
Algorithm 1 Algorithm for SpLoC_test

Require: model

$sz \leftarrow r \times (r - 1)/2$

$Ustat \leftarrow \text{zeros}(sz, 1)$

for i=1: sz **do**

$Ustat(i) \leftarrow lutein_PCA * brain_features^T / (SSE) * (n - 1)$

end for

$U, V \leftarrow \text{Permutation_test}()$

$T_burden \leftarrow (\text{sum}(Ustat)^2) / (\text{sum}(\text{sum}(V)))$

$prob \leftarrow 0.3$

$ver_nbd \leftarrow \text{Binomial}(1, prob * \text{ones}(r))$

$pair_nbd \leftarrow \text{functionaloverlap}(ver_nbd)$

$func_overlap \leftarrow pair_nbd$

$func_dist \leftarrow 1 / func_overlap$

$\text{Gamma} \leftarrow [0.51, 21.52, 5510]$

for rad=1:size(Gamma,2) **do**

for k=1:sz **do**

$w \leftarrow rnn(k, \text{Gamma}(\text{rad}), func_dist)$

if (**then** $w.' * V * w > 0$

$test_stat(k, \text{rad}) \leftarrow (w.' * Ustat)^2 / (w.' * V * w)$

else

$test_stat(k, \text{rad}) \leftarrow -1$

end if

end for

end for

$Sp_stat \leftarrow \max(\max(test_stat))$

T_perm gets $\max(\max(\text{permuted_test_stat}))$

Algorithm 2 Algorithm to Calculate Type 1 error

Require: model

```
 $r \leftarrow 4$   
 $c \leftarrow r$   
 $h \leftarrow 21$   
 $Iter \leftarrow 100$   
 $count \leftarrow 0$   
  
for  $i \leq Iter$  do  
   $Y_{sim} \leftarrow \text{Multivariate Normal}(0, I)_{h \times 1}$   
   $demographics\_Table\_null.lutein\_PCA1 \leftarrow Y_{sim}$   
   $SSE, n \leftarrow BWAS()$   
   $pval \leftarrow SpLoc\_test()$   
  if  $pval \leq 0.05$  then  
     $count \leftarrow count + 1$   
  end if  
end for
```

Algorithm 3 Algorithm to Calculate Type 2 error

Require: model

```
 $r \leftarrow 4$   
 $c \leftarrow r$   
 $h \leftarrow 21$   
 $Iter \leftarrow 100$   
 $count \leftarrow 0$   
 $\beta_0 \leftarrow (3, 3, 3, 0, 0, 0)$   
  
for  $i \leq Iter$  do  
   $Y_{sim} \leftarrow brain\_features * \beta_0 + \text{Multivariate Normal error}(0, I)_{h \times 1}$   
   $demographics\_Table\_null.lutein\_PCA1 \leftarrow Y_{sim}$   
   $SSE, n \leftarrow BWAS()$   
   $pval \leftarrow SpLoc\_test()$   
  if  $pval \leq 0.05$  then  
     $count \leftarrow count + 1$   
  end if  
end for
```

Type 1 error for 500 Iterations=0.0540

Type 2 error for 500 Iterations with $\beta = (1, 1, 1, 0, 0, 0) = 0.5200 / 0.5600$

Type 2 error for 500 Iterations with $\beta = (2, 2, 2, 0, 0, 0) = 0.04$

Type 2 error for 500 Iterations with $\beta = (3, 3, 3, 0, 0, 0) = 0$

Type 2 error for 100 Iterations with $\beta \sim 3^*$ Erdos Renyi (4, 0.2) = 0.05
Type 2 error for 500 Iterations with $\beta \sim 3^*$ Erdos Renyi (4, 0.2) = 0.1840
Type 2 error for 500 Iterations with $\beta \sim 3^*$ Erdos Renyi (4, 0.4) = 0.1840

no of ROIs(r) height	error
4	0.0800
20	0.2029

Table 1: Type 1 error for different sizes with 500 Iterations

no of ROIs(r)	error
5	0.12
10	0.0800

Table 2: Type 1 error for different sizes with 100 Iterations

no of ROIs(r)	error
5	0.06

Table 3: Type 1 error for different sizes with 200 Iterations

p	error
0.08	0.0867
0.1	
0.2	0.1467 0.4
0 0.6	0.8
1	

Table 4: Type 2 error for r=5 for different probabilities of the the ER random graph and 200 Iterations

1 Errors under different set-ups

r	Iter	type1	Type 2	Type 2	ER-prob=0.08	ER-prob=0.005
15	100	0.1	0.63		0.91	

Table 5: Caption