1 Definitions

Let:

- Adjacency matrix of network: $\mathbf{A}$
- Neighbourhood of a given node: B
- Size of neighbourhood: $k = |B|$:
- Number of node in network: n.
- Action vector: $\mathbf{x}$.
- Private belief: p
- Social belief: $q = \sum_{i \in B} x_i / k = \mathbf{A}\mathbf{x}/k$.
- Threshold function: $f(p, q)$ such that:

$$x = \begin{cases} 1 & \text{if } f(p,q) > 0.5 \\ 0 & \text{if } f(p,q) < 0.5 \end{cases} \tag{1}$$

We distinguish between three cases for the threshold function:

1. "Equal":
$$f(p, q) = 0.5(p + q). \tag{2}$$
2. "Neighbor":
$$f(p, q) = \frac{1}{k+1} p + \frac{k}{k+1} q. \tag{3}$$
3. "Rel Neighbor":
$$f(p, q) = \left(1 - \frac{k}{n-1}\right) p + \frac{k}{n-1} q. \tag{4}$$

2 Figures for exogenous network case

Notation	Description	Value		
		1 - (I)	2 - (U)	3 - (U)
N	Number of agents	100	100	100
μ_0	Average signal for $\theta = 0$	0.4	0.49	0.3
μ_1	Average signal for $\theta = 1$	0.6	0.51	0.7
σ_0	Standard deviation of signal for $\theta = 0$	$\sqrt{0.1}$	$\sqrt{0.1}$	$\sqrt{0.1}$
σ_1	Standard deviation of signal for $\theta = 1$	$\sqrt{0.1}$	$\sqrt{0.1}$	$\sqrt{0.1}$
T	Number of iterations of updater	100	100	100
ρ	Density of ER network	$[0, 0.95]$	$[0, 0.95]$	0.5
p	Probability of being informed	NA	NA	$[0.1, 0.9]$
S	Number of simulations per parameter configuration	1000	1000	1000

Table 1: Parameters for runs for the three cases above.

Throughout this document we assume that the state of the world is $\theta = 1$. We consider three cases below:

1. Informed agents (I): a population of 100 agents connected via an ER network and a signal $\mu_0 = 0.4$

2. Uninformed agents (U): a population of 100 agents connected via an ER network and a signal $\mu_0 = 0.49$

3. Heterogeneous agents (H): a population of 100 agents connected via an ER network. With probability p agents are either informed ($\mu_0 = 0.3$) or uninformed ($\mu_0 = 0.49$).

Cases 1 and 2 we further sub-divide into the three cases for the threshold function ("equal","neighbor","rel neighbor").

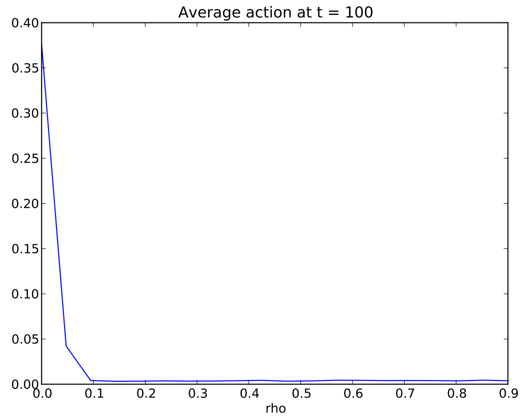

Figure 1: Case 1 - (I) - equal: Average final action of agents as a function of the density of the ER network (rho). The average final action is defined as: $x_F = \sum_j x_j(T)/N$. Data shown is averaged over S (1000) simulations per network density ρ. For $\rho = 0$ the agents' actions do not synchronize as they are solely determined by their private belief. As the network density is increased social learning leads to action synchronization.

