ECEN 3400 Spring 2014	Homework #2	Vince Coghlan

1) **5.6:**

A very large flat plate of thickness d is uniformly charged with volume charge density ρ. Find the electric field strength at all points. Determine the potential difference between the two boundary planes, and between the plane of symmetry of the plate and a boundary plane.

Above and below the plate, the elecric feild from the plate is the integral:

$$\int_{-d/2}^{d/2} \frac{\rho}{2\epsilon_0} dy = \frac{\rho d}{2\epsilon_0}$$

Inside the plate, the field is the superposition of the fields from the top and bottom of the plate. The field from the above boundary plate is:

$$\frac{\rho(y - d/2)}{2\epsilon_0}$$

And from below:

$$\frac{\rho(y + d/2)}{2\epsilon_0}$$

Where y is the distance from the center of the plate. The sum of this is the field that at the point y:

$$\frac{\rho y}{\epsilon_0}$$

The potential difference between the boundary plates is:

$$\int_{-d/2}^{d/2} \frac{\rho y}{\epsilon_0} dy = \frac{\rho(d/2)^2}{2\epsilon_0} - \frac{\rho(-d/2)^2}{2\epsilon_0} = 0$$

To move it out to the boundary plane we modify the bounds of the integral:

$$\int_{0}^{d/2} \frac{\rho y}{\epsilon_0} dy = \frac{\rho(d/2)^2}{2\epsilon_0} = \frac{\rho d^2}{8\epsilon_0}$$

2) **5.14:**

Repeat problem P5.13 assuming that the two cylinders carry unequal charges per unit length, when these charges are (1) of the same sign, and (2) of opposite signs. Plot your results and compare to problem P5.13.

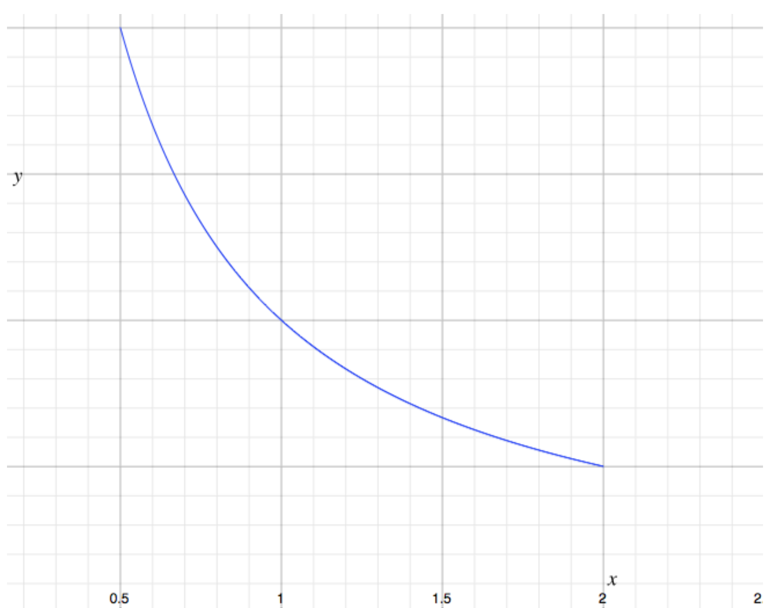
First I will do problem 5.13, which asks me to find an expression for the electric field strength and potential between and outside two long coaxial cylinders of radii a and b ($b > a$), carrying charges Q' and $-Q'$ per unit length. (This structure is known as a coaxial cable, or coaxial line.) Plot your resuls. Determine the voltage between the two cylinders. The electric field is going to be the superposition of the fields from both wires. For the inner wire:

$$\oint_S E \cdot dS = E(r)2\pi rh = \frac{Q}{\epsilon_0} \Rightarrow E(r) = \frac{Q'}{2\pi\epsilon_0 r}$$

The outer cylinder contributes the opposite field. This creates a net field of 0 outside of both cylinders. In between the cylinders, the field will only be the field contributed by the inner wire, this is because of how Gauss' law allows us to make our integrating surface in the middle of the wires and ignore the outside one. The electric field is therefore:

$$\frac{Q'}{2\pi\epsilon_0 r}$$

The plot if $a = 0.5$ and $b = 4$ looks like:



This allows us to find the potential difference by integrating the electric field:

$$\int_a^b \frac{Q'}{2\pi\epsilon_0 r} dr = \frac{Q'}{2\pi\epsilon_0} \ln\frac{b}{a}$$

