Genericity through Dependently Constrained Types

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Abstract

We present a general framework for *generic constrained types* that captures the notion of value-dependent and type-dependent (generic) type systems for object-oriented languages. Constraint systems formalize systems of partial information. Constrained types are formulas `C{c}` where `C` is the name of a class or an interface and `c` is a constraint on the immutable state of an instance of `C` (the *properties*).

The basic idea is to formalize the essence of nominal object-oriented types as a constraint system, and to permit both value and type properties and parameters. Type-generic dependence is now expressed through constraints on these properties and parameters. Type-valued properties are required to have a run-time representation—the run-time semantics is not defined through erasure.

Many type systems for object-oriented languages developed over the last decade can be thought of as constrained type systems in this formulation. This framework is parametrized by an arbitrary constraint system $\mathcal{C}$ of interest. It permits the development of languages with pluggable type systems, and supports dynamic code generation to check casts at run-time.

The paper makes the following contributions: (1) We show how to accommodate generic object-oriented types within the framework of constrained types. (1) We illustrate the type system with the development of a formal calculus GFX and establish type soundness. (3) We discuss the design and implementation of the type system for X10, a modern object-oriented language, based on constrained types. The type system integrates and extends the features of nominal types, virtual types, and Scala's path-dependent types, as well as representing generic types.

1. Introduction

todo: Awkward, repetitive

todo: More positioning, relative to: DML, HM(X), constrained types (Trifonov, Smith) and subtyping constraints, Java generics, GJ, PolyJ, C# generics, virtual types, liquid types

todo: Possible claim: first type system that combines genericity and dep types in some vague general way.

todo: Incorporate some text from OOPSLA paper on deptypes.

todo: Cite liquid types and whatever it cites

Modern object-oriented type systems provide many features to improve productivity by allowing programmers to express program invariants as types that are checked by the compiler, without sacrificing the ability to reuse code. We present a dependent type system that extends a class-based language with statically-enforced constraints on types and values. This type system supports several features of modern object-oriented language through natural extensions of the core dependent type system: generic types, virtual types, and self types among them.

The key idea is to define *constrained types*, a form of dependent type defined on predicates over types and over the immutable state of the program. The type system is parametrized on a constraint system. We have formalized the type system in an extension of Featherweight Java [26] and provide proof of soundness. By augmenting the default constraint system, the type system can serve as a core calculus for formalizing extensions of a core object-oriented language.

This work is done in the context of the X10 programming language [46]. In X10, objects may have both value members (fields) and type members. The immutable state of an object is captured by its *value properties*: public final fields of the object. For instance, the following class declares a two-dimensional point with properties `x` and `y` of type `float`:

```
class Point(x: float, y: float) { }
```

A constrained type is a type `C{e}`, where `C` is a class—called the *base class*—and `e` is a *constraint*, or list of constraints, on the properties of `C` and the final variables in scope at the type. For example, given the above class definition, the type `Point{x*x+y*y<1}` is the type of all points within the unit circle.

Constraints on properties induce a natural subtyping relationship: `C{c}` is a subtype of `D{d}` if `C` is a subclass of `D` and `c` entails `d`. Thus, `Point{x==1,y==1}` is a subtype of `Point{x>0}`, which in turn is a subtype of `Point{true}`—written simply as `Point`.

In previous work [46, 45], we considered only value properties. In this paper, to support genericity these types are generalized to allow *type properties*, type-valued instance members of an object. Types may be defined by constraining the type properties as well as the value properties of a class.

The following code declares a class `Cell` with a type property named `T`.

```
class Cell[T] {
    var value: T;
    def get(): T = value;
    def set(v: T) = { value = v; }
}
```

The class has a mutable field `value` of type `T`, and has `get` and `set` methods for accessing the field.

This example shows that type properties are in many ways similar to type parameters as provided in object-oriented languages such as Java [23] and Scala [38] and in functional languages such as ML [35] and Haskell [30].

As the example illustrates, type properties are types in their own right: they may be used in any context a type may be used, including in `instanceof` and cast expressions. However, the key distinction between type properties and type parameters is that type properties are instance members. Thus, for an expression `e` of type `Cell`, `e.T` is a type, equivalent to the concrete type to which `T` was initialized when the object `e` was instantiated. To ensure soundness, `e` is restricted to final access paths. Within the body of a class, the unqualified property name `T` resolves to `this.T`.

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All properties of an object, both type and value, must be bound at object instantiation and are immutable once bound. Thus, the type property T of a given Cell instance must be bound by the constructor to a concrete type such as String or Point{x>=0}.

As with value properties, type properties may be constrained by predicates to produce *constrained types*. Many features of modern object-oriented type systems fall out naturally from this type system.

Generic types. Constraints on type properties lead directly to a form of generic class. The Cell defined class above is a generic class. X10 supports equality constraints, written T_1==T_2, and subtyping constraints, written T_1<:T_2, on types. For instance, the type Cell{T==float} is the type of all Cells containing a float. For an instance c of this type, the types c.T and float are equivalent. Thus, the following code is legal.

```
val x: float = c.get();
c.set(1.0);
```

Subtyping constraints enable *use-site variance* [**?**]. The type Cell{T<:Collection} constrains T to be a subtype of Collection. All instances with this type must bind T to a subtype of Collection. Variables of this type may contain Cells of Collection, Cell of List, or Cell of Set, etc. Subtyping constraints provide similar expressive power as Java wildcards. We describe an encoding of wildcards in Section **??**.

