

Yo he venido a hablar de mi libro (o cómo hacer inferencia Bayesiana con INLA)

Virgilio Gómez Rubio
Universidad de Castilla-La Mancha



Virgilio.Gomez@uclm.es



@precariobecario



<https://becarioprecario.github.io>



Castilla-La Mancha

Una manera
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Fondo Europeo de
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DE ESPAÑA

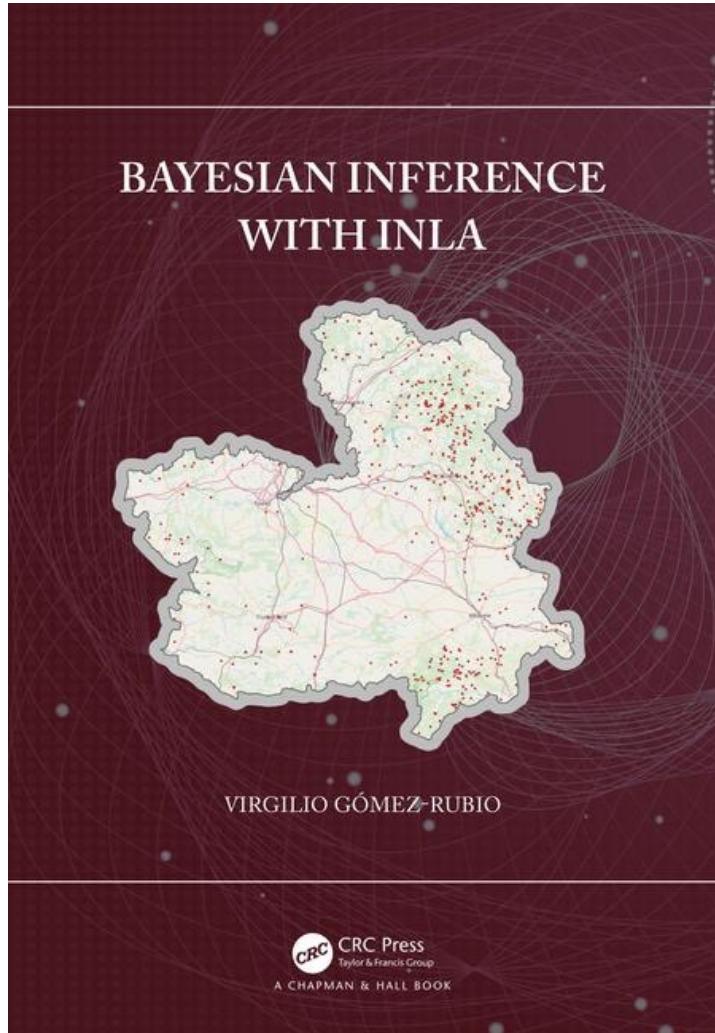
MINISTERIO
DE CIENCIA, INNOVACIÓN
Y UNIVERSIDADES



Introducción

- Aprendizaje Bayesiano.
- Inferencia Bayesiana.
- Métodos computacionales.
- Inferencia Bayesiana con INLA.
- Ejemplo: estimación del exceso de defunciones en 2020.

Bayesian inference with INLA



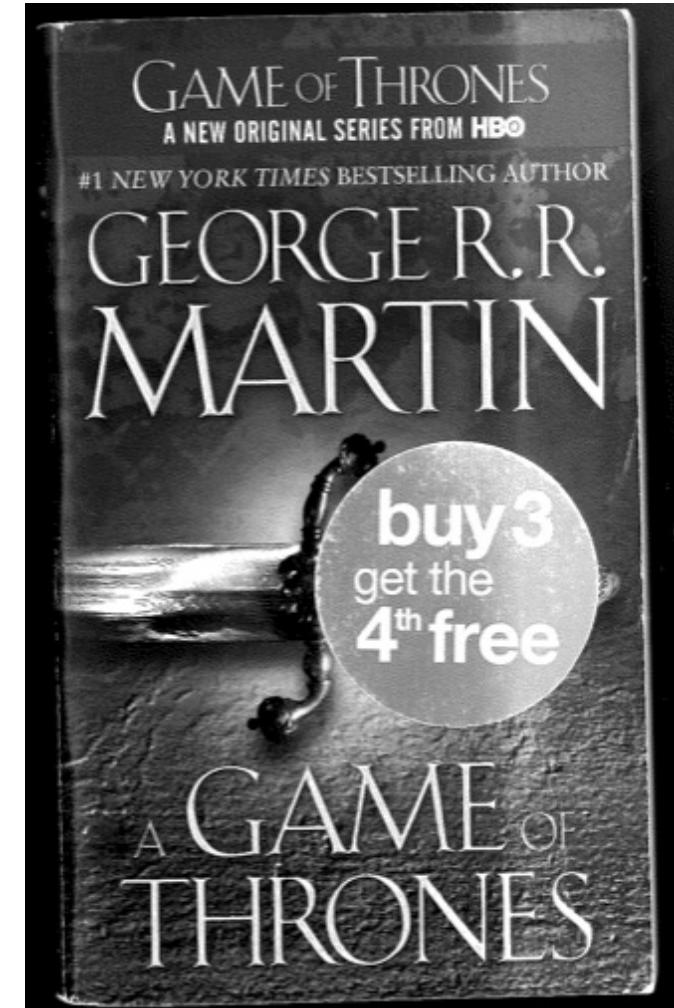
Fundación
BBVA



Juego de Tronos

- ¿Cuántas veces aparece la letra u en una página de Juego de Tronos?
- Usaremos la edición en inglés de Juego de Tronos.
- Solo las páginas del prólogo (excepto la primera y la última).
- 31 páginas en total (de distintos libros).
- Se trata de un conteo:

$$O_i \sim Po(\lambda), \quad i = 1, \dots, 31$$



Juego de Tronos



PROLOGUE

"We should start back," Gared urged as the woods began to grow dark around them. "The wildlings are dead."

"Do the dead frighten you?" Ser Waymar Royce asked with just the hint of a smile.

Gared did not rise to the bait. He was an old man, past fifty, and he had seen the lordlings come and go. "Dead is dead," he said. "We have no business with the dead."

"Are they dead?" Royce asked softly. "What proof have we?"

"Will saw them," Gared said. "If he says they are dead, that's proof enough for me."

Will had known they would drag him into the quarrel sooner or later. He wished it had been later rather than sooner. "My mother told me that dead men sing no songs," he put in.

"My wet nurse said the same thing, Will," Royce replied. "Never believe anything you hear at a woman's tit. There are things to be learned even from the dead." His voice echoed, too loud in the twilit forest.

"We have a long ride before us," Gared pointed out. "Eight days, maybe nine. And night is falling."

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GEORGE R.R. MARTIN

Ser Waymar Royce glanced at the sky with disinterest. "It does that every day about this time. Are you unmanned by the dark, Gared?"

Will could see the tightness around Gared's mouth, the barely suppressed anger in his eyes under the thick black hood of his cloak. Gared had spent forty years in the Night's Watch, man and boy, and he was not accustomed to being made light of. Yet it was more than that. Under the wounded pride, Will could sense something else in the older man. You could taste it, a nervous tension that came perilously close to fear.

Will shared his unease. He had been four years on the Wall. The first time he had been sent beyond, all the old stories had come rushing back, and his bowels had turned to water. He had laughed about it afterward. He was a veteran of a hundred rangings by now, and the endless dark wilderness that the southron called the haunted forest had no more terrors for him.

