Homework Assignment 2

1. Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Give NFAs that recognize the following languages. The answer must be a state diagram. Ensure your NFA has nondeterministic or ϵ-transitions.

   (a) $L_1 = \{w \mid w \text{ starts with } \mathtt{ab} \text{ and ends with } \mathtt{ba}\}$ using three states

   (b) $L_2 = \{w \mid w \text{ contains neither } \mathtt{aa} \text{ nor } \mathtt{bb}\}$ using five states

   (c) $L_3 = \{\mathtt{ab}\}$ using three states

2. Convert the NFA $N_1 = (\{q_1, q_2, q_3, q_4\}, \{\mathtt{a}, \mathtt{b}\}, \delta, q_1, \{q_2, q_4\})$ into a DFA, where δ is defined below. Show only the reachable states. The answer must be a state diagram.

$$\delta(q_1, \epsilon) = \{q_2\}$$
$$\delta(q_1, \mathtt{a}) = \{q_2\}$$
$$\delta(q_1, \mathtt{b}) = \{q_3\}$$
$$\delta(q_3, \mathtt{a}) = \{q_2, q_3\}$$
$$\delta(q_3, \mathtt{b}) = \{q_4\}$$
$$\delta(q, c) = \emptyset \qquad \text{otherwise}$$

3. Let $N_2 = (\{q_1, q_2, q_3\}, \{\mathtt{a}, \mathtt{b}\}, \delta, q_1, \{q_3\})$ where

$$\delta(q_1, \mathtt{a}) = \{q_1, q_2\}$$
$$\delta(q_1, \mathtt{b}) = \{q_1\}$$
$$\delta(q_2, \mathtt{a}) = \{q_3\}$$
$$\delta(q, c) = \emptyset \qquad \text{otherwise}$$

   The answers must be given as state diagrams.

   (a) Using the method from class, give an NFA that recognizes $N_2 \cdot N_1$

   (b) Using the method from class, give an NFA that recognizes $N_1 \cup N_2$

   (c) Using the method from class, give an NFA that recognizes $N_2^\star$