ECEN 3400 Spring 2014	Homework #3	Vince Coghlan

1) **14.9:**

A rectangular wire loop with sides of lengths a and b is moving away from a straight wire with a current I (Fig. P14.9). The velocity of the loop, v, is constant. Find the induced emf in the loop. The reference direction of the loop is shown in the figure. Assume that at $t = 0$ the position of the loop is defined by $x = a$.

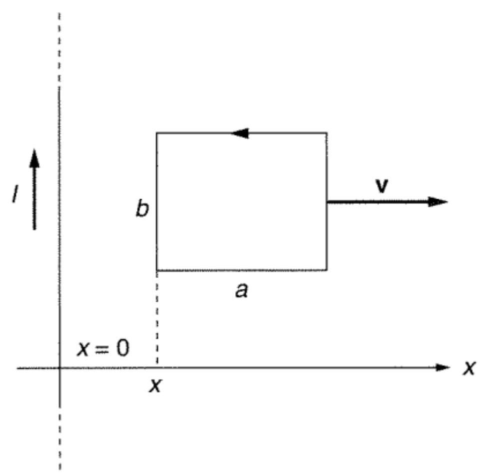

Figure P14.9 A moving frame in a magnetic field

We begin by finding the emf:

$$e = -\frac{d}{dt}\int_S B \cdot dS$$

$$e = \frac{d}{dt} b \int_{x=vt}^{x=a+vt} \frac{\mu_0 I}{2\pi x} \cdot dx$$

$$e = \frac{d}{dt}\frac{\mu_0 I b}{2\pi}(\ln(a+vt) - \ln(vt))$$

$$e = \frac{\mu_0 I b}{2\pi}(\frac{v}{a+vt} - \frac{1}{t})$$

This should be positive since the induced current will try and oppose the magnetic field provided. Since B field provided goes into the page, the induced emf will be counterclockwise to create a B field out of the page. Since there is a little arrow in the picture, that signifies the direction of the surface vector (or so I pressumed).

2) **15.8:**

Assume that within a certain time interval the current in circuit 1 in Fig. P15.8 grows linearly, $i_1(t) = I_0 + It/t_1$. Will there be any current in circuit 2 during this time? If yes, what is the direction and magnitude of the current? The number of turns of the two coils is the same.

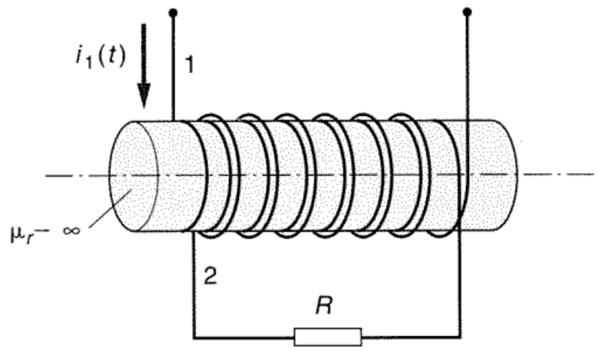

Figure P15.8 Two coupled circuits

There will be a current in circuit 2. Since the magnetic field is identical within the two coils, the induced emf in the second wire will be such as to generate an identical current. The direction will be as to oppose the induced field. If we say that our current moves from left to right of the resistor then the current will be:

$$i_2(t) = -i_1(t)$$

3) **15.12:**

The core of a toroidal coil of N turns consists of two materials of respective permeabilities μ_1 and μ_2, as in each part of Fig. P15.12. Find the self-inductance of the toroidal coil and the mutual inductance between the coil and the loop positioned as in Fig. P15.1 if (1) the ferrite layers are of equal thicknesses, $h/2$, in Fig. P15.12a, and (2) the ferrite layers are of equal heights h and the radius of the surface between them is c ($a < c < b$), in Fig. P15.12b.

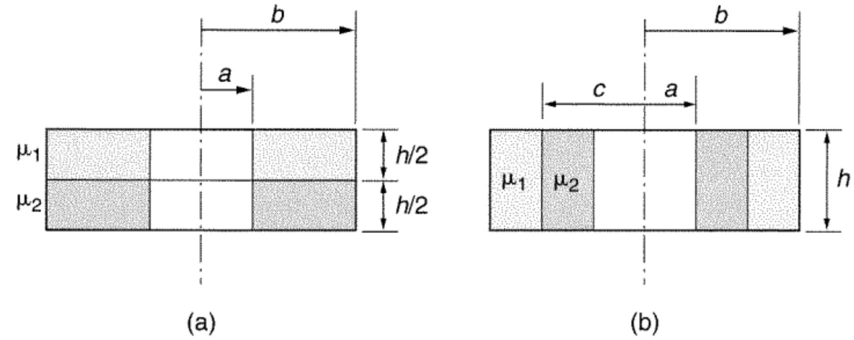

Figure P15.12 Two toroidal coils with inhomogeneous cores

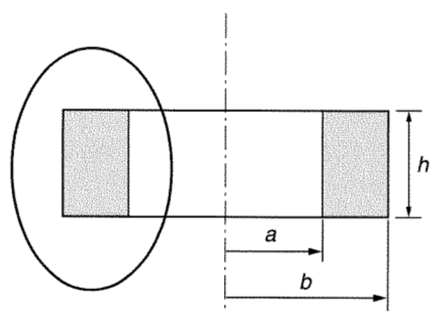

Figure P15.1 A toroidal coil and wire loop

(a) The flux can be found:

$$d\Phi_{21}(r) = B(r)dS = \frac{\mu_1 N I_2}{2\pi r}h_1 + \frac{\mu_2 N I_2}{2\pi r}h_2 dr$$

$$\Phi_{21} = (\frac{\mu_1 N I_2 h_1}{2\pi} + \frac{\mu_2 N I_2 h_2}{2\pi})\int_a^b \frac{dr}{r} = (\frac{\mu_1 N I_2 h_1}{2\pi} + \frac{\mu_1 N I_2 h_1}{2\pi})\ln\frac{b}{a}$$