Until tonight. Something was different tonight. There was an edge to this darkness that made his hackles rise. Nine days they had been riding, north and northwest and then north again, farther and farther from the Wall, hard on the track of a band of wildling raiders. Each day had been worse than the day that had come before it. Today was the worst of all. A cold wind was blowing out of the north, and it made the trees rustle like living things. All day, Will had felt as though something were watching him, something cold and implacable that loved him not. Gared had felt it too. Will wanted nothing so much as to ride hellbent for the safety of the Wall, but that was not a feeling to share with your commander.

Especially not a commander like this one.

Ser Waymar Royce was the youngest son of an ancient house with too many heirs. He was a handsome youth of eighteen, grey-eyed and graceful and slender as a knife. Mounted on his huge black destrier, the knight towered above Will and Gared on their smaller garrons. He wore black leather boots, black woolen pants, black moleskin gloves, and a fine supple coat of gleaming black ringmail over layers of black wool and boiled leather. Ser Waymar had been a Sworn Brother of the Night's Watch for less than half a year, but no one could say he had not prepared

A GAME OF THRONES

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for his vocation. At least insofar as his wardrobe was concerned.

His cloak was his crowning glory; sable, thick and black and soft as sin. "Bet he killed them all himself, he did," Gared told the barracks over wine, "twisted their little heads off, our mighty warrior." They had all shared the laugh.

It is hard to take orders from a man you laughed at in your cups, Will reflected as he sat shivering atop his garron. Gared must have felt the same.

"Mormont said as we should track them, and we did," Gared said. "They're dead. They shan't trouble us no more. There's hard riding before us. I don't like this weather. If it snows, we could be a fortnight getting back, and snow's the best we can hope for. Ever seen an ice storm, my lord?"

The lordling seemed not to hear him. He studied the deepening twilight in that half-bored, half-distracted way he had. Will had ridden with the knight long enough to understand that it was best not to interrupt him when he looked like that. "Tell me again what you saw, Will. All the details. Leave nothing out."

Will had been a hunter before he joined the Night's Watch. Well, a poacher in truth. Mallister freeriders had caught him red-handed in the Mallisters' own woods, skinning one of the Mallisters' own bucks, and it had been a choice of putting on the black or losing a hand. No one could move through the woods as silent as Will, and it had not taken the black brothers long to discover his talent.

"The camp is two miles farther on, over that ridge, hard beside a stream," Will said. "I got close as I dared. There's eight of them, men and women both. No children I could see. They put up a lean-to against the rock. The snow's pretty well covered it now, but I could still make it out. No fire burning, but the firepit was still plain as day. No one moving. I watched a long time. No living man ever lay so still."

"Did you see any blood?"

"Well, no," Will admitted.

"Did you see any weapons?"

"Some swords, a few bows. One man had an axe.

VIBASS 6



VIBASS6
VALÈNCIA INTERNATIONAL
BAYESIAN SUMMER SCHOOL
July 10 - 14 2023

Mon 10th - Tue 11th

Introduction to Bayesian Learning
Basic course - 12 hours

Instructors:

Carmen Armero	Mark Brewer
David Conesa	BioSS (Scotland)
Anabel Forte	Virgilio Gómez-Rubio
Miguel Angel Martinez Beneito	Universidad de Castilla - La Mancha (Spain)
Universitat de València (Spain)	
Facundo Muñoz	
CIRAD (France)	

Wed 12th - Thu 13th

Bayesian capture-recapture inference with hidden Markov models
Advanced course - 12 hours

Instructor:

Olivier Gimenez	Centre d'Ecologie Fonctionnelle & Evolutive, Montpellier (France)
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Fri 14th

6th VIBASS Workshop

Invited speakers:

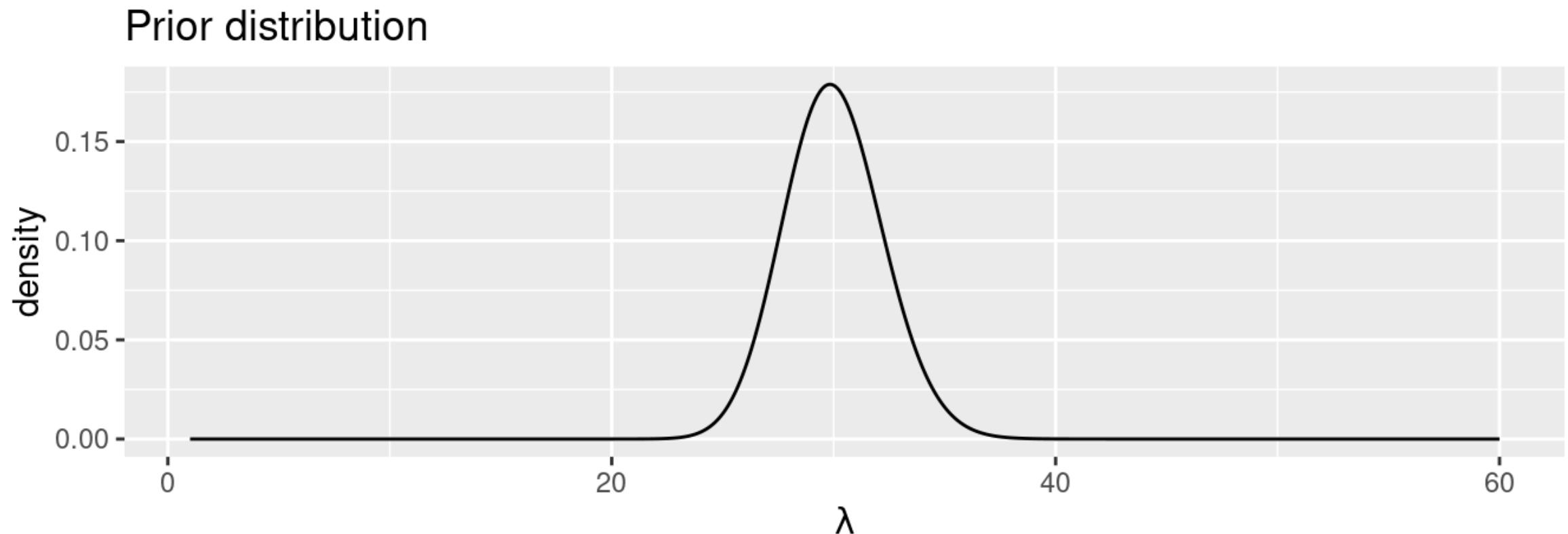
Concepción Ausín	Universidad Carlos III de Madrid (Spain)
Duncan Lee	University of Glasgow (Scotland)



Inferencia Bayesiana

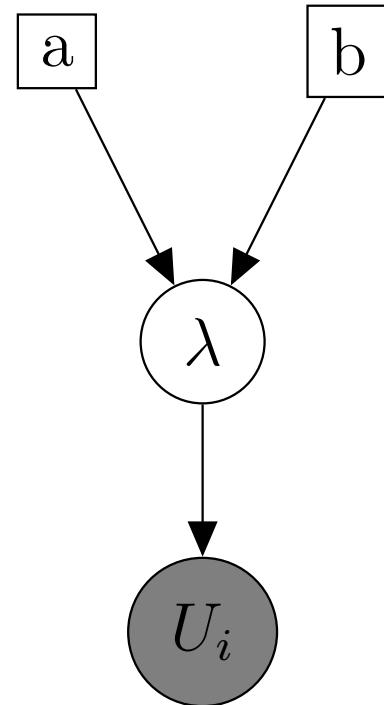
- Habría que incluir una distribución previa para λ .
 - No informativa (¿tiene sentido?).
 - Informativa (¿qué sabemos sobre la distribución de la letra u en inglés?).
- Previa informativa:
 - No tiene sentido que la previa dé valores negativos a λ .
 - La distribución de valores tiene que ser “razonable”:
 - No tiene sentido que de media haya 1.000 úes en una página.
 - Podría ser razonable que hubiese entre 20 y 40, por ejemplo.
 - Podemos usar una previa Gamma de media 30 y varianza 5: $\text{Ga}(180, 6)$.