Other type systems. While this paper focuses mainly on supporting generic types through dependent types, the formalism generalizes other object-oriented type systems, e.g., self types [8, 9], virtual types [33, 32, 17], and structually constrained types [6, 24].

1.1 Contributions

todo: We need some!

1.2 Implementation

Type properties are a powerful mechanism for providing genericity in X10. Unlike existing existing proposals for generic types in Java-like languages [23, 7, 39, 6, 50, 38], which are implemented via type erasure, our design supports run-time introspection of generic types.

Another problem with many of these proposals is inadequate support for primitive types, especially arrays. The performance of primitive arrays is critical for the high-performance applications for which X10 is intended. These proposals introduce unnecessary boxing and unboxing of primitives. Our design does not require primitives be boxed.

Outline. The rest of the paper is organized as follows. Section 6 discusses related work. An informal overview of generic constrained types in X10 is presented in Section 2. Section 3 presents a formal semantics and a proof of soundness. The implementation of generics in X10 by translation to Java is described in Section 4. Finally, Section 7 concludes.

todo: Fix this

2. X10 language overview

This section presents an informal description of generic constrained types in X10. The type system is formalized in a simplified version of X10, GFX (Generic Featherweight X10), in Section 3.

When presenting syntax, we follow the usual conventions for Featherweight Java: we write $\overline{\mathtt{t}}$ for the list $\mathtt{t}_1,\ldots,\mathtt{t}_n$; terms with list subterms are considered a single list of terms (e.g., we write $\overline{\mathtt{x}}:\overline{\mathtt{T}}$ for the list $\mathtt{x}_1:\mathtt{T}_1,\ldots,\mathtt{x}_n:\mathtt{T}_n$).

```
class List[T](length: int) {
  var head: T;
  val tail: List[T];

  def get(i: int) = {
    if (i == 0) return head;
    else return tail.get(i-1);
  }

  def this[S](hd: S, tl: List[S]): List[S](tl.length+1) = {
    property[S](tl.length+1);
    head = hd; tail = tl;
  }
}
```

Figure 1. List example, simplified

X10 is a class-based object-oriented language. The language has a sequential core similar to Java or Scala, but constructs for concurrency and distribution, as well as constrained types, described here. Like Java, the language provides single class inheritance and multiple interface inheritance.

A constrained type in X10 is written C{e}, where C is the name of a class and e is a constraint on the properties of C and the final variables in scope at the type. The constraint e may refer to the value being constrained through the special variable self, which has type C in the constraint. Constraints are drawn from a constraint language that, syntactically, is a subset of the boolean expressions of X10.

The compiler checks that constraints are expressions of type boolean and that they can be statically checked by the compiler's constraint solver. X10 supports conjunctions of equalities over final variables and compile-time constants, and equalities and subtyping constraints over types. Compiler plugins may be installed to handle richer constraint systems such as Presburger arithmetic or set constraints.

For brevity, the constraint may be omitted and interpreted as true. The syntax C[$\overline{\mathtt{T}}$]($\overline{\mathtt{e}}$) is sugar for C[$\overline{\mathtt{X}}$==$\overline{\mathtt{T}}$]($\overline{\mathtt{x}}$==$\overline{\mathtt{e}}$) where X$_i$ are the type properties and x$_i$ are the value properties of C. If either list of properties is empty, it may be omitted.

To illustrate the features of dependent types in X10, we develop a List class. We will present several versions of List as we introduce new features. A List class with a type property T and an int property length is declared as in Figure 1. Classes in X10 may be declared with any number of type properties and value properties.

Like in Scala, fields are declared using the keywords var or val. The List class has a mutable head field with type T (which resolves to this.T), and an immutable (final) tail field with type List[T], that is, with type List{self.T==this.T}. Note that this occurring in the constraint refers to the instance of the enclosing List class, and self refers to the value being constrained—this.tail in this case.

Methods are declared with the def keyword. The method get takes a final integer i argument and returns the element at that position.

Objects in X10 are initialized with constructors, which must ensure that all properties of the new object are initialized and that the class invariants of the object's class and its superclasses and superinterfaces hold. X10 uses method syntax with the name this for constructors. In X10, constructors have a "return type, which constrains the properties of the new object. The constructor in Figure 1 takes a type argument S and two value arguments hd and tl. The constructor return type specifies that the constructor

```
class List[T](length: int){length >= 0} {
  var head{length>0}: T;
  val tail{length>1}: List[T](length-1);

  def get(i: int{0 <= i, i < length}){length > 0} = {
    if (i == 0) return head;
    return tail.get(i-1);
  }

  def map[S](f: (T)=>S): List[S] = {
    if (length==0)
      return new List[S](0);
    else if (length==1)
      return new List[S](f(head));
    else
      return new List[S](f(head), tail.map[S](f));
  }

  def this[S](): List[S](0) = property[S](0);
  def this[S](hd: S): List[S](1) = {
    property[S](1); head = hd;
  }
  def this[S](hd: S, tl: List[S]) : List[S](tl.length+1) = {
    property[S](tl.length+1);
    head = hd; tail = tl;
  }
}
```

Figure 2. List example, with more constraints

initializes the object to have type `List[S](tl.length+1)`, that is, `List{self.T==S, self.length==tl.length+1}`. The formal parameter types and return types of both methods and constructors may refer to final parameters of the same declaration.

