RATIONAL FUNCTION

OBJECTIVES

- ❑ REPRESENTS A RATIONAL FUNCTION THROUGH ITS:
  - TABLE OF VALUES
  - GRAPHS
  - EQUATION
- ❑ IDENTIFIES INTERCEPTS, ZEROS, AND ASYMPTOTES THROUGH GIVEN FUNCTIONS AND GRAPHS

REVIEW!!!

GIVE THE DOMAIN & RANGE OF THE FOLLOWING RATIONAL FUNCTION

$$\mathrm{f(x)} = \frac{3x-2}{x+5}$$

$$x+5 \neq 0$$

$$x \neq -5$$

D: $\{x/x$ is a set of all real numbers, $x \neq -5\}$

$$\mathrm{y}(x+5) = 3x+2$$

$$\mathrm{xy} + 5y = 3x+2$$

$$\mathrm{xy} - 3x = -5y+2$$

$$\frac{x(y-3)}{y-3} = \frac{-5y+2}{y-3}$$

$$x = \frac{-5y+2}{y-3}$$

$$y-3 \neq 0$$

$$y \neq 3$$

R: $\{y/y$ is a set of all real numbers, $x \neq 3\}$

REVIEW!!! GIVE THE DOMAIN & RANGE OF THE FOLLOWING RATIONAL FUNCTION

$$f(x) = \frac{x-4}{3x+5}$$

$$3x + 5 \neq 0$$

$$\frac{3x}{3} \neq \frac{-5}{3}$$

$$x = -\frac{5}{3}$$

D: x/x is a set of real numbers, $x \neq -5/3\}$

$$y(3x + 5) = x - 4$$

$$3xy + 5y = 3x - 4$$

$$3xy - 3x = -5y - 4$$

$$x(3y - 3) = -5y - 4$$

$$\frac{x(3y - 3)}{3y - 3} = \frac{-5y - 4}{3y - 3}$$

$$x = \frac{-5y - 4}{3y - 3}$$

$$3y - 3 \neq 0$$

$$3y \neq 3$$

$$\frac{3y}{3} \neq \frac{3}{3}$$

$$y \neq 1$$

R: y/y is a set of real numbers, $y \neq 1\}$

REPRESENTATION OF RATIONAL FUNCTION

THROUGH ITS

TABLE OF VALUES

GRAPHS

EQUATIONS

Represents the following Rational function through table of values

A

$$f(x) = \frac{1}{2x}$$

B

$$g(x) = \frac{x}{x-1}$$

C

$$h(x) = \frac{2}{x+1}$$

A

$$f(x) = \frac{1}{2x}$$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
f(x)	$-\frac{1}{10}$	$-\frac{1}{8}$	$-\frac{1}{6}$	$-\frac{1}{4}$	$-\frac{1}{2}$	Und.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$

TABLE OF VALUES

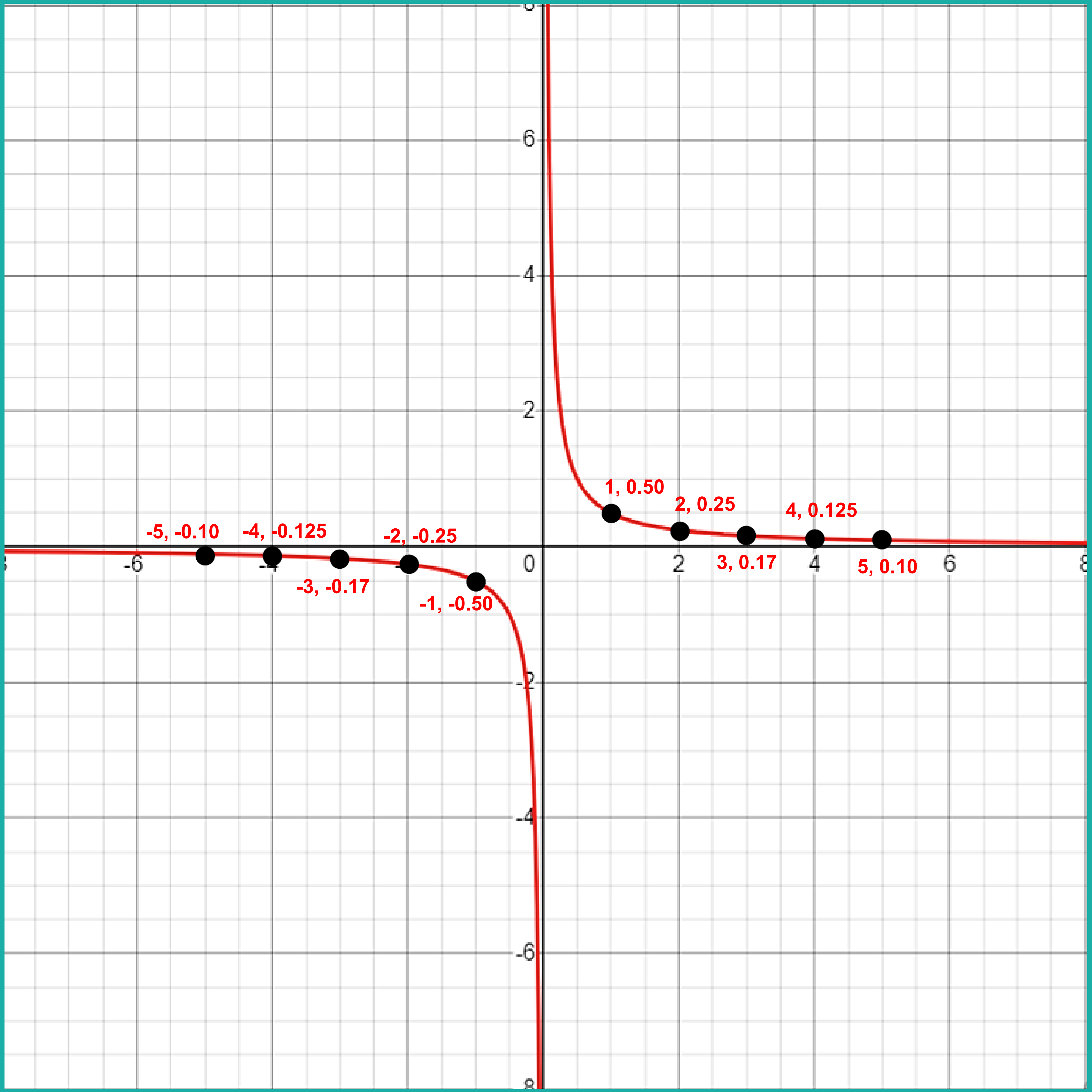
$$f(x) = \frac{1}{2(-5)}$$
$$f(x) = \frac{1}{-10}$$

