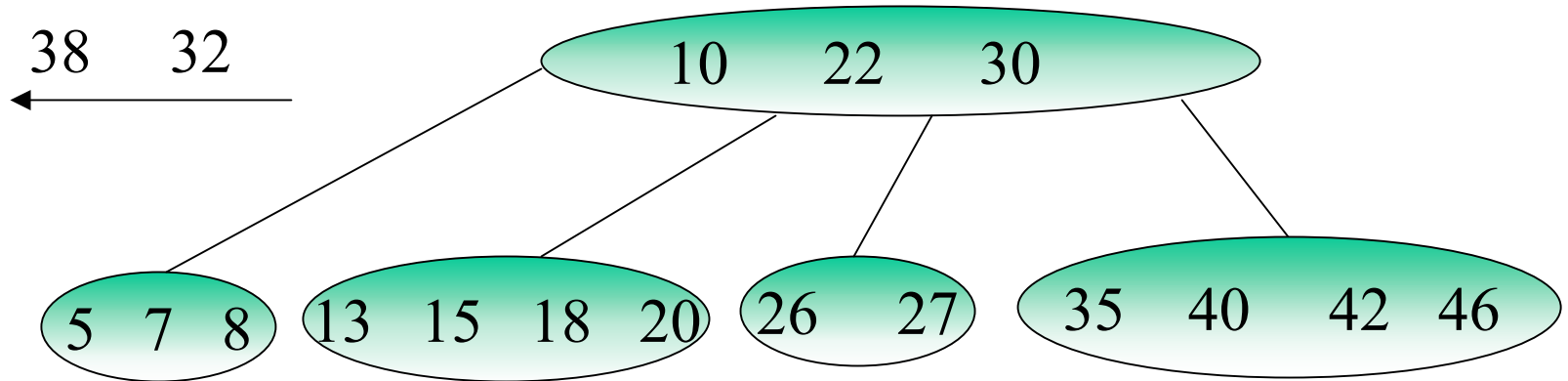


## Deletion examples ...



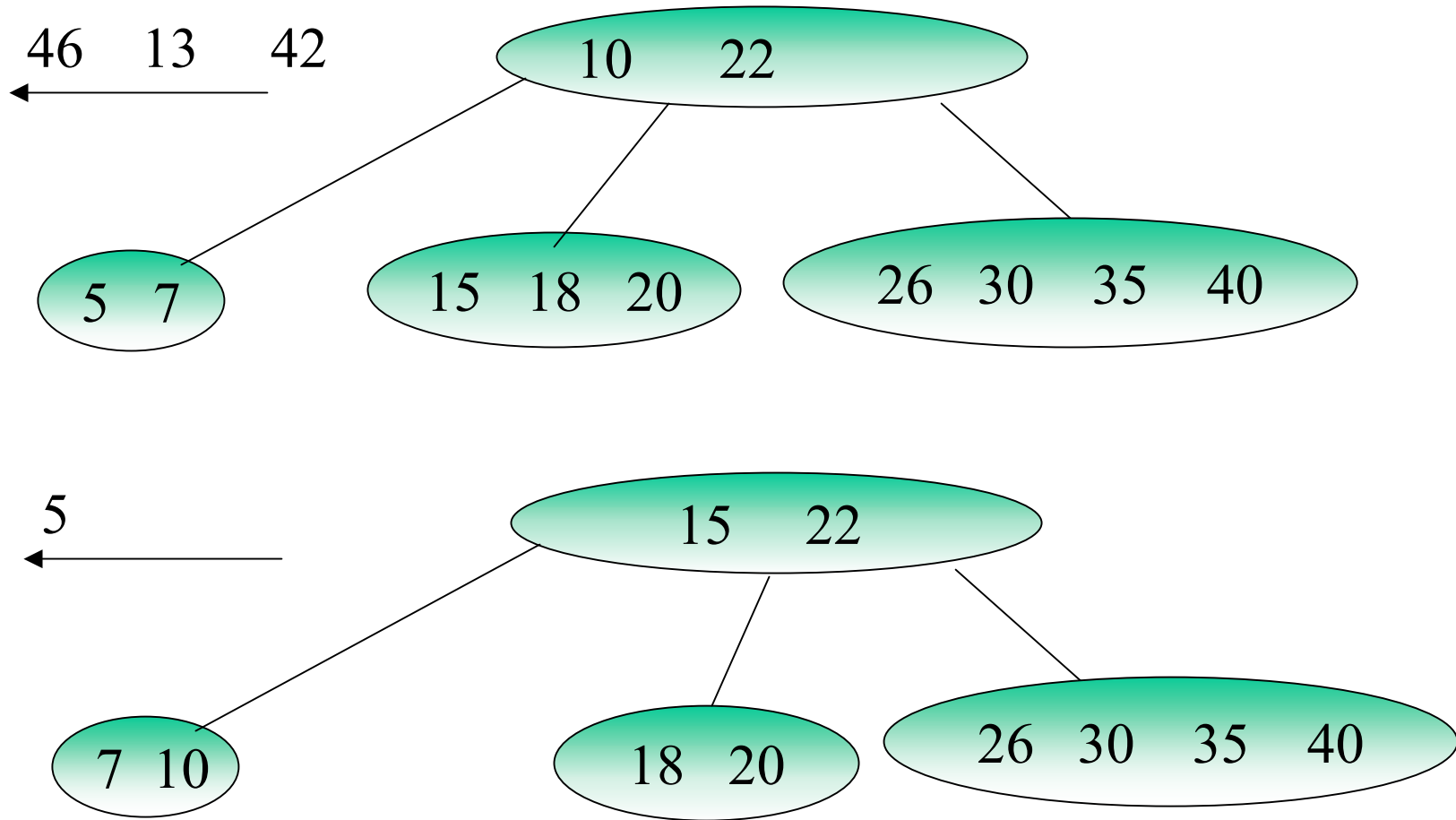


## Deletion examples ...