The body of the constructor begins with a `property` statement that initializes the properties of the new instance. All properties are initialized simultaneously and it is required that the property assignment entail the constructor return type. The remainder of the constructor assigns the fields of the instance with the constructor arguments.

We next present a version of `List` where we write invariants to be enforced statically. Consider the new version in Figure 2.

2.1 Class invariants

Properties of a class may be constrained with a *class invariant*. The `List` declaration's class invariant specifies that the length of the list be non-negative.

2.2 Field, method, and constructor constraints

Field, method, and constructor declarations may have additional constraints that must be satisfied for access.

The field declarations in Figure 2 each have a *field constraint*. The field constraint on `head` requires that the `this.length>0` to access `head`; that is `this.head` may not be dereferenced unless `this` has type `List{length>0}`. Similarly, `tail` cannot be accessed unless the tail is non-empty. The compiler is free to generate optimized representations of instances of `List` with a given length: it may remove the the `head` and `tail` fields for empty lists, for instance. Similarly, the compiler may specialize instances of `List` with a given concrete type for `T`. This specialization is described in Section **??**.

The method `get` in Figure 2 has a constraint on the type of `i` that requires that it be within the list bounds. The method also has a *method constraint*. A method with a method constraint is called a *conditional method*. The method constraint on `get` requires that the actual receiver's `length` field must be non-zero—calls to `get` on empty lists are not permitted. This constraint ensures that the field constraint on `tail` is satisfied in the method body. In the method body, the `head` of the list is returned for position `0`; otherwise the call recurses on the `tail`. Note that the type-checker must be flow sensitive to be able go guarantee that `i` is within the loop bounds of `tail`. The constraint solver must prove that `tail.length > 0` and `0 <= i-1` and `i-1 < tail.length` from the facts: `this.length > 0`, `tail.length == this.length-1`, `0 <= i`, `i < length`

Method overriding is similar to Java: a method of a subclass with the same name and parameter types overrides a method of the superclass. An overridden method may have a return type that is a subtype of the superclass method's return type. A method constraint may be weakened by an overriding method; that is, the method constraint in the superclass must entail the method constraint in the subclass.

Methods may also have both type parameters. For instance, the `map` method in Figure 2 has a type parameter `S` and a value parameter that is a function from `T` to `S`. A parametrized method can is invoked by giving type arguments before the expression arguments, as is done by the recursive call to `map`. Actual type arguments can be inferred from the types of the value arguments—we do not describe this here.

`List` also defines three constructors: the first constructor takes no value arguments and initializes the length to `0`. Note that `head` and `tail` are not assigned since they are inaccessible. The second constructor takes an argument for the head of the list; the third takes both a head and tail.

2.3 Type constraints and variance

Type properties and subtyping constraints may be used in X10 to provide use-site variance constraints [28].

Consider the following subtypes of the `List` from Figure 2.

- `List`. This type has no constraints on the type property `T`. Any type that constrains `T`, is a subtype of `List`. The type `List` is equivalent to `List{true}`. For a `List l`, the return type of the `get` method is `l.T`. Since the property `T` is unconstrained, the caller can only assign the return value of `get` to a variable of type `l.T` or of type `Object`. In the following code, `y` cannot be passed to the `set` method because it is not known if `Object` is a subtype of `c.T`.

  ```
  val x: l.T = l.get(0);
  val y: Object = l.get(1);
  l.set(x); // legal
  l.set(y); // illegal
  ```

- `List{T==float}`. The type property `T` is bound to `float`. Assuming `l` has this type, then following code is legal:

  ```
  val x: float = l.get();
  l.set(1.0);
  ```

  The type of `l.get()` is `l.T`, which is equivalent to `float`.

- `List{T<:Collection}`. This type constrains `T` to be a subtype of `Collection`. All instances of this type must bind `T` to a subtype of `Collection`; for example `List[Set]` (i.e., `List{T==Set}` is a subtype of `List{T<:Collection}` because `T==Set` entails `T<:Collection`. If `l` has the type `List{T<:Collection}`, then `l.get(0)` has type `l.T`, which is an unknown but fixed subtype of `Collection`; the return value can be assigned into a variable of type `Collection`.

- `List{T:>String}`. This type bounds the type property `T` from below. The `set` method may be called with any supertype of

```
class SortedList(compare: (T,T)=>int) extends List {
    def this[T](hd: T, tl: List[T],
                compare: (T,T)=>int)
        : SortedList[T](compare) = {
        super[T](hd, tl);
        property(compare);
    }

    def this[T](hd: T, tl: List[T]){T <: Comparable}
        : SortedList[T](T.compare.(Object)) = {
        this[T](hd, tl, T.compareTo.(Object));
    }

    def add(x: T) = {
        ... compare(x, y) ...
    }
}
```

Figure 3. A `SortedList` class with function-typed value properties

`String`; the return type of the `get` method is known to be a supertype of `String` (and implicitly a subtype of `Object`).