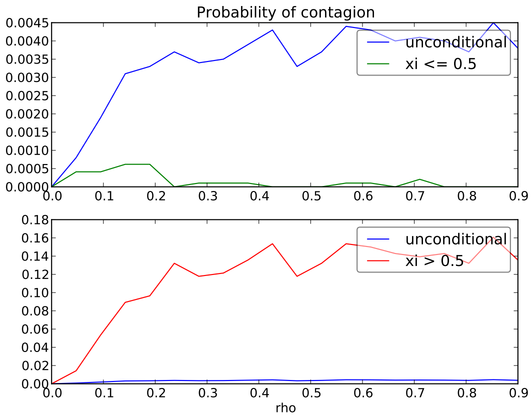

Figure 2: Case 1 - (I) - equal : Fraction of simulations per parameter configuration S (1000) in which agents synchronize on the state non matching action (more than 80% of agents choose state non-matching action) as a function of network density (rho). We distinguish three cases: (1) unconditional: we compute the fraction based on the full sample S. (2) conditional $xi \leq 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N \leq 0.5$, i.e. when the agents start with a state matching action. (3) conditional $xi > 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N > 0.5$, i.e. when the agents start with a state non matching action.

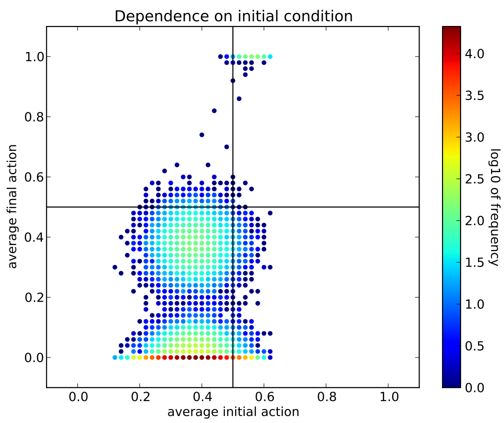

Figure 3: Case 1 - (I) - equal: We plot the average initial action $x_i = \sum_j x_j(0)/N$ vs. the average final action $x_F = \sum_j x_j(T)/N$. Data points are averages over S (1000) simulations and all network densities ρ (20 values equally distributed over the interval $[0, 0.95]$. The color code indicates the frequency with which a point occurs in the sample (total size 20×1000), the scale of the color code is logarithmic of base 10.

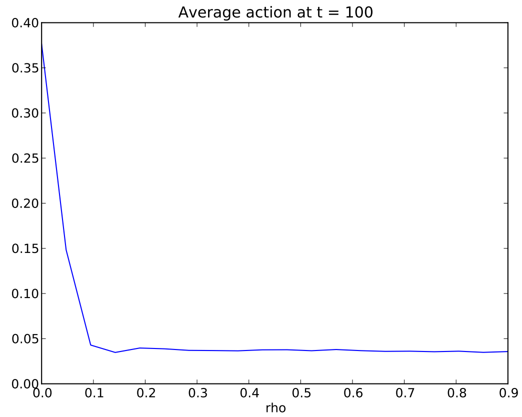

Figure 4: Case 1 - (I) - neighbor: Average final action of agents as a function of the density of the ER network (rho). The average final action is defined as: $x_F = \sum_j x_j(T)/N$. Data shown is averaged over S (1000) simulations per network density ρ. For $\rho = 0$ the agents' actions do not synchronize as they are solely determined by their private belief. As the network density is increased social learning leads to action synchronization.

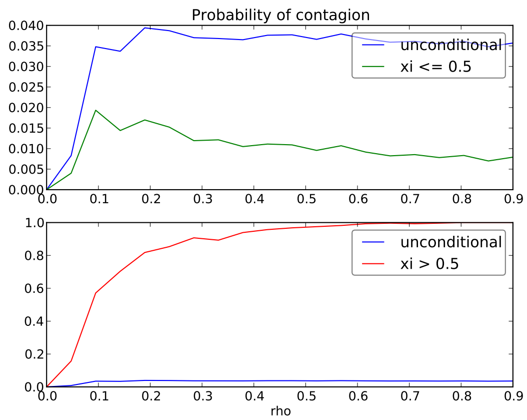

Figure 5: Case 1 - (I) - neighbor: Fraction of simulations per parameter configuration S (1000) in which agents synchronize on the state non matching action (more than 80% of agents choose state non-matching action) as a function of network density (rho). We distinguish three cases: (1) unconditional: we compute the fraction based on the full sample S. (2) conditional $xi \leq 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N \leq 0.5$, i.e. when the agents start with a state matching action. (3) conditional $xi > 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N > 0.5$, i.e. when the agents start with a state non matching action.