$$f(x) = \frac{1}{2(-4)}$$
$$f(x) = \frac{1}{-8}$$

$$f(x) = \frac{1}{2(0)}$$
$$f(x) = \frac{1}{0} = 0$$

$$f(x) = \frac{1}{2(1)}$$
$$f(x) = \frac{1}{2}$$

$$f(x) = \frac{1}{2(2)}$$
$$f(x) = \frac{1}{4}$$



1, 0.50
2, 0.25
3, 0.17
4, 0.125
5, 0.10
-1, -0.50
-2, -0.25
-3, -0.17
-4, -0.125
-5, -0.10

TABLE OF VALUES

B $g(x) = \dfrac{x}{x-1}$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
g(x)	$\frac{5}{6}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	0	Und.	2	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$

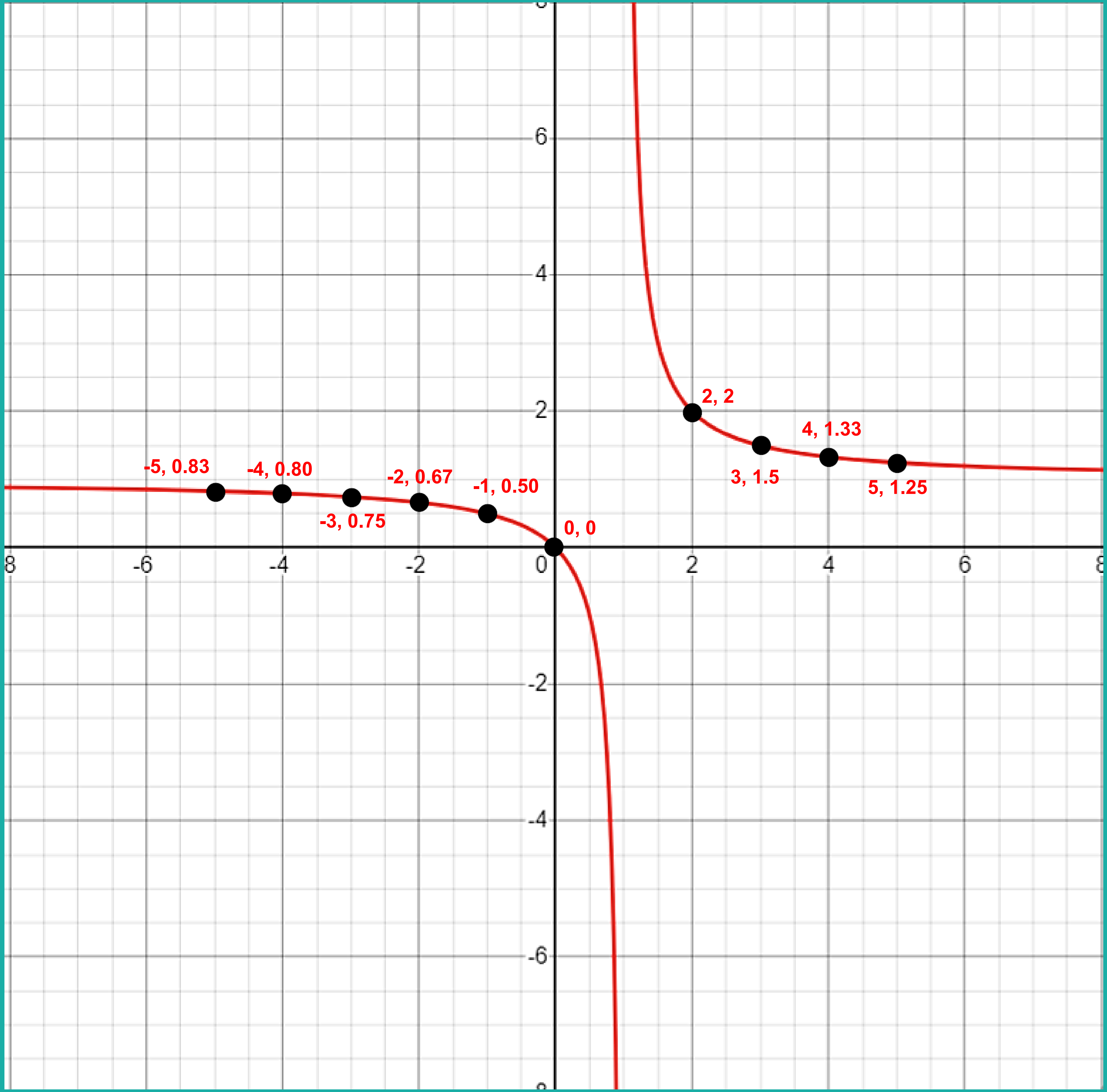
$$g(x) = \frac{-5}{(-5)-1}$$
$$= \frac{-5}{-6} \text{ or } \frac{5}{6}$$