In the shortened syntax for types (e.g., `List[T](n)`), an actual type argument `T` may optionally be annotated with a *use-site variance tag*, either + or -: if `X` is a type property, then the syntax `C[+T]` is sugar `C{X<:T}` and `C[-T]` is sugar `C{X:>T}`; of course, `C[T]` is sugar `C{X==T}`.

2.4 Function-typed properties

X10 supports first-class functions. Function-typed properties are a useful feature for generic collection classes. Consider the definition of the `SortedList` class in Figure 3. The class has a property `compare` of type `(T,T)=>int`—a function that takes two `T`s and returns an `int`. The class declares two constructors, one that takes a function to bind to the `compare` property, and another that binds `T`'s `compare` method to the property. The `compare` method uses the `equals` function to compare elements.

Using this definition, one can create lists with distinct types of, for example, case-sensitive and case-insensitive strings:

```
val unixFiles
  = new SortedList[String]
        (String.compareTo.(String));
val windowsFiles
  = new SortedList[String]
        (String.compareToIgnoreCase.(String));
```

The lists `unixFiles` and `windowsFiles` are constrained by different comparison functions. This allows the programmer to write code, for instance, in which it is illegal to pass a list of UNIX files into a function that expects a list of Windows files, and vice versa.

3. Formal semantics

We present a core calculus, GFX, for X10 with generics. GFX is based on Constrained Featherweight Java [45].

todo: Add method overriding rules: covariant return, contravariant args, weaker constraints

The grammar for GFX is shown in Figure 4. The calculus elides features of the full X10 language not relevant to this paper.

Figure 5 extends the grammar with syntactic sugar for subtyping constraints and existential types. The subtyping constraint $\mathtt{t_1} <: \mathtt{t_2}$ is atomic formula. The existential type $\exists \mathtt{x}:\mathtt{T}.\ \mathtt{R\{c\}}$ is sugar for $\mathtt{R}\{\exists \mathtt{x}.\ \sigma(\mathtt{x}:\mathtt{T}), \mathtt{c}\}$.

program	P	::=	$\overline{\mathtt{L}}$
classes	L	::=	$\mathtt{class\ C}[\overline{\mathtt{X}}](\overline{\mathtt{x}}:\overline{\mathtt{T}})\{\mathtt{c}\}$ $\mathtt{extends\ T\ \{\ }\overline{\mathtt{M}}\ \mathtt{\}}$
base types	R		
classes		::=	C
type variables		|	X
type members		|	e.X
type type		|	type
types	T	::=	R{c}
methods	M	::=	$\mathtt{def\ m}[\overline{\mathtt{X}}](\overline{\mathtt{x}}:\ \overline{\mathtt{T}})\{\mathtt{c}\}\mathtt{:\ T = e}$
expressions	e		
literals		::=	true | false | null | n
variables		|	x
field access		|	e.x
call		|	$\mathtt{e}_0\mathtt{.m}[\overline{\mathtt{T}}](\overline{\mathtt{e}})$
new		|	$\mathtt{new\ C}[\overline{\mathtt{T}}](\overline{\mathtt{e}})$
cast		|	e as T
constraint terms	t		
self		::=	self
variables		|	x
properties		|	t.x
atoms		|	$\mathtt{g}(\mathtt{t}_1,\ldots,\mathtt{t}_n)$
new		|	$\mathtt{new\ C}(\mathtt{t}_1,\ldots,\mathtt{t}_n)$
constraint	c		
true		::=	true
equality		|	$\mathtt{t}_1 == \mathtt{t}_2$
existentials		|	$\exists \mathtt{x}.\ \mathtt{c}$
conjunction		|	$\overline{\mathtt{c}}$
predicates		|	$\mathtt{p}(\mathtt{t}_1,\ldots,\mathtt{t}_n)$
environments	Γ	::=	ε
		|	$\Gamma, \mathtt{c}$
		|	$\Gamma, \mathtt{x}:\mathtt{T}$
		|	$\Gamma, \mathtt{X}:\mathtt{type}$

Figure 4. GFX grammar

types	T	::=	...
		|	$\exists \mathtt{x}:\mathtt{T}_0.\ \mathtt{T}$
		|	$\exists \mathtt{X}:\mathtt{type}.\ \mathtt{T}$
constraint terms	t	::=	...
literals			true | n | C
type variables		|	X
type properties		|	t.X
constraint	c	::=	...
		|	$\mathtt{t}_1 <: \mathtt{t}_2$
		|	cons(T,z)

Figure 5. GFX grammar with subtyping constraints

We assume a fixed but unknown constraint system $\mathcal{D}$. A program P is written using constraints from $\mathcal{D}$, We assume classes defined in P do not have a cyclic inheritance structure.

$$\frac{\mathtt{extends}^+ \text{ acyclic}}{\vdash \overline{\mathtt{L}}\ \mathrm{ok}} \qquad \text{(PROGRAM OK)}$$

3.1 The object constraint system, O

From P and $\mathcal{D}$ we generate an *object constraint system* O, shown in Figure 6, as follows. Let C and D range over names of classes in P, f over field names, m over method names, T over types, and c over constraints in the underlying data constraint system $\mathcal{D}$.

