ECEN 3400 Spring 2014 **Vince Coghlan**

Homework #4

1) **12.13:**

Evaluate the magnetic flux density at point A in Fig. P12.13.

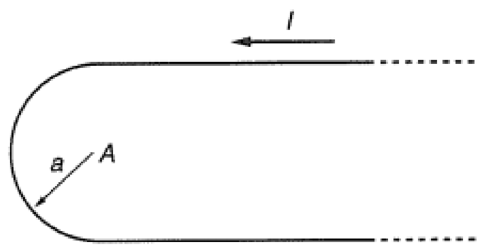


Figure P12.13 Short-circuited two-wire line

We can find this by using an integral. Since l and u_r are always normal to eachother, the values of all the little dBs will compound together in the same direction, and the cross products will be 1. We are basically evaluating the length of the wire from 0 to π. Since the circumference of a circle is $2\pi a$ then $\frac{1}{2}$ of the circumference, and the value of the integral is just πa. We get the following:

$$B_A = \int_0^\pi \frac{\mu_0}{4\pi} \frac{I}{a^2} dl = \frac{\mu_0 I}{4a}$$

We then add this to the magnetic field from a wire:

$$B_A = \frac{\mu_0 I}{4a} + \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I(2+\pi)}{4\pi a}$$

2) **12.38:**

A current $I = 0.5$ A flows through the torus winding shown in Fig. P12.38. Find the magnetic flux density at points A_1, A_2, and A_3 inside the torus. There are $N = 2500$ turns, $a = 5$ cm, $b = 10$ cm, and $h = 4$ cm.

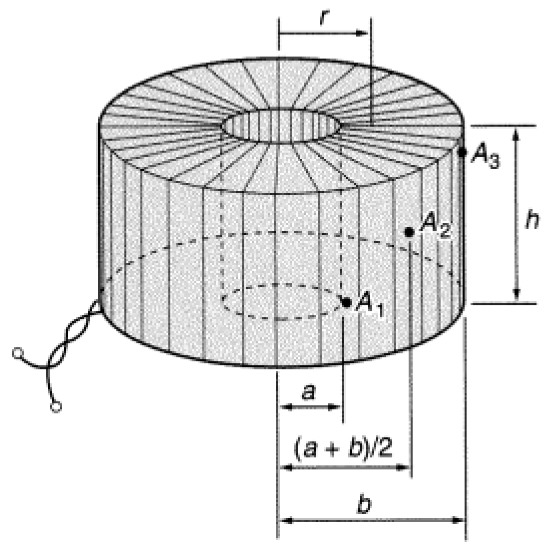


Figure P12.38 A densely wound thick toroidal coil

We can utilize ampere's law to find the magnetic field generally:

$$\oint_C B \cdot dl = \mu_0 \int_S J \cdot S \Rightarrow 2\pi r B = \mu_0 N I$$

$$B_{A_1} = \frac{\mu_0 N I}{2\pi a},\ B_{A_2} = \frac{\mu_0 N I}{\pi(a+b)},\ B_{A_3} = \frac{\mu_0 N I}{2\pi b}$$

3) **13.5:**

The ferromagnetic toroidal core sketched in Fig. P13.5a has an idealized initial magnetization curve as shown in Fig. P13.5b. Determine the magnetic field strength, the magnetic flux density, and the magnetization at all points of the core, if the core is wound uniformly with $N = 628$ turns of wire with current of intensity (1) 0.5 A, (2)

