Natural Numbers, Natural Language: Architecting the Semantic Web

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Abstract

Two disparate themes are brought together, to underwrite Semantic Web architecture. A bit-string- (as opposed to quantity-) based Number Theory gives body to the study of zero-divisors (ZDs) in infinite-dimensional extensions of hypercomplex numbers, via the Cayley-Dickson Process (CDP). Their Cayley-Table-like Emanation Tables (ETs) of ZD products – representable as infinitely extensible spreadsheets – have patterns of empty cells defining fractals ... and "fractality" is, per Sir Tim Berners-Lee, the fundamental feature of World Wide Web geometry. But ETs themselves decompose into integral numbers of ZD "atoms" called Box-Kites, octahedral wire-frame figures whose 6 vertices represent planes whose diagonals are saturated with all and only "primitive" ZDs. Their basic circuits and scaffolding provide a "representation theory" unifying two distinct semantic models, derived by Jean Petitot from Singularity Theory mathematics, and applied to the semiotics of literary and mythological structures, of Algirdas Greimas and Claude Lévi-Strauss respectively.

1. Introduction: Box-Kite Basics

As argued in [1][2][3] and extensively visualized in [4], CDP's dimension-doubling algorithm recursively builds the Imaginary Numbers from the Reals in 2-D, the non-commutative Quaternions (basis of vector algebra) in 4-D, and the non-associative Octonions (beloved by string theorists) in 8-D. All of these support familiar things like fields and division rings, but both of these vanish forever once doubling proceeds to the 16-D Sedenions. Here, entities long deemed "pathological" abound which, while not null themselves, nevertheless have zero *products* with certain other numbers of their ilk. There are simple patterns, however, governing their behavior, made evident with appropriate XOR-based notation and a few simple rules.

For any N, denote the 2^N-1 imaginary indices of the 2^N-ion units, with consecutive integers; index the Real unit with 0. For all N, associative triplets isomorphic to Quaternion i, j, k are generated by asserting the index of the product of any two units is the XOR of their indices. The 4 units of the Quaternions proper are thus written (0, 1, 2, 3).

What isn't thus determined is the *sign*: for all units for any N, sign is fixed by two rules, which boil down to the essence of CDP. Triplets are put in CPO (for "cyclical positive order") thus: if (per the usual Quaternion labeling) $i_1 * i_2 = + i_3$, $i_2 * i_3 = + i_1$, and $i_3 * i_1 = + i_2$, then (1,2,3), (2,3,1) and (3,1,2) are equivalently CPO. To build the next set of units, append one whose index is the next power of 2. To get Octonions from Quaternions, this is 4, with N = 3. Abstractly labeling this new unit G, we state Rule 1: for any unit of index L < G, its product with G on the right generates the CPO triplet (L, G, G+L). Put in CPO with lowermost index leftmost, Rule 1 gives these 3 associative triplets ("trips") in the Octonions: (1,4,5); (2,4,6); (3,4,7).

To obtain the other 4 Octonion trips, start with Rule 0, which says trips already generated for lower N still obtain: hence, (1, 2, 3) is also an *Octonion* trip. For the rest, we need Rule 2: for any "Rule 0" trip, add G to two of the units and *switch their positions*, retaining the position and value of the third. Fixing 1, 2, 3 in that order, we get the other Octonion trips: (1,7,6); (2,5,7); (3,6,5). These rules are necessary and sufficient to generate all trips for all N. And, for any N, simple combinatorics tell us there are exactly $(2^N-1)(2^N-2)/6$ such trips: 1 for Quaternions, 7 for Octonions, 35 for Sedenions, and 155 for the 32-D Pathions, where fractals are first adumbrated. The nature of this adumbration is dependent upon bits in high positions of an integer called the "strut constant": an invariant of the ZD "atom" (or, for N > 4, ET ensembles of same) called a Box-Kite. Its comprehension requires first understanding the nature of ZD clustering that motivates this formalism.

In the Sedenions, ZD patterns have a simple clarity: take any unit L whose index < G, and consider the diagonals in any of the 42 planes ("Assessors") spanned by such an L and any unit U > G and *not the XOR of G with L* (and hence not G + L, since G's singleton bit will always be further left than all bits in L). Then, for real scalar k, all points on the lines $k(L \pm U)$ are ZDs. However, none of these have zero product with any of the other ZDs in the same Assessor.