$$g(x) = \frac{-4}{(-4)-1}$$
$$= \frac{-4}{-5} \text{ or } \frac{4}{5}$$

$$g(x) = \frac{0}{(0)-1}$$
$$= \frac{0}{-1} \text{ or } 0$$

$$g(x) = \frac{1}{(1)-1}$$
$$= \frac{1}{0} \text{ or } 0$$

$$g(x) = \frac{2}{(2)-1}$$
$$= \frac{2}{1} \text{ or } 2$$



-5, 0.83
-4, 0.80
-3, 0.75
-2, 0.67
-1, 0.50
0, 0
2, 2
3, 1.5
4, 1.33
5, 1.25

C

$$h(x) = \frac{2}{x+1}$$

TABLE OF VALUES

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
h(x)	$\frac{1}{2}$	$\frac{2}{3}$	-1	-2	Und.	2	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$

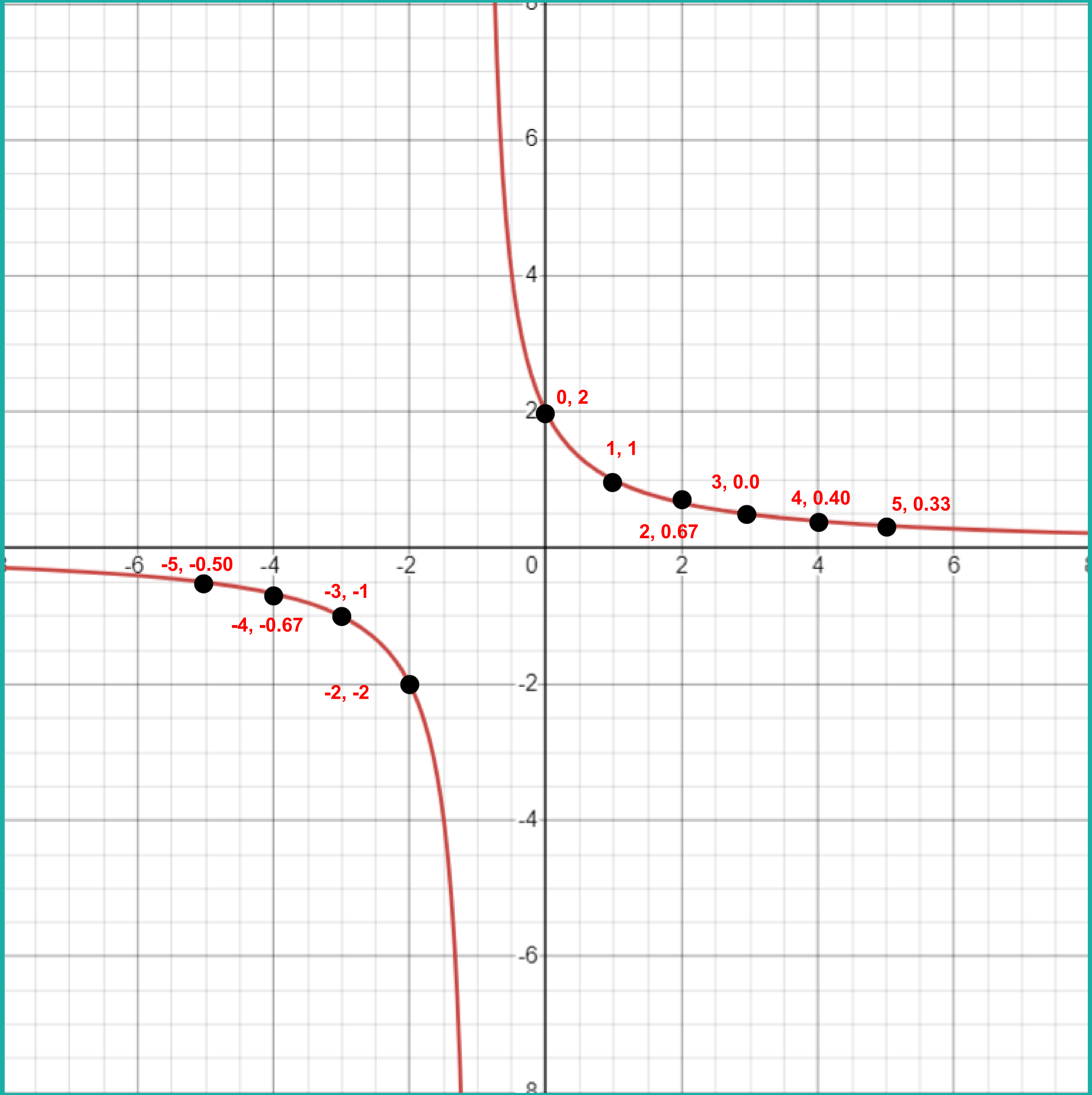
$$h(x) = \frac{2}{(-5)+1} = \frac{2}{-4} \text{ or } -\frac{1}{2}$$

