Tile EM Model

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1 Problem Formulation

$\mathcal{T} = \{t_k\}$ is the set of all non-overlapping tiles for an object i, where T is the ground truth tile set and T' is some combination of tiles chosen from $\mathcal{T}$.

We define the ground truth as the tile combination T' that maximizes the probability that it would be ground truth, given the set of worker qualities Q_j and tile indicator labels l_{kj}. The indicator label l_{kj} is one when worker votes on the tile t_{kj}(i.e. the bounding box that he draws contains t_{kj}), and zero otherwise. Given these definitions, the objective function of the segmentation problem is formulated as:

$$T = \arg\max_{T' \subseteq \mathcal{T}} P(T = T' | l_{kj}, Q_j) \tag{1}$$

Using Bayes rule we can rewrite this in terms of the posterior probability of the tile-based values(l_{kj}) or worker-based values(Q_j), which we can use for the E and M step equations respectively.

2 Inference

For the E step, we fix T and estimate the Q_j parameters. We can rewrite Eq.1 as:

$$p(T'|Q_j, l_{kj}) = \frac{p(Q_j|l_{kj}, T')\cancel{P(T')}}{\cancel{p(Q_j)}} \tag{2}$$

Our worker error model follows a Bernoulli distribution by describing worker quality $Q_j = q_j$ as the probability that the worker makes a correct vote for a tile (w=1), otherwise $Q_j = 1 - q_j$ for an incorrect guess.

$$p(w) = q_j p(w=1) + (1 - q_j) p(w=0) \tag{3}$$

$$\tag{4}$$

There is a closed form of the maximum likelihood solution for the Bernoulli distribution.

$$\mathcal{L} = \ln p(w) = p(w=1) \ln q_j + p(w=0) \ln(1 - q_j) \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial q_j} = \frac{p(w=1)}{\hat{q}_j} - \frac{p(w=0)}{1 - \hat{q}_j} = 0 \tag{6}$$

$$\tag{7}$$

Solving for $\hat{q}_j$:

$$\hat{q}_j = \frac{p(w=1)}{p(w=0)+p(w=1)} = \frac{\frac{n_{w=1}}{\cancel{n_{total}}}}{\frac{n_{w=0}}{\cancel{n_{total}}} + \frac{n_{w=1}}{\cancel{n_{total}}}} \tag{8}$$

$$\boxed{\hat{q}_j = \frac{n_{correct}}{n_{total}}} \tag{9}$$

For the M step, we maximize the likelihood of the tile combination T' for a fixed set of worker qualities, $\{Q_j\}$. Following Eq.1,

$$p(T'|Q_j, l_{kj}) = \frac{p(l_{kj}|Q_j, l_k)\cancel{P(T')}}{\cancel{p(l_{kj})}} = p(l_{kj}|Q_j, l_k) \tag{10}$$

We can decompose T' into its tile components:

$$= \prod_{t_k \in T'} p(t_k \in T|Q_j, l_k) \prod_{t_k \notin T'} p(t_k \notin T|Q_j, l_k) \tag{11}$$

We can evaluate these quantities by the observed indicator labels. The worker makes a correct response in the scenario where they vote for a tile that actually lies within T or they don't vote for a tile that is excluded from T. In that case, the probability of the tile being in T is the probability that the worker had made the correct response: $p(l_k) = q_j$. Likewise, a worker makes a incorrect response when their vote contradicts with the inclusion of the tile in T ($\{t_k \in T \& l_{kj} = 0\}, \{t_k \notin T \& l_{kj} = 1\}$), in which case the tile is assigned a probability of $p(l_k) = 1 - q_j$.