

1.1:

- (a) What is the duty ratio,
- D
- , of the converter?

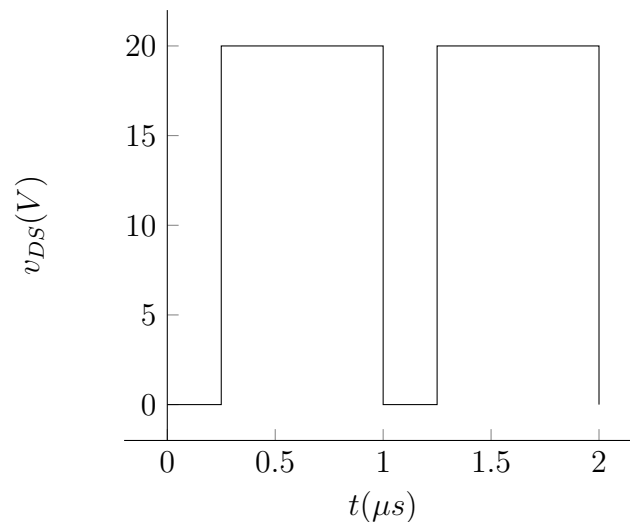
since:

$$V = \frac{V_G}{1 - D}$$

The duty ratio is $D = \frac{-V_G}{V} + 1 = -\frac{5}{20} + 1 = \frac{3}{4}$.

- (b) Sketch the waveform of the MOSFET drain-to-source voltage,
- v_{DS}
- . Label the numerical values of all relevant times and voltages.

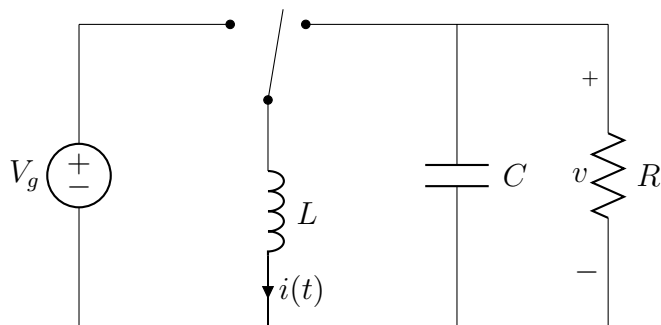
First we assume that the MOSFET is on, this means that the voltage across the inductor is $V_L = V_{IN}$. And that means that $V_{DS} = 0V$. When the MOSFET is off, we can see that $V_L = V_{IN} - V_{OUT} = -15V$. In this case, $V_{DS} = 20V$



- (c) Find the DC component of the voltage waveform of art (b). How does this value relate to the value of
- V_{IN}
- ? Does this make sense and why?

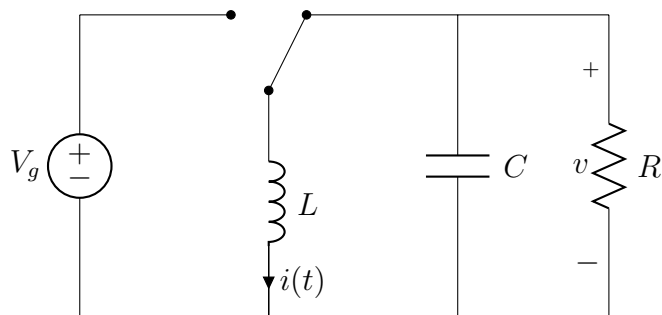
The DC component of this waveform is $15V$. This input voltage is $5V$. We can do this because our output current will become lower than before. This fact maintains the same power.

1.2:



- (a) Find the dependence of the equilibrium output voltage V and the inductor current i on the duty ratio D_s , input voltage V_g , and load resistance R . You may assume that the inductor current ripple and capacitor voltage ripple are small.

