ECEN 3400 Spring 2014 **Vince Coghlan**

Homework #1

1) **P1.9:**

A small body charged with $Q = -10^{-10}$ C finds itself in a uniform electric and magnetic field as shown in Fig. P1.9. The electric field vector and the magnetic flux density vector are **E** and **B**, respectively, everywhere around the body. If the magnitude of the electric field is $E = 100\,\mathrm{N/C}$, and the magnetic flux density magnitude is $B = 10^{-4}\,\mathrm{N \cdot s/C \cdot m}$, find the force on the body if it is moving with a velocity **v** as shown in the figure, where $v = 10\,\mathrm{m/s}$ (the speed of a slow car on a mountain road). How fast would the body need to move to maintain its direction of motion?

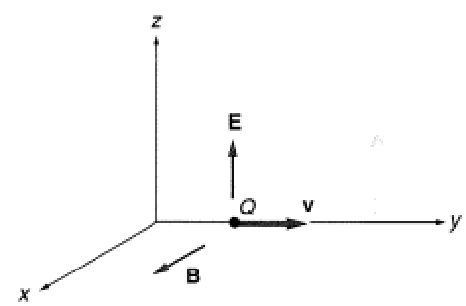


Figure P1.9 Point charge in an electric and magnetic field

To find the force we can use two simple equations:

$$\bar{F_m} = Q\bar{v} \times \bar{B} \text{ and } \bar{F_e} = \bar{E} \cdot q$$

We can work these out:

$$\bar{F_m} = -10^{-10} \cdot (10 \cdot 10^{-4})(-\hat{z}) = 10^{-13}\hat{z}$$

$$\bar{F_e} = 100 \cdot -10^{-10}(\hat{z}) = -10^{-8}\hat{z}$$

The vectors are in opposite directions and thus the force felt by the charged body is mostly from the electric field, which would be: $\bar{F} \approx -10^{-8}\hat{z}$. In order for the object to maintain its direction:

$$|\bar{F_m}| = |\bar{F_e}| \Rightarrow -10^{-10} \cdot (v \cdot 10^{-4}) = 100 \cdot -10^{-10} \Rightarrow v = 1 \cdot 10^{6} \text{ m/s}$$

2) **P2.1:**

The capacitance of a switch ranges from a fraction of a picofarad to a few picofarads. Assume that a generator of variable angular frequency ω is connected to a resistor of resistance of 1 MΩ, but that the switch is open. Assuming a switch capacitance of 1 pF, at what frequency is the open switch reactance equal to the resistor resistance?

The resistance of a capacitor is $\frac{1}{C\omega}$, we can setup a simple equation to solve this problem:

$$\frac{1}{1 \cdot 10^{-12} \cdot \omega} = 1 \cdot 10^{6} \Rightarrow \omega = 1 \text{ MHz}$$

3) A surface-mount chip capacitor package has an unknown parasitic inductance L. You need to use the capacitor at some high frequency and therefore you need to find out what L is, i.e. is the capacitor starting to act as an inductor at your design frequency. To find the parasitic inductance, which is assumed to be the same for all capacitors with the same package, you can do the following experiment: order several capacitors of different values - 1pF, 40pF, 80pF. Then connect each to ground and measure the input impedance as a function of frequency. The measurement shows resonant frequencies of 1.01, 1.44 and 8.001GHz (how do you know what the resonant frequency is from an impedance measurement?). What is your best estimate of the parasitic inductance of this surface-mount package? What is your estimate on the tolerance in the value of the spec-sheet capacitance?

The resonant frequency of a LC circuit is $\frac{1}{2\pi\sqrt{LC}}$, thus we can solve for L.

$$1.01 \cdot 10^{9} = \frac{1}{2\pi\sqrt{L \cdot 1 \cdot 10^{-12}}} \Rightarrow L = 2.48 \cdot 10^{-8} L$$

$$1.44 \cdot 10^{9} = \frac{1}{2\pi\sqrt{L \cdot 40 \cdot 10^{-12}}} \Rightarrow L = 3.05 \cdot 10^{-10} L$$

$$8.001 \cdot 10^{9} = \frac{1}{2\pi\sqrt{L \cdot 80 \cdot 10^{-12}}} \Rightarrow L = 4.95 \cdot 10^{-12} L$$

My estimate for the parasitic inductance will be right in the middle, at 300pH. The tolerance on the capacitance is probably maximized at 80 pF when we use this parasitic inductance. We find it to be $\pm$1.3pF, this is $\pm$1.6%.