¿Cómo valoro lo que ya sé?

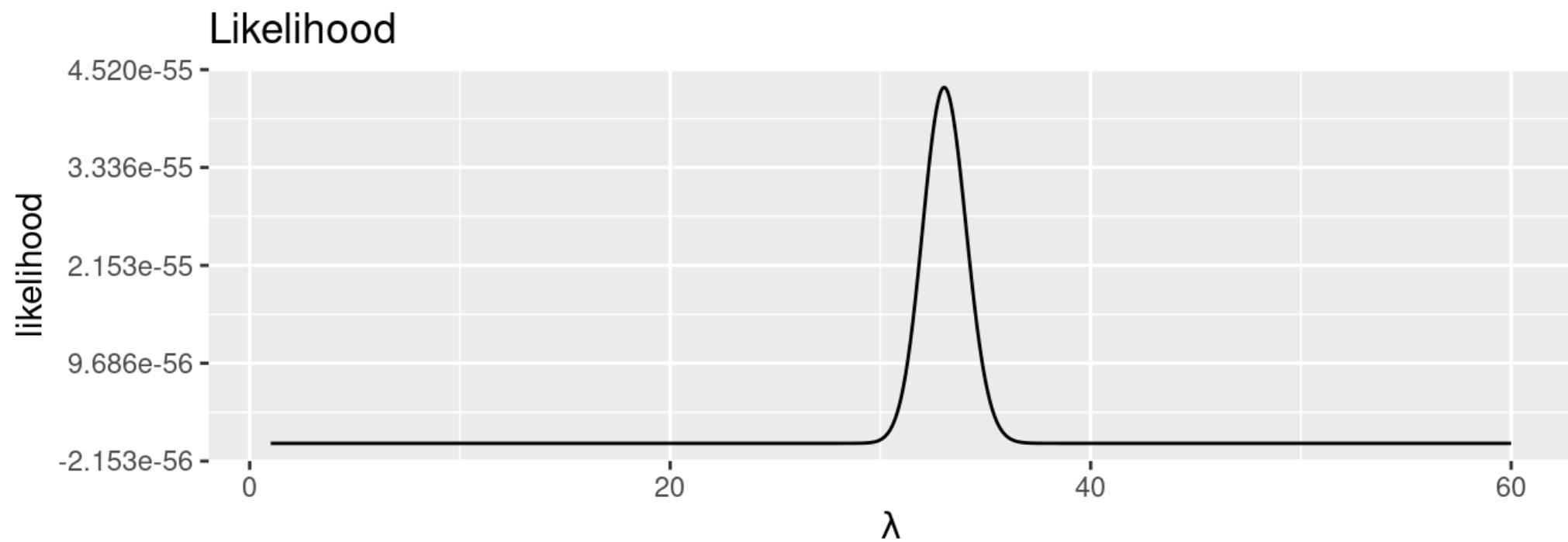


Modelo inicial

$$U_i \sim Po(\lambda), \quad i = 1, \dots, 31$$
$$\lambda \sim Ga(180, 5)$$

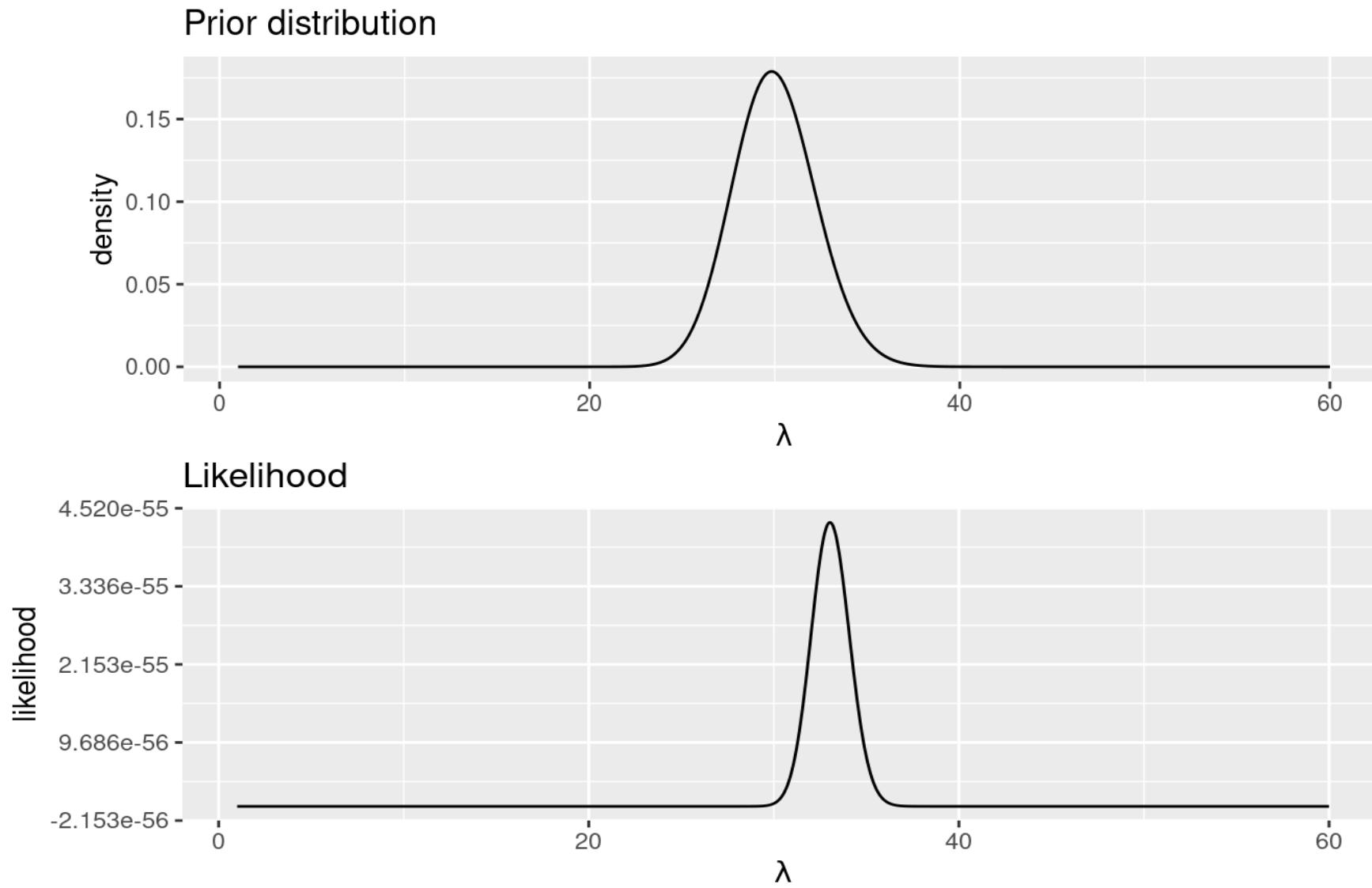


Recogida de datos

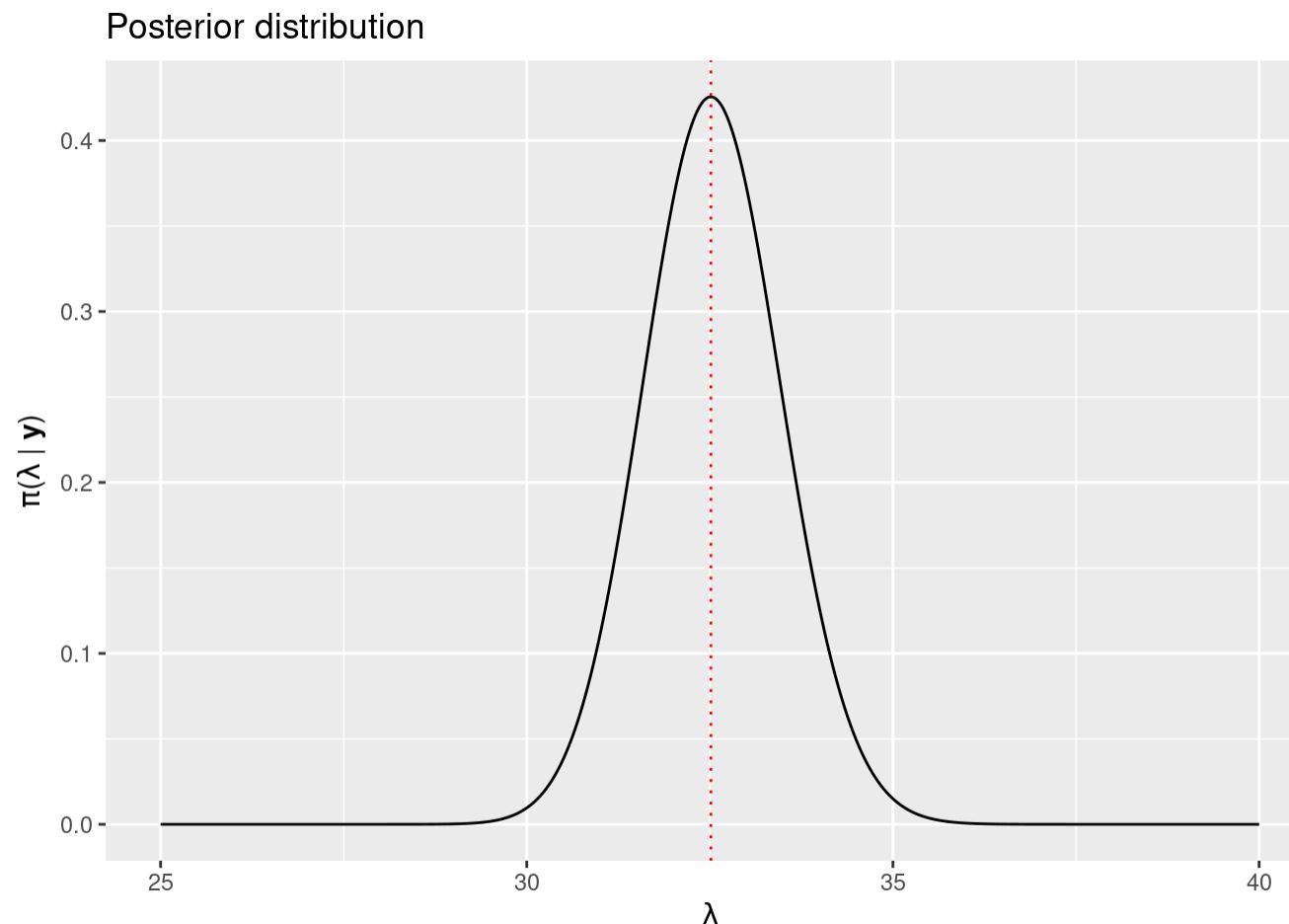


$$\bar{x} = 33.03, n = 31$$

¿Cómo actualizo mi información previa?