$$\begin{array}{rcl} \text{constraint} \quad \mathtt{c} & ::= & \mathtt{class(C)} \\ & | & \mathtt{C\ extends\ D} \\ & | & \mathtt{fields(x,}\overline{\mathtt{f}}\mathtt{:}\overline{\mathtt{T}}\mathtt{)} \\ & | & \mathtt{mtype(x,m,[}\overline{\mathtt{X}}\mathtt{(}\overline{\mathtt{x}}\mathtt{:}\overline{\mathtt{T}}\mathtt{)\{c\}} \end{array}$$

$$\frac{\mathtt{class\ C[}\overline{\mathtt{X}}\mathtt{](}\overline{\mathtt{f}}\mathtt{:}\overline{\mathtt{T}}\mathtt{)\{c\}\ extends\ D\{d\}\ \{}\overline{\mathtt{M}}\mathtt{\}} \in \mathtt{P}}{\begin{array}{c}\vdash_{\mathcal{O}} \mathtt{class(C)} \\ \vdash_{\mathcal{O}} \mathtt{C\ extends\ C} \\ \vdash_{\mathcal{O}} \mathtt{C\ extends\ D}\end{array}}$$

$$\frac{\mathtt{class\ C[}\overline{\mathtt{X}}\mathtt{](}\overline{\mathtt{f}}\mathtt{:}\overline{\mathtt{T}}\mathtt{)\{c\}\ extends\ Object\ \{}\overline{\mathtt{M}}\mathtt{\}} \in \mathtt{P}}{\Gamma, \mathtt{z:C}\{d\} \vdash_{\mathcal{O}} \mathtt{fields(z,}\overline{\mathtt{f}}\mathtt{:}\overline{\mathtt{T}}\mathtt{)}} \text{ (FIELDS)}$$

$$\frac{\begin{array}{c}\vdash_{\mathcal{O}} \mathtt{C\ extends\ D} \\ \mathtt{class\ C[}\overline{\mathtt{X}}\mathtt{](}\overline{\mathtt{f}}\mathtt{:}\overline{\mathtt{T}}\mathtt{)\{c\}\ extends\ D\{d\}\ \{}\overline{\mathtt{M}}\mathtt{\}} \in \mathtt{P} \\ \Gamma, \mathtt{z:D\{d\}} \vdash_{\mathcal{O}} \mathtt{fields(z,}\overline{\mathtt{f}}_0\mathtt{:}\overline{\mathtt{T}}_0\mathtt{)}\end{array}}{\Gamma, \mathtt{z:D\{d\}} \vdash_{\mathcal{O}} \mathtt{fields(z,}\overline{\mathtt{f}}_0\mathtt{:}\overline{\mathtt{T}}_0\mathtt{,}\overline{\mathtt{f}}\mathtt{:}\overline{\mathtt{T}}\mathtt{)}}$$
(FIELDS-EXTENDS)

$$\frac{\begin{array}{c}\mathtt{class\ C[}\overline{\mathtt{X}}\mathtt{](}\overline{\mathtt{f}}\mathtt{:}\overline{\mathtt{T}}\mathtt{)\{c\}\ extends\ Object\ \{}\overline{\mathtt{M}}\mathtt{\}} \in \mathtt{P} \\ \mathtt{M}_i = \mathtt{def\ m}_i\mathtt{[}\overline{\mathtt{X}}\mathtt{](}\overline{\mathtt{x}}\mathtt{:}\overline{\mathtt{T}}\mathtt{)\{c\}:\ T\ =\ e}\end{array}}{\Gamma, \mathtt{z:C\{d\}} \vdash_{\mathcal{O}} \mathtt{mtype(z,m}_i\mathtt{,[}\overline{\mathtt{X}}\mathtt{](}\overline{\mathtt{x}}\mathtt{:}\overline{\mathtt{T}}\mathtt{)\{c\}}\rightarrow\mathtt{T)}} \text{ (MTYPE)}$$

Figure 6. The constraint system $\mathcal{O}$

In the method signature $\mathtt{[}\overline{\mathtt{X}}\mathtt{(}\overline{\mathtt{x}}\mathtt{:}\overline{\mathtt{T}}\mathtt{)\{c\}}$, the type variables $\overline{\mathtt{X}}$ and data variables $\overline{\mathtt{x}}$ are considered bound; formulas with bound variables are considered equivalent up to α-renaming.

The constraint system satisfies the axioms and inference rules in Figure 6. The `class`, `extends`, `fields`, and `mtype` constraints are given directly from the program `P`.

The constraint system $\mathcal{C}$ is the disjoint conjunction $\mathcal{D}, \mathcal{O}$ of the constraint systems $\mathcal{D}$ and $\mathcal{O}$. (This requires the assumption that $\mathcal{D}$ does not have any constraints in common with $\mathcal{O}$.

3.2 Structural and logical rules

All judgments are intuitionistic. In particular, this means that all constraint systems satisfy the rules and axioms in Figure 7.

3.3 Well-formedness rules

We use the judgment for well-typedness for expressions to represent well-typedness for constraints. That is, we posit a special type `o` (traditionally the type of propositions), and regard constraints as expressions of type `o`.