Meaning that the mutual inductance is:

$$L_{12} = L_{21} = (\frac{\mu_1 N h_1}{2\pi} + \frac{\mu_2 N h_2}{2\pi})\ln\frac{b}{a}$$

We have the flux though a cross section of the torus, this means we can easiliy find the self-inductance (since the flux exists though N turns of the wire, we find:)

$$L = \frac{(\mu_1 + \mu_2)N^2 h}{2\pi} \ln \frac{b}{a}$$

(b) For this problem we need to use a different integral:

$$\Phi_{21} = \frac{NIh}{2\pi}(\int_a^c \frac{\mu_2 dr}{r} + \int_c^b \frac{\mu_1 dr}{r}) = \frac{NIh}{2\pi}(\mu_2 \ln \frac{c}{a} + \mu_1 \ln \frac{b}{c})$$

The mutual inductance:

$$L_{12} = L_{21} = \frac{Nh}{2\pi}(\mu_2 \ln \frac{c}{a} + \mu_1 \ln \frac{b}{c})$$

The self inductance is similarily:

$$L = \frac{N^2 h}{2\pi}(\mu_2 \ln \frac{c}{a} + \mu_1 \ln \frac{b}{c})$$

4) **15.17:**

A thin toroidal core of permeability μ, mean radius R, and cross-sectional area S is densely wound with two coils of thin wire with N_1 and N_2 turns, respectively. The windings are wound one over the other. Determine the self- and mutual inductances of the coils and the coefficient of coupling between them.

Just like in the last problem, we know the self inductances to be:

$$L_1 = \frac{\mu N_1^2 S}{2\pi R} \text{ and } L_2 = \frac{\mu N_2^2 S}{2\pi R}$$

The mutual inductance is given on page 268 of the textbook:

$$L_{12} = L_{21} = \frac{\mu N_1 N_2 h}{2\pi R}$$

Interestingly, $\sqrt{L_1 \times L_2} = L_{12}$ Which tells us that $k = 1$.

5) **16.4:**

A thin ferromagnetic toroidal core is made of a material that can be characterized approximately by a constant permeability $\mu = 4000\mu_0$. The mean radius of the core is $R = 10\,\text{cm}$ and the core cross-sectional area is $S = 1\,\text{cm}^2$. A current of $I = 0.1\,\text{A}$ is flowing through $N = 500$ turns wound around the core. Find the energy spent on magnetizing the core. Is this equal to the energy contained in the magnetic field in the core?

We know that the inductance of a torroidal core is:

$$L = \frac{\mu N^2 S}{2\pi R}$$

The magnetic energy in the coil is therefor:

$$W_m = \frac{1}{2}LI^2 = \frac{I^2 N^2 \mu S}{4\pi R} = 1mJ$$

The energy it takes to magnitize this device is equal to the amount of energy in theorem magnetic field, due to conservation of energy. This is also $1mJ$.

6) **16.17:**

An electromagnet and the weight it is supposed to lift are shown in Fig. P16.17. The dimensions are $S = 100\,\text{cm}^2$, $l_1 = 50\,\text{cm}$, $l_2 = 20\,\text{cm}$. Find the current through the winding of the electromagnet and the number of turns in the winding so that it can lift a load that is $W = 300$ kiloponds (a kp is 9.81 N) heavy. The electromagnet is made of a material whose magnetization curve can be approximated by $B(H) = 2H/(400 + H)$, where B is in T and H is in A/m.

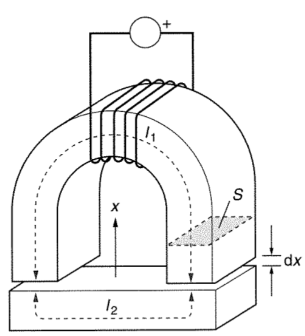

Figure P16.17 An electromagnet

The force is equal to:

$$F = \frac{1}{2}\frac{B^2}{\mu}2S$$

We need

$$300 * 9.81 = \frac{2H^2}{\mu(400 + H)}2(1)$$

In order to lift this we need an H field of $0.6086H$. To generate A field of this magnetude we will need a current of:

$$0.61 = \frac{I}{2A} \Rightarrow I = 0.61 * 2S = 0.0122A$$

7) **17.1:**

What is the minimum magnitude of a magnetic flux density vector that will produce the same magnetic force on an electron moving at 100 m/s that a 10-kV/cm electric field produces?

The force from this E-field is:

$$F = QE = -0.2pN$$

This mus be equal to:

$$-0.2pN = Qv \times B$$

If the B field is in the same direction

$$B = \frac{-0.2p}{Qv} = 10kT$$

d8) **18.1:**

Given a high-frequency RG-55/U coaxial cable with $a = 0.5$ mm, $b = 2.95$ mm, $\epsilon_r = 2.25$ (polyethylene), and $\mu_r = 1$, find the values for the capacitance and inductance per unit length of the cable.

These are from the textbook:

$$C' = \frac{2\pi\epsilon}{\ln(b/a)}$$

$$L' = \frac{\mu_r}{2\pi}\ln\frac{b}{a}$$

9) **18.5:**

Noting that $c = 1/\sqrt{\epsilon\mu}$ for all transmission lines in Table 18.1, prove that for these lines the inductance per unit length and the characteristic impedance of a lossless transmission line can be expressed in terms of c and C'.

Some alegebra and we can see:

$$L' = \frac{1}{c^2C'} \text{ and } Z_0 = \frac{1}{cC'}$$

For all of these.