Aprendizaje Bayesiano

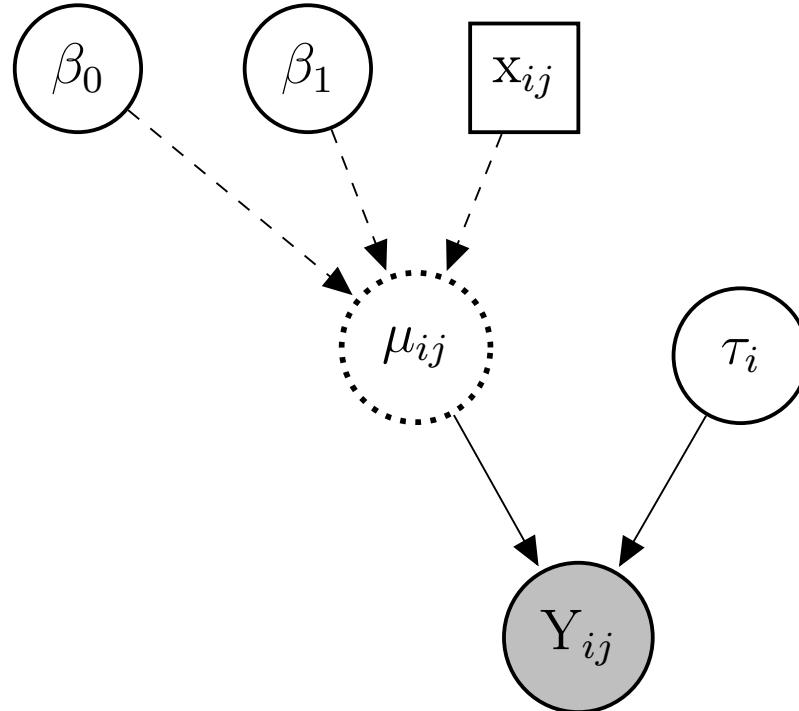


$$\lambda | \text{datos} \sim Ga(180 + 31 * 33.03, 5 + 31)$$

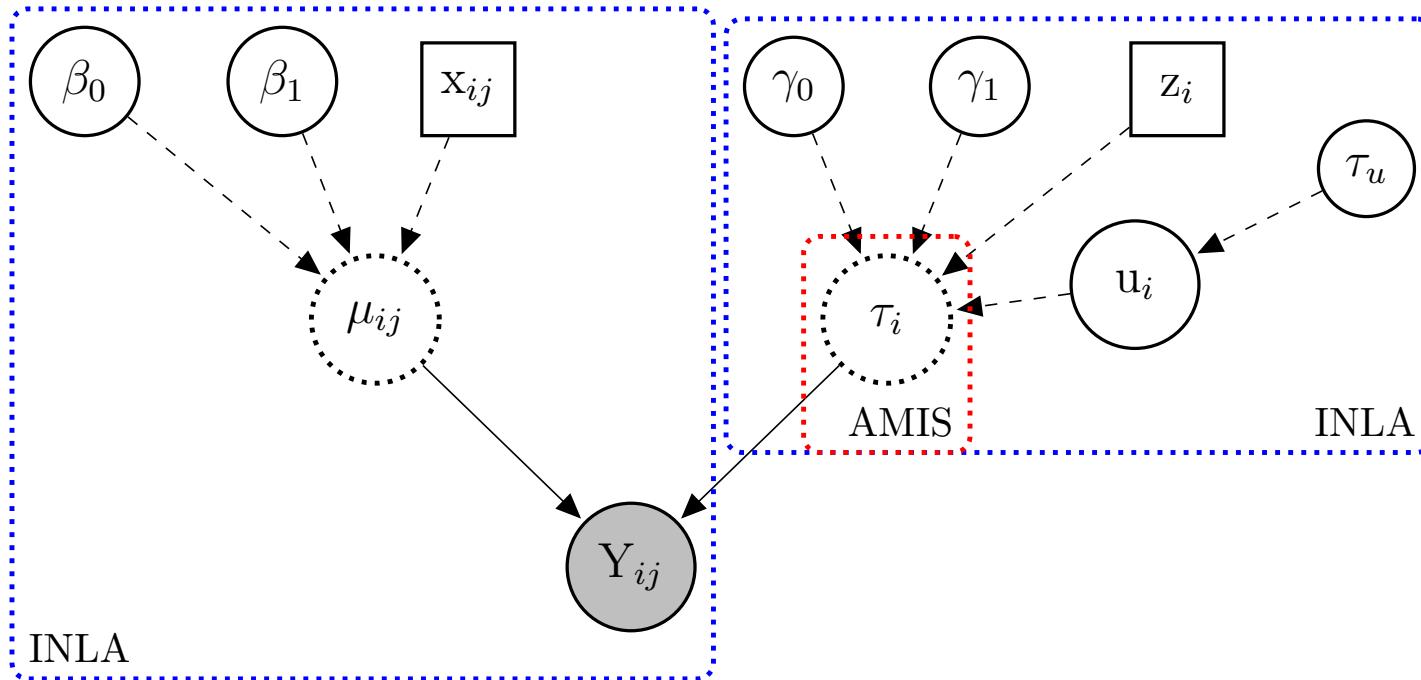
Vientos de invierno

- ¿Qué va a ocurrir cuándo salga la siguiente novela de Juego de Tronos?
- ¿Cómo puedo predecir el número de úes en una página del prólogo?
 - Distribución predictiva:
$$N_u \mid \text{datos} = \int Po(N_u \mid \lambda) f(\lambda \mid \text{datos}) d\lambda \equiv \text{Binomial negativa}$$
- ¿Cómo puedo incorporar la nueva evidencia y actualizar lo que sé sobre λ ?
 - Usaremos $Ga(180 + 31 * 33.03, 5 + 31)$ como “previa”.
 - Combinamos con la verosimilitud de los nuevos datos...
 - ¡¡Y tenemos una actualización de la distribución de λ que es Gamma!!

Modelos jerárquicos



Modelos doblemente jerárquicos

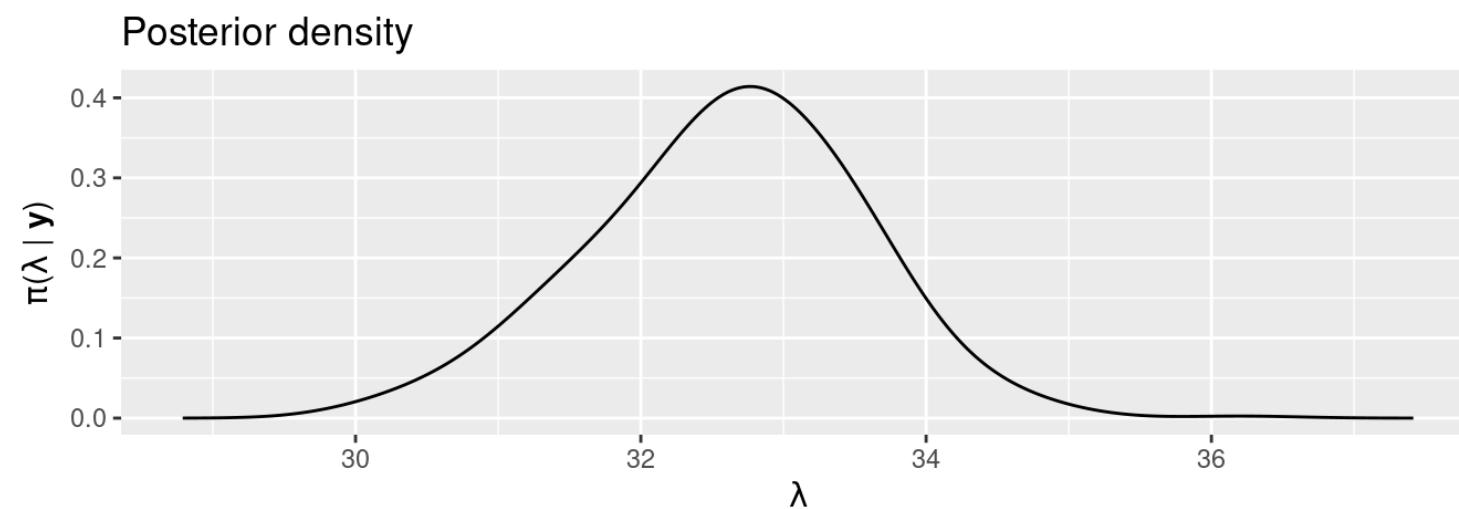
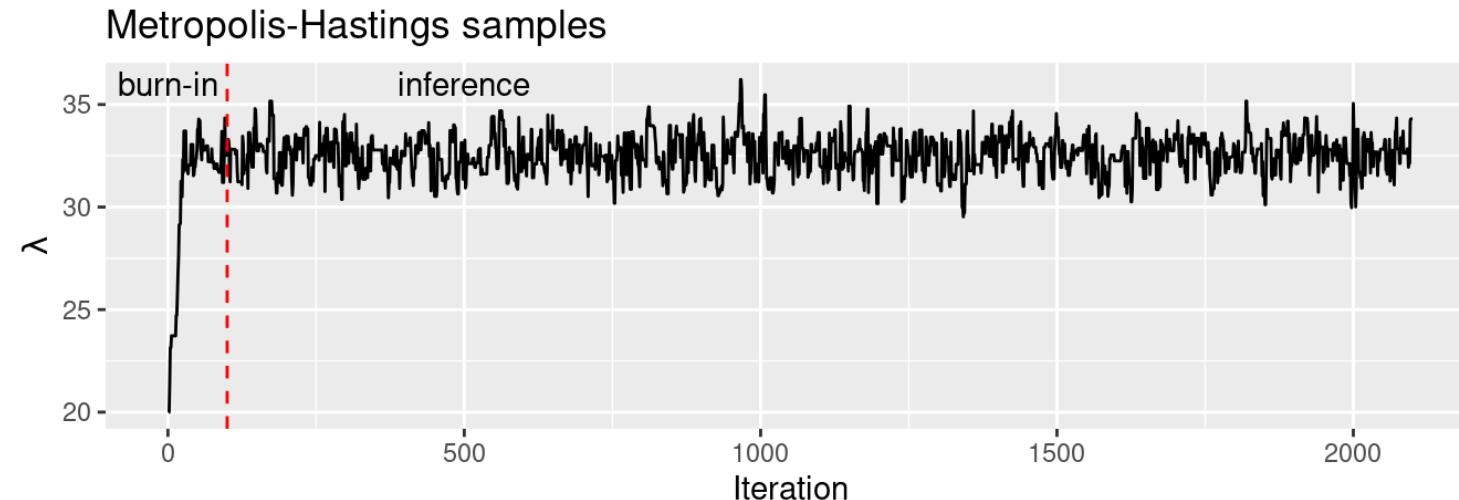


Fuente: Morales-Otero, M., Gómez-Rubio, V. & Núñez-Antón, V. *Fitting double hierarchical models with the integrated nested Laplace approximation*. Stat Comput 32, 62 (2022). <https://doi.org/10.1007/s11222-022-10122-1>

Métodos computacionales

- Markov Chain Monte Carlo
 - Metropolis-Hastings:
 - Gibbs Sampling
 - Hamiltonian Monte Carlo.
 - ...

Métodos computacionales



Integrated nested Laplace approximation

- Enfoque basado en aproximaciones numéricas de las marginales de los parámetros y efectos del modelo.
- Tres niveles:
 - Verosimilitud: no está restringida a la familia exponencial.
 - Efectos latentes: estructura de Gaussian Markov Random Field.
 - Hiperparámetros, con una previa asignada.
- En la práctica, los efectos latentes deben tener una estructura de normal multivariante con matriz de precisión dispersa.
- Los efectos son aditivos para modelizar la media de la verosimilitud.
- Los hiperparámetros no pueden depender de nada.

