

# Chapter 1

## Library imp

Theorem *imp* :  $\forall (a\ b\ c : \text{Prop}), ((a \rightarrow b) \wedge (a \rightarrow c)) \rightarrow a \rightarrow (b \wedge c)$ .

Proof.

```
intros a b c H.  
intro Ha.  
split.  
destruct H as (H1 & H2).  
apply H1. assumption.  
destruct H as (H1 & H2).  
apply H2. assumption.
```

Qed.

Theorem *et\_refl*:  $\forall (a\ b:\text{Prop}), a \wedge b \rightarrow b \wedge a$ .

Proof.

```
intros a b H.  
split.  
destruct H as [Ha Hb].  
assumption.  
destruct H as [Ha Hb].  
assumption.
```

Qed.

Print *et\_refl*.

Theorem *hilbertS* :  $\forall (a\ b\ c:\text{Prop}), (a \rightarrow b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow (a \rightarrow c)$ .

Proof.

```
intros a b c.  
intros h1 h2 h3.  
apply h1.  
exact h3.  
Show Proof.  
apply h2.  
exact h3.
```

Show Proof.

Qed.

Print *or*.

Theorem *or\_elim*:  $\forall (a\ b\ c:\text{Prop}), (a \rightarrow c) \rightarrow (b \rightarrow c) \rightarrow (a \vee b) \rightarrow c$ .

Proof.

  intros *a b c h1 h2 h3*.

  destruct *h3* as [*ha* | *hb*].

  apply *h1*. exact *ha*.

  apply *h2*. exact *hb*.

Qed.

Print *or\_elim*.

Qed.