ECEN 3400 Spring 2014	Homework #6	Vince Coghlan

1) **P18.9:**

A short and then an open load are connected to a 50-Ω transmission line at $z = 0$. Make a plot of the impedance, normalized voltage ("normalized" means that you divide the voltage by its maximal value to get a maximum normalized voltage of 1), and normalized current along the line up to $z = -3\lambda/2$ for the two cases.

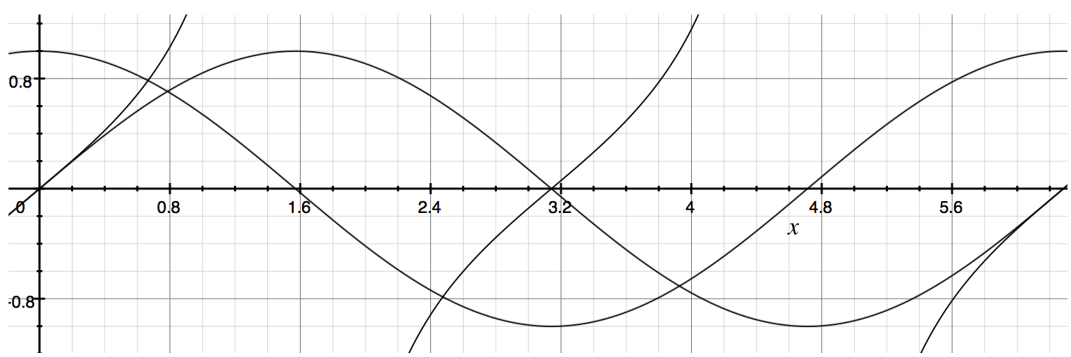
While this line is short-circuited, the impedence can be seen in example 18.6 from the textbook:

$$Z(z) = jZ_0 \tan(\beta|z|)$$

We know that $Z = \frac{V}{I}$. This means that the sinusoidal components of V and I must make a tangent. The solution to this is:

$$V(z) = jZ_0 I(0) \sin(\beta|z|),\ I(z) = I(0)\cos(\beta|z|)$$

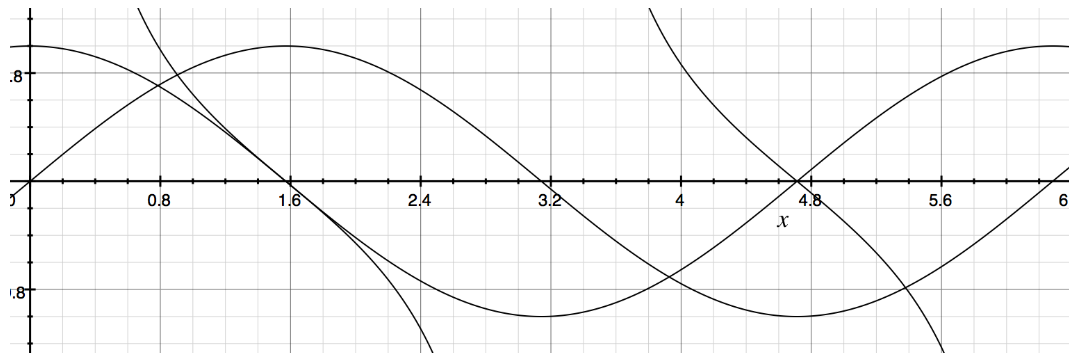
We can plot this:



The cosine curve is real, and the sine curve and tangent curve are imaginary.

Similarily, the textbook also tells us the loaded impedence and such:

$$Z(z) = \frac{Z_0}{j\tan(\beta|z|)},\ V(z) = V(0)\cos(\beta|z|),\ I(z) = \frac{jV(0)}{Z_0}\sin(\beta|z|)$$



only the cosine curve is real, the rest are imaginary.

2) **P18.10:**

A *lumped* capacitor is inserted into a transmission-line section, as shown in Fig. P18.10. Find the reflection coefficient for a wave incident from the left. Assume the line is terminated to the right so that there is no reflection off the end of the line. Find a simplified expression that applies when C is small. The characteristic impedance of the line is Z_0.