Modelos jerárquicos con INLA

- Ajuste implementado en el paquete R-INLA:
 - Verosimilitud.
 - Efectos latentes: series temporales, efectos espaciales, ...
 - Previas.
 - Criterios de selección de modelos: DIC, WAIC, etc.
- Limitaciones:
 - Previas univariantes (en su mayoría).
 - No se pueden modelizar los hiperparámetros.
 - No puede haber valores ausentes en los efectos latentes.
 - Inferencia multivariante mediante muestreo.

R-INLA



```
# Prior of precision
prec.prior <- list(prec = list(param = c(0.001, 0.001)))

brain.st <- inla(Observed ~ 1 + f(Year, model = "rw1",
  hyper = prec.prior) +
  f(as.numeric(ID), model = "besag", graph = adj.mat,
  hyper = prec.prior),
  data = brainst@data, E = Expected, family = "poisson",
  control.predictor = list(compute = TRUE, link = 1))
summary(brain.st)
```

Mortalidad en pandemia

- ¿Cuántos muertos ha causado la pandemia de COVID-19?
 - Fallecidos por COVID-19.
 - Fallecidos por la saturación del sistema, diagnósticos tardíos, ...
- ¿Cómo podemos estudiar el exceso de defunciones?
 - ¿Qué habría pasado en 2020 si no hubiera habido pandemia?

Mortalidad en pandemia



ARTICLE

<https://doi.org/10.1038/s41467-022-28157-3>

OPEN

Check for updates

Regional excess mortality during the 2020 COVID-19 pandemic in five European countries

Garyfallos Konstantinoudis¹ , Michela Cameletti², Virgilio Gómez-Rubio³ , Inmaculada León Gómez^{4,5}, Monica Pirani⁶ , Gianluca Baio⁶, Amparo Larrauri^{4,5}, Julien Riou⁷, Matthias Egger^{7,8}, Paolo Vineis¹ & Marta Blangiardo¹



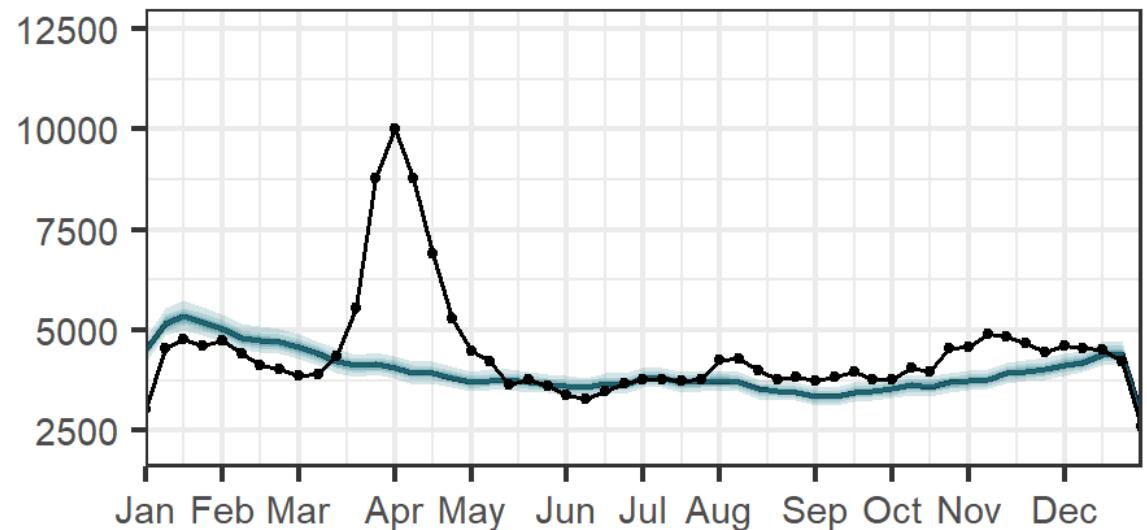
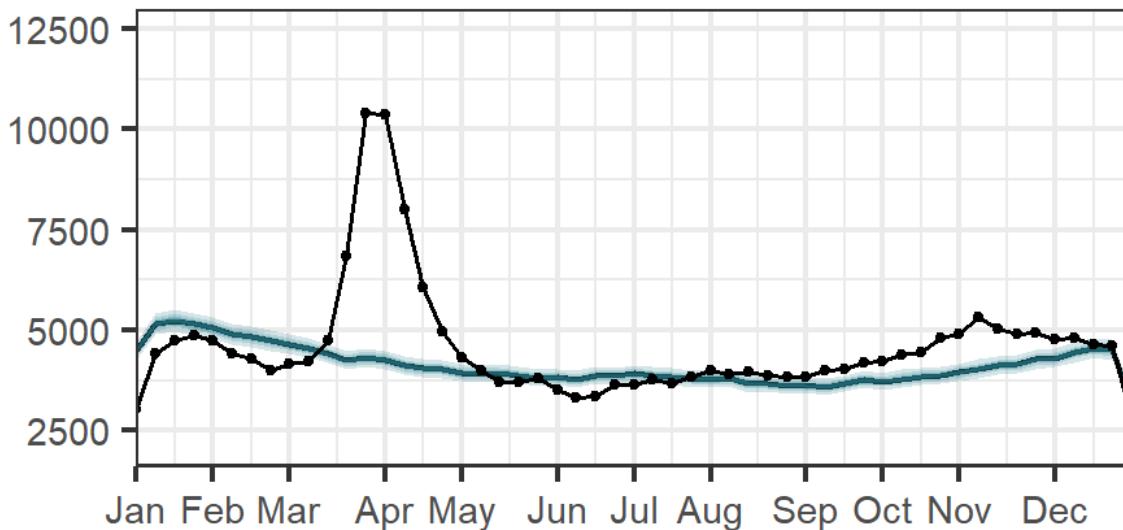
Mortalidad en pandemia