Further, we change the formulation slightly so that there are no constraints of the form $\mathtt{p(t}_1\mathtt{,\ldots,t}_n\mathtt{)}$; rather instance method invocation syntax is used to express invocation of pre-defined constraints. This logically leads to the step of simply marking certain classes as "predicate" classes—all the (instance) methods of these classes whose return type is `o` then correspond to "primitive constraints". Syntactically, we continue to use the symmetric syntax $\mathtt{p(t}_1\mathtt{,\ldots,t}_n\mathtt{)}$ rather than $\mathtt{t}_1\mathtt{.p(t}_2\mathtt{,\ldots,t}_n\mathtt{)}$. The alternative is tor introduce static methods and static method invocations in the expression language. This is not difficult, but is annoying to have to repeat most of the formulation of instance methods.

This means that the only cases left to handle are all the simple ones, expression the availability of certain constraints and operations of type `o`.

$$\frac{}{\Gamma, \mathtt{c} \vdash \mathtt{c}} \text{ (ID)}$$

$$\frac{\Gamma \vdash \mathtt{c} \qquad \Gamma, \mathtt{c} \vdash \mathtt{d}}{\Gamma \vdash \mathtt{d}} \text{ (CUT)}$$

$$\frac{\Gamma \vdash \phi \qquad \Gamma \vdash \mathtt{T:type} \qquad \mathtt{x} \notin \mathit{var}(\Gamma)}{\Gamma, \mathtt{x:T} \vdash \phi} \text{ (WEAK-1)}$$

$$\frac{\Gamma \vdash \phi \qquad \Gamma \vdash \mathtt{c:o}}{\Gamma, \mathtt{c}:\phi} \text{ (WEAK-2)}$$

$$\frac{\Gamma, \psi_0, \psi_1 \vdash \phi}{\Gamma, (\psi_0, \psi_1) \vdash \phi} \text{ (AND-L)}$$

$$\frac{\Gamma \vdash \psi_0 \qquad \Gamma \vdash \psi_1}{\Gamma \vdash (\psi_0, \psi_1)} \text{ (AND-R)}$$

$$\frac{\Gamma \vdash \phi[\mathtt{t}/\mathtt{x}]}{\Gamma \vdash_{\mathcal{C}} \exists \mathtt{x}.\ \phi} \text{ (EXISTS-R)}$$

$$\frac{\Gamma, \mathtt{x:T}, \psi \vdash \phi \qquad \mathtt{x}\ \text{fresh}}{\Gamma, \exists \mathtt{x:T}.\ \psi \vdash \phi} \text{ (EXISTS-L)}$$

Figure 7. Logical rules

$$\frac{}{\Gamma \vdash \mathtt{true:o}} \text{ (TRUE)}$$

$$\frac{\Gamma \vdash \mathtt{t}_0\mathtt{:T}_0 \qquad \Gamma \vdash \mathtt{t}_1\mathtt{:T}_1 \qquad (\Gamma \vdash \mathtt{T}_0 \mathtt{<:} \mathtt{T}_1 \vee \Gamma \vdash \mathtt{T}_1 \mathtt{<:} \mathtt{T}_2)}{\Gamma \vdash \mathtt{t}_0 \mathtt{==} \mathtt{t}_1\mathtt{:o}} \text{ (EQUALS)}$$

$$\frac{\Gamma \vdash \mathtt{c}_0\mathtt{:o} \qquad \Gamma \vdash \mathtt{c}_1\mathtt{:o}}{\Gamma \vdash (\mathtt{c}_0, \mathtt{c}_1)\mathtt{:o}} \text{ (AND)}$$

$$\frac{\Gamma \vdash \mathtt{t:T} \qquad \Gamma \vdash \mathtt{c}[\mathtt{t}/\mathtt{x}]\mathtt{:o}}{\Gamma \vdash \exists \mathtt{x:T}.\ \mathtt{c:o}} \text{ (SOME)}$$

$$\frac{\Gamma \vdash \mathtt{class(C)} \qquad \Gamma, \mathtt{self:C} \vdash \mathtt{c:o}}{\Gamma \vdash \mathtt{C\{c\}:type}} \text{ (TYPE)}$$

Figure 8. Well-formedness rules

3.4 Subtyping constraints

3.5 Constraint projection

First, for a type environment Γ, we define the *constraint projection*, $\sigma(\Gamma)$ thus:

$$\begin{aligned} \sigma(\varepsilon) &= \mathtt{true} \\ \sigma(\Gamma, \mathtt{x:T}) &= \sigma(\Gamma), \mathit{cons}(\mathtt{T}, \mathtt{x}) \\ \sigma(\Gamma, \mathtt{c}) &= \sigma(\Gamma), \mathtt{c} \end{aligned}$$