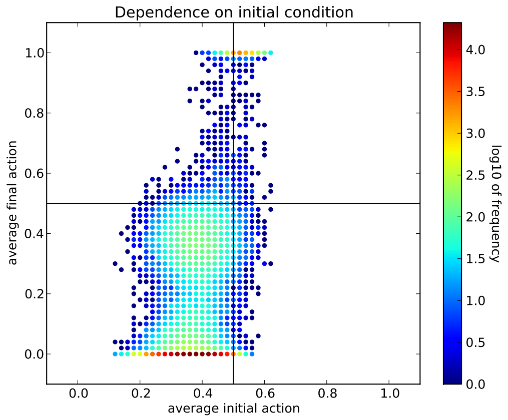

Figure 6: Case 1 - (I) - neighbor: We plot the average initial action $x_i = \sum_j x_j(0)/N$ vs. the average final action $x_F = \sum_j x_j(T)/N$. Data points are averages over S (1000) simulations and all network densities ρ (20 values equally distributed over the interval $[0, 0.95]$. The color code indicates the frequency with which a point occurs in the sample (total size 20×1000), the scale of the color code is logarithmic of base 10.

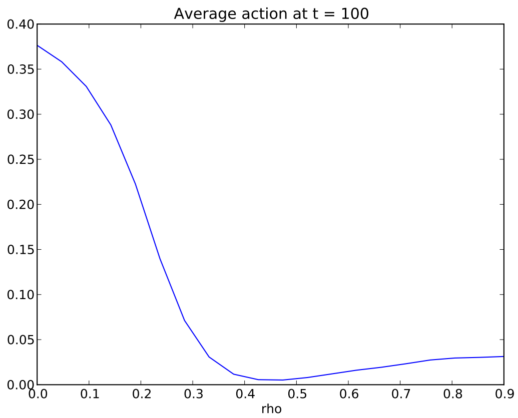

Figure 7: Case 1 - (I) - rel neighbor: Average final action of agents as a function of the density of the ER network (rho). The average final action is defined as: $x_F = \sum_j x_j(T)/N$. Data shown is averaged over S (1000) simulations per network density ρ. For $\rho = 0$ the agents' actions do not synchronize as they are solely determined by their private belief. As the network density is increased social learning leads to action synchronization.

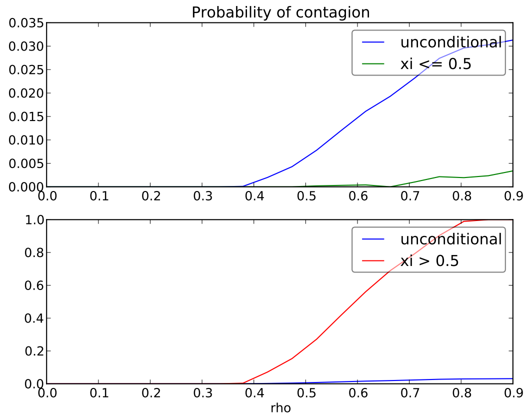

Figure 8: Case 1 - (I) - rel neighbor: Fraction of simulations per parameter configuration S (1000) in which agents synchronize on the state non matching action (more than 80% of agents choose state non-matching action) as a function of network density (rho). We distinguish three cases: (1) unconditional: we compute the fraction based on the full sample S. (2) conditional $xi \leq 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N \leq 0.5$, i.e. when the agents start with a state matching action. (3) conditional $xi > 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N > 0.5$, i.e. when the agents start with a state non matching action.

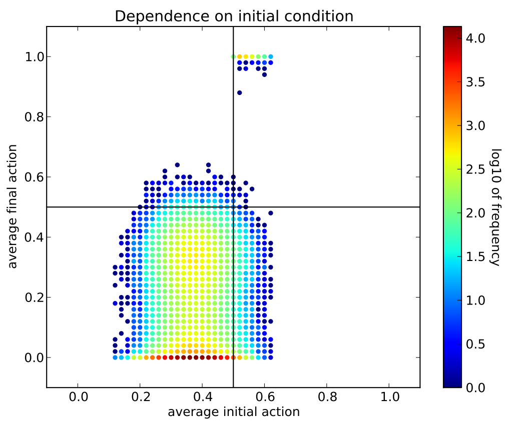

Figure 9: Case 1 - (I) - rel neighbor: We plot the average initial action $x_i = \sum_j x_j(0)/N$ vs. the average final action $x_F = \sum_j x_j(T)/N$. Data points are averages over S (1000) simulations and all network densities ρ (20 values equally distributed over the interval $[0, 0.95]$. The color code indicates the frequency with which a point occurs in the sample (total size 20×1000), the scale of the color code is logarithmic of base 10.

