

Homework Assignment 6

Suggested textbook Theorems: 1.45, 1.47, 1.49, 2.32. You can also assume the results in Exercise 2.16 and that context-free languages are Turing recognizable.

1. Prove or disprove the following statements. Your answer should have two parts: (i) whether or not the statement holds; (ii) a formal proof that backs your claim.
 - (a) If $L_1 \subseteq L_2$ and L_2 is regular, then L_1 is regular.
 - (b) If L_1 is regular and L_2 is context-free, then $L_1 \cup L_2$ is context-free.
 - (c) If $L_1 \cdot L_2$ is not context free, then either L_1 is not context free or L_2 is not context free.
 - (d) If L is context-free, then L is regular.
 - (e) If L^* is not context free, then L is not context free.
 - (f) If L is not regular, then L is not context-free.
 - (g) For any language L_1 there exists a regular language L_2 such that $L_2 \subseteq L_1$.
 - (h) A Turing machine can write the empty character in the tape.
 - (i) In a Turing machine, the input Σ and the tape alphabet Γ must differ.
 - (j) A Turing machine contains at least 3 different states.
2. Given that $\{a^n b^n c^{n+1} \mid n \geq 0\}$ is not context-free and without using the Pumping Lemma or the Theorem of Non-Context-Free Languages from Lecture 11, show that $\{a^n b^n c^n \mid n \geq 0\}$ is not context-free.
3. Let M be the Turing Machine of Example 2 in Lecture 12. Show the configuration history that rejects string 001.
4. Consider language $L_1 = \{w\#w \mid w \in \{a, b\}^*\}$. Check all that apply.
 - ☐ regular
 - ☐ not regular
 - ☐ context free
 - ☐ not context-free
 - ☐ Turing recognizable