Número de defunciones

Hombres

Mujeres

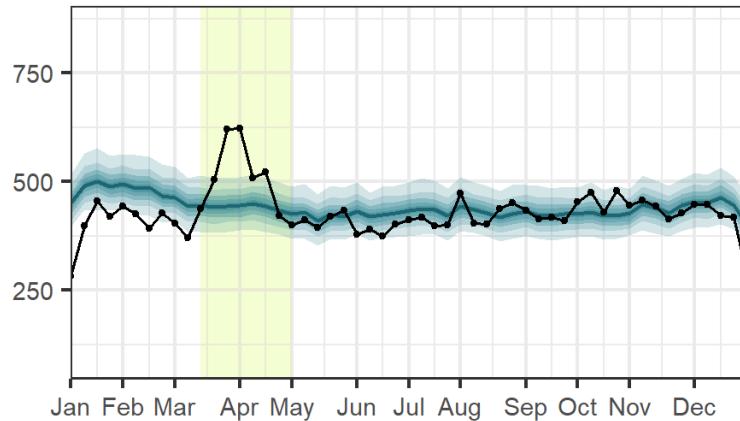
Spain



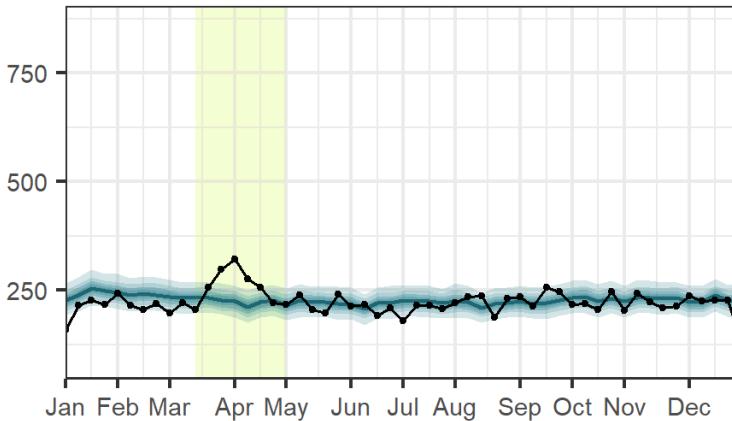
Mortalidad en pandemia

Número de defunciones

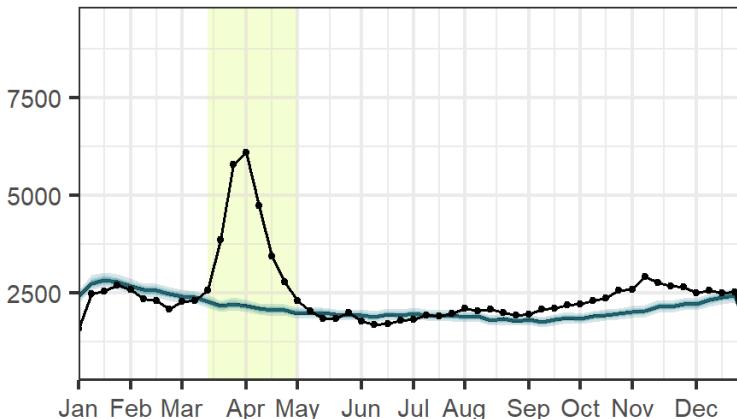
Males 40-59



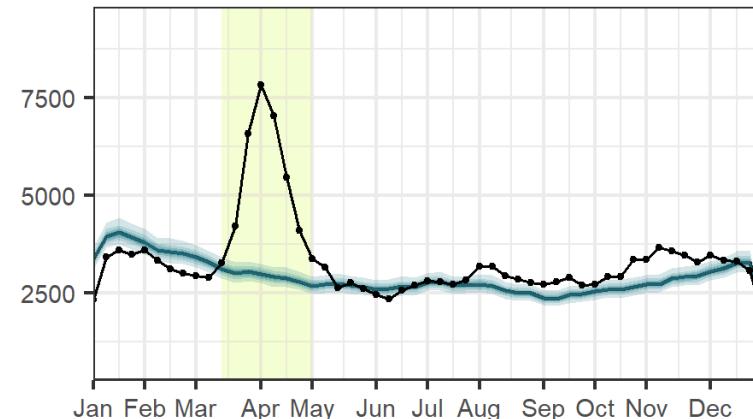
Females 40-59



Males 80+



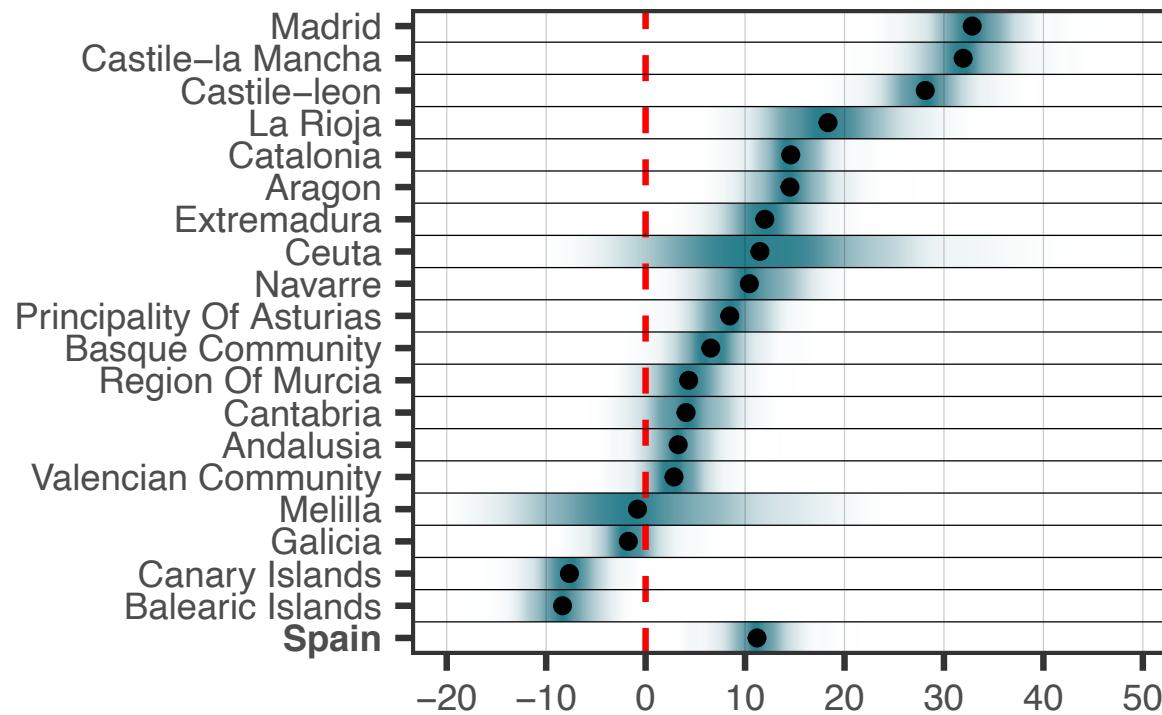
Females 80+



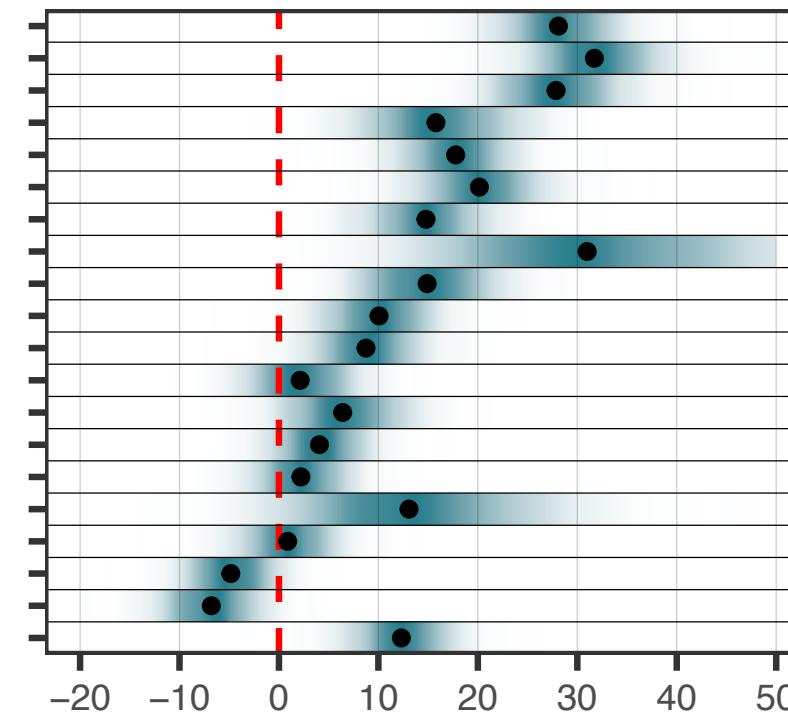
Mortalidad en pandemia

