ECEN 5018 Spring 2014	Homework #7	Vince Coghlan

1) Consider the following multiagent system (vehicle target assignment problem) with the following elements:

- Set of vehicles: $\mathcal{V} = \{1, 2, 3\}$
- Vehicle detection probability p_1, p_2, $p_3 \in [0, 1]$
- Set of targets $\mathcal{T} = \{x, y\}$
- Set of possible assignments for each vehicle: $\mathcal{A}_i = \{x, y\}$, i.e., each vehicle can select only on of the two targets
- Target specific welfare functions: for any set of vehicles $S \subseteq \mathcal{V}$

$$W_x(S) = v_x \left[1 - \prod_{j \in S}(1 - p_j)\right]$$

$$W_y(S) = v_y \left[1 - \prod_{j \in S}(1 - p_j)\right]$$

- Global objective: Maximize total welfare

$$W(a) = W_x(\{a\}_x) + W_y(\{a\}_y)$$

  where $\{a\}_x = \{i \in \mathcal{V} : t \in a_x\}$.

Part #1: Model the above multiagent system as a game with player set $\mathcal{V}$ and the wonderful life utility.

(a) What is the payoff matrix?
We can find the utility:

$$U_i(a) = \sum_{r \in a_i}(W_r(\{a\}_r) - W_r(\{a\}_r \backslash \{i\}))$$

For example, if player 1 goes to x, and players two and three are not there:

$$U_1(x) = v_x \left[1 - \prod_{j \in S}(1 - p_1)\right] - 1 = v_x p_1 - 1$$

If player 2 is there:

$$U_1(x) = v_x\left[1 - (1-p_2)(1-p_1)\right] - v_x\left[1-(1-p_2)\right]$$

If player 2 and 3 are there:

$$U_1(x) = v_x\left[1 - (1-p_3)(1-p_2)(1-p_1)\right] - v_x\left[1-(1-p_2)(1-p_3)\right]$$

Whereas at this specific point player 2 would see:

$$U_2(x) = v_x\left[1 - (1-p_3)(1-p_2)(1-p_1)\right] - v_x\left[1-(1-p_3)(1-p_1)\right]$$

This is way too long to fit in a payoff matrix, so ill save writing down the payoff matrix until part (c).

(b) Is the game a potential game? If so, what is the potential function?
Yes it is a potential game, with a potential function W:

$$\Phi = W_x(\{a\}_x) + W_y(\{a\}_y)$$

(c) From this point on, i.e., all future questions in Part #1, set $v_x = 2$, $v_y = 1$, $p_1 = 1$, $p_2 = 1/2$, and $p_3 = 1/4$. What is the payoff matrix for this specific setting?
The payoff matrix:

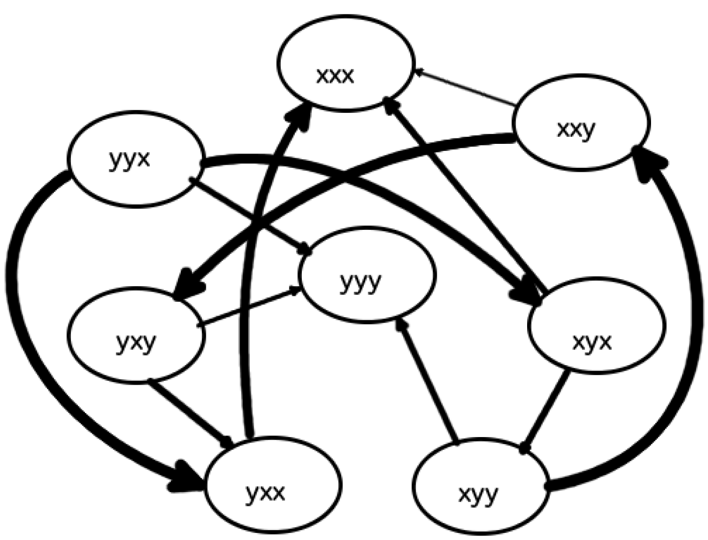
	x	y
x	3/4, 0, 0	3/2, -1/2, 0
y	0, 3/4, 1/4	0, 1/2, -3/2

x

	x	y
x	0, 1, -3/4	0, 3/8, 1/8
y	3/4, -1, 0	3/8, 0, 0

y

(d) What is the better reply graph?



(e) What are the N.E.? yyy and xxx are the NE.

(f) If we apply log-linear learning, what is analytical stationary distrobution for $T = 10$, $T = 1$, and $T = 0.1$?
Since this is a potential game, we know that there will only be one stationary distrobution. At $T = 10$ this looks like:

$$p_i = \frac{e^{\frac{1}{T}u_i(a_i)}}{\sum_{\tilde{a}\in\mathcal{A}_i} e^{\frac{1}{T}u_i(\tilde{a},a_{-i})}}$$

So if we are at xxx, then player 1 will see the following:

$$p_1(x) = \frac{e^{\frac{1}{10}3/4}}{e^{\frac{1}{10}3/4} + e^{\frac{1}{10}0}} = 0.52$$

and indeed for all players:

$$p = \begin{pmatrix} .5187 \\ .5125 \\ .5187 \end{pmatrix}$$

for $T = 1$:

$$p = \begin{pmatrix} .6792 \\ .6225 \\ .6792 \end{pmatrix}$$

and for $T = 0.1$:

$$p = \begin{pmatrix} .9994 \\ .9933 \\ .9994 \end{pmatrix}$$

And as $t \to 0$ the stationary distrobution becomes $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ for all players.

Part #2: Model the above mutiagent system as a game with player set $\mathcal{V}$ and the Shapley value utility.

(a) What is the payoff matrix?
We know that a utility for a specific player will look like:

$$U_i(a) = \sum_{r\in a_i} \sum_{T\subseteq\{a\}_r\setminus\{i\}} \frac{|T|!(|a|_r - |T| - 1)!}{(|a|_r)!}(W_r(T \cup \{i\}) - W_r(T))$$

We will calculate these when we get to the numeric values.

(b) From this point on, i.e., all future questions in Part #2, set $v_x = 2$, $v_y = 1$, $p_1 = 1$, $p_2 = 1/2$, and $p_3 = 1/4$. What is the payoff matrix for this specific setting?
We can now find the payoff matrix:

	x	y
x	4/3, 11/24, 5/24	7/4, 1/2, 1
y	1, 19/12, 17/12	1/2, 3/4, 1/2

x

	x Text	y
x	1, 1, 1/4	2, 5/16, 5/16
y	1/2, 1, 1/2	2/3, 11/48, 5/48

y

(c) Is the game a potential game? If so, what is the potential function? (Hint: Set $\Phi(x, x, x) = 0$).

	x	y
x	0	1/24
y	-1/3	-1/8

x

	x	y
x	1/24	x
y	x	x

y