PCA - Principal Component Analysis

Table of Contents

What is PCA? - Intro

Given n dimensional data sample $\{x_i : \ i = 1, \ 2, \ ..., \ m\}$ from sample space X with zero mean, find $k, \ k \leq n$ orthonormal vectors $q_j, \ j = 1, \ 2, \ ..., \ k$ which can represents x_i as linear combination of q_j *'sufficiently' close*

In short, we want to do a *dimensionality reduction*

PCA? - *'Sufficient'?*

We can always represent vector x_i as linear combination of orthonormal basis, i.e. $\{e_j :\ j = 1,\ 2,\ ...,\ m\}$
Now Consider the "error"

$$\mathbf{E}[\ ||x_i\ -\ \langle x_i,\ e_j \rangle\ e_j||^2]$$

It is clear that this error term of e_j for data X is minimized when the variance of projected data on e_j

$$\mathbf{E}[\langle x_i,\ e_j \rangle^2]$$

is maximized

PCA? - Toy Example

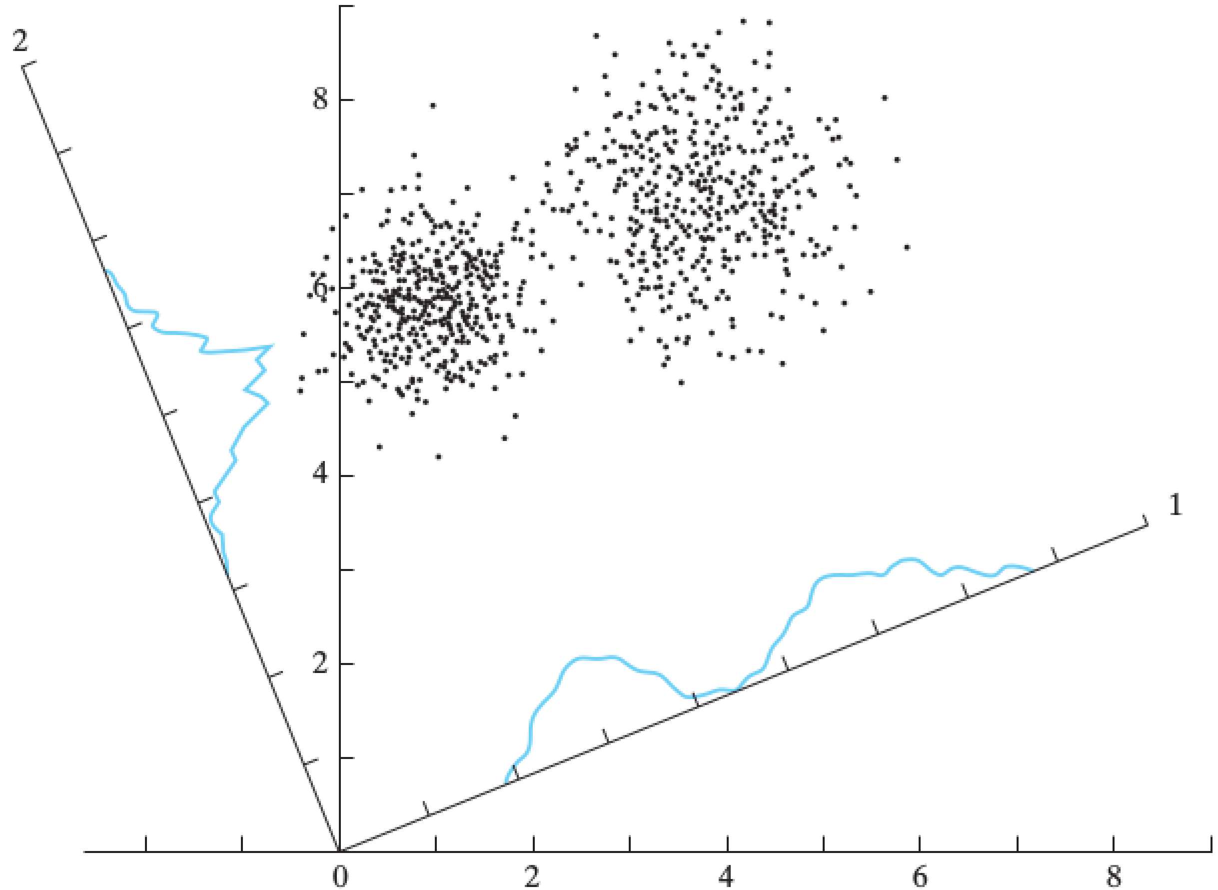


FIGURE 8.4 A cloud of data points is shown in two dimensions, and the density plots formed by projecting this cloud onto each of two axes, 1 and 2, are indicated. The projection onto axis 1 has maximum variance and clearly shows the bimodal, or clustered, character of the data.

PCA? - Eigenvalue Problem

TODO: maximize the varience of projected data on q_j restricted by its norm, $||q_j|| = 1$

Thus, we have to solve

$$J(q_j) = \mathbf{E}[\langle x_i,\ q_j \rangle^2] - \lambda \langle q_j,\ q_j \rangle$$

$$\nabla_{q_j} J(q_j) = 0$$

Solving this equation we get

$$Rq_j = \lambda q_j$$