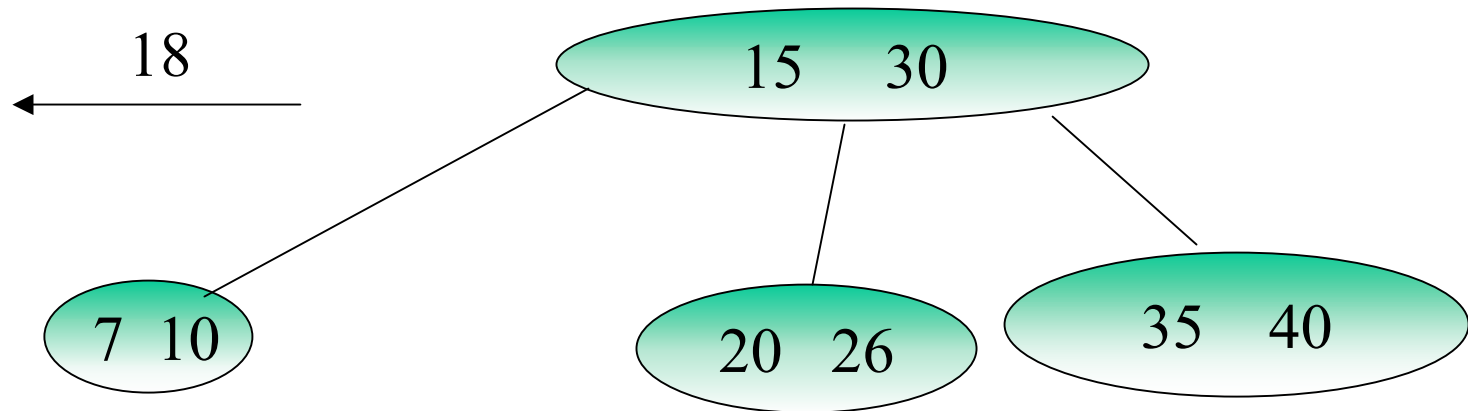
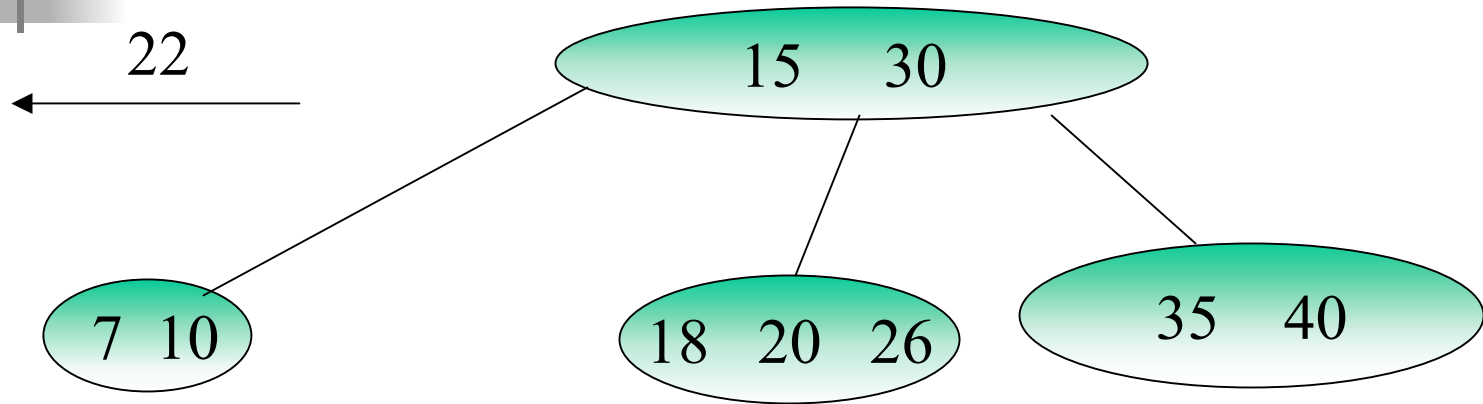


## Deletion examples ...





## Deletion examples ...



Delete 26 , 7 , 35 , 15