The Box-Kite wire-frame figure gives an easily visualized way to see which ZDs in one Assessor have zero products with which ZDs in any other. The 6 vertices of its octahedron are Assessor planes, each of which has L one of the 7 Octonions, with the seventh – the "strut constant" S alluded to above – being "ZD-free" in the given Box-Kite, of which there are 7 (one for each Octonion found absent from the vertices). For a given S, the L indices on opposite vertices form a trip with it, with 3 such trips always containing any such S in the Octonions. The 3 lines joining such opposite vertices are called "Struts," and *no ZDs belonging to Assessors on opposite ends of a Strut can have zero products with each other*.

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All ZD points in an Assessor *sharing an edge* with another, however, will "make zero" with those in *exactly one* of the other's diagonal lines. If we indicate indices greater or less than G with upper and lower case of the same letter, using uppercase also as shorthand for the Assessor as such, we can express two general instances thus: for real scalars k and q, $k(m + M)*q(n + N) = 0$, or $k(m + M)*q(n – N) = 0$. (We assume throughout that indices written in such equations be understood as shorthands for the usual, but quite tedious, letter i for "imaginary unit" with subscript equal to the index.) If we dub these two instances cases of /*/ and /*\ respectively, we can also say the same for their duals *\ and */ -- e.g, $k(m – M)*q(n – N) = 0$ when the first instance just given holds.

Where both diagonals slope the same way, we draw the edge blue, or mark it with a plus in brackets [+] on the Box-Kite diagram; if diagonals have *opposite* slope, we draw the edge joining their Assessors in red, or mark it with a minus in brackets [–]. For any Box-Kite, for any N, there will be 6 red and 6 blue edges, with the reds clustered in 2 disjoint triangles forming opposite faces on the Box-Kite's octahedron. For one of these 2 red triangles, the L units form a trip; by convention, we label its vertices A, B, C so that (a,b,c) is CPO, a < b and a < c. It is one of 4 triangles whose L-units form trips, collectively called Sails; the all-red-edged Sail is the Zigzag, so named due to the 6-cycle sequence of diagonals traversed as we go around the perimeter twice: starting with a /, we get / \ / \ / \; else, \ / \ / \ /.

The other 3 "Trefoil" Sails each share exactly one vertex with the Zigzag, and one with each of the others: they are arrayed, then, like same-colored squares on a checkerboard. The 4 "other-colored" triangles are "vents" through which the wind blows, lifting up the Box-Kite whose structural integrity is vouchsafed by the ZD-free Struts (wood or plastic dowels, perhaps). The vertices opposite the Zigzag's are labeled in "nested parentheses" order, so that the Struts join Assessors labeled AF, BE, CD. Trefoils can thus be written ADE, BFD, CEF. (Note: L-trips (a,d,e); (b,f,d); (c,e,f) are CPO, but leftmost indices are not necessarily lowermost.)

For a Zigzag, we know not only that the L-indices form a trip (a,b,c); more, there are 3 similarly oriented "U-trips": by a Rule 2 related logic, (a, B, C); (A, b, C); (A, B, c) are also CPO triplets. The same is generally true for Trefoil Sails, though, only up to sign: for the U-trip containing the L-index of the Assessor shared with the Zigzag, the L- and U- trips spanning the Trefoil have similar orientation; for the other two U-trips, though, *L- and U- trip orientations are reversed*. We say the Zigzag has "trip-sync" symmetry, broken when some current initially circuiting its Assessors moves, for reasons of its own, through one of them into a Trefoil. This is the first dynamic fact that suggests the morphogenetics of unfolding singularities (specifically, of Umbilics) in Box-Kite formalism. But it is not the simplest such fact to indicate possibilities for dynamic modeling.

2. Strut Dynamics and "Semiotic Squares"

Within the Sedenions, three simple laws govern the mutual interactions of the 4 units comprising any Strut. As one Assessor bounding it will belong to the Zigzag, while the other will belong to the Vent opposite it, we can term them Z and V respectively, leading to the "Three Viziers":

VZ1: $v*z = V*Z = S$ **VZ2:** $Z*v = V*z = G$
VZ3: $V*v = z*Z = (G + S) \equiv X$

Only the second is truly universal; the others hold for all N beyond the (N=4) Sedenions only if unsigned – or, if we consider only "Type I" box-kites. ("Type II" first show up in 32-D, and have two struts reversed, a side-effect of their Zigzags being built by applying Rule 2 to another of Type I; they can thereby be derived from Type I by simple operations detailed in [14], which allow all symbolic expressions to be written using VZ1 and VZ3 anyway.) If we place the indices of each Assessor's L- and U- units at the ends of separate and parallel horizontal lines, in left to right order, the letter X forms trips with V on top and Z below. If we then join the L-units with a vertical, and do the same for the U-units, we see that each of these is operated on by S, with which the ends form trips in both cases. Finally, we can cite VZ2 and join the diagonally opposite units, whose XOR, in both cases, is G.

