A Lazy Precalculus Refresher

Slava Akhmechet

September 16, 2024

As I'm working through Spivak Calculus I'm discovering there is a ton of high school math I've forgotten (or simply never learned). The purpose of these notes is to relearn the necessary prerequisites as quickly as possible.

For these notes I pull in material "lazily". There is no predefined set of topics; nothing that "should" or "shouldn't" be a part of precalculus. I don't want to go through a 500 page book or take a year long class, so when I need to know something to get through Spivak's chapter but don't already know it, it goes into these notes.

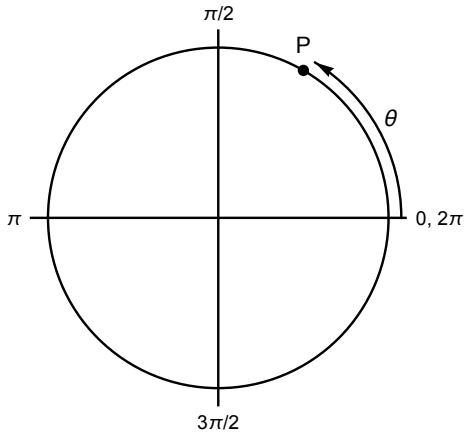
When I do need to (re-)learn a topic, I'm mostly pulling material from "Paul's Online Notes" excellent Algebra Trig Review. He definitely nails the topic list– the stuff his calculus students forgot (or never learned) lines up really well with what I need.

Contents

1 Trigonometry

1.1 The Basics

Let P be a point on a unit circle $x^2+y^2=1$. Let θ be the length of the arc from $(1,0)$ to P, measured counterclockwise along the circle. Then the coordinates of P are $(\cos\theta, \sin\theta)$.[1]



The measure of angles by the length of the arc is in units called *radians*. Recall the circumference of a circle is $C = 2\pi r$, and so the circumference of a unit circle is 2π. Thus π represents a 180° angle. Some common angles in radians are $2\pi, \pi, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$, and $\frac{3\pi}{2}$. To convert these to degrees simply replace π with 180, and compute the fraction.

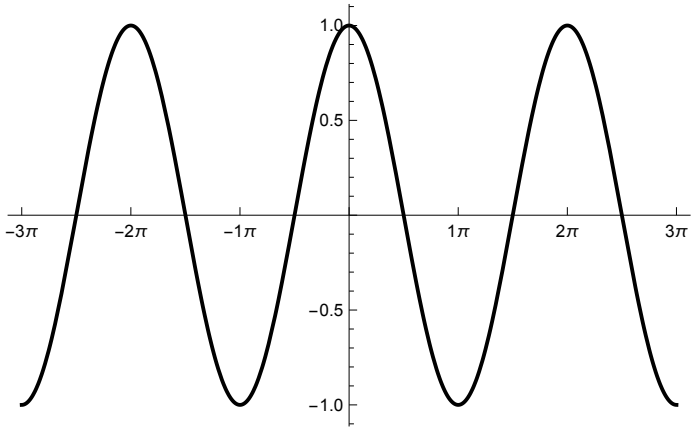
It should be self-evident that adding 2π to an angle results in the angle itself; and that adding $\frac{\pi}{2}$ to an angle shifts it by 90°. Further:

$$(\cos 0, \sin 0) = (1,0) \qquad \text{and} \qquad (\cos\frac{\pi}{2}, \sin\frac{\pi}{2}) = (0,1)$$

[1] The order is easy to remember– it's alphabetical.

1.2 Plotting

It is not too difficult to plot trigonometric functions. Consider some properties of cosine we've already seen (or can easily deduce): $\cos 0 = 1$, $\cos \frac{\pi}{2} = 0$, $\cos \pi = -1$. We've also seen that $\cos(x + 2\pi) = \cos x$. The x-axis below covers $[-3\pi, 3\pi]$ (i.e. a total length of 6π). Since cosine repeats every 2π, we should expect the graph to repeat thrice. And this is exactly what we see.



We can easily increase the frequency by plotting $y = \cos cx$. Here we double the frequency with $c = 2$: