

1) Consider the following multiagent system (vehicle target assignment problem) with the following elements:

- Set of vehicles: $\mathcal{V} = \{1, 2, 3\}$
- Vehicle detection probability $p_1, p_2, p_3 \in [0, 1]$
- Set of targets $\mathcal{T} = \{x, y\}$
- Set of possible assignments for each vehicle: $\mathcal{A}_i = \{x, y\}$, i.e., each vehicle can select only on of the two targets
- Target specific welfare functions: for any set of vehicles $S \subseteq \mathcal{V}$

$$W_x(S) = v_x \left[1 - \prod_{j \in S} (1 - p_j) \right]$$

$$W_y(S) = v_y \left[1 - \prod_{j \in S} (1 - p_j) \right]$$

- Global objective: Maximize total welfare

$$W(a) = W_x(\{a\}_x) + W_y(\{a\}_y)$$

where $\{a\}_x = \{i \in \mathcal{V} : t \in a_x\}$.

Part #1: Model the above multiagent system as a game with player set \mathcal{V} and the wonderful life utility.

(a) What is the payoff matrix?

We can find the utility:

$$U_i(a) = \sum_{r \in a_i} (W_r(\{a\}_r) - W_r(\{a\}_r \setminus \{i\}))$$

For example, if player 1 goes to x, and players two and three are not there:

$$U_1(x) = v_x \left[1 - \prod_{j \in S} (1 - p_1) \right] - 1 = v_x p_1 - 1$$

If player 2 is there:

$$U_1(x) = v_x [1 - (1 - p_2)(1 - p_1)] - v_x [1 - (1 - p_2)]$$

If player 2 and 3 are there:

$$U_1(x) = v_x [1 - (1 - p_3)(1 - p_2)(1 - p_1)] - v_x [1 - (1 - p_2)(1 - p_3)]$$

Whereas at this specific point player 2 would see:

$$U_2(x) = v_x [1 - (1 - p_3)(1 - p_2)(1 - p_1)] - v_x [1 - (1 - p_3)(1 - p_1)]$$

This is way too long to fit in a payoff matrix, so ill save writing down the payoff matrix until part (c).

- (b) Is the game a potential game? If so, what is the potential function?

Yes it is a potential game, with a potential function W :

$$\Phi = W_x(\{a\}_x) + W_y(\{a\}_y)$$

- (c) From this point on, i.e., all future questions in Part #1, set $v_x = 2$, $v_y = 1$, $p_1 = 1$, $p_2 = 1/2$, and $p_3 = 1/4$. What is the payoff matrix for this specific setting?

The payoff matrix:

| | | | | | |
|-----|-------------|--------------|-----|------------|-------------|
| | x | y | | x | y |
| x | 3/4, 0, 0 | 3/2, -1/2, 0 | x | 0, 1, -3/4 | 0, 3/8, 1/8 |
| y | 0, 3/4, 1/4 | 0, 1/2, -3/2 | y | 3/4, -1, 0 | 3/8, 0, 0 |
| | x | | | y | |