$$h(x) = \frac{2}{(-4)+1} = \frac{2}{-3} \text{ or } -\frac{2}{3}$$

$$h(x) = \frac{2}{(0)+1} = \frac{0}{-1} \text{ or } 0$$

$$h(x) = \frac{2}{(1)+1} = \frac{2}{2} \text{ or } 1$$

$$h(x) = \frac{2}{(2)+1} = \frac{2}{3}$$


0, 2
1, 1
2, 0.67
3, 0.0
4, 0.40
5, 0.33
-5, -0.50
-4, -0.67
-3, -1
-2, -2

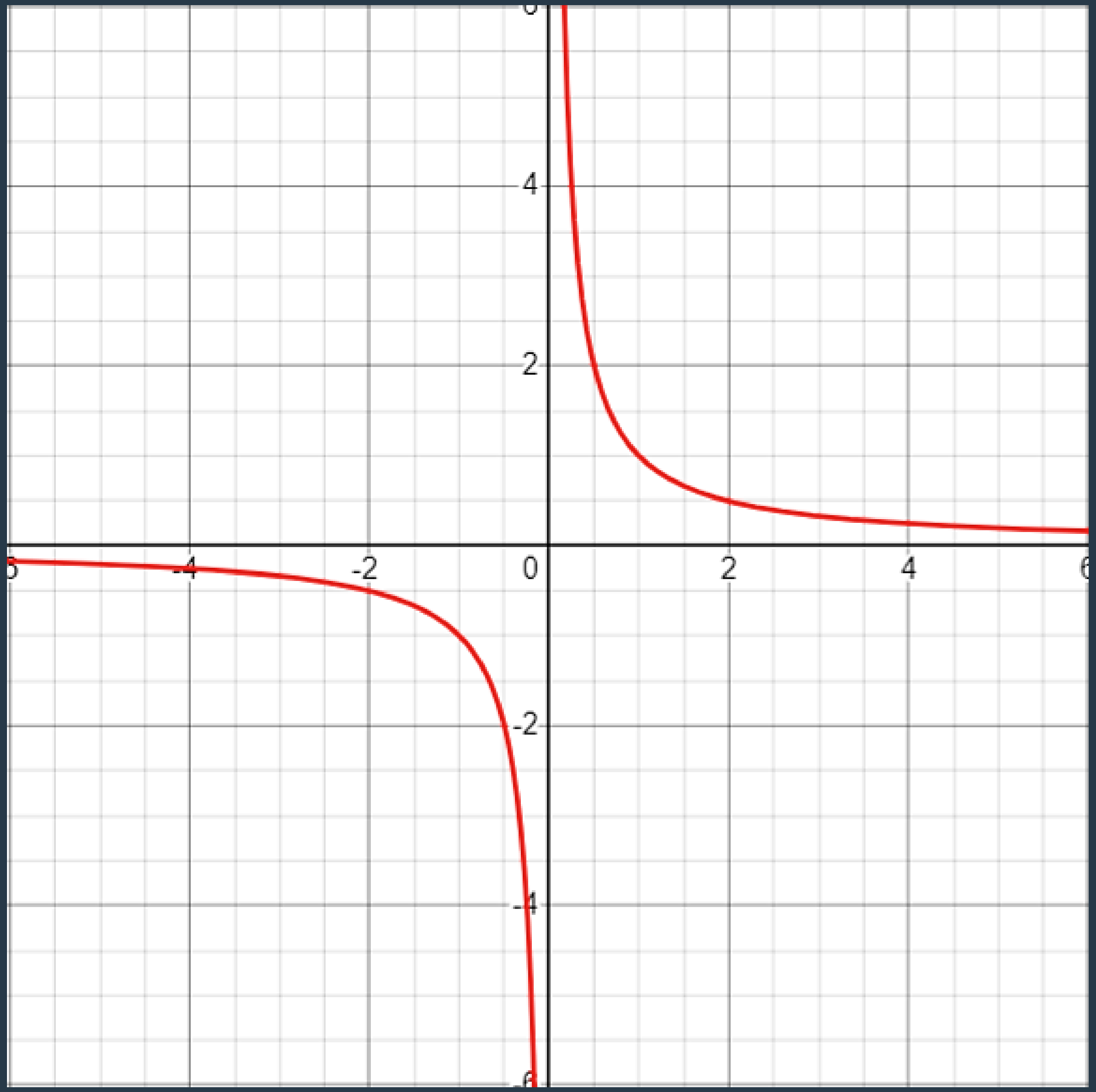
Represents Rational Function through Graphs

