ECEN 5018 Spring 2014	Homework #7	**Vince Coghlan**

1) Consider the following multiagent system (vehicle target assignment problem) with the following elements:

- Set of vehicles: $\mathcal{V} = \{1, 2, 3\}$
- Vehicle detection probability p_1, p_2, $p_3 \in [0, 1]$
- Set of targets $\mathcal{T} = \{x, y\}$
- Set of possible assignments for each vehicle: $\mathcal{A}_i = \{x, y\}$, i.e., each vehicle can select only on of the two targets
- Target specific welfare functions: for any set of vehicles $S \subseteq \mathcal{V}$

$$W_x(S) = v_x \left[1 - \prod_{j \in S}(1 - p_j)\right]$$

$$W_y(S) = v_y \left[1 - \prod_{j \in S}(1 - p_j)\right]$$

- Global objective: Maximize total welfare

$$W(a) = W_x(\{a\}_x) + W_y(\{a\}_y)$$

where $\{a\}_x = \{i \in \mathcal{V} : t \in a_x\}$.

Part #1: Model the above multiagent system as a game with player set $\mathcal{V}$ and the wonderful life utility.

(a) What is the payoff matrix?
We can find the utility:

$$U_i(a) = \sum_{r \in a_i}(W_r(\{a\}_r) - W_r(\{a\}_r \backslash \{i\}))$$

For example, if player 1 goes to x, and players two and three are not there:

$$U_1(x) = v_x \left[1 - \prod_{j \in S}(1 - p_1)\right] - 1 = v_x p_1 - 1$$

If player 2 is there:

$$U_1(x) = v_x \left[1 - (1 - p_2)(1 - p_1)\right] - v_x \left[1 - (1 - p_2)\right]$$

If player 2 and 3 are there:

$$U_1(x) = v_x \left[1 - (1 - p_3)(1 - p_2)(1 - p_1)\right] - v_x \left[1 - (1 - p_2)(1 - p_3)\right]$$

Whereas at this specific point player 2 would see:

$$U_2(x) = v_x \left[1 - (1 - p_3)(1 - p_2)(1 - p_1)\right] - v_x \left[1 - (1 - p_3)(1 - p_1)\right]$$

This is way too long to fit in a payoff matrix, so ill save writing down the payoff matrix until part (c).

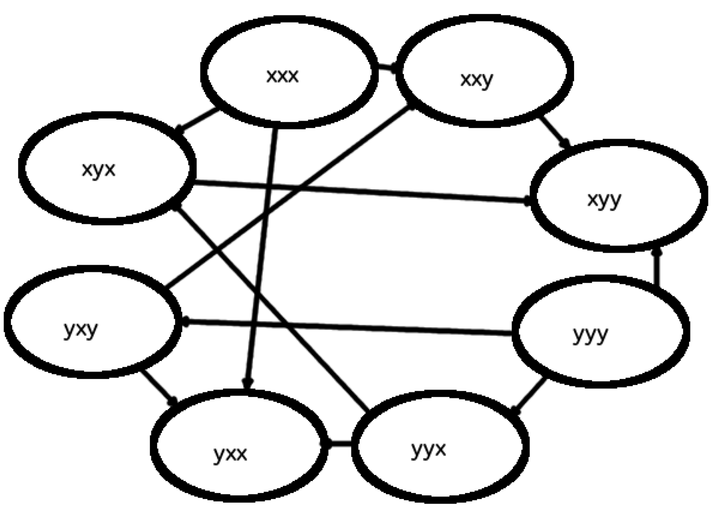
(b) Is the game a potential game? If so, what is the potential function?
Yes it is a potential game, with a potential function W:

$$\Phi = W_x(\{a\}_x) + W_y(\{a\}_y)$$

(c) From this point on, i.e., all future questions in Part #1, set $v_x = 2$, $v_y = 1$, $p_1 = 1$, $p_2 = 1/2$, and $p_3 = 1/4$. What is the payoff matrix for this specific setting?
The payoff matrix:

	x	y		x	y
x	3/4, 0, 0	3/2, 1/2, 0	x	1, 0, 1/4	2, 3/8, 1/8
y	1, 3/4, 1/4	1/2, 0, 1/2	y	3/4, 1, 0	1, 0, 0
	x			y	

(d) What is the better reply graph?