where R is a covariance matrix for X

(explicit derivation would be given at ML SiG)

PCA? - Remarks

Typically, solving eigenvalue problem is done by SVD or eigendecomposition(spectral decomposition)
However, I don't want to treat them in this SiG (actually I can't)

There is a probabilistic model nearly equivalent to PCA, which can enable us to select a good value for k (google Minka's MLE)

It is fruitful to know *factor analysis* also

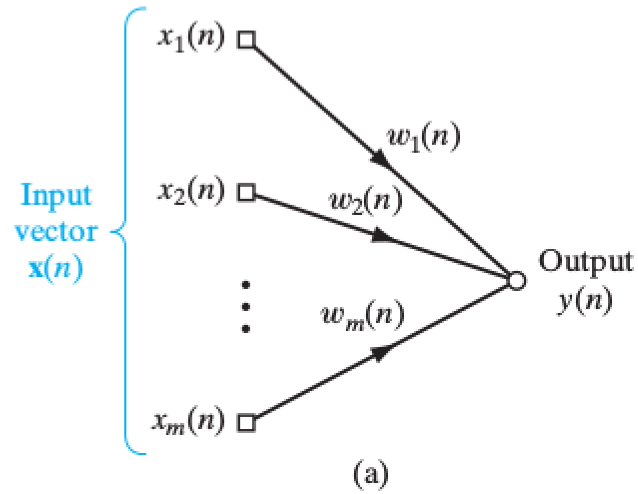
Code Implementation

Neural Network for PCA - Intro

There are four principles for self-organization, namely

1. Self-amplification (Hebbian Learning)
2. Competition
3. Cooperation
4. Structural Information

NN, PCA - Maximum Eigenfilter



Now consider a neural network consist of a single output neuron,

$$y(n) = \sum_{i=1}^{m} w_i(n) x_i(n)$$

with a Hebb's rule

$$\Delta w_i(n) = \eta y(n) x_i(n)$$

NN, PCA - Maximum Eigenfilter (conti.)

However, using Hebb's rule without competition, weigths eventually diverge

Thus, we restrain weight vector $||\mathbf{W}(n)|| = 1$ and updating rule becomes

$$w_i(n+1) = \frac{w_i(n) + \eta y(n) x_i(n)}{(\sum_{j=1}^{m} (w_j(n) + \eta y(n) x_i(n))^2)^{1/2}}$$

Assuming $\eta \ll 1$, we get

$$w_i(n+1) = w_i(n) + \eta y(x_i(n) - y(n) w_i(n))$$

NN, PCA - Maximum Eigenfilter (conti.)

It is pretty hard to show that

$$\mathbf{W}(n) \to q_1,\ \mathbf{E}[y^2] \to \lambda_{max}\ as\ n \to \infty$$