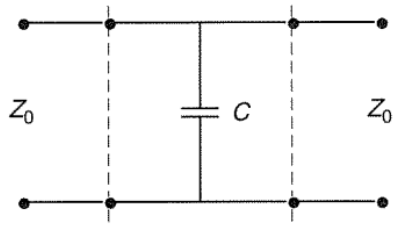


Figure P18.10 A shunt capacitor in a line

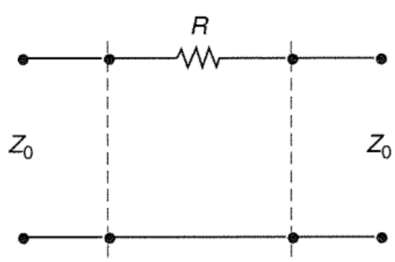


Figure P18.11 A series resistor in a line

The equivalent impedence

$$Z_e = \frac{Z_0/(1/j\omega C)}{Z_0 + 1/(j\omega C)} = \frac{Z_0}{1 + j\omega C Z_0}$$

This gives us the reflection coefficient

$$\rho = \frac{Z_e - Z_0}{Z_e + Z_0} = \frac{-j\omega C Z_0}{2 + j\omega C Z_0} = -\frac{\omega^2 C^2 Z_0^2}{4 + \omega^2 C^2 Z_0^2} - \frac{2j\omega C Z_0}{4 + \omega^2 C^2 Z_0^2}$$

When C is small

$$\rho \approx \frac{\omega C Z_0}{2j}$$

3) **P18.12:**

Repeat problem P18.10 assuming that a lumped inductor is inserted into a transmission-line section as shown in Fig. P18.12. Find a simplified expression that applies when L is small.

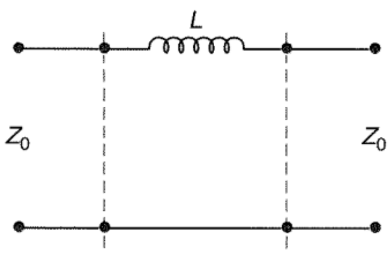


Figure P18.12 A series coil in a line

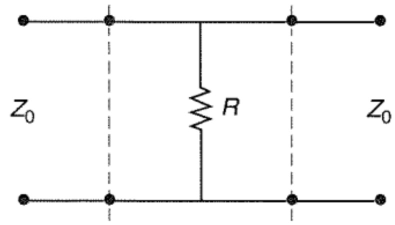


Figure P18.13 A shunt resistor in a line

The equivalent impedence

$$Z_e = Z_0 + Lj\omega$$

This gives us the reflection coefficient

$$\rho = \frac{Z_e - Z_0}{Z_e + Z_0} = \frac{Lj\omega}{Lj\omega + Z_0}$$

When L is small:

$$\rho \approx \frac{Lj\omega}{Z_0}$$

4) **P18.17:**

Find the reflection and transmission coefficients for the transmission line in Fig. P18.17. Because the reflection coefficient is defined by voltage, the power is given by its square. What are the reflected and transmitted power equal to? Does the power balance make sense?

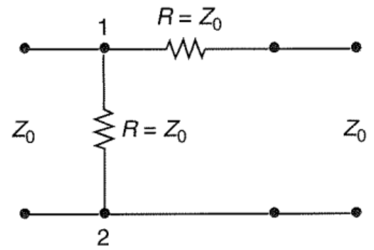


Figure P18.17 Two resistors in a line

This is a transmission line where $Z_e = \frac{2}{3}Z_0$. We can for an equation:

$$\rho = \frac{\frac{-1}{3}Z_0}{\frac{5}{3}Z_0} = -0.2Z_0$$

$$\tau = \rho + 1 = 0.8Z_0$$

This means that the power reflected is 0.04 the power transmitted is 0.64. The difference is the power loss through the impedence.

