MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

I SEM - Engg.Mathematics I
MAT -1151 (I sessional)

Time: 1 Hr. **Date: 07.09.2019** **Time: 4.15PM-5.15PM** **Max.Marks: 15**

Q1. Integrating factor for the equation $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ is

1. $\frac{-1}{x+1}$ 2. ** $\frac{1}{x+1}$ 3. $\frac{1}{x}$ 4. $\frac{-1}{x}$

Q2. If y= $e^{3x}cosx$ is a solution to $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + ky = 0$, the value of k is

1. **10 2. -10 3. 6 4. 7

Q3. $\nabla^3 y_4 =$ ________

1. **$\Delta^3 y_1$ 2. $\nabla^2 y_4$ 3. $\Delta^4 y_3$ 4. $\Delta^2 y_3$

Q4. The value of $\Delta^9((1-2x^3)(1+8x^2)(1+4x^4))$ with $h=1$ is

1. $9! \times 64 \times x^9$ 2. Zero 3. **$9! \times (-64)$ 4. 9!

Q5. The particular integral of the differential equation $y'' + 4y = cos2x$ is

1. $\frac{sin2x}{2}$ 2. $\frac{xsin2x}{2}$ 3. $\frac{xcos2x}{2}$ 4.** $\frac{xsin2x}{4}$

Q6. The solution of $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$ is

1. $C_1e^{-3t} + C_2e^{3t}$ 2. $C_1e^{3t} + C_2te^{-3t}$ 3. ** $C_1e^{-3t} + C_2te^{-3t}$ 4. $(C_1 + C_2t)e^{3t}$

Q7. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = c\frac{d^2y}{dx^2}$ is

1. 1 2. **2 3. 3 4. 4

Q8. The Wronskian of the equation $y'' - 2y' + 1 = (x+1)e^{2x}$ is

1. xe^x 2. e^{2x} 3. xe^{2x} 4. ** $2e^{2x}$

Q9. If h is the interval of differences, then $(\Delta - \nabla)x^2$ equals to

1. 2h 2. **$2h^2$ 3. $2h^3$ 4. h^4

Q10. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is

1. $y = logx + x$ 2. $y = xe^{(x-1)}$ 3. $y = logx + x^2$ 4.**$y = xlogx + x$

Q11. Use Lagrange's interpolation formula for the following data to evaluate the value of x when y = 20.

x	1	2	3	4
y	1	8	27	64

Solution: Using inverse Lagrange's formula,

$$x = \frac{12 * -7 * -44}{-7 * -26 * -63} * 1 + \frac{19 * -7 * -44}{7 * -19 * -56} * 2 + \frac{19 * 12 * -44}{26 * 19 * -37} * 3 + \frac{19 * 12 * -7}{63 * 56 * 37} * 4.$$

$y(20) = 2.8462.$ (2M)

Q12. Solve the differential equation $y'' + y = \frac{1}{1+sinx}$ using the method of variation of parameter.

Solution: $y_c = c_1 cosx + c_2 sinx$ and $W = 1$ (0.5M)

$$y_p = -y_1 \int y_2 \frac{R(x)}{W} dx + y_2 \int y_1 \frac{R(x)}{W} dx$$

$$y_p = -cosx \int \frac{sinx+1-1}{1+sinx} dx + sinx \int \frac{cosx}{1+sinx} dx$$

$$= -cosx \int 1 - \frac{1-sinx}{\cos^2 x} \; dx + sinx \ln(1+sinx)$$ (0.5M)

$$= -cosx \left\{ x - \int (\sec^2 x - secxtanx) dx \right\} + sinx \ln(1+sinx)$$

$$= -cosx\{x - tanx + secx\} + sinx \ln(1+sinx)$$

$$y_p = -xcosx + sinx - 1 + sinx \ln(1+sinx)$$ (0.5M)

$$y = y_c + y_p$$ (0.5M)

Q13. Solve $x^2 y'' - 3xy' + 4y = \sin(logx)$.

Solution: Put $x = e^t, t = logx$. (0.5M)

$(D(D-1) - 3D + 4)y = sint$

$y_c = (C_1 + C_2 t)e^{2t}$ (0.5M)

$y_p = \frac{1}{D^2-4D+4} sint = \frac{1}{3-4D} sint = \frac{(3+4D)}{9-16D^2} sint = \frac{4cost+3sint}{25}.$ (0.5M)

$y = y_c + y_p$ where $x = e^t$ and $t = logx$. (0.5M)

Q14. Solve the differential equation $x\frac{dy}{dx} = y(logy - logx + 1)$ (2Marks)

Solution: $\frac{1}{y}\frac{dy}{dx} - \frac{logy}{x} = \frac{1-logx}{x}$

Put logy = t , $\frac{1}{y} = e^{-t}$ (0.5M)

then $\frac{dt}{dx} - \frac{1}{x}t = \frac{1-logx}{x}$, I.F = $\frac{1}{x}$ (0.5M)

Solution, $\frac{1}{x}$ t = $\int(\frac{1}{x^2} - \frac{logx}{x^2})$dx

$\frac{logy}{x} = \frac{-1}{x}$ - $\int ve^{-v}\,dv$, by putting log x =v

$\frac{logy}{x} = \frac{-1}{x} + \frac{logx}{x} + \frac{1}{x}$ +c

log $(\frac{y}{x})$ = cx. (1M)

Q15. From the following table estimate the number of students who obtained marks between 40 and 45

Marks	30-40	40-50	50-60	60-70	70-80
No. Students	31	42	51	35	31

Solution: Using Forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

(1M)

The value of $p = \frac{45-40}{10} = 0.5$

By Using Appropriate interpolation formula

$y\,(\,45\,) = 47.8672 = 48$ (0.5M)

The number of students obtained marks between 40 and 45 is $48 - 31 = 17.$

(0.5M)