GRAPHS

EXAMPLE 1

Represent Each rational function by its graph. (Use Desmos graphing calculator to easily graph the rational functions)

$$f(x) = \frac{1}{x}$$

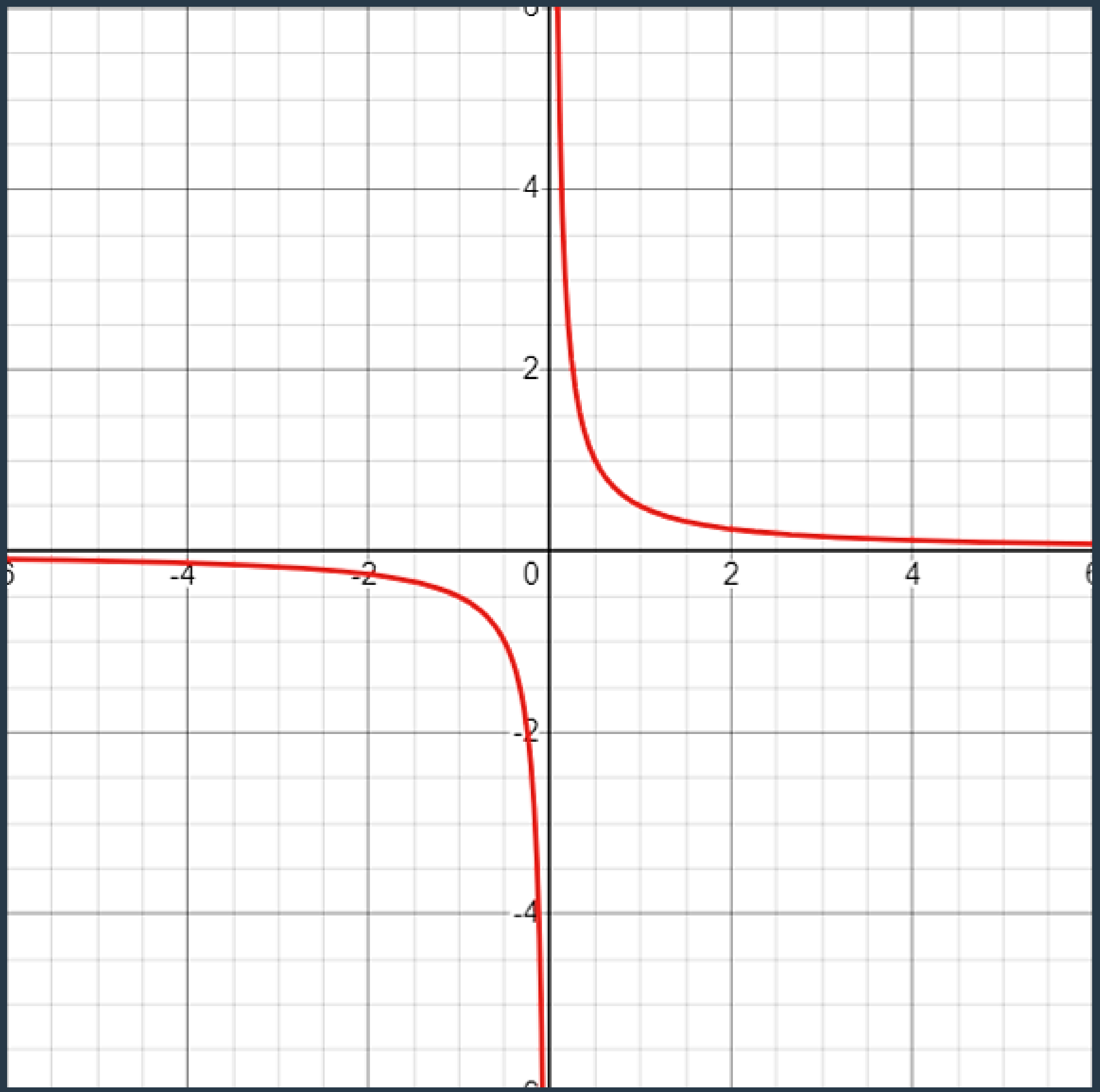


GRAPHS

EXAMPLE 2

Represent Each rational function by its graph. (Use Desmos graphing calculator to easily graph the rational functions)