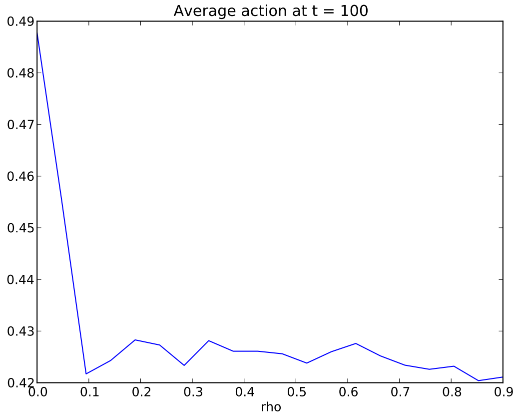

Figure 10: Case 2 - (U) - equal: Average final action of agents as a function of the density of the ER network (rho). The average final action is defined as: $x_F = \sum_j x_j(T)/N$. Data shown is averaged over S (1000) simulations per network density ρ. For $\rho = 0$ the agents' actions do not synchronize as they are solely determined by their private belief. As the network density is increased social learning leads to action synchronization.

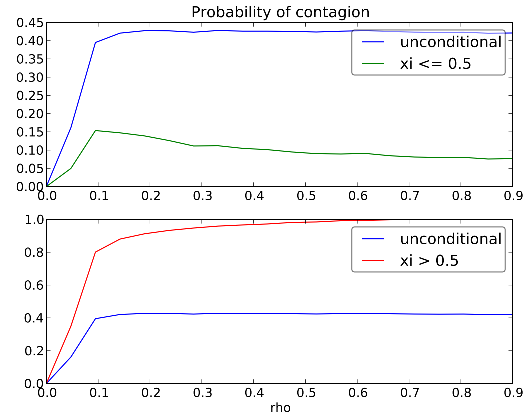

Figure 11: Case 2 - (U) - equal : Fraction of simulations per parameter configuration S (1000) in which agents synchronize on the state non matching action (more than 80% of agents choose state non-matching action) as a function of network density (rho). We distinguish three cases: (1) unconditional: we compute the fraction based on the full sample S. (2) conditional $xi \leq 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N \leq 0.5$, i.e. when the agents start with a state matching action. (3) conditional $xi > 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N > 0.5$, i.e. when the agents start with a state non matching action.

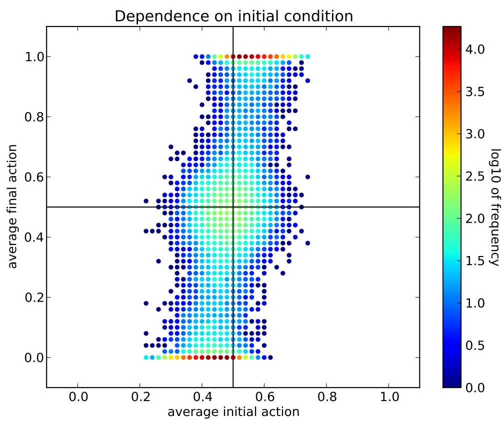

Figure 12: Case 2 - (U) - equal: We plot the average initial action $x_i = \sum_j x_j(0)/N$ vs. the average final action $x_F = \sum_j x_j(T)/N$. Data points are averages over S (1000) simulations and all network densities ρ (20 values equally distributed over the interval $[0, 0.95]$. The color code indicates the frequency with which a point occurs in the sample (total size 20×1000), the scale of the color code is logarithmic of base 10.

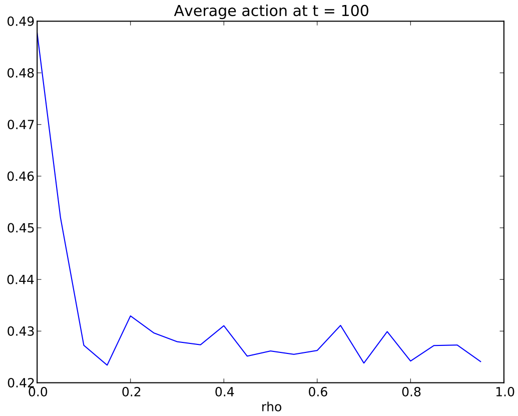

Figure 13: Case 2 - (U) - neighbor: Average final action of agents as a function of the density of the ER network (rho). The average final action is defined as: $x_F = \sum_j x_j(T)/N$. Data shown is averaged over S (1000) simulations per network density ρ. For $\rho = 0$ the agents' actions do not synchronize as they are solely determined by their private belief. As the network density is increased social learning leads to action synchronization.

