

# Chapter 1

## Library cantor

Require Import *Bool*.

Section *Cantor*.

Lemma *negb\_prop* :  $\forall a:\text{bool}, \text{negb } a = a \rightarrow \text{False}$ .

Proof.

intros.

unfold *negb* in *H*.

induction *a*. inversion *H*. inversion *H*.

Qed.

Definition *surjective* {*X*:Type} (*f* : *nat* → *X*) : Prop :=  $\forall y, \exists x, f \ x = y$ .

Theorem *cantor* :  $\neg \exists f : \text{nat} \rightarrow \text{nat} \rightarrow \text{bool}, \text{surjective } f$ .

Proof.

intros [*f SURJ*].

pose (*g* := fun *b* => *negb b* ).

pose (*h* := fun *x* => *g (f x x)*).

destruct (*SURJ h*) as [*x B*].

assert (*C*: *h x* = *f x x*).

{

rewrite *B*. reflexivity.

}

unfold *h* in *C*.

unfold *g* in *C*.

apply *negb\_prop* in *C*.

assumption.

Qed.

End *Cantor*.