$$f(x) = \frac{1}{2x}$$

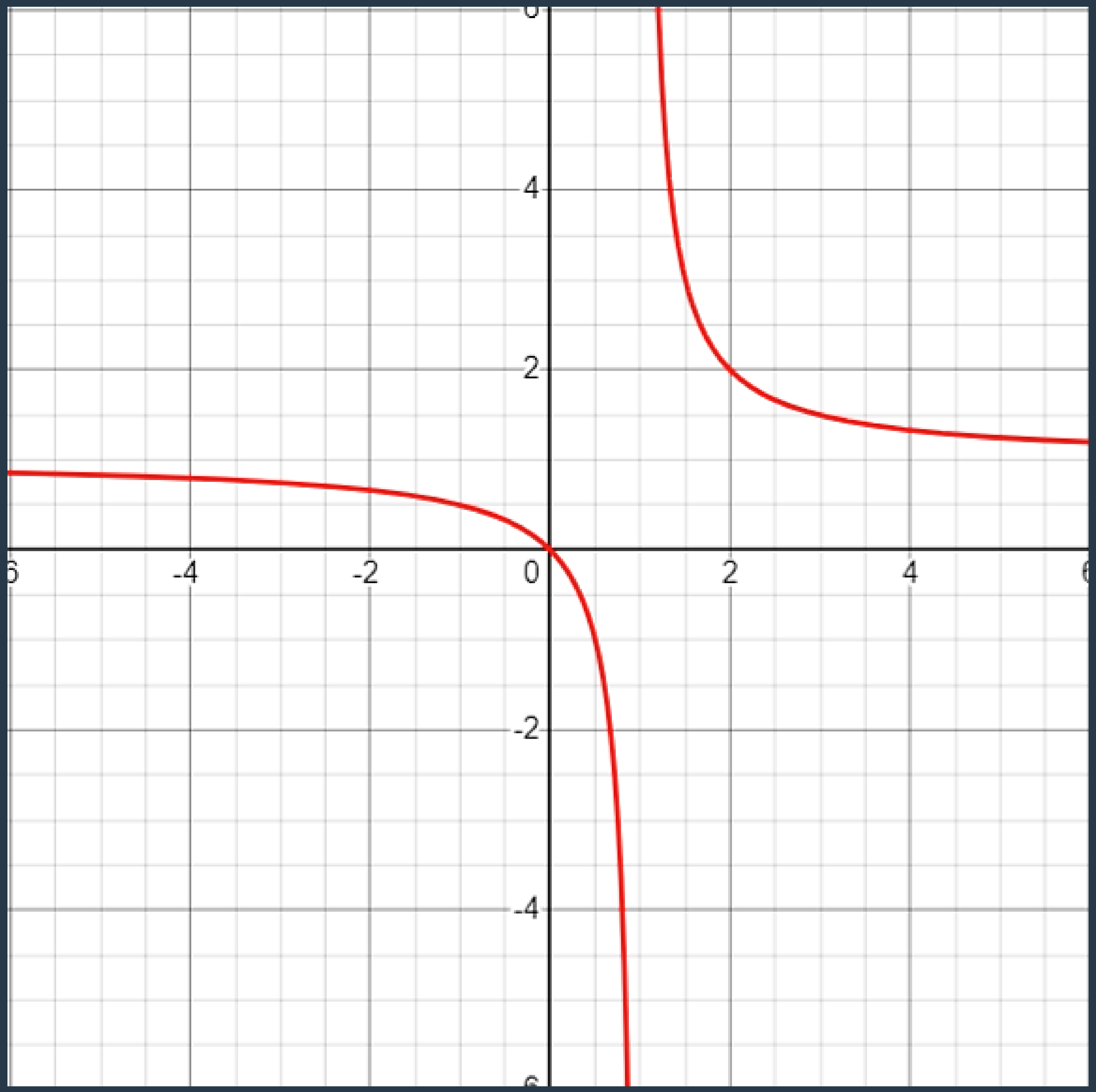


EXAMPLE 3

GRAPHS

Represent Each rational function by its graph. (Use Desmos graphing calculator to easily graph the rational functions)