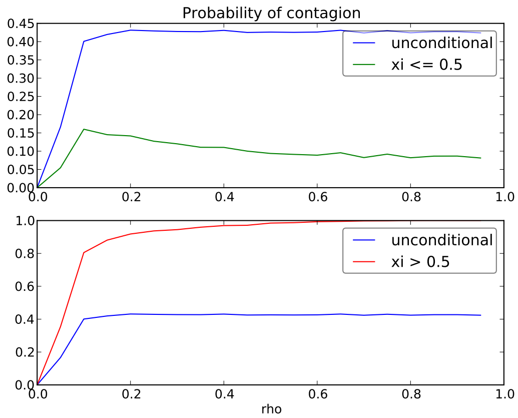

Figure 14: Case 2 - (U) - neighbor: Fraction of simulations per parameter configuration S (1000) in which agents synchronize on the state non matching action (more than 80% of agents choose state non-matching action) as a function of network density (rho). We distinguish three cases: (1) unconditional: we compute the fraction based on the full sample S. (2) conditional $xi \leq 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N \leq 0.5$, i.e. when the agents start with a state matching action. (3) conditional $xi > 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N > 0.5$, i.e. when the agents start with a state non matching action.

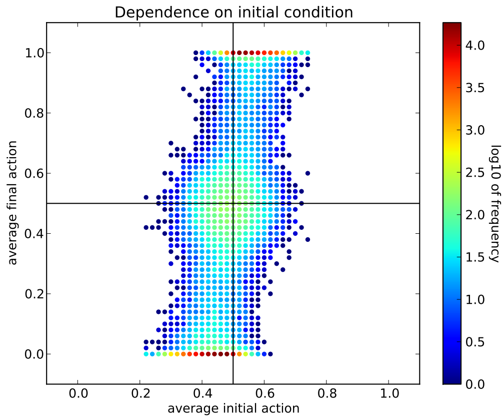

Figure 15: Case 2 - (U) - neighbor: We plot the average initial action $x_i = \sum_j x_j(0)/N$ vs. the average final action $x_F = \sum_j x_j(T)/N$. Data points are averages over S (1000) simulations and all network densities ρ (20 values equally distributed over the interval $[0, 0.95]$. The color code indicates the frequency with which a point occurs in the sample (total size 20×1000), the scale of the color code is logarithmic of base 10.

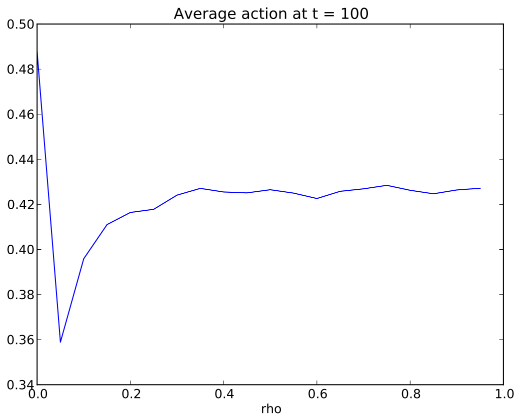

Figure 16: Case 2 - (U) - rel neighbor: Average final action of agents as a function of the density of the ER network (rho). The average final action is defined as: $x_F = \sum_j x_j(T)/N$. Data shown is averaged over S (1000) simulations per network density ρ. For $\rho = 0$ the agents' actions do not synchronize as they are solely determined by their private belief. As the network density is increased social learning leads to action synchronization.