5) **P18.23:**

Find the input impedance for the circuit in Fig. P18.23.

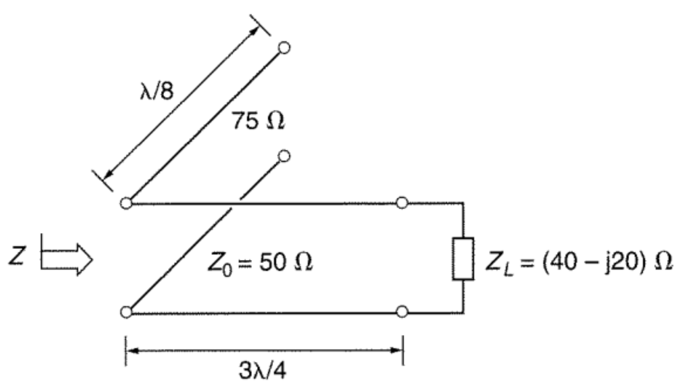


Figure P18.23 Impedance of a line with a shunt stub

We know the impedence to be:

$$Z = Z_0 \frac{Z_L + jZ_0 \tan(\beta\zeta)}{Z_0 + jZ_L \tan(\beta\zeta)} = Z_0 \frac{Z_L + jZ_0 \tan(\frac{2\pi}{\lambda}\frac{3\lambda}{4})}{Z_0 + jZ_L \tan(\frac{2\pi}{\lambda}\frac{3\lambda}{4})}$$

$$= Z_0 \frac{75 + jZ_0 \tan(\frac{3}{2})}{Z_0 + j75 \tan(\frac{3}{2})}$$

6) **P18.24:**

> A coaxial transmission line with a characteristic impedance of 150 Ω is 2 cm long and is terminated in a load impedance of $Z = 75 + j150$ Ω. The dielectric in the line has a relative permittivity of $\epsilon_r = 2.56$. Find the input impedance and VSWR on the line at $f = 3$ GHz.

We can find that the electrical length is $\zeta = \frac{2}{\lambda} = \frac{2}{10} = \frac{1}{5}$. This gives us the input impedence: $(\beta = \frac{\pi}{5})$

$$Z = 150 \frac{75 + j150 + j(150)\tan(\pi/25)}{150 + j(75 + j150)\tan(\pi/25)}$$

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 + 150j - 150}{75 + 150j + 150} = \frac{1 + 8j}{13}$$

$$\text{VSWR } = -\frac{2 + 8i}{8i} = \frac{j}{4} - 1$$

7) **P18.31:**

> Trace the procedure for solving problem P18.8 by means of the Smith chart.

the normalized load impedence of the first line $Z_{1L} = \frac{Z_l}{Z_{01}}$. We begin at this point then rotate wavelength clockwise by $\frac{\zeta_1}{\lambda}$. The normalized impedance along the first line is thus $z_1(\zeta_1)$ Which tells us the actual impedance $Z_1(\zeta_1) = Z_{01}z_1(\zeta_1)$. We then repeat this for line two, to find the new impedance $Z_2(\zeta_2) = Z_{02}z_2(\zeta_2)$. Once we have the impedance we can find the corresponding reflection coefficient.

$$\rho = \frac{Z_L - Z_1 - Z_2}{Z_L + Z_1 + Z_2}$$

$$\text{VSWR } = \frac{1 + |\rho|}{1 - |\rho|}$$

8) **P18.35:**

What circuit element corresponds to the point on the Smith chart that is defined by the intersection of the circle $r = 1$ and the arc $jx = -j0.4$ at 500 MHz, if the normalizing impedance is 50 Ω?

This is going to be a combination of a negative imaginary part and a positive real part. This is a resistor in series capacitor. The value of the resistor is the impedance, 50Ω. The value of the capacitor can be found:

$$C \Rightarrow \frac{1}{j2\pi 500 \cdot 10^6 C} = -0.4j$$

$$\frac{1}{\pi * 10^9 C} = 0.4 \Rightarrow C = 318.3pF$$