With minimal interpretation, we can further relate S, X, and G to the three fundamental kinds of "binary oppositions" first elicited by phonologist Roman Jakobson, and later generalized to the canonical pattern informing countless so-called "structuralist" disciplines in their methods of defining the basic relationships their analytics would investigate. In this latter aspect, the figure just limned verbally (with a prehistory extending through Apuleius of Madaura to Aristotle himself [5]) was drawn and labeled explicitly as the "Semiotic Square" by Algirdas Greimas [6].

Inspired by René Thom's Catastrophe Theory (CT) approach to linguistic modeling, Greimas' Square has been expressed in simple Cuspoid models by Jean Petitot, whose doctoral research was conjointly guided by Greimas and Thom both [7]. We would appropriate Petitot's analysis to our own ZD-based "representation theory," in the hope of solving some problems he leaves unresolved, not the least of which entails a merger with Petitot's quite separate (Double Cusp based) analysis of the "Canonical Law of Myths" [8]. This empirical tool, claimed by Lévi-Strauss as the unfailing guide of his approach for more than half a century, was focused on at book-length [9], some 30 years after he announced it – the trigger of Petitot's own work toward comprehending it. Petitot also treats "reciprocal double exchange" via the highest Cuspoid stratum in the Double Cusp -- an artificial tactic, disliked by its own author [12, 388-394]; the much simpler ZD behavior of "twist products" swaps units between Assessor dyads [14].

For Greimas, our VZ3 corresponds to a two-level conception of "contraries" (the horizontals), with each level itself forming a plane spanned by competing "actants" with two-ply interpretations. Their semantic loads, "joined at the hip," instantiate Hjelmslev's idea of "reciprocal presupposition" (cat vs. mouse, say), without whose mutual referral a unifying theme of "predation" cannot be posited. Read dynamically, we have at least a Cusp Catastrophe or its dual, both of which participate in a single behavior, guided by two "controls." These regulate capture or release of one "attractor" by the other (or, in the "dual" case where the sign on behavior is reversed, one state of equilibrium between two pan-balance contents, corresponding to a zone of inaccessibility – a maximum or saddle -- between two locally parabolic minima in the non-dual reading): think troughs of the letter "W" separated by a peak in the middle, or an "M" for the dual. In ZD algebra, either Assessor diagonal can be taken as a line of (anti-) synchronization, or "balance," between tandem controls. More, CT itself is inherently two-tiered, concerned with mappings between "control" and "behavior" spaces, both 2-D for the Cusp and its dual: $f(x) = \pm(X^4 - uX^2 - vX)$. In this canonical form, negative leading sign indicates the dual; controls u and v govern changes $V \leftrightarrow W$ (or, dually, $\Lambda \leftrightarrow M$) in the $(X, f(X))$ plane. A 45° coordinate-frame rotation can associate a ZD diagonal with the $v=0$ "splitting" line, along which balance between attractors (or pan-balance maxima) obtains.

The Cuspoids are an infinite family A_n of such generic equations, guiding the "universal unfolding" of seed-forms (degenerate singularities) into "blossoms," abstractly writable as germ X^{n+2}, followed by n "harmonics" X^k, $1 \le k \le n$, each regulated by control variable a_k. For n=0, this reduces to the trivial linear inverse square law; for other n even, we get a "bucket brigade" of 1 to (n+2)/2 minima depending on controls. For n odd, the chain has a leading or terminating tail extending to $\pm \infty$ in "behavior space": pulling up a tent peg against a cone of earth (weight $\sim X^3$), has control u working on X ($\sim$ tensile strength of the rope), providing the "figure of regulation" determining whether the rope will snap before the peg's extracted. For Cuspoid A_3 (germ X^5), a Cusp competition's resolution can be followed by "throwing the baby out with the bathwater" (the merged pair of attractors fall off to $\pm \infty$), sometimes called the "Suicide Catastrophe." In Petitot, the next (and even) Cuspoid, A_4, (germ X^6) is crucial: this "Butterfly" (or its dual) provides the container for the Semiotic Square itself in his model.