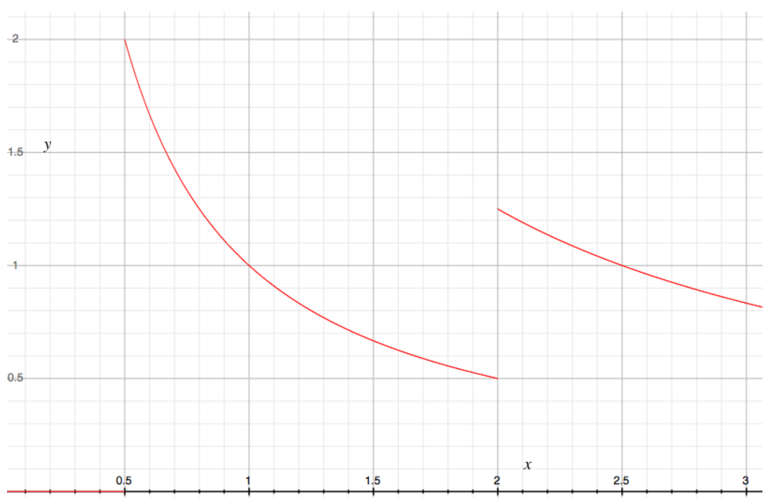
Now we will look at unequal charges, first of the same sign. These will be represented as λ_a and λ_b. In this case the electric field inside will be the electric field from the inner wire, or $\frac{\lambda_a}{2\pi\epsilon_0 r}$. Outside of both wires these fields will sum together and look like the following:

$$\frac{\lambda_a + \lambda_b}{2\pi\epsilon_0 r}$$

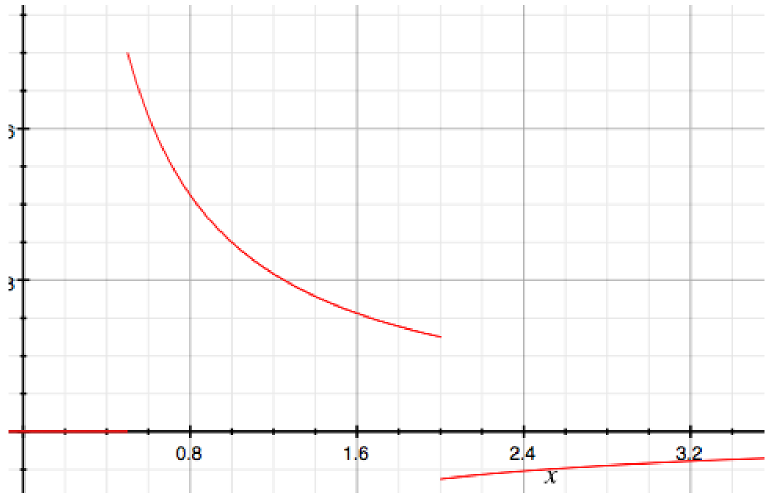
The potential is found in the same way:

$$\frac{\lambda_a + \lambda_b}{2\pi\epsilon_0} \ln\frac{b}{a}$$

Our plot is going to change slightly:



If the charges are opposite signs, then that will be encompased inside the same equations as above. The plot will change slightly:



Depending on which sign is larger this could look different. The big thing to notice is that the electric field inside is going to remain the same regardless of the outside charge. Gauss' law explains this phenomena with how it defines the integrating surface.

3) **6.11:**

Three coaxial conducting hollow cylinders have radii $a = 0.5\,\text{cm}$, $b = 1\,\text{cm}$, and $c = 2\,\text{cm}$, and equal lengths $d = 10\,\text{m}$. The middle cylinder is charged with a charge $Q = 1.5 \cdot 10^{-10}\,\text{C}$, and the other two are uncharged. Determine the voltages between the middle cylinder and the other two. Neglect effects at the ends of the cylinders.

The electric field inside the middle cylinder is going to be 0, since there is no enclosed charge. This makes the voltage also 0, since the integral of the electric field over this difference is 0. The electric field from the middle cylinder to the outer one can be found using a similar method to that of the last problem. We find it to be:

$$\frac{1.5 \cdot 10^{-10}/10}{2\pi\epsilon_0 r}$$

We can use this to find the voltage:

$$V = \int_{0.01}^{0.02} \frac{1.5 \cdot 10^{-10}/10}{2\pi\epsilon_0 r} dr = \frac{1.5 \cdot 10^{-10}/10}{2\pi\epsilon_0} \ln \frac{0.02}{0.01} = 0.1869V$$

4) **6.14:**

A point charge Q is at a point $(a, b, 0)$ of a rectangular coordinate system. The half-planes $(x \geq 0, y = 0)$ and $(x = 0, y \geq 0)$ are conducting. Determine the electric field at a point $(x, y, 0)$, where $x > 0$ and $y > 0$.

This point charge is going to feel the effects on each side as if another charge was on the other side of each plane. We can sum the fields from each "mirror image" charge to find the field at any point.

$$E(x, y, 0) = \frac{kQ}{(x-a)^2 + (y-b)^2} + \frac{kQ}{(x+2a)^2 + (y-b)^2} + \frac{kQ}{(x-a)^2 + (y+2b)^2}$$

5) **7.7:**

The permittivity of an infinite dielectric medium is given as the following function of the distance r from the center of symmetry: $\epsilon(r) = \epsilon_0(1 + a/r)$. A small conducting sphere of radius R, carrying a charge Q, is centered at $r = 0$. Determine and plot the electric field strength and the electric scalar potential as functions of r. Determine the volume density of polarization charges.

