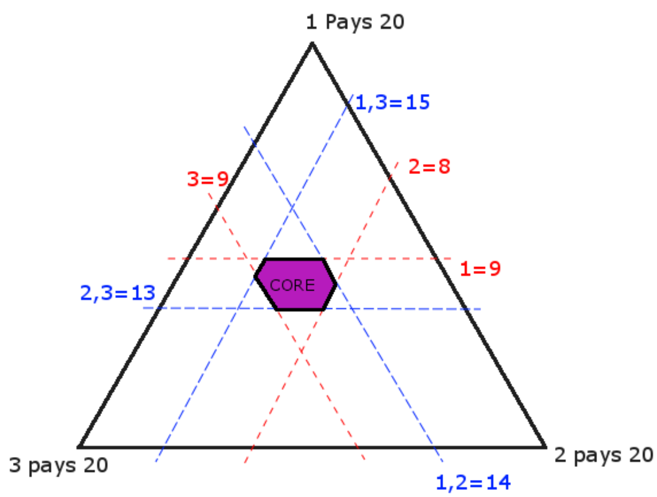
ECEN 5018 Spring 2014	Homework #5	**Vince Coghlan**

1) Consider the following cost sharing problem:

- Player set: $N = \{1, 2, 3\}$
- Opportunity costs: $c : 2^N \to R$

$$c(\{1\}) = 9, \quad c(\{2\}) = 8, \quad c(\{3\}) = 9$$
$$c(\{1,2\}) = 14,\ c(\{1,3\}) = 15,\ c(\{2,3\}) = 13$$
$$c(\{1,2,3\}) = 20 \qquad\qquad c(\{\emptyset\}) = 0$$

(a) Identify the core graphically



(b) Is the core nonempty?
Yes, as can be seen in the picture.

(c) Compute the marginal contribution for each player.
The marginal cost of each player will be the marginal cost to the full coalition, for player 1:

$$c(\{1,2,3\}) - c(\{2,3\}) = 7$$

for player 2:

$$c(\{1,2,3\}) - c(\{1,3\}) = 5$$

and player 3:

$$c(\{1,2,3\}) - c(\{1,2\}) = 6$$

(d) Compute the Shapley value for each player using equation in notes
The equation in the notes is:

$$Sh(i, S; c) = \sum_{T \subseteq S \setminus \{i\}} \frac{|T|!(|S| - |T| - 1)!}{|S|!} (c(T \cup \{i\}) - c(T))$$

For player 1:

$$Sh(1, \{1, 2, 3\}; c) = \frac{2}{6}(c(\{1, 2, 3\}) - c(\{2, 3\})) + \frac{1}{6}(c(\{1, 2\}) - c(\{2\})) +$$

$$\frac{1}{6}(c(\{1, 3\}) - c(\{3\})) + \frac{2}{6}(c(\{1\}) - c(\{\emptyset\})) = 7\frac{1}{3}$$

For player 2:

$$Sh(2, \{1, 2, 3\}; c) = \frac{2}{6}(c(\{1, 2, 3\}) - c(\{1, 3\})) + \frac{1}{6}(c(\{1, 2\}) - c(\{1\})) +$$

$$\frac{1}{6}(c(\{2, 3\}) - c(\{3\})) + \frac{2}{6}(c(\{2\}) - c(\{\emptyset\})) = 5\frac{5}{6}$$

For player 3:

$$Sh(3, \{1, 2, 3\}; c) = \frac{2}{6}(c(\{1, 2, 3\}) - c(\{1, 2\})) + \frac{1}{6}(c(\{1, 3\}) - c(\{1\})) +$$

$$\frac{1}{6}(c(\{2, 3\}) - c(\{2\})) + \frac{2}{6}(c(\{3\}) - c(\{\emptyset\})) = 6\frac{5}{6}$$

(e) Compute the Shapley value for each player using ordering approach in notes
The marginal contribution over all oderings can be easily calculated from the marginal values. These must be found for each ordering, as seen below:

$$3 \leftarrow 2 \leftarrow 1 \Rightarrow c(\{1, 2, 3\}) - c(\{2, 3\}) = 7$$

$$2 \leftarrow 3 \leftarrow 1 \Rightarrow c(\{1, 2, 3\}) - c(\{2, 3\}) = 7$$

$$2 \leftarrow 1 \leftarrow 3 \Rightarrow c(\{1, 2\}) - c(\{2\}) = 6$$

$$3 \leftarrow 1 \leftarrow 2 \Rightarrow c(\{1, 3\}) - c(\{3\}) = 6$$

$$1 \leftarrow 3 \leftarrow 2 \Rightarrow c(\{1\}) - c(\{\emptyset\}) = 9$$

$$1 \leftarrow 2 \leftarrow 3 \Rightarrow c(\{1\}) - c(\{\emptyset\}) = 9$$

For player 2:

$$3 \leftarrow 1 \leftarrow 2 \Rightarrow c(\{1, 2, 3\}) - c(\{1, 3\}) = 5$$

$$1 \leftarrow 3 \leftarrow 2 \Rightarrow c(\{1, 2, 3\}) - c(\{1, 3\}) = 5$$

$$3 \leftarrow 2 \leftarrow 1 \Rightarrow c(\{2, 3\}) - c(\{3\}) = 4$$

$$1 \leftarrow 2 \leftarrow 3 \Rightarrow c(\{1, 2\}) - c(\{1\}) = 5$$

$$2 \leftarrow 3 \leftarrow 1 \Rightarrow c(\{2\}) - c(\{\emptyset\}) = 8$$

$$2 \leftarrow 1 \leftarrow 3 \Rightarrow c(\{2\}) - c(\{\emptyset\}) = 8$$

For player 3:

$$2 \leftarrow 1 \leftarrow 3 \Rightarrow c(\{1,2,3\}) - c(\{1,2\}) = 6$$

$$1 \leftarrow 2 \leftarrow 3 \Rightarrow c(\{1,2,3\}) - c(\{1,2\}) = 6$$

$$2 \leftarrow 3 \leftarrow 1 \Rightarrow c(\{2,3\}) - c(\{2\}) = 5$$

$$1 \leftarrow 3 \leftarrow 2 \Rightarrow c(\{1,3\}) - c(\{1\}) = 6$$

$$3 \leftarrow 2 \leftarrow 1 \Rightarrow c(\{3\}) - c(\{\emptyset\}) = 9$$

$$3 \leftarrow 1 \leftarrow 2 \Rightarrow c(\{3\}) - c(\{\emptyset\}) = 9$$

We can then calculate the shapley value for each player, for player 1:

$$\frac{1}{6}(7+7+6+6+9+9) = 7\frac{1}{3}$$

for player 2:

$$\frac{1}{6}(5+5+4+5+8+8) = 5\frac{5}{6}$$

for player 3:

$$\frac{1}{6}(6+6+5+6+9+9) = 6\frac{5}{6}$$

Note that when we add these together we get 20.

(f) Verify approaches in (d) and (e) result in the same answer.
Yes it is.