(e) What are the N.E.? yxx and xyy are the NE.

(f) If we apply log-linear learning, what is analytical stationary distrobution for $T = 10$, $T = 1$, and $T = 0.1$?
Since this is a potential game, We need to find the potential function:

	x	y
x	3/4	5/4
y	1	1/4

x

	x	y
x	1	3/4
y	3/4	-1/4

y

At $T = 10$ we can see that:

	x	y
x	0.1257	0.1321
y	0.1288	0.1195

x

	x	y
x	0.1288	0.1257
y	0.1257	0.1137

y

At $T = 1$ we can see that:

	x	y
x	0.1221	0.2013
y	0.1568	0.0740

x

	x	y
x	0.1568	0.1221
y	0.1221	0.0449

y

At $T = .1$ we can see that:

	x	y
x	0.0057	0.8443
y	0.0693	0.0000

x

	x	y
x	0.0693	0.0057
y	0.0057	0.0000

y

As $T \to 0$ we can see that:

	x	y
x	0	1
y	0	0

x

	x	y
x	0	0
y	0	0

y

Part #2: Model the above mutiagent system as a game with player set $\mathcal{V}$ and the Shapley value utility.

(a) What is the payoff matrix?
We know that a utility for a specific player will look like:

$$U_i(a) = \sum_{r \in a_i} \sum_{T \subseteq \{a\}_r \setminus \{i\}} \frac{|T|!(|a|_r - |T| - 1)!}{(|a|_r)!} (W_r(T \cup \{i\}) - W_r(T))$$

We will calculate these when we get to the numeric values.

(b) From this point on, i.e., all future questions in Part #2, set $v_x = 2$, $v_y = 1$, $p_1 = 1$, $p_2 = 1/2$, and $p_3 = 1/4$. What is the payoff matrix for this specific setting?
We can now find the payoff matrix:

	x	y		x	y
x	4/3, 11/24, 5/24	7/4, 1/2, 1/4	x	3/2, 1/2, 1/4	2, 7/16, 3/16
y	1, 7/8, 3/8	3/4, 1/4, 1/2	y	7/8, 1, 1/8	2/3, 11/48, 5/48
	x			y	

(c) Is the game a potential game? If so, what is the potential function? (Hint: Set $\Phi(x, x, x) = 0$).

	x	y		x	y
x	0	1/24	x	1/24	-1/48
y	-1/3	-23/24	y	-7/12	-65/48
	x			y	

(d) If we apply log-linear learning, what is analytical stationary distrobution for $T = 10$, $T = 1$, and $T = 0.1$?
At $T = 10$ we can see that:

	x	y		x	y
x	0.1299	0.1304	x	0.1304	0.1296
y	0.1256	0.1180	y	0.1225	0.1134
	x			y	

At $T = 1$ we can see that:

	x	y		x	y
x	0.1672	0.1743	x	0.1743	0.1638
y	0.1198	0.0641	y	0.0933	0.0432
	x			y	

At $T = .1$ we can see that:

	x	y
x	0.2047	0.3106
y	0.0073	0.0000

x

	x	y
x	0.3106	0.1662
y	0.0006	0.0000

y

As $T \to 0$ we can see that:

	x	y
x	0	1/2
y	0	0

x

	x	y
x	1/2	0
y	0	0

y

Part #3: Model the above mutiagent system as a game with player set $N = \{\{v_1, v_3\}, v_2\}$ and the marginal contribution utility. This is the setting where there are only two decision makers, i.e., vehicles v_1 and v_3 are designed as a SINGLE decision maker and vehicle v_2 is the other decision maker.

(a) What is the payoff matrix?
Once again, this payoff matrix would be quite complicated to write out in terms of the given parameters, you can see the payoff matrix when I answer problem (b).

(b) From this point on, i.e., all future questions in Part #2, set $v_x = 2$, $v_y = 1$, $p_1 = 1$, $p_2 = 1/2$, and $p_3 = 1/4$. What is the payoff matrix for this specific setting?
The payoff matrix is identical to that in Part #1.