$$f(x) = \frac{x}{x-1}$$

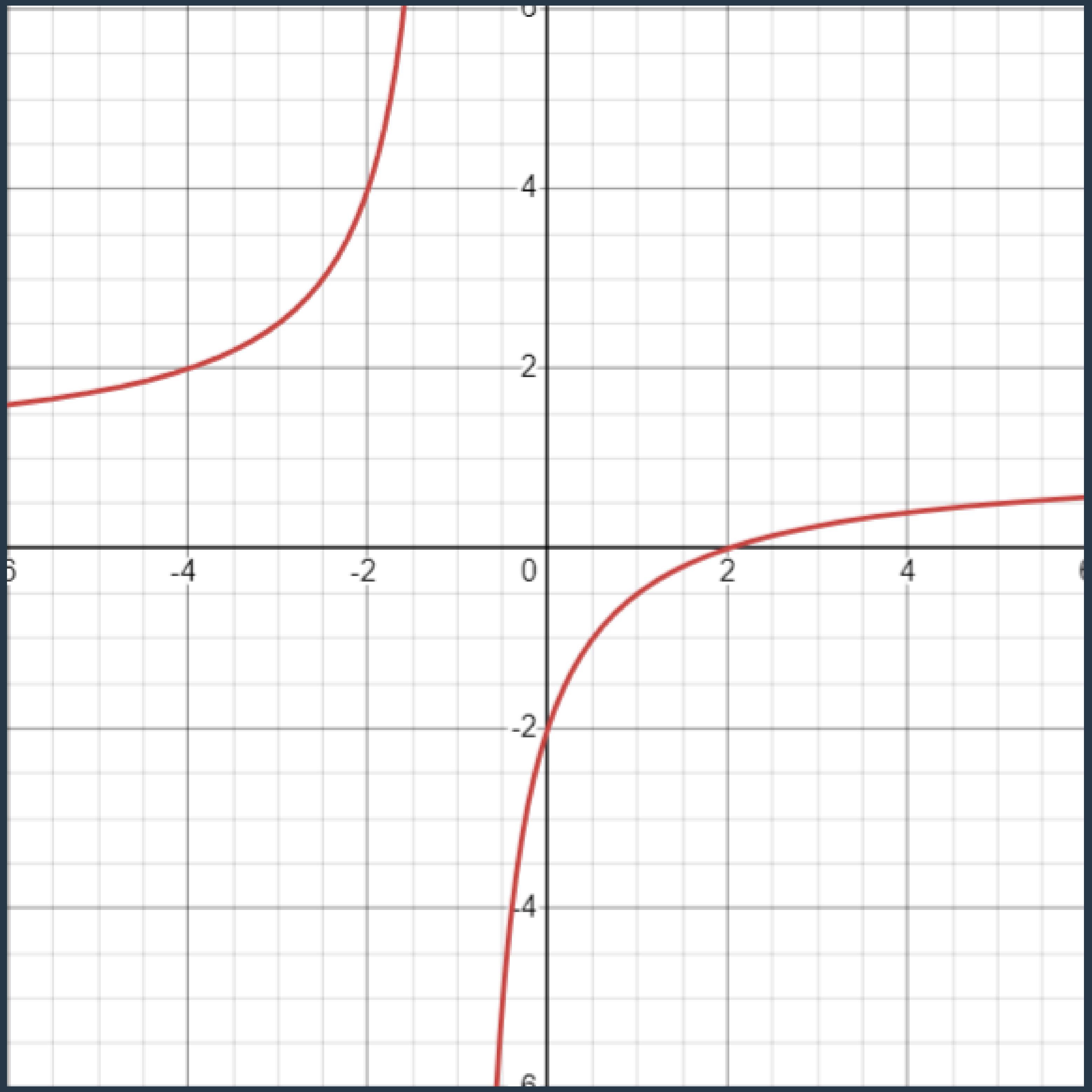


GRAPHS

EXAMPLE 4

Represent Each rational function by its graph. (Use Desmos graphing calculator to easily graph the rational functions)

$$f(x) = \frac{x-2}{x+1}$$



Represents Rational Function through Equations

EQUATIONS

EXAMPLE 1

Represent this problem in a rational function, then answer what is asked

In an inter-barangay basketball league, the team from barangay 1 has won 9 out of 20 games a winning percentage of 45%. What would be their winning percentage if they will win 5 games consecutively.

SOLUTIONS:

LET X BE THE NUMBER OF WINS BARANGAY 1 NEEDS TO WIN IN A ROW. THEN THE FUNCTION P IS A FUNCTION OF THE NUMBER OF WINS THAT THE TEAM NEEDS TO WIN. THE FUNCTION IS

$$p(x) = \frac{9 + x}{20 + x}$$

$$p(x) = \frac{9 + 5}{20 + 5} = \frac{14}{25}$$

$$p(x) = 0.56 \text{ or } 56\%$$

Therefore, the winning percentage of Barangay 1, if they win 5 games in a row is 56%.

EQUATIONS

EXAMPLE 2

Represent this problem in a rational function, then answer what is asked

Consider a 100-meter track used for foot races. The speed of the runner can be computed by taking the time it will take him to run the track. Applying to it the formula of average speed $s = d/t$, what is the rational function represented by the speed as a function of time? What is the speed of the runner in 20 seconds?

SOLUTIONS:

Let x the time it takes the runner to run 100 meters. Then the function S is a function of the time it takes the runner to run 100 meters. The function is

$$s(x) = \frac{100}{x}$$

$$s(20) = \frac{100}{20} = 5 \text{ meter/second}$$

Therefore, the speed of the runner for 20 minutes is 5m/s

EQUATIONS

EXAMPLE 3

Represent this problem in a rational function, then answer what is asked

Let's say you are taking an exam. You already got 18 questions correctly out of 23, which is a grade percentage of 78%. What would be your grade percentage if you got the last 2 consecutive questions correctly?

SOLUTIONS:

Let x be the additional number of consecutive questions correctly answered. Then the function f is a function of the number of questions that you need to answer to get correctly. The function is

$$f(x) = \frac{18 + x}{23 + x}$$

$$f(x) = \frac{18 + 2}{23 + 2} = \frac{20}{25} = 0.8 \text{ or } 80\%$$

therefore, your grade percentage,
if you got the least 2 questions correctly is 80%

SOLVING
THE INTERCEPTS

INTERCEPTS

Intercepts are x or y coordinates of the points at which a graph crosses the x- axis or y-axis respectively.

RULE TO SOLVE FOR THE INTERCEPTS

- To find the y-intercept ,substitute 0 for x and solve for y or $f(x)$.
- To find the x-intercept ,substitute 0 for y or $f(x)$ and solve for x.