4) **P3.14:**

An electric dipole consists of two equal and opposite point charges Q and $-Q$ that are a distance d apart, Fig. P3.14. (1) Find the electric field vector along the x axis in the figure. (2) Find the electric field vector along the y axis. (3) How does the electric field strength behave at distances $x \gg d$ and $y \gg d$ away from the dipole? How does this behavior compare to that of the field of a single point charge?

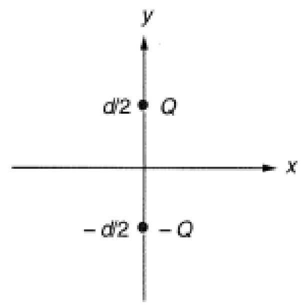


Figure P3.14 An electric dipole consists of two equal charges of opposite signs.

(1) The electric field vector along the x axis will be in the $-y$ direction along the entire axis. The magnitude can be found like so:

$$\bar{E} = -\frac{kQ}{x^2 + (\frac{d}{2})^2}\hat{y}$$

(2) Along the y axis it varies. For $\frac{-d}{2} > y > \frac{d}{2}$ then the force points in the positive direction. The field would be the sum of the two fields from the charges:

$$\bar{E} = (\frac{kQ}{(y - d/2)^2} - \frac{kQ}{(y + d/2)^2})\hat{y}$$

(3) As $x >> d$ and $y >> d$ then the two charges begin to behave as one, and the field from each charge cancels out. This is not true of a single point charge, where the force would be much stronger at that distance.

5) **P3.23:**

A dielectric cube with sides of length a is charged over its volume with a charge density $\rho(x) = \rho_0 x/a$, where x is the normal distance from one side of the cube. Determine the charge of the cube.

This is a geometry problem:

$$\int_V p(x)dV = \int_0^a \int_0^a \int_0^a p_0 \frac{x}{a} dxdydz = p_0 \frac{a^3}{2}$$

6) **P4.5:**

A volume of a liquid conductor is sprayed into N equal spherical drops. Then, by some appropriate method, each drop is given a potential V with respect to the reference point at infinity. Finally, all these small drops are combined into a large spherical drop. Determine the potential of the large drop.

The potential of all the drops together would be $N \cdot V$. This is because the voltage from infinity will be defined as:

$$V_A = \frac{Q}{4\pi\epsilon_0 r}$$

And since the the radius and constant don't change, each drop has a measurable charge. When we put the drops together the charge adds on itself, and we are left with more charge. N drops means:

$$V_{A\text{ new}} = \frac{k \cdot N \cdot Q}{r} = N \cdot V_A$$

7) **P4.12:**

An insulating disk of radius $a = 5\,\mathrm{cm}$ is charged by friction uniformly over its surface with a total charge of $Q = -10^{-8}\,\mathrm{C}$. Find the expression for the potential of the points which lie on the axis of the disk perpendicular to its surface. Plot your result. What are the numerical values for the potential at the center of the disk, and at a distance

$z = a$ from the center, measured along the axis? What is the voltage between these two points equal to?

The potential equation can be found using an integral:

$$V = \int_A \frac{k \cdot Q}{R} dA$$

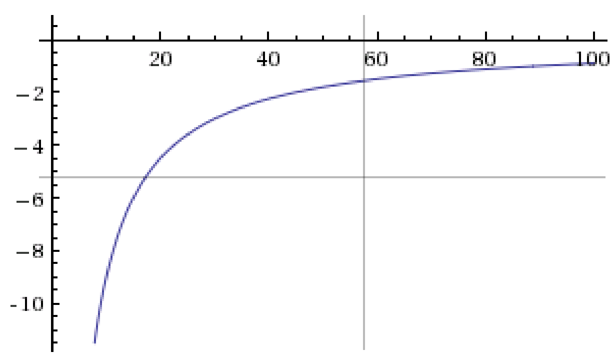
The nifty way of doing this problem is in radial coordinates(adding up a bunch of charged rings). If the area is 5cm and we know the charge, then the charge density is $-1.273 \cdot 10^{-6}$ C/m. The charge on any one of the rings $dq = \sigma(2\pi r dr)$. The potential from a ring is:

$$dV = k\frac{dq}{\sqrt{z^2 + r^2}} = 2\pi\sigma k \frac{rdr}{\sqrt{z^2 + r^2}}$$

We can now add these up:

$$V(z) = 2\pi\sigma k \int_0^{.05} \frac{rdr}{\sqrt{z^2 + r^2}} = 2\pi\sigma k(\sqrt{z^2 + (0.05)^2} - z)$$

This looks like:



For $z = a$:

$$V = 2\pi\sigma k(\sqrt{a^2 + (0.05)^2} - a)$$