The electric field strength can be found as a simple functino of r:

$$E(r) = \frac{Q}{4\pi\epsilon_1 r^2} \Rightarrow \frac{Q}{4\pi\epsilon_0(1 + a/r)r^2} \text{ for } r > R \text{ else } E = 0$$

To find to scalar potential, we can preform the following integral:

$$V = \int_R^r \frac{Q}{4\pi\epsilon_0(a + r)r} dr = \frac{Q}{4\pi\epsilon_0}(\ln(\frac{r}{r+1}) - \ln(\frac{R}{R+1}))$$

The volume density of polarization charges is the charge Q devided by the total volume of the sphere:

$$\frac{Q}{4/3\pi R^3}$$

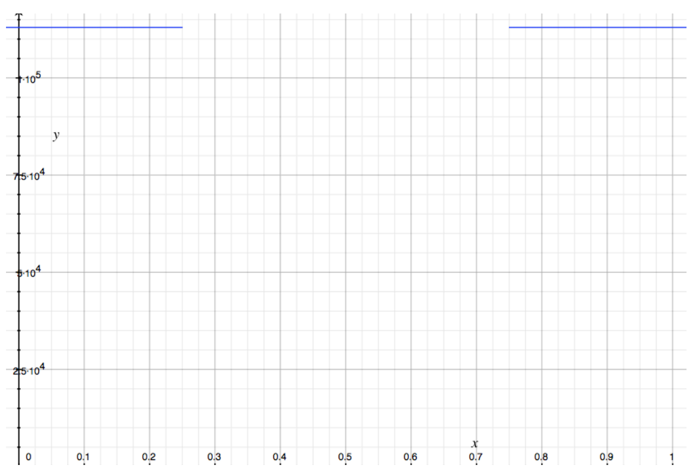
6) **8.15:**

Two flat parallel conductive plates of surfaces $S = 0.05\,\mathrm{m}^2$ are charged with $Q_1 = 5 \cdot 10^{-8}\,\mathrm{C}$ and $Q_2 = -Q_1$. The distance between the plates is $D = 1\,\mathrm{cm}$. Find the electric field strength vector at all points if a third, uncharged metal plate, $d = 5\,\mathrm{mm}$ thick, is placed between the two plates $a = 2\,\mathrm{mm}$ away from one of the charged plates and parallel to it. Plot the electric field strength before and after the third plate is inserted. Compare and explain. Find the capacitance between the charged plates without and with the third plate between them.

Without the charged plate in between the other two, we can find the Electric field quite easily:

$$E = \frac{\sigma}{\epsilon_0} \text{ where } \sigma = \frac{Q}{A} \Rightarrow E = 113000N/C$$

With the new uncharged plate, the electric field remains unchaged. This is because the perfect conductor will move a charge Q towards the plate 1 and a charge $-Q$ towards the other plate. The only difference is that the electric field in that 5mm where the plate is becomes 0. The plot would look like so:



The capacitance has changed, since the sum of the electric field over the distance is now different. In the original case, the voltage is:

$$V = E \cdot d = 1130Volts$$

And in the second case:

$$V = E \cdot (0.25cm) + E \cdot (0.25cm) = 565Volt$$

7) **8.20:**

A coaxial cable has two dielectric layers with relative permittivities $\epsilon_{1r} = 2.5$ and $\epsilon_{2r} = 4$. The inner conductor radius is $a = 5$ mm, and the inner radius of the outer conductor is $b = 25$ mm. (1) Find how the dielectrics need to be placed and how thick they need to be so that the maximum electric field strength will be the same in both layers. (2) What is the capacitance per unit length of the cable in this case? (3) What is the largest voltage that the cable can be connected to if the dielectrics have a breakdown field of 200 kV/cm?

Gauss' law tells us that the electric field on the surface of the inner dielectric is $\frac{Q'}{2\pi\epsilon_{1r}r}$. Note that the thinckness and position of this inner dielectric doesnt matter. The electric field in the outer dielectric is 0. The outer and inner electric can never be the same unless there is no charge running through the cable. The capacitance per unit length can be found quite easily, first we find the voltage:

$$V = \frac{Q'}{2\pi\epsilon_{1r}} \ln(\frac{b}{a})$$

$$C' = \frac{Q'}{V} = \frac{2\pi\epsilon_{1r}}{\ln\frac{b}{a}} = 9.76F/m$$

The largest voltage that can be applied is:

$$V = 200kV/cm \cdot 20mm = 400kV$$

8) **9.15:**

A two-wire line has conductors with radii $a = 3$ mm and the wires are $d = 30$ cm apart. The wires are connected to a voltage generator such that the voltage between them is on the verge of initiating air ionization. (1) Find the electric energy per unit length of this line. (2) Find the force per unit length acting on each of the line wires.

The field per unit length can be found in the textbook as

$$E' = \frac{Q'}{2\pi\epsilon_0(30cm)} \text{ where } Q' = \pi\epsilon_0 V/\ln(30cm/3mm) = 506569V/m$$

The force per unit lenght can also be found by an equation in the book:

$$F = -Q'E' = 1.071N/m$$