Examples and Solutions

Find the x and y intercept of the given rational function

$$a.)\ f(x) = \frac{x+2}{x-6}$$

For **y- intercept** substitute x to 0 and solve for y

$$y = \frac{0+2}{0-6} \qquad y = -\frac{1}{3}$$

$$\boldsymbol{y\ intercept}: \left(\mathbf{0}, -\frac{\mathbf{1}}{\mathbf{3}}\right)$$

For **x- intercept** substitute y to 0 and solve for x

$$0 = \frac{x+2}{x-6}$$

$$0 = x + 2$$

$$\boxed{\boldsymbol{x = -2}} \qquad \boldsymbol{x\ intercept}: (\mathbf{-2}, \mathbf{0})$$

Examples and Solutions

Find the x and y intercept of the given rational function

$$b.\ f(x) = \frac{x^2 + 4x + 4}{x + 2}$$

For **y- intercept** substitute x to 0 and solve for y

$$y = \frac{(x+2)(x+2)}{x+2}$$

$$y = x + 2$$

$$y = 0 + 2$$

$$\boxed{\boldsymbol{y = 2}} \quad \boldsymbol{y\ intercept: (0, 2)}$$

For **x- intercept** substitute Y to 0 and solve for x

$$0 = \frac{(x+2)(x+2)}{x+2}$$

$$0 = x + 2$$

$$\boxed{\boldsymbol{x = -2}}$$

$$\boldsymbol{x\ intercept: (-2, 0)}$$

SOLVING

THE ZEROES OF RATIONAL EQUATIONS

RULE TO SOLVE ZEROES OF THE RATIONAL EQUATION

- Get the factors of both numerator and denominator.
- Identify the restriction. (denominator ≠ 0)
- Identify the values of x that make the numerator equal to zero.
- Identify the zeroes of f(x) or y.

Examples and Solutions

Find the zeroes of the given rational function

a. $f(x) = \frac{x^2-3x}{x-2}$

Get the factors of both numerator and denominator

$$\boxed{f(x) = \frac{x(x-3)}{x-2}}$$

Identify the restriction

$x - 2 = 0$

$\boxed{x = 2}$

Equate to 0 the factors of numerator

$\boxed{x = 0}$

$x - 3 = 0$

$\boxed{x = 3}$

Identify the zeroes:

$\boxed{x = 0}$ $\boxed{x = 3}$

Examples and Solutions

Find the zeroes of the given rational function

$$b.\, f(x) = \frac{x^2 + 5x + 6}{x^2 + 8x + 15}$$

Get the factors of both numerator and denominator

$$\boxed{f(x) = \frac{(x+2)\cancel{(x+3)}}{(x+5)\cancel{(x+3)}}}$$

Identify the restriction

$$x + 5 = 0$$

$$\boxed{x = -5}$$

Equate to 0 the factors of numerator

$$x + 2 = 0$$

$$\boxed{x = -2}$$

Identify the zeroes

$$\boxed{x = -2}$$

SOLVING
THE VERTICAL AND HORIZONTAL ASYMPTOTES

RULE TO FIND THE VERTICAL ASYMPTOTE

- Reduce the rational function to lowest terms by cancelling out the common factor/s in the numerator and denominator.
- Find the values a that will make the denominator of the reduced rational function equal to zero.
- The line $x = a$ is a vertical asymptote.

RULE TO FIND THE HORIZONTAL ASYMPTOTE

The line $y = c$ is the horizontal asymptote for the of a function f if $f(x) \to c \; as \; x \to \infty \; or \; x \to -\infty$

HORIZONTAL ASYMPTOTES RULES

Let n is the highest degree of the leading variable in numerator
Let m is the highest degree of the leading variable in denominator

First case: $\mathbf{n < m, y = 0}$

Second case: $\mathbf{n = m, y = \frac{a}{b}}$

Third case: $\mathbf{n > m,}$ **there is no horizontal asymptote.**

Examples and Solutions

Find the vertical and horizontal asymptotes of following rational function.

$$f(x) = \frac{2}{x+1}$$

To find the vertical asymptotes

$$x + 1 = 0$$

$$x = -1$$

The vertical asymptotes
Is $x = -1$

To find the Horizontal asymptotes

$\boldsymbol{n} < \boldsymbol{m}, y = 0 \quad \boldsymbol{n} = \boldsymbol{m}, y = \frac{a}{b}$

$\boldsymbol{n} > \boldsymbol{m}, there\ is\ no\ hirizontal$ asymptote.

$$f(x) = \frac{2}{x+1} \qquad y = 0$$

The Horizontal asymptotes
Is $y = 0$

Examples and Solutions

Find the vertical and horizontal asymptotes of following rational function.

$$f(x) = \frac{4x^2 + 9x + 2}{x^2 + 4x + 3}$$

To find the vertical asymptotes

$$f(x) = \frac{(4x + 1)(x + 2}{(x + 3)(x + 1)}$$

$$x + 3 = 0 \qquad x + 1 = 0$$

$$x = -3 \qquad x = -1$$

The vertical asymptotes
Is $x = -3\ and\ -1$

To find the Horizontal asymptotes

$\boldsymbol{n} < \boldsymbol{m}, y = 0$ $\boldsymbol{n} = \boldsymbol{m}, y = \frac{a}{b}$

$\boldsymbol{n} > \boldsymbol{m}, there\ is\ no\ hirizontal$ asymptote.

$$f(x) = \frac{4x^2 + 9x + 2}{x^2 + 4x + 3}$$

$$y = \frac{4}{1} \quad y = 4$$

The Horizontal asymptotes
Is $y = 4$

ASSESSMENT

Directions Find the zeroes ,intercepts and asymptotes of the given Rational Equation.

$$1.)\ f(x) = \frac{x^2 + 3x}{x - 2}$$

a. Find x and y intercepts

b. Find the zeroes of rational function

C. Find the Vertical and Horizontal Asymptotes

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THANKYOU! WAY TO GO

Stay Safe and healthy! Godbless!!