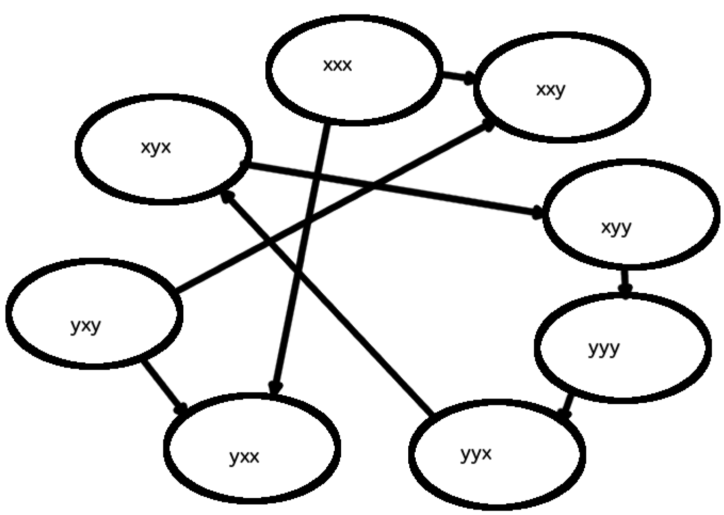
	x	y
x	3/4, 0, 0	3/2, 1/2, 0
y	1, 3/4, 1/4	1/2, 0, 1/2

x

	x	y
x	1, 0, 1/4	2, 3/8, 1/8
y	3/4, 1, 0	1, 0, 0

y

(c) What is the better reply graph?



(d) What are the N.E.?
yxx and xyy are the NE.

(e) How does the price of anarchy of this game compare to the setting in Part #1? This game's BRG has a cycle, meaning that there is a situation where no Nash equilibrium is reached. Since this is not an equilibrium, however, it has no bearing on the PoA. Both games have the same N.E. and the same utilitarian cost function, they must also have the same PoA.

2) Consider the following two player game with the (utility) payoff matrix:

	a_2	b_2
a_1	3,3	0,4
b_1	4,0	1,1

(a) What are the analytical stationary distrobutions when $T = 0.1, 1, 10$, and 100 for log-linear learning?
This is a potential game with potential function Φ:

	a_2	b_2
a_1	0	1
b_1	1	2

We know that for a potential game like this, finding the stationary distrobution is quite easy, for $T = 0.1$:

	a_2	b_2
a_1	0.0000	0.0000
b_1	0.0000	0.9999

for $T = 1$:

	a_2	b_2
a_1	0.0723	0.1966
b_1	0.1966	0.5344

for $T = 10$:

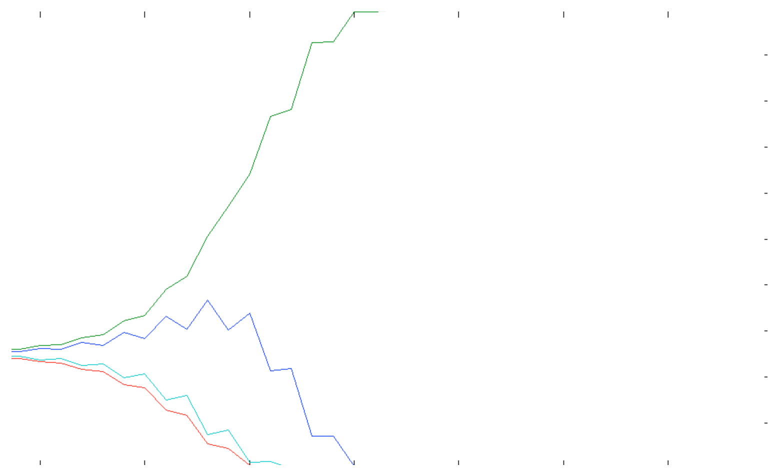
	a_2	b_2
a_1	0.2256	0.2494
b_1	0.2494	0.2756

for $T = 100$:

	a_2	b_2
a_1	0.2475	0.2500
b_1	0.2500	0.2525

(b) Write a Matlab script to simulate the above game using Log-Linear Learning. Verify that the empirical distrobution approaches your analytical predictions for the previous problem.(plot empirical distrobution vs. iteration for each of the four joint action)

The matlab script shows us the empirical distrobution vs. iteration for each joint action as $T \to 0$:



the green line is $b_1 b_2$, the eventual joint distrobution.

3) Write a Matlab script to solve 4×4 Sudoku using Log-Linear Learning. If you haven't already, Insert a "kick out" test that terminates the iterations upon solution of the puzzle.