Stack one Cusp atop another: associate one with a hysteresis loop regulating blade/water entrainment of a mill wheel; the other, with a low-gradient diversion canal creating an artificial waterfall, beneath which the mill wheel's effectiveness is amplified. This model of Thom's [10, pp. 51-64] contains a constrained subset of the Butterfly proper; expanding its setup to the full-blown model yields the "gift morphology" wherein two $V \leftrightarrow W$ dyads, chained with minimal overhead, effect an exchange: A gives B to C.

The two VZ3 horizontals, each housing a "contrary" pair, are (Jakobson's second primordial mode of binary opposition) "contradictory" with respect to each other: mutual exclusiveness obtains on a term-by-term reading, so that each diagonal entails a *privation*. In phonemics, 'g' is voiced, but 'k' is not … and in VZ2, the singleton G bit is likewise 'on' (right column) or 'off' (left column). VZ1's verticals, meanwhile, are deemed "complementary": the upper terms are "implications" (in the strict set-theoretic sense) of the lower, alternately called cultural or arbitrary, since they reference convention rather than anything inherent in them. (Hjelmslev on color-name boundaries in English vs. Welsh, on pp. 52-3 of [11], is the classic example.) Put another way, the "absence" displayed by the lower horizontal *vis à vis* the upper can provide the site for the attaching of multiple "contradictory" possibilities once N > 4, when any given horizontal can belong to multiple V or Z L-trips in different same-S,G Box-Kites. The real interest in our approach, then, will only reveal itself for high N. It is obvious, for instance, that any power of $2 \ge G$ will work to similar effect along the diagonals – and, indeed, for the Square as a whole, provided X (= G + S) be similarly augmented. It is not obvious, however, that appending an odd number of such G-bits simultaneously *turns on*, whereas appending an even number *turns off,* whole Box-Kites.

For detailed computer-graphical analysis of the relations between slices of A_3 and A_4 unfoldings and Petitot's model, see Chapter VII of [12]. Our interest now will focus on how to embed any such models recursively in higher-order ZD "representations" for large N. We start by noting that VZ3, regardless of signing, tells us this: for N and S fixed, X is fixed also; hence, all products along edges or across struts are determined by the L-terms of Assessors involved. For a basic Sedenion Box-Kite, then, we can build a multiplication table of ZDs, with L-index labels C (R) arrayed in "nested parentheses" order, left to right across the top (top to bottom along the left), with cells of the resulting 6 x 6 "spreadsheet" *left empty* if the ZDs indicated by R and C do *not* mutually zero-divide, else *filled* with the L-index of the third Assessor in their Sail (since their zero product will equal the sum of oppositely signed copies of its units). And since no ZDs housed in the same or strut-opposite Assessors can "make zero" together, both long diagonals of this smallest "Emanation Table" (ET) will be empty, while all 24 remaining cells will contain integers P = R *xor* C: one entry for each lane of 2-way traffic along the 12 edges. (This implies a convention for mapping a*b and b*a, say, to the distinct flows along their shared edge – a detail of no concern here.) ETs also prefix non-empty cell entries with a dash (–) if the edge implied is *blue* (making it easy to read off the L-indices of Zigzags, which will all be unmarked: for details, see [2]). For N > 4, an ET's square will have edge-length G–2 and long diagonals each housing as many empty cells; hence, filled cells for N=5,6,7… *may* equal 168, 840, 3720… – but *only if* S < 8 or a power of 2.

