TRIGONOMETRIC IDENTITIES

• Reciprocal identities

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u}$$

$$\tan u = \frac{1}{\cot u} \quad \cot u = \frac{1}{\tan u}$$

$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u}$$

• Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1$$
$$1 + \tan^2 u = \sec^2 u$$
$$1 + \cot^2 u = \csc^2 u$$

• Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

• Co-Function Identities

$$\sin(\frac{\pi}{2} - u) = \cos u \quad \cos(\frac{\pi}{2} - u) = \sin u$$

$$\tan(\frac{\pi}{2} - u) = \cot u \quad \cot(\frac{\pi}{2} - u) = \tan u$$

$$\csc(\frac{\pi}{2} - u) = \sec u \quad \sec(\frac{\pi}{2} - u) = \csc u$$

• Parity Identities (Even & Odd)

$$\sin(-u) = -\sin u \quad \cos(-u) = \cos u$$

$$\tan(-u) = -\tan u \quad \cot(-u) = -\cot u$$

$$\csc(-u) = -\csc u \quad \sec(-u) = \sec u$$

• Sum & Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

• Double Angle Formulas

$$\sin(2u) = 2\sin u \cos u$$

$$\cos(2u) = \cos^2 u - \sin^2 u$$

$$= 2\cos^2 u - 1$$

$$= 1 - 2\sin^2 u$$

$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

• Power-Reducing/Half Angle Formulas

$$\sin^{2} u = \frac{1 - \cos(2u)}{2}$$
$$\cos^{2} u = \frac{1 + \cos(2u)}{2}$$
$$\tan^{2} u = \frac{1 - \cos(2u)}{1 + \cos(2u)}$$

• Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

• Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2} \left[\cos(u - v) - \cos(u + v) \right]$$

$$\cos u \cos v = \frac{1}{2} \left[\cos(u - v) + \cos(u + v) \right]$$

$$\sin u \cos v = \frac{1}{2} \left[\sin(u + v) + \sin(u - v) \right]$$

$$\cos u \sin v = \frac{1}{2} \left[\sin(u + v) - \sin(u - v) \right]$$