

Fractions continues

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27/12/2019

$$\sqrt{2} = 1 + \sqrt{2} - 1 = 1 + \frac{(\sqrt{2}-1)(\sqrt{2}+1)}{\sqrt{2}+1} = 1 + \frac{1}{1+\sqrt{2}}$$

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{\ddots}}}}}}} \quad (1)$$

```
racine2 <- function(n=1, limit=20) {  
  if (n==limit) {return(1)}  
  else  
  {  
    return(1+(1/(1+racine2(n+1, limit))))  
  }  
}
```

```
options(digits = 20)
print("la valeur de racine 2"); print (2^(1/2))
```

```
## [1] "la valeur de racine 2"
```

```
## [1] 1.4142135623730951
```

```
print("l'approximation par la fraction continue"); print(racine2())
```

```
## [1] "l'approximation par la fraction continue"
```

```
## [1] 1.4142135623730965
```

```
racine2latex <- function(n=1, limit=20) {
  if (n==limit) {return("...")}
  else
  {
    return(c("\\cfrac{1}{2+}", racine2latex(n+1, limit), "}" ))
  }
}
cat(c("\\sqrt{2} = 1 +", racine2latex()))
```

[illegible]

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}}}}}}}}}}}}$$

```
eu <- function(a, b) {
  euclide <- matrix(ncol=4, nrow=10)
  colnames(euclide) <- c("a", "b", "q", "r")
  euclide[1,"a"] <- a
  euclide[1,"b"] <- b
  euclide[1,"q"] <- a%%b
  euclide[1,"r"] <- a%%b
  i <- 1
  while (euclide[i, "r"] != 0)
  {
    i <- i+1
    euclide[i, "a"] <- euclide[i-1,"b"]
    euclide[i, "b"] <- euclide[i-1,"r"]
    euclide[i, "q"] <- euclide[i,"a"] %/% euclide[i,"b"]
    euclide[i, "r"] <- euclide[i,"a"] %% euclide[i,"b"]
  }

  output <- c("\\frac{", a, "{", b, "}" , "=")
  i <- 1
  while (euclide[i, "r"] != 0)
  {
    output <- c(output, euclide[i, "q"], "+\\cfrac{1}{", euclide[i,"b"], "}" , "=")
    i <- i+1
  }
  output <- c(output, euclide[i, "q"] , "+\\cfrac{1}{", euclide[i,"b"], "}" , "=")
  for (j in (1:(i-1)))
  { output <- c(output, "}")
  }
}
```

```

print(euclide)
cat(output)

}

```

```
eu(840,611)
```

```

##          a    b    q    r
## [1,] 840 611  1 229
## [2,] 611 229  2 153
## [3,] 229 153  1  76
## [4,] 153  76  2   1
## [5,]  76   1 76   0
## [6,] NA  NA NA  NA
## [7,] NA  NA NA  NA
## [8,] NA  NA NA  NA
## [9,] NA  NA NA  NA
## [10,] NA  NA NA  NA
## \frac{ 840 }{ 611 } = 1 + \cfrac{1}{ 2 + \cfrac{1}{ 1 + \cfrac{1}{ 2 + \cfrac{1}{ 76 } } } } }

```

$$\frac{840}{611} = 1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{76}}}}$$

```
eu(11,8)
```

```

##          a    b    q    r
## [1,] 11  8  1  3
## [2,]  8  3  2  2
## [3,]  3  2  1  1
## [4,]  2  1  2  0
## [5,] NA NA NA NA
## [6,] NA NA NA NA
## [7,] NA NA NA NA
## [8,] NA NA NA NA
## [9,] NA NA NA NA
## [10,] NA NA NA NA
## \frac{ 11 }{ 8 } = 1 + \cfrac{1}{ 2 + \cfrac{1}{ 1 + \cfrac{1}{ 2 } } } }

```

$$\frac{11}{8} = 1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2}}}$$

```
eu(15625,6842)
```

```

##          a    b    q    r
## [1,] 15625 6842  2 1941
## [2,]  6842 1941  3 1019
## [3,]  1941 1019  1  922
## [4,]  1019  922  1   97

```

```
## [5,] 922 97 9 49
## [6,] 97 49 1 48
## [7,] 49 48 1 1
## [8,] 48 1 48 0
## [9,] NA NA NA NA
## [10,] NA NA NA NA
## \frac{15625}{6842} = 2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{9 + \frac{1}{1 + \frac{1}{1 + \frac{1}{48}}}}}}}
```

$$\frac{15625}{6842} = 2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{9 + \frac{1}{1 + \frac{1}{1 + \frac{1}{48}}}}}}}$$