When the switch is in the closed position (to the right), the circuit will look like:



We know that

$$\frac{dI_L}{dt} = \frac{V_g}{L}$$

The change in I_L can thus be approximated by:

$$\Delta I_L = \int_0^{DT} \frac{V_g}{L} = \frac{V_g DT}{L}$$

When the circuit is in the open position, the current will flow through the inductor:

$$\frac{dI_L}{dt} = \frac{V}{L}$$

and similarly the change in the current will be:

$$\Delta I_L = \int_0^{(1-D)T} \frac{V(1-D)}{L} = \frac{V(1-D)T}{L}$$

Since we are in periodic steady state we know that these must be equal, and:

$$\frac{V(1-D)T}{L} + \frac{V_g DT}{L} = 0 \Rightarrow \frac{V(1-D)T}{L} = -\frac{V_g DT}{L}$$

$$\frac{V}{V_g} = \frac{-D}{1-D} \Rightarrow V = \frac{DV_g}{D-1}$$

To find the current we must first find the current through the capacitor $i_C(t)$. In the closed and open position these are:

$$i_c(t) = \frac{-V}{R}, \text{ and } i_c(t) = -i_L - \frac{V}{R}$$

Respectively. Since we are in periodic steady state we know that the average current through the capacitor is going to be 0. This means that

$$\int_0^{DT} \frac{-V}{R} dt = \int_0^{(1-D)T} i_L + \frac{V}{R} dt$$

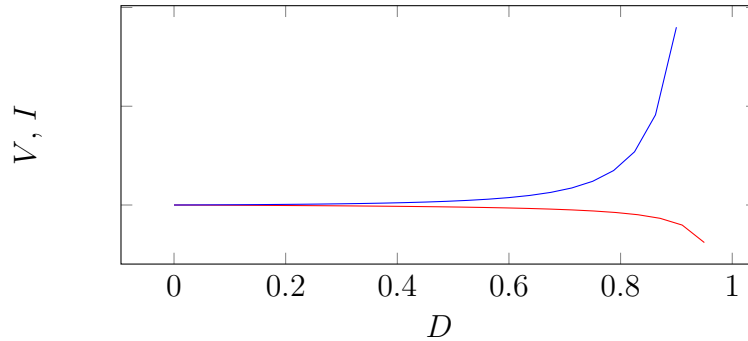
$$\frac{-VDT}{R} = \frac{V(1-D)T}{R} + (1-D)Ti_L$$

$$i_L = -\frac{VD + V(1-D)}{R(1-D)} \Rightarrow i_L = -\frac{V}{R(1-D)}$$

and using our expression for the output voltage V we find:

$$i_L = \frac{VD}{R(1-D)^2}$$

(b) Plot your results of part (a) over the range $0 \leq D \leq 1$



Where the red value is the voltage and the blue value is the current.

(c) DC design: for the specifications:

$$\begin{aligned} V_g &= 30V & V &= -20V \\ R &= 4\Omega & f_{\Delta} &= 40kHz \end{aligned}$$

(i) Find D and I .

We can use what we know from the last problem:

$$V = \frac{DV_g}{D-1}$$

$$D = \frac{V}{V - V_g} = \frac{-20}{-50} = \frac{2}{5}$$

To find I :

$$I \approx i_L = \frac{VD}{R(1-D)^2} = \frac{-20 \cdot \frac{2}{5}}{4 \cdot \frac{9}{25}} = \frac{-50}{9}$$

(ii) Calculate the value of L that will make the peak inductor current ripple Δi equal to ten percent of the average inductor current I .

Using our result:

$$\Delta I_L = \frac{V_g DT}{2L}$$

$$(0.10) \frac{50}{9} = \frac{30 \cdot \frac{2}{5}}{2L \cdot 40000} \Rightarrow L = 270\mu H$$

(iii) Choose C such that the peak output voltage ripple Δv is $0.1V$.

First we find i_c :

$$i_c = C \frac{dV_c}{dt} \Rightarrow \Delta V_c = \frac{IDT_s}{2C} \Rightarrow 0.1 = \frac{50}{9} \frac{\frac{2}{5}}{2C \cdot 40000}$$

$$C = 278\mu F$$