1 Problem 47, part(a)

The choosen function are (c) and (d) in the problem $6-16$. In case of using Runge-Kutta method, the error bound of $\max_{x\in[a,b]}|y(x)-y_h(x)|$ from above can be written as $\frac{1}{L}\left(\exp\left((b-a)L\right)-1\right)\tau(h)$, where

- $(a,b)=(0,4)$,
- $L=1$ is the Lipschitz constant of this function,
- h is the step size for the Euler method,
- $\tau(h)=\max_{x_n}\left|\frac{1}{h}\left(y(x_{n+1})-y(x_n)\right)-F(x_n,y(x_n),h;f)\right|$,
- $F(x_n,y(x_n),h;f)=\frac{1}{6}\left(V_1+2V_2+3V_3+V_4\right)$ defined by the Runge-Kutta method.

We confirmed that the numerical errors are bounded by the theoretical estimations as shown in the tables.
We also compared the numerical results with the Runge-Kutta method and the Trapezoidal method obtained in the problem 6-17. In all cases, the values of $\max_{x\in[a,b]}|y(x)-y_h(x)|$ with Runge-Kutta are smaller than the ones with the Trapezoidal method. However, even if we use the Runge-Kutta method, the approximations fail when the step size h is not small enough (for example, $h=.5$).

2 Problem 47, part(b)

We compare the numerical results with the Runge-Kutta method to the fourth order Taylor method obtained in the problem 6-46. When we choose $h = .5$, the results with the Taylor method does not seem to converge correctly. On the other hand, the Runge-Kutta method works even in the case of $h = .5$. The theoretical bounds from above by the Richardson extrapolation are added in the rightmost columns on the tables.