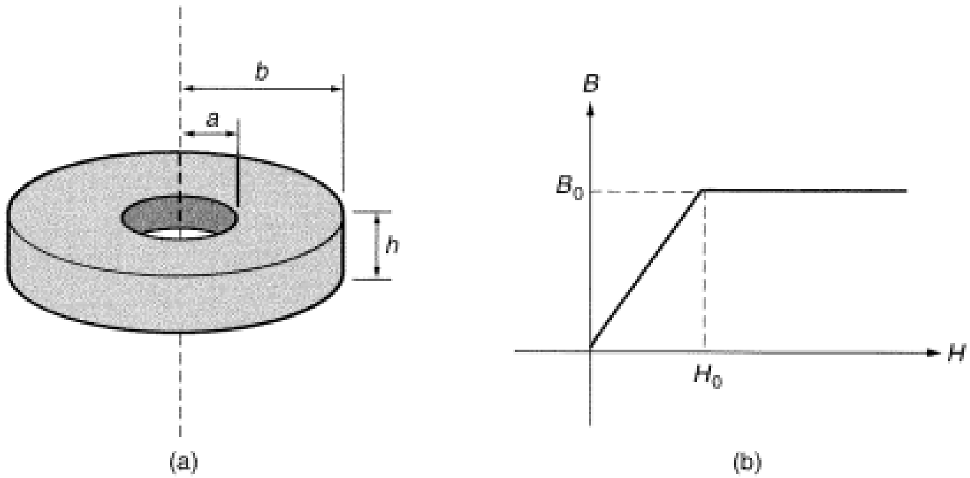


Figure P13.5 (a) A ferromagnetic core, and (b) its idealized initial magnetization curve

0.75 A, or (3) 1 A. The core dimensions are $a = 5$ cm, $b = 10$ cm, and $h = 5$ cm, and the constants of the magnetization curve are $H_0 = 1000$ A/m and $B_0 = 2$ T. For the three cases determine the magnetic flux through the core's cross section. Assume that the core was not magnetized prior to turning on the current in the winding.

We will begin with Ampere's law (so we can find H).

$$\oint_C H \cdot dl = \int_S J \cdot dS$$

I will first find the edges of where the magnetization curve applies. If we are on the left side of the magnetization curve, we can pull H out of the integral and solve:

$$H 2\pi r = NI \Rightarrow H = \frac{NI}{2\pi r},\ H < H_0$$

Lets test to see if we are indeed on that side of the equation (using 0.5A):

$$H = \frac{NI}{2\pi a} = 999.5$$

Since this will be the point of largest H, we know that for this case, we will never have to deal with M. I will finish out this case and then move on to the more complicated ones. We know the magnetic field strength to be:

$$H = \frac{NI}{2\pi r}$$

The Magnetic flus density B is:

$$B = \frac{\mu NI}{2\pi r}$$

The magnetization is:

$$M = \frac{B}{\mu} - H = 0$$

The flux Φ is:

$$\Phi = \int\int_S H \cdot dS = h\int_a^b \frac{NI}{2\pi r} \cdot dr = \frac{hNI\ln(\frac{b}{a})}{2\pi} = 1.73Wb$$

Note that we can consider the field to be constant in the vertical direction, and so we only consider the field as it changes around the radius r. We will continue on to the next current, 1A:

$$H = \frac{NI}{2\pi b} = 999.5$$

This is not that much smaller than 1000, so I will consider this to be the upper portion of the magnetization curve. This means that our value for B is going to remain constant at 2T while our value for H is going to increase with M.

$$B = 2T$$

$$H = \frac{NI}{2\pi r}$$

$$M = \frac{2T}{\mu} - H = \frac{2T}{\mu} - \frac{NI}{2\pi r}$$

$$\Phi = \int\int_S H \cdot dS = h\int_a^b \frac{NI}{2\pi r} \cdot dr = \frac{hNI\ln(\frac{b}{a})}{2\pi} = 3.46Wb$$

And for .75A we are going to have to use both equations for some point in between r where:

$$H = \frac{NI}{2\pi r} = 1000$$

We can solve and see that r = 0.0749 m. This means that we must have two equations:

$$B = \begin{cases} \frac{\mu NI}{2\pi r} & : 0.05 < r < 0.0749 \\ 2T & : 0.0749 < r < 0.10 \end{cases}$$

$$M = \begin{cases} 0 & : 0.05 < r < 0.0749 \\ \frac{2T}{\mu} - \frac{NI}{2\pi r} & : 0.0749 < r < 0.10 \end{cases}$$

At this point the flux is going to be using the same equation, since H is not dependent on these things, and encompases only the field generated using Ampere's law, which is immune to this magnetisim tomfoolery.

$$\Phi = \int\int_S H \cdot dS = h \int_a^b \frac{NI}{2\pi r} \cdot dr = \frac{hNI \ln(\frac{b}{a})}{2\pi} = 2.60Wb$$

4) **13.25:**

The thick toroidal core sketched in Fig. P13.25 is made out of the ferromagnetic material from problem P13.23. There are $N = 200$ turns wound around the core, and the core dimensions are $a = 3$ cm, $b = 6$ cm, and $h = 3$ cm. Find the magnetic flux through the core for $I = 0.2$ A and $I = 1$ A in two different ways: (1) using the mean radius; and (2) by dividing the core into 5 layers and finding the mean magnetic field in each of the layers.

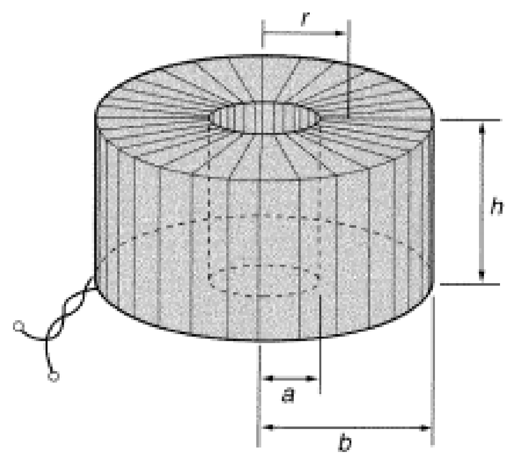


Figure P13.25 A thick toroidal coil

We will begin, as we often do, with Ampere's law:

$$\oint_C H \cdot dl = \int_S J \cdot dS$$

$$H = \frac{NI}{2\pi r}$$

At I = 0.2 we can find the flux in two ways: The first way:

$$\Phi = H \cdot A = \frac{NI}{\pi(b-a)} h(b-a) = 0.38Wb$$

The other way is:

$$\frac{1}{6} \sum_{r=3.25,3.75,4.25,4.75,5.25,5.75} \frac{NI}{2\pi r} = 1.47Wb$$

At I = 1

$$\Phi = H \cdot A = \frac{NI}{\pi(b-a)} h(b-a) = 1.91Wb$$

$$\frac{1}{6} \sum_{r=3.25,3.75,4.25,4.75,5.25,5.75} \frac{NI}{2\pi r} = 7.35Wb$$

5) **14.6:**

A current $i(t) = I_m \sin(2\pi f t) = 2.5 \sin 314t$ A is flowing through the solenoid in Fig. P14.6, where frequency is in hertz and time is in seconds. The solenoid has $N_1 = 50$ turns of wire, and the coil K shown in the figure has $N_2 = 3$ turns. Calculate the emf induced in the coil, as well as the amplitude of the induced electric field along the coil turns. The dimensions indicated in the figure are $a = 0.5$ cm, $b = 1$ cm, and $L = 10$ cm. Plot the induced emf as a function of N_1, N_2, and f.