Note the sums listed are multiples of 24: for N=5, 6, 7, filled cell counts of up to 7, 35, 155 Box-Kites can reside in each ET … *the count of trips, in each case, for N-2*. This surprising result is a corollary of Theorem 8 in [2], which mandates the "all or nothing" rule: ZDs "make zero" along all edges of a Box-Kite, or not at all. (We'll see what empty Box-Kites signify shortly.) The "*may* equal" caveat of last paragraph refers to the on/off behavior referenced in the paragraph prior, when varying counts of high-bits ≥ 8 belong to S. In the 32-D Pathions, $8 < S < 16$ determines 7 ETs which each have 3, not 7 Box-Kites' worth of cells. Such sequences of 7 (or 8, when the upper bound is not a power of 2) form what pre-cartoon animators would call a "flip-book." If each increment of S be mapped to a unit of time passed for N fixed, a dynamic sequence of changes is readily observed, obeying a bit-twiddling logic. When S=9, all 12 cells *not* on the long diagonals are filled on all 4 sides, with the remaining 24 cells filled with all and only the ET's 1s and 8s (the S and G, respectively, for the Sedenions' S=1 (= 9–8) Box-Kite), symmetrically arrayed along the long diagonals. As S increments, the nearly filled lines (NFLs) move in, one step at a time, from the perimeter, and the 1s and 8 rearrange along long diagonals, until (S=15) we see 4 NFLs forming crossbars with a 2 x 2 hole in the ET's center, and 1s and 8s forming diagonal "suspension cables" joining the horizontal and vertical extrema of the crossbars.   (See Slides 25-31 of [4]) In all 7 cases, a simple formula involving each cell's row and column labels R and C and their XOR P, '|' for logical or, and g for G/2 = 2^{N-2}, obtains: a cell is filled only if

$$R \mid C \mid P = g \mid (S \bmod g)$$

Readily algorithmized variations (see [3]) on this sample bit of "recipe theory" generate fill/hide patterns for ETs associated with any S, hence literally demarcate a pure Number Theory peculiar to ZD behavior, pertaining to bit-string "placeholder" considerations not quantities, with powers of 2 playing a role analogous to that of primes in the approach that has held sway since ancient Greece.

Such recipes also extend to indefinitely large ETs when S is held fixed and N is incremented: such "balloon rides" take one through a sequence of nested "sky-boxes" by a recursive process whose key is realizing that the "label lines" (the extra row and column on top and left), when their contents are echoed in reverse order on opposite sides (so that strut opposites mirror each other) in an appended row and column of "mirror label lines"), create 2^{N-1}-celled edges of a square, with 4 empty corner cells. In the implied boxes-within-boxes recursion, cell *labels* in one iteration become cell *contents* on the next. Other indices fill by variations on recipe equations; and, as N grows indefinitely, the empty ET cells are seen to approach a fractal limit, with cells mapped to pixels, and total dimension (half the total cell count) approaching infinity.

Taking the S=15 Pathion instance just discussed as template, the limit-case is a fractal Mandelbrot calls the Cesàro double-sweep. (See Fig. 1 in [3]) Easily adapted code, written in a version of Visual Basic called "Lotus Script" customized for the Lotus Notes object hierarchy, can be found in the appendix to [13]: all Excel spreadsheets of ETs in [4]'s Powerpoint slideshow were output by it. (Mathematica notebooks and Java libraries incorporating this much and more are now being built.)

As pure bit-logic, constructing ET meta-fractals with an infinite-dimensional Sky as the limit is quite easy. The dynamics underlying such formalism merits examining as well – and can tell us much a more formal approach will miss. The seminal fact is "Box-Kite explosion": assume some kind of background turbulence whose nature needn't concern us induces L and U units to suffer scission, leading each to seek higher-index partners from the next-higher realm of 2^N-ions. As detailed in Sect. 5 of [2], G and S become L-units of strut opposites B and E in a trio of inter-linked Box-Kites, sharing this strut but no edges. The trio of strut-opposites in the starter Box-Kite each generate the other pair of strut-opposite Assessors in one of the result-set Box-Kites (with VZ3 saying starter X must be result-set S). Meanwhile, all 4 Sail-fixing L-trips in the starter do *not* generate viable Box-Kites on the next level. This can be visualized easily by re-using the Box-Kite wire-frame as a housing for the L-trips and their U-index partners: on ABC, place (a,b,c), and place U-indices (not f,e,d) in strut-opposite positions: "Trefoils" are now the Zigzag's U-trips, but the orientations of their orbits are improper (which is why the implied Box-Kite is a failure). Perform the same procedure using Trefoil L-trips, to same effect (albeit differing "failure modes"). As studied in our most recent monograph [14], Box-Kites explode to yield quartets of hidden box-kites (HBKs) bereft of ZD edges, called "spandrels." The HBK spawned by the starter's Zigzag L-trip is not only ZD-free along edges, but contains a ZD-free copy of the Octonions – meaning, it can be treated as the basis of a recursive process generating a universe of ZD indices independent of those defining its locus in the starter's setup. This suggests a next topic for study: since Chaotic attractors are essentially woven from multiple fractal "threads," can a "representation theory" of Chaotic forms be constructed by such "spandrelizing"?

