ECEN 5018 Spring 2014 Homework #2	**Vince Coghlan**

1) **The Pass Line:** The pass line in craps is one of the most popular bets in vegas. Craps is a dice game that utilizes two die. Here are the rules:

- The first roll is called the *come out roll*. Two die are rolled and if the sum is 7 or 11 you win. If the sum is 2, 3, or 12 you lose. Otherwise, the number you roll {4, 5, 6, 8, 9, 10} is called your point.
- If the outcome of the game is not determined on the first roll then the game continues until either one of two events happen:
  - If I roll my point before I roll a 7 I win.
  - If I roll a 7 before I roll my point I lose.

There is no limit on the number of rolls in any given game.

The pass line in craps is an even money bet, meaning that if I bet \$5 and win, I win \$5. If I lose, I lose \$5. What is the probability of winning given that I bet on the pass line? If I bet \$5 on the pass line, how much should I expect to have at the end of the game?

There are 36 possible rolls. The probability of rolling a 7 or 11 on your come out roll is going to equal the probability of rolling a 7 ($\frac{6}{36}$) and the probability of rolling an 11 ($\frac{2}{36}$). This means the probability of winning on the come out roll is $\frac{8}{36}$. After that we need to go through each number and find the probability of winning if that number is the point. Ill start with 4. The probability of rolling a 4 as your point is $\frac{3}{36}$. The probability of rolling a 4 again before rolling a 7 can be seen below:

$$\sum_{n=0}^{\infty} \frac{3}{36}\left(\frac{27}{36}\right)^n = \frac{1}{3}$$

This is because you either get your 4, or you get something other than a 4 or a 7. The probability of winning with this point value is $\frac{1}{3}\frac{3}{36} = \frac{1}{36}$. Since 10 has the same probabilities of being rolled as 4, it has the same total probability of winning too. We can find similar values for the other possible points:

$$P(4,10) = \frac{1}{36},\ P(5,9) = \frac{2}{45},\ P(6,8) = \frac{25}{396}$$

Thus the probability of winning on the pass line is:

$$P(\text{win}) = \frac{6}{36} + \frac{2}{36} + 2 \cdot (\frac{1}{36}) + 2 \cdot (\frac{2}{45}) + 2 \cdot (\frac{25}{396}) \approx 0.493$$

These are pretty good odds for vegas, The expected return would be:

$$0.493 \cdot 10 + (1 - 0.493) \cdot 0 = \$4.93$$

2) **Consider Two Events:** A and B, with $\Pr(A) > 0$ and $\Pr(B) > 0$. Is the following sentance TRUE or FALSE or DEPENDENT ON A and B:

If A and B are disjoint, then A and B must be independent.

Since A and B are disjoint, $\mathrm{P}(A \cap B) = 0$, since they can never both happen at the same time. If they were independent then $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$. This is not possible unless $\mathrm{P}(A)$ or $\mathrm{P}(B)$ is 0, which is against the presumption in the question prompt. The answer is FALSE.

3) **The Random Variable:** X takes on values $\{0,1,2,3\}$ with probabilities $\{0.4,0.2,0.1,?\}$, respectively. Compute the following:

(a) $\Pr(X = 3)$
$\mathrm{P}(X = 3) = 1 - 0.4 - 0.2 - 0.1 = 0.3$

(b) $\Pr(X \text{ is odd})$
$\mathrm{P}(X \text{ is odd}) = 0.2 + 0.3 = 0.5$

(c) $\mathrm{E}[X]$
$\mathrm{E}[X] = 0 \cdot 0.4 + 1 \cdot 0.2 + 2 \cdot 0.1 + 3 \cdot 0.3 = 1.3$

(d) $\mathrm{E}[1/(X + 1)]$
$\mathrm{E}[1/(X + 1)] = 0.4/1 + 0.2/2 + 0.1/3 + 0.3/4 = \frac{73}{120}$

4) **The Probabilities:** of the outcome of two coin tosses are:

outcome	probability
HH	2/9
HT	1/9
TH	4/9
TT	2/9

Compute teh following:

(a) Pr(first toss is H) Pr(first toss is H) $= 3/9$

(b) Pr(second toss is T) Pr(second toss si T) $= 3/9$

(c) Pr(first toss is H | second toss is T) Pr(first toss is H | second toss is T) $= 5/9$

(d) Are the two coin tosses independent? No, since the probability that the second toss is a T is entirely dependent on what the first toss was. P(HT) $\neq$ P(TT).

5) **The Table Below:** can be interpreted as tossing a pair of 3-sided dice, labeled X and Y.

	3	0	0	2/8
Y	2	0	2/8	1/8
	1	2/8	1/8	0
		1	2	3
			X	

(a) $\Pr(X = 2) = 3/8$

(b) $\Pr(Y = 2) = 3/8$

(c) $\Pr(Y = 3 \mid X = 3)$
$\Pr(Y = 3 \mid X = 3) = 2/8$

(d) E[max(X,Y)]
$\mathrm{E}[\max(X,Y)] = 3 \cdot 2/8 + 3 \cdot 1/8 + 2 \cdot 2/8 + 2 \cdot 1/8 + 1 \cdot 2/8 = 17/8$

(e) Are the two tosses independent?

No they are not, since X cannot be 1 unless Y is 1.

6) **For Each of the Following 2 $\times$ 2 Games:** the column player is using a randomized strategy of L with probability p and R with probability $1 - p$. The row player seeks to optimize the expected payoff.

- BoS:

	B	S
B	2,1	0,0
S	0,0	1,2

- Stag hunt:

	Stag	Hare
Stag	2,2	0,1
Hare	1,0	1,1

- Typewriter:

	Alt	Std
Alt	3,3	0,0
Std	0,0	1,1

(a) Determine the best response of the row player as a function of p.

(b) For what value of p is the row player indifferent between its "top" action versus its "bottom" action?

Lets start with the BoS game, the best response of the row player is going to be contingent on the probability p that the column player is going to play B. This is going to be the maximum of:

$$p(q \cdot 2 + (1-q) \cdot 0) + (1-p)(q \cdot 0 + (1-q) \cdot 1)$$

$$2q < (1-q) \text{ when } q < 1/3$$

$$2q > (1-q) \text{ when } q > 1/3$$

$$B_{ROW}(q) = \begin{cases} 1 & q > 1/3 \\ 0 & q < 1/3 \\ [0,1] & q = 1/3 \end{cases}$$

1/3 is the value at which the row player is indifferent.

Now we can look at the Stag hunt game:

$$p(q \cdot 2 + (1-q) \cdot 0) + (1-p)(q \cdot 1 + (1-q) \cdot 1)$$

$$2q < 1 \text{ when } q < 1/2$$

$$2q > 1 \text{ when } q > 1/2$$

$$B_{ROW}(q) = \begin{cases} 1 & q > 1/2 \\ 0 & q < 1/2 \\ [0,1] & q = 1/2 \end{cases}$$

1/2 is the value at which the row player is indifferent.

Now we can look at the Typewriter game:

$$p(q \cdot 3 + (1-q) \cdot 0) + (1-p)(q \cdot 0 + (1-q) \cdot 1)$$

$$3q < (1-q) \text{ when } q < 1/4$$

$$3q > (1-q) \text{ when } q > 1/4$$

$$B_{ROW}(q) = \begin{cases} 1 & q > 1/4 \\ 0 & q < 1/4 \\ [0,1] & q = 1/4 \end{cases}$$

1/4 is the value at which the row player is indifferent.