The auxiliary function *cons* specifies the constraint for a type `T` with `self` bounds to `x`. The constraint projection uses an atomic

$$\frac{\overline{c} \vdash_{\mathcal{C}} T_1 == T_2}{\overline{c} \vdash_{\mathcal{C}} \texttt{cons}(T_1, z) == \texttt{cons}(T_2, z)} \quad \text{(CONS-EQ)}$$

$$\frac{\overline{c} \vdash_{\mathcal{C}} T_1 <: T_2}{\overline{c}, \texttt{cons}(T_1, z) \vdash_{\mathcal{C}} \texttt{cons}(T_2, z)} \quad \text{(CONS-SUB)}$$

$$\vdash_{\mathcal{C}} \texttt{cons}(C, z) \quad \text{(CONS)}$$

$$\frac{\overline{c} \vdash_{\mathcal{C}} \overline{s} == \overline{t}}{\overline{c} \vdash_{\mathcal{C}} f(\overline{s}) == f(\overline{t})} \quad \text{(EQ-ATOM)}$$

$$\overline{c} \vdash_{\mathcal{C}} t == t \quad \text{(EQ-REFL)}$$

$$\frac{\overline{c} \vdash_{\mathcal{C}} t_1 == t_2 \qquad \overline{c} \vdash_{\mathcal{C}} t_2 == t_3}{\overline{c} \vdash_{\mathcal{C}} t_1 == t_3} \quad \text{(EQ-TRANS)}$$

$$\frac{\overline{c} \vdash_{\mathcal{C}} t_1 == t_2}{\overline{c} \vdash_{\mathcal{C}} t_2 == t_1} \quad \text{(EQ-SYM)}$$

$$\frac{\overline{c} \vdash_{\mathcal{C}} T_1 <: T_2 \quad \overline{c} \vdash_{\mathcal{C}} T_2 <: T_1}{\overline{c} \vdash_{\mathcal{C}} T_1 == T_2} \quad \text{(EQ-SUB)}$$

$$\frac{\overline{c} \vdash_{\mathcal{C}} C\{c\} : \texttt{type} \qquad \overline{c}, c \vdash_{\mathcal{C}} d}{\overline{c} \vdash_{\mathcal{C}} C\{c\} <: C\{d\}} \quad \text{(SUB-CONS)}$$

$$\frac{C[\overline{X}](\overline{x}:\overline{T})\{c\}\ \texttt{ext}\ T\ \{\ K\ \overline{M}\ \overline{F}\ \}}{\vdash_{\mathcal{C}} C <: T} \quad \text{(SUB-SUPER)}$$

$$\vdash_{\mathcal{C}} T <: \texttt{Object} \quad \text{(SUB-OBJECT)}$$

$$\frac{\overline{c} \vdash_{\mathcal{C}} T_1 == T_2}{\overline{c} \vdash_{\mathcal{C}} T_1 <: T_2} \quad \text{(SUB-EQ)}$$

$$\frac{\overline{c} \vdash_{\mathcal{C}} T_1 <: T_2 \qquad \overline{c} \vdash_{\mathcal{C}} T_2 <: T_3}{\overline{c} \vdash_{\mathcal{C}} T_1 <: T_3} \quad \text{(SUB-TRANS)}$$

Figure 9. Equality and subtyping rules

formula `cons`, which is equated to the constraint of `T` if `T` is not a type variable.

$$\begin{aligned} \mathit{cons}(C, z) &= \texttt{cons}(C, z) \\ \mathit{cons}(C\{c\}, z) &= c[z/\texttt{self}], \texttt{cons}(C\{c\}, z) == c[z/\texttt{self}] \\ \mathit{cons}(p.X, z) &= \texttt{cons}(p.X, z) \\ \mathit{cons}(X, z) &= \texttt{cons}(X, z) \end{aligned}$$

Thus, for example, the constraint projection of the environment:

```
b: D, a: C{self.X==D{d},self.Y<:b.Z}
```

is:

```
a.X==D{d}, a.Y<:b.Z
```

$$\frac{C[\overline{X}](\overline{x}:\overline{T})\{c\}\ \texttt{ext}\ T\ \{\ \overline{M}\ \overline{F}\ \}}{\vdash C : \texttt{type}}$$

$$\frac{\Gamma \vdash T : \texttt{type} \qquad \Gamma, \texttt{self}:T \vdash c : \texttt{Boolean} \qquad \sigma(\Gamma) \vdash_{\mathcal{C}} c\ \text{OK}}{\Gamma \vdash T\{c\} : \texttt{type}}$$

$$\frac{\Gamma \vdash p : T \qquad \Gamma \vdash T\ \texttt{has}\ X}{\Gamma \vdash p.X : \texttt{type}}$$

$$\Gamma, X : \texttt{type} \vdash X : \texttt{type}$$

Figure 10. Type well-formedness

$$\frac{C[\overline{X}](\overline{x}:\overline{T})\{c\}\ \texttt{ext}\ T\ \{\ K\ \overline{M}\ \overline{F}\ \}}{\begin{array}{c}\vdash C\ \texttt{has}\ K \\ \vdash C\ \texttt{has}\ X_i \\ \vdash C\ \texttt{has}\ x_i : T_i \\ \vdash C\ \texttt{has}\ M_i \\ \vdash C\ \texttt{has}\ F_i\end{array}} \quad \text{(HAS-CLASS)}$$

$$\frac{Z \neq K \qquad \Gamma \vdash T_1\ \texttt{has}\ Z \qquad \sigma(\Gamma) \vdash T_2 <: T_1}{\Gamma \vdash T_2\ \texttt{has}\ Z} \quad \text{(HAS-SUB)}$$

Figure 11. Structural constraints

3.6 Type well-formedness

3.7 Type inference rules

3.7.1 Constraint rules

3.7.2 Expression typing judgment

The cast rule T-CAST requires that the cast type be well-formed.