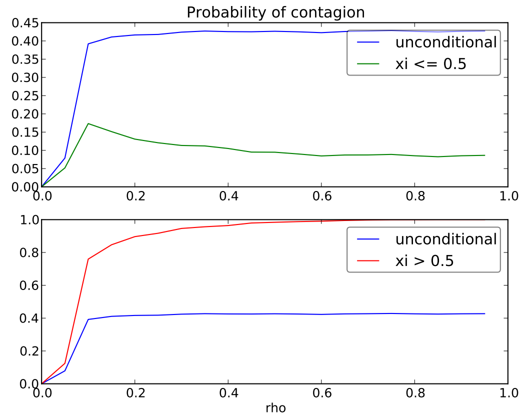

Figure 17: Case 2 - (U) - rel neighbor: Fraction of simulations per parameter configuration S (1000) in which agents synchronize on the state non matching action (more than 80% of agents choose state non-matching action) as a function of network density (rho). We distinguish three cases: (1) unconditional: we compute the fraction based on the full sample S. (2) conditional $xi \leq 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N \leq 0.5$, i.e. when the agents start with a state matching action. (3) conditional $xi > 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N > 0.5$, i.e. when the agents start with a state non matching action.

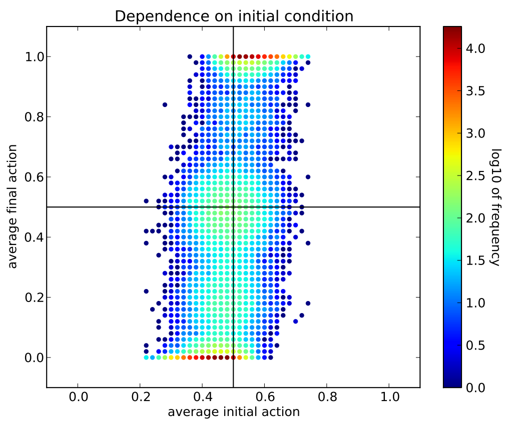

Figure 18: Case 2 - (U) - rel neighbor: We plot the average initial action $x_i = \sum_j x_j(0)/N$ vs. the average final action $x_F = \sum_j x_j(T)/N$. Data points are averages over S (1000) simulations and all network densities ρ (20 values equally distributed over the interval $[0, 0.95]$. The color code indicates the frequency with which a point occurs in the sample (total size 20×1000), the scale of the color code is logarithmic of base 10.

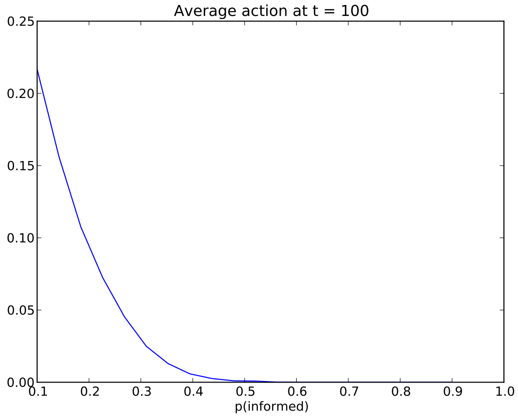

Figure 19: Case 3 - (H) - het: Average final action of agents as a function of the probability of being informed p. The average final action is defined as: $x_F = \sum_j x_j(T)/N$. Data shown is averaged over S (1000) simulations per probability of being informed p.

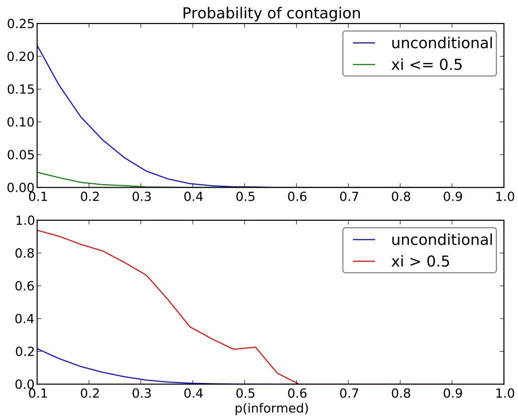

Figure 20: Case 3 - (H) - het: Fraction of simulations per parameter configuration S (1000) in which agents synchronize on the state non matching action (more than 80% of agents choose state non-matching action) as a function of the probability of being informed p. We distinguish three cases: (1) unconditional: we compute the fraction based on the full sample S. (2) conditional $xi \leq 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N \leq 0.5$, i.e. when the agents start with a state matching action. (3) conditional $xi > 0.5$: we compute the fraction based on the sub-set of simulations in which the average initial action $x_i = \sum_j x_j(0)/N > 0.5$, i.e. when the agents start with a state non matching action.