Exceso relativo de defunciones

Hombres



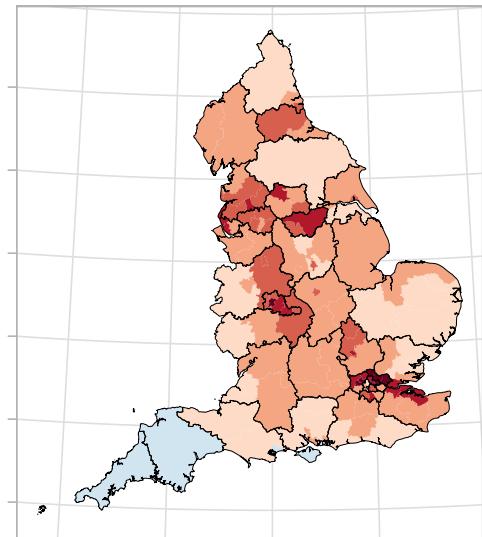
Mujeres



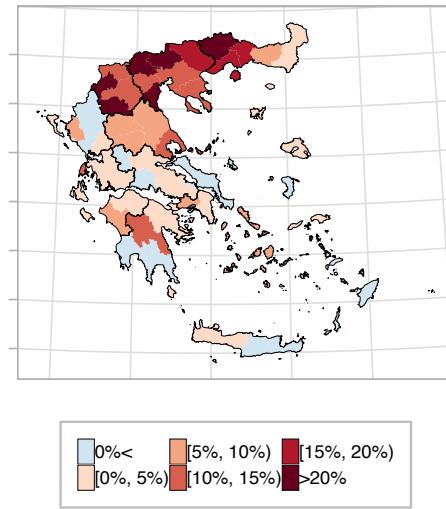
Epidemiología espacial

Exceso relativo de defunciones

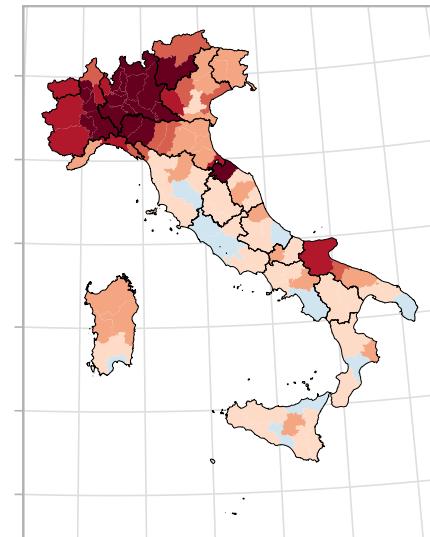
England



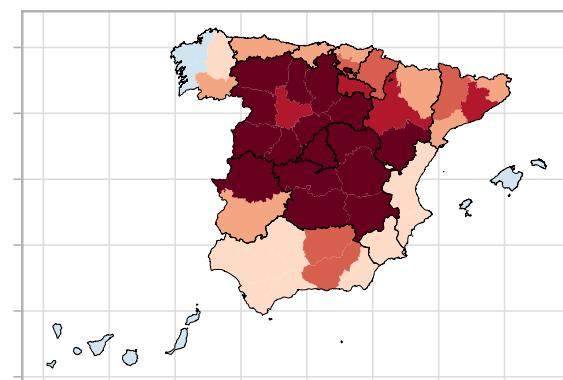
Greece



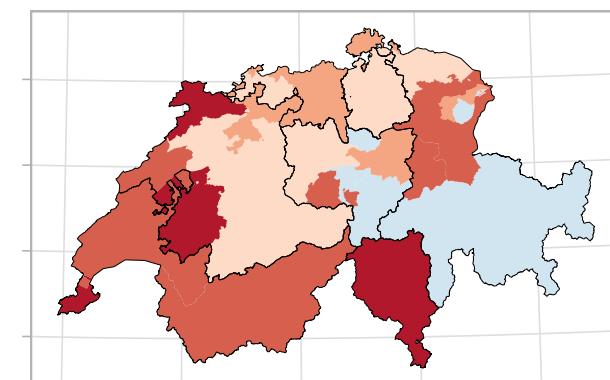
Italy



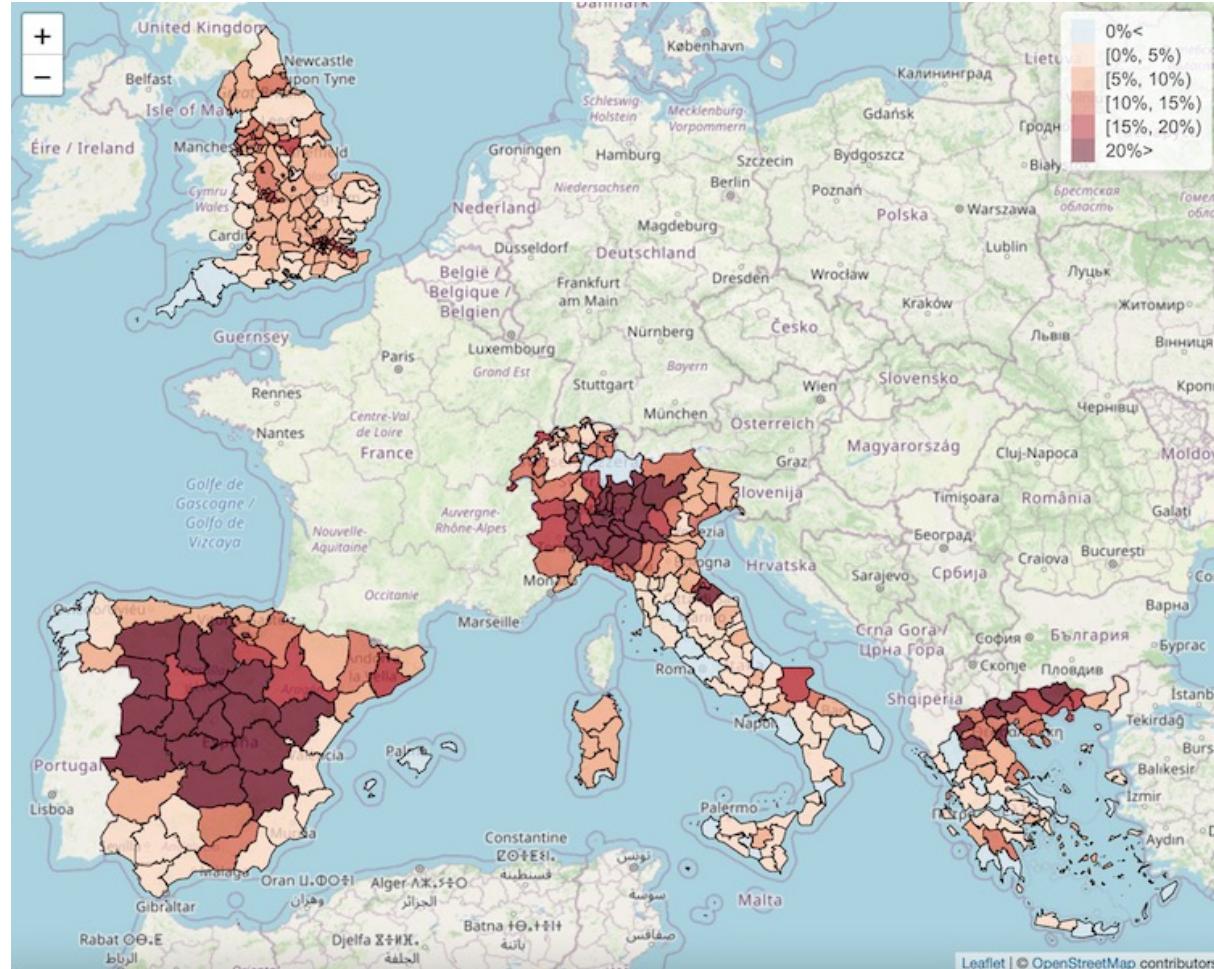
Spain



Switzerland



Epidemiología espacial



MUCHAS GRACIAS