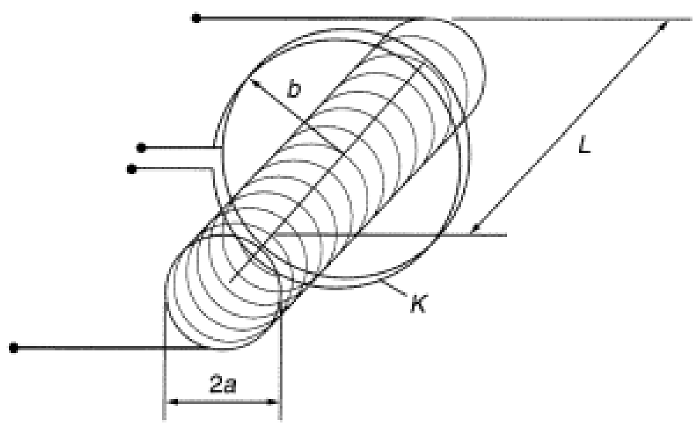


Figure P14.6 A solenoid and a coil

We will use Faraday's law:

$$V = -\frac{d}{dt}\int_S B \cdot dS$$

$$V = -\frac{d}{dt}\frac{\mu_0 N_1 N_2 i(t) 2\pi a}{L}$$

$$V = -\frac{d}{dt}\frac{\mu_0 N_1 N_2 I_m \sin(2\pi f t) 2\pi a}{L}$$

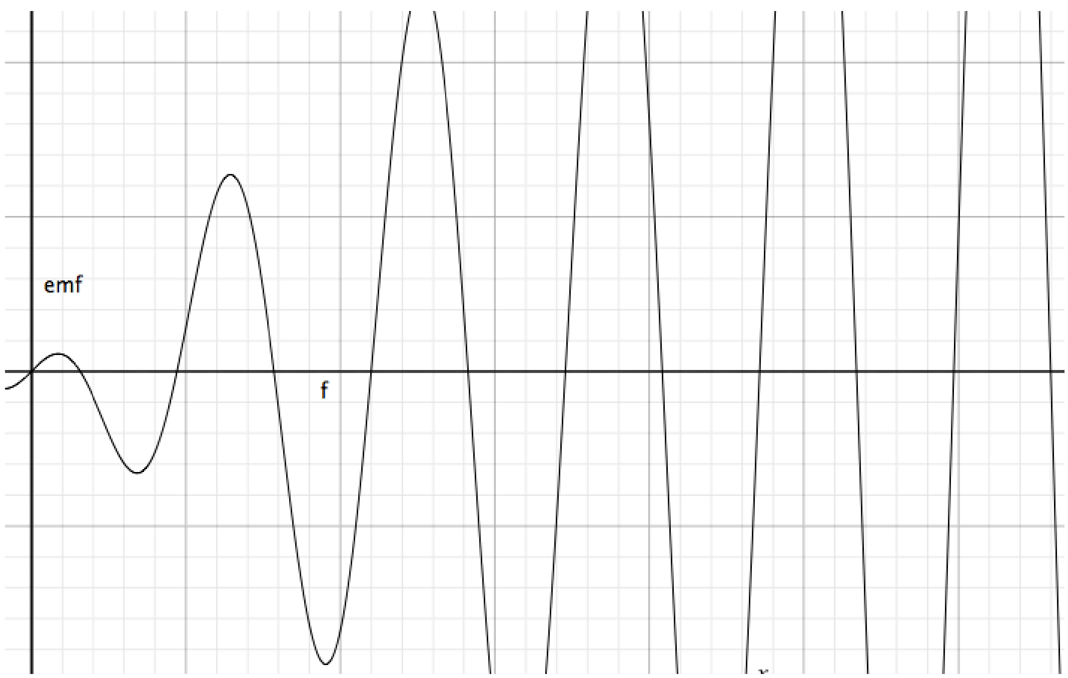
$$V = -\mu_0 N_1 N_2 I_m \cos(2\pi f t) 4\pi^2 a f \frac{1}{L} \approx -9V cos(314t)$$

The induced emf will induce an electric field proportional to the resistance of the wire, so the question cannot be answered numerically, but symbollically:

$$E_{max} = \frac{\mu_0 N_2 |V|}{2Rb}$$

We know $|V|$ to be 9, but R was not given in the problem. The plots will be the same with respect to N_1 and N_2... so:

And with frequency:



6) **14.16:**

A circular loop of radius a rotates with an angular velocity ω about the axis lying in its plane and containing the center of the loop. It is situated in a uniform magnetic field of flux density $\mathbf{B}(t)$ normal to the axis of rotation. Determine the induced emf in the loop. At $t = 0$, the position of the loop is such that $\mathbf{B}$ is normal to its plane.