Returning to the present, we note one other feature: the splitting of 2^N-ion Assessors into 2^{N+1}-ion strut-opposite L indices has an obvious side-effect on the Semiotic Square: horizontal "contrary" lines are transformed into vertical "implications"! We assert this is the fundamental move which transforms Greimas's apparatus into Lévi-Strauss's … something Greimas, as we'll soon see, would not find surprising, but which Petitot's approach leaves unstudied. To reach the Double Cusp, we must move up from Cuspoids into higher strata: for starters, the D_n ("Umbilic") singularities.

3. Klein Groups, Umbilics, Lanyard Circuits

In remarks echoed elsewhere in his corpus, Greimas tells us his Square is "isomorphic to … the structures called, in mathematics, the Klein group, and, in psychology, the Piaget group," as well as underwriting "the model of myth propounded by Claude Lévi-Strauss." ([6], pp. 88, 89) This last is our target; let's start with the Klein group. As Greimas says of his Square, it appears "as the correlating of two categories of opposites, the correlation itself being defined as a relation of homologized contradictions." But the Klein group appears also as the *factor group* of the Quaternions: the 4-element group formed by its *unsigned units*. This has direct bearing on our 3 Viziers: the ZD-free quartet formed by their 3 symbols plus 0 (for the Real unit) constitutes the Klein group as what Java coders might call the "abstract class" of a Box-Kite ensemble, instantiated by L-trip contents -- except (S,G,X) can *be* such an L-trip, for *higher* N!

We note that Lévi-Strauss's own analysis, in this regard, is subtler than Greimas': he does not identify his canonical law of myths with the Klein group; rather, the latter is used as a tool to generate series of themes, the overall trafficking among them being what the canonical law regulates. A telling passage in the "Finale" of the last volume of his magnum opus makes this clear: Klein groups' "interlocking four-term structures, retaining a relationship of homology with each other," are never independent or self-sufficient, but inhabit nodes of a "brain envisaged as a network." The "ordered series of variants" threading it "does not return to the initial term after running through the first cycle of four: as through an effect of slippage … comparable to that of the gear-change of a bicycle, the logical chain is jolted loose and engages with the initial term of the immediately following interlocking group" – a process "repeated right through to the end." ([15], pp. 649, 627, 649-50) The most efficient read-out of a Box-Kite's 24 lanes of traffic along its 12 edges (no ZD line touched twice, with all engaged) we dub the "bicycle chain" after this passage. It must be traversed twice, starting with / or \ from the same point:
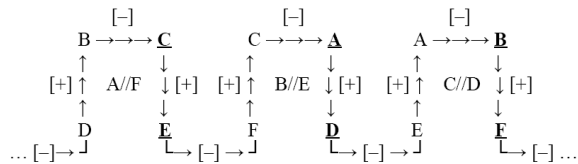

Figure 1. Bicycle-Chain (tell / from \ by underscores)

The Klein group can be embedded in the 24-element "unit quaternions," which, as basis of 4-D closest-sphere packing, is isomorphic to the Lie-theorist's D_4 structure that underwrites the first Umbilic singularities. Such abstract correspondences motivate a search for concrete ways to "represent" the Umbilics via Box-Kite dynamics.

To elicit Umbilics, we first assert their correspondence to segments of the "bicycle chain" as shown: the "hop, skip and jump" of Zigzag-edge sequences along the top horizontal provides half of the Elliptic Umbilic (EU) model; the "J" shapes appending [+] signs marked mid-vertical to the flow beginning with rightmost [-] segments, then down and left, provide 3 half-copies of the complementary Hyperbolic Umbilic (HU) model. This 3-to1 relation is built in to the EU/HU model in fundamental ways – both topologically in CT, as the relative count of Dual Cusps embedded in their respective "control space" deployments; and, in terms of [-] vs. [+] edge-currents in the ZD theory. (And, of course, it is the trademark of the simplest non-cyclic or "Klein" group too.) Note that while Cuspoid representations were based on paths connecting L- and U- units within an Assessor, the scheme we'll now consider doubles that basis: it is the join *between* pairs of Assessors (sharing an edge), not unit-pairs *within* an Assessor (opposed to another along a strut). And note, too, that pursuing this comparison breaks down, at least within the framework of the bicycle-chain: for we do *not* see a "double articulation" of the Semiotic Square; rather, we have an "open square" in each J, which would need to have their bottom horizontals shifted once to the right – resulting in three disjoint squares instead of one chain.
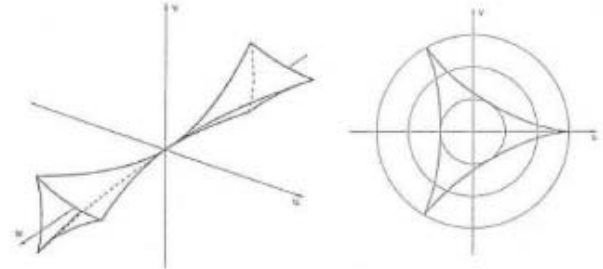

