MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

I SEM - Engg.Mathematics I
MAT -1151 (II sessional Scheme)

Q1. An interval that contains a root of the equation $xe^x = 3$ is (0.5)

1. [0, 1] 2. **[1, 1.1] 3. [1.1, 1.2] 4. [1.2, 1.3]

Q2. Using Runge-Kutta method of second order, the value of $y(0.1)$ for $\frac{dy}{dx} = y - x$ given $y(0) = 2, h = 0.1$ is (0.5)

1. 0.2 2. 0.21 3. 0.2050 4. **2.2050

Q3. If $(x_0, y_0) = (1,3), (x_1, y_1) = (3,4)$ and $(x_2, y_2) = (4,5)$. Then second divided difference of the arguments (x_0, x_1, x_2) is (0.5)

1. **$1/6$ 2. $3/2$ 3. 0 4. $1/2$

Q4. The rank of a non-singular matrix of order 5 is (0.5)

1. 0 2. > 5 3. **5 4. < 5

Q5. The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 8 & 9 \end{bmatrix}$ is (0.5)

1. 3 2. 1 3. 0 4. **2

Q6. Cube root of 14 using Newton Raphson method (one iteration) with $x_0 = 2$ is (0.5)

1. 2 2. **2.5 3. 0 4. 1.6666

Q7. The minimum no. of iteration required to approximate the root of $x^5 - 3x^2 + 1$ in [1,2] to get an accuracy of 0.05 using bisection method is (0.5)

1. 3 2. **5 3. 1 4. 2

Q8. We can conclude surely that a real root of $f(x) = 0$ lies between x_0 and x_1 if, (0.5)

1. $f(x_0) + f(x_1) < 0$
2. $\frac{f(x_0)}{f(x_1)} > 0$
3. $f(x_0)f(x_1) > 0$
4. ** $f(x_0)f(x_1) < 0$

Q9. Given that $\int_0^2 y\,dx = 8$, when evaluated using Trapezoidal Rule with

h = 1, if y(0) = 2 and y(2) = 4, then y(1) is (0.5)

1. 7   2. 6   3. **5   4. 3

Q10. The $(i,j)^{th}$ entry of the inverse of the Identity matrix, $i \neq j$ is (0.5)

1. **0   2. 1   3. 2   4. 3

Q11. Test for consistency and solve

$5x + 3y + 7z = 4,\ \ 3x + 26y + 2z = 9,\ \ 7x + 2y + 10z = 5.$ (2)

$$\text{Writing } [A:B] = \begin{bmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{bmatrix}$$

$$R_2 \to R_2 - \frac{3}{5}R_1 \qquad \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & \frac{121}{5} & \frac{-11}{5} & \frac{33}{5} \\ 7 & 2 & 10 & 5 \end{bmatrix}$$

$$\mathrm{R}_3 \to \mathrm{R}_3 - \frac{7}{5}\mathrm{R}_1 \qquad \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & \frac{121}{5} & \frac{-11}{5} & \frac{33}{5} \\ 0 & \frac{-11}{5} & \frac{1}{5} & \frac{-3}{5} \end{bmatrix} \qquad (0.5\mathrm{M})$$

$$R_2 \to R_2\left(\frac{5}{121}\right) \text{ and } R_3 \to R_3\left(\frac{5}{11}\right) \qquad \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 1 & \frac{-1}{11} & \frac{3}{11} \\ 0 & -1 & \frac{1}{11} & -\frac{3}{11} \end{bmatrix} \qquad (0.5\mathrm{M})$$

$$\text{We have, } \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 1 & \frac{-1}{11} & \frac{3}{11} \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (0.5\mathrm{M})$$

Thus, $r(A) = r(A:B) = 2$. System is consistent and has infinitely many solutions. Let $z = k$, Correspondingly we have $x = \frac{7-16k}{11}, y = (3+k)/11$.

(0.5M)

Q12. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's 3/8 rule (Take h=1). (2)

x	0	1	2	3	4	5	6
f(x)	1	0.5	1/5	1/10	1/17	1/26	1/37
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

(1M)

$\int_0^6 \frac{dx}{1+x^2} = 1.35708$ (1M)

Q13. The population of a certain town is shown in the following table;

year	1951	1961	1971	1981	1991
Population in thousand	19.96	39.65	58.81	77.21	94.61

Estimate the rate of growth of the population in the year 1951. (2)

Soln:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1951	19.96				
		19.69			
1961	39.65		-0.53		
		19.16		-0.23	
1971	58.81		-0.76		-0.01
		18.4		-0.24	
1981	77.21		-1		
		17.4			
1991	94.61				

(1M)

BY Newton's forward difference formulae

$$\left(\frac{dy}{dx}\right)_{x=1951} = 1.98808$$ (1M)(

Q14. Using Runge-Kutta method of fourth order, taking $h = 0.2$, compute $y(0.2)$, given $\frac{dy}{dx} = 3x + \frac{y}{2}$ $and\ y = 1, when\ x = 0$. (2)

Solution: $k_1 = 0.1, k_2 = 0.165, k_3 = 0.1683, k_4 = 0.2368$ (1M)
$k = 0.1672, y_1 = y(0.2) = y_0 + k = 1.1672.$ (1M)

Q15. Find the root of the equation $xe^{-x} = \cos x$ using the Regula – falsi method correct to four decimal places. (2)

Solution: $f(x)=xe^{-x} - \cos x$

f(0)=-1

f(1)= -0.1724 f(2) =0.68682. Root lies between [1, 2] (0.5M)

x1=1.200066 (0.5M)

x2=1.2011 (0.5M)

x3=1.2011 (0.5M)

OR

Marks can be given even for the selection of different intervals containing the root, and four iterations.