First we find the flux to be:

$$\Phi = B(t)cos(\omega t)\pi a^2$$

Then we can find the induced emf to be

$$V = -\frac{d}{dt}B(t)\cos(\omega t)\pi a^2$$

7) **15.3:**

A cable-car track runs parallel to a two-wire phone line, as in Fig. P15.3. The cable-car power line and track form a two-wire line. The amplitude of the sinusoidal current through the cable-car wire is I_m and its angular frequency is ω. All conductors are very

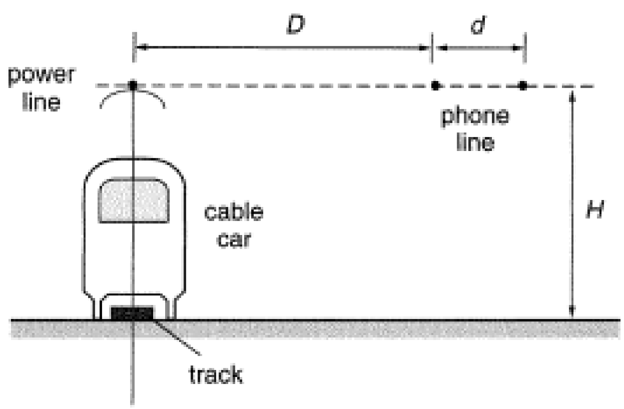


Figure P15.3 Cable-car track parallel to phone line

thin compared to the distances between them. Find the amplitude of the induced emf in a section of the phone line b long.

First we find the flux per unit length:

$$\Phi_1' = \frac{\mu_0 i(t)}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\mu_0 i(t)}{2\pi}\ln(\frac{r_2}{r_1})$$

where $r_1 = D$ and $r_2 = \sqrt{D^2 + H^2}$. On the other line it is:

$$\Phi_2' = \frac{\mu_0 i(t)}{2\pi}\int_{r_3}^{r_4}\frac{dr}{r} = \frac{\mu_0 i(t)}{2\pi}\ln(\frac{r_4}{r_3})$$

where $r_3 = D + d$ and $r_4 = \sqrt{(D+d)^2 + H^2}$. The total flux per unit length is thus:

$$\frac{\mu_0 i(t)}{2\pi}\ln(\frac{r_2}{r_1}) + \frac{\mu_0 i(t)}{2\pi}\ln(\frac{r_4}{r_3}) = \frac{\mu_0 i(t)}{2\pi}\ln(\frac{r_2 r_4}{r_1 r_3})$$

and our emf is:

$$-\frac{d}{dt}\frac{\mu_0 i(t)}{2\pi}\ln(\frac{r_2 r_4}{r_1 r_3}) = -\frac{\omega\mu_0 I_m\cos(\omega t)}{2\pi}\ln(\frac{r_2 r_4}{r_1 r_3})$$

8) **15.10:**

A coaxial cable has conductors of radii a and b. The inner conductor is coated with a layer of ferrite d thick ($d < b - a$) and of permeability μ. The rest of the cable is air-filled. Find the external self-inductance per unit length of the cable. What should your expression reduce to (1) when $d = 0$ and (2) when $d = b - a$?

We know from the textbook that the self inductance per unit length is:

$$L' = \frac{\mu_0}{2\pi}\ln(\frac{b}{a})$$

With our new surface we will have to modify our equation to:

$$L' = \frac{\mu}{2\pi}\ln(\frac{a+d}{a}) + \frac{\mu_0}{2\pi}\ln(\frac{b}{a+d})$$

It works much like a dielectric when capacitance is in question.