Figure 2. EU controls (3-D and conical cross-section)

The trio of Cusps are self-evident in (u,v) cross-section, which tapers to a point at the origin along the *w* axis. There are two distinct behaviors, not one as with Cuspoids proper, conventionally labeled X and Y, with respect to which *u* and *v* are just "loading" factors of simplest-possible Folds (viz., the tent-rope tensile strength control in the peg-pulling example). The *w* controls the two-behavior mix, as epitomized in the conic-section signature which gives the form its name, $(X^2 + Y^2)$: a simple circle, deformable into an *ellipse* without changing the CT form. The full equation is written $X^3 - XY^2 + w(X^2 + Y^2) - uX - vY$. The key is the pair of "germ" terms on the left, defining the classic "monkey saddle." The first is just the germ of the simplest Cuspoid, while the Y^2 acts like a mirror on this initial X behavior, as manifest in the (*w*=0) reflection plane. This is where the D in the D_n series comes from: it stands for "dihedral," the name for groups which append mirror operators to rotational subgroups, assumed to be an (n-1)-gon, or triangle for D_4: the simplest dihedral symmetry is

contained in the Klein group, while mirrors through each face of the tetrahedral 12-group (containing both a 3-gon or triangle *and* a Klein group) would give a 24-element dihedral arrangement – the right D_4 count, as we've seen.

One of the few works cited as a seminal influence by Thom and Lévi-Strauss both is D'Arcy Wentworth Thompson's *On Growth and Form*; and here, in his discussion of hexagonal cell-packing in honeycombs, he notes (p. 104, [16]) the generality of such patterning in nature:

A circle surrounded by six similar circles, the whole bounded by a circle of three times the radius of the original one, forms a unit, so to speak, next in order after the circle itself. A round pea or grain of shot will pass through a hole of its own size; but peas or shot will not *run out* of a vessel through a hole less than *three times* their own diameter. There can be no freedom of motion among the close-packed grains when confronted by a smaller orifice.

This "puncture and squirt" instantaneous cross-section is a hypocycloid of 3 cusps – the *classica*l figure traced by a point on a circle as it rolls along the inside of another with triple its diameter, and CT's *regulatory* figure of *emission*.

The behavior space is ultra-simple, and mappable by a single bit: the left half-cone is stable, with one minimum; the right is unstable, with none. (Think syringe injection; pollen disseminating,;a snake engulfing its prey at the stable end, excreting what remains after digestion is done at the unstable end.) Fluid dynamics experiments simulate the EU ideal behavior to near perfection with a system of six rollers, arrayed in a circle with alternating clockwise and anticlockwise motion. Three motors drive pairs of oppositely turning rollers to generate the (u,v) control plane, while mixing in a polymer simulates a "viscosity controller" for w. (Chapter 7, [17]) A hexagon of alternating rollers suggests a Zigzag 6-cycle of oppositely sloping diagonals.

Assume the cusp lines on the figure's left pass through the Zigzag's ABC (whose L-trip signifies *stability*), then lead through the origin (S, at the 0 of the wire-frame octahedron) and out DEF (which, as L-tripless vent, is *unstable*). Each edge indicates ZD trafficking, and in the Zigzag this means a Dual Cusp balance-pan of alternately signed copies of the third Assessor in the Sail. Simple assumption: symmetry-breaking of trip-sync toggles the "behavior bit," in context of an underlying, highly mobile "tension of attention," as with "change blindness" in the brain's visual system.

Algorithmically, Umbilic "edge-current" models imply recursive invocations of the branching possibilities toward a third Assessors' diagonals: a pair-creation NFA (non-deterministic finite automaton), which splits into multiple copies of itself and follows *all* possibilities in parallel as it reads symbols in an input string, killing off copies of itself

when a symbol doesn't appear on any arrows exiting the current state, then issuing a final "accept" or "deny" bit when done processing the string. Regardless of mobility of backdrop, then, an NFA conception of processes lets us frame volatile contexts in "closed," even classical, algebraic terms (e.g., Sails *qua* dihedral groups). ([18], p. 48)
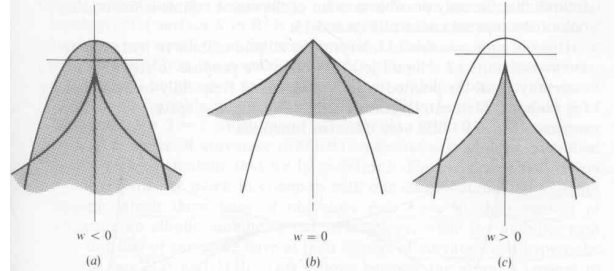

Figure 3. HU fixed-*w* cross-sections for (*u,v*) controls

For w=0, we have the breaker-line on an ocean wave, 3 cross-sections of which appear in (u,v) space as shown. The canonical equation comprises 2 Fold (A_2) unfoldings, in X and Y behaviors, with the w control working on their hyperbolic mixture: $X^3 + Y^3 – uX – vY + wXY$. As with the standard hyperbola, the control-space surface has two mirror-image sheets, puckered into a Cusp on one end, with a gradually flattening curve on the other. Waves fall over themselves as the Cusp of one sheet rises up with respect to the other sheet's smoothly curving end, the latter ($w > 0$) then dropping into a Cusp while the former shrouds it in an empty arc, where surfers get free rides with shore in sight: the figure of *tethered transport* with source unspecified, the complement of EU *message emission* with undefined target.

EU and HU are, in fact, the same forms in Complex co-ordinates, only splitting into two complementary ones when unfoldings are considered in exclusively Real spaces: they are both based on cubic solutions, with either all 3 solutions real (hence, the EU's Dual Cusp count – and the [--]-edge tally for Zigzags) or 1 real with the others being complex conjugates (hence, the HU's 2 [+]-edges to 1 [--]). They are both, then, designated as type D_4, with the subscript having a minus (EU) or plus (HU) sign affixed to it.

The most natural ways to deploy them in ZD models would exploit trip traffic leading to "trip sync" flow-reversals (with one on/off bit in either case); or, would exploit the trips as fundamental units of navigation in search algorithms, with storage in spandrels (rather than ET long diagonals, which Semiotic Squares would use). We see both tactics as interlinked and indispensable. The first strips a box-kite to bare essentials: a bit-grabbing and/or bit-passing mouth/sphincter dyad ... with its inputs and outputs registering in the fourfold HBK array of a spandrel which mirrors the EU/HU dichotomy in its own flow-structure. The Zigzag-based HBK has all flows save that of the Zigzag circle itself reversed, whereas the Trefoil-based HBKs each reverse only the side whose midpoint contains a Zigzag term and the strut which ends in it.

It is easy to imagine numerous "dual" readings, mapping the EU's 3 cusps to the 3 Trefoil HBKs, else to the Zigzag HBK which contains all 3 reversal patterns, etc. Of paramount interest: the so-called "pathology" that lets us store collections of objects along HBK diagonals, say, while exploiting the imperviousness to metrics of the ZD navigation.

In CT, the EU/HU umbilics are "non-compact": their narratives are open-ended. In [14], we show Zigzags of Type I HBKs (and Type II ADE Trefoils) always contain a "cowbird's nest" tethered to the source box-kite from which its spandrel was exploded, within which CDP can propagate its own triplet-indices Hence, the trio of "Semiotic Squares" whose ensemble makes a box-kite can always be fed "out-of-context" surprises through these insertion points for infiltrating contexts, whose L-indices can serve to co-opt the prefabricated pathways of explosion their locations would seem to indicate, leading to "unintended consequences," metaphoric leaps, or acts of ventriloquy.

It is standard in W3C circles to link maximal uptake of web content to data exposed in RDF, by dint of the "principle of least power": the less expressive the language, the more reusable the data. It is ever more commonplace to see RDF triplets (and hence their underlying URLs) signified by pure integers in Web Service mapping-files. Our ZD triplets then suggest themselves as the last word possible in "least power" arguments: a pure number theory underwriting web fractality, triadic logic [19], indefinitely extensible search protocols and parsing algorithms [20]. Just what's called for, once the Brave New World of "cloud computing" is upon us, and the Grid simultaneously inaugurates the "planned obsolescence" of the World Wide Web itself.

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