The field access rule T-FIELD differs from the rule in the paper in that there is no need to substitute a fresh variable for the receiver. Note that `this` may be free in S—that would be a reference to the current object in the code in which `e.f` occurs, not a reference to the receiver of the `e.f` field selection (i.e., the object obtained by evaluating `e`).

if we allow adding constraints to arbitrary types—do we?

TODO: type parameters!

Now we consider the rule for method invocation. Assume that in a type environment Γ the expressions $e_0, \ldots, e_n$ have the types $T_0, \ldots, T_n$. Since the actual values of these expressions are not known, we shall assume that they take on some fixed but unknown values $z_0, \ldots, z_n$ of types $T_0, \ldots, T_n$. Now, for z_0 as receiver, let us assume that the type T_0 has a method named `m` with signature $[\overline{Z}](\overline{z}:\overline{S})\{c\} \to U$ (Let $T_0 = C\{d\}$. If there is no method named `m` for the class `C` then this method invocation cannot be type-checked. Without loss of generality, we may assume that the type parameters of this method are named $Z_1, \ldots, Z_k$, and the value parameters are named $z_1, \ldots, z_n$ since we are free to choose variable names as we wish.) Now, for the method to be invokable, it must be the case that the types $T_1, \ldots, T_n$ are subtypes of $S_1, \ldots, S_n$. (Note that there may be no occurrences of `this` in $S_1, \ldots, S_n$—they have been replaced by z_0.) Further, it must be the case that for these parameter values, the constraint `c` is entailed. Given all these assumptions it must be the case that the return type is `U`, with all the parameters $z_0, \ldots, z_n$ existentially quantified.

$$\frac{\Gamma \vdash \mathtt{e}:\mathtt{S} \qquad \sigma(\Gamma) \vdash_{\mathcal{C}} \mathtt{S} <: \mathtt{T} \qquad \Gamma \vdash \mathtt{T}:\mathtt{type}}{\Gamma \vdash \mathtt{e}:\mathtt{T}} \quad \text{(T-SUB)}$$

$$\frac{}{\vdash \mathtt{true}:\mathtt{Boolean\{self==true\}} \quad \vdash \mathtt{false}:\mathtt{Boolean\{self==false\}}} \quad \text{(T-BOOL)}$$

$$\frac{}{\vdash n:\mathtt{Int\{self==}n\mathtt{\}}} \quad \text{(T-INT)}$$

$$\frac{\Gamma \vdash \mathtt{e}_1:\mathtt{T}_1 \qquad \Gamma \vdash \mathtt{e}_2:\mathtt{T}_2}{\Gamma \vdash \mathtt{e}_1 == \mathtt{e}_2 : \exists \mathtt{z}_1:\mathtt{T}_1, \mathtt{z}_2:\mathtt{T}_2.\ \mathtt{Boolean\{self==(z_1==z_2)\}}} \quad \text{(T-EQ)}$$

$$\frac{\Gamma \vdash \mathtt{T}_1:\mathtt{type} \qquad \Gamma \vdash \mathtt{T}_2:\mathtt{type}}{\Gamma \vdash \mathtt{T}_1 == \mathtt{T}_2 : \mathtt{Boolean}} \quad \text{(T-TEQ)}$$

$$\frac{\Gamma \vdash \mathtt{T}_1:\mathtt{type} \qquad \Gamma \vdash \mathtt{T}_2:\mathtt{type}}{\Gamma \vdash \mathtt{T}_1 <: \mathtt{T}_2 : \mathtt{Boolean}} \quad \text{(T-TSUB)}$$

$$\frac{}{\Gamma, \mathtt{x}:\mathtt{T} \vdash \mathtt{x}:\mathtt{T}} \quad \text{(T-VAR)}$$

$$\frac{\Gamma \vdash \mathtt{e}:\mathtt{S} \qquad \Gamma \vdash \mathtt{T}:\mathtt{type}}{\Gamma \vdash \mathtt{e\ as\ T}:\mathtt{T}} \quad \text{(T-CAST)}$$

$$\frac{\begin{array}{c}\Gamma \vdash \mathtt{e}:\mathtt{T} \\ \mathtt{T\ has\ f\{c\}:U} \\ \sigma(\Gamma, \mathtt{this}:\mathtt{T}) \vdash_{\mathcal{C}} \mathtt{c}\end{array}}{\Gamma \vdash \mathtt{e.f} : \exists \mathtt{this}:\mathtt{T}.\ \mathtt{U\{self==this.f\}}} \quad \text{(T-FIELD)}$$

$$\frac{\begin{array}{c}\Gamma \vdash \mathtt{e}_0:\mathtt{T}_0 \qquad \Gamma \vdash \overline{\mathtt{e}}:\overline{\mathtt{T}} \\ \mathtt{T}_0\ \mathtt{has\ def\ m[}\overline{\mathtt{X}}\mathtt{](}\overline{\mathtt{x}}:\overline{\mathtt{S}}\mathtt{)\{c\}:U = e} \\ \Gamma' = \Gamma, \overline{\mathtt{X}}:\mathtt{type}, \mathtt{this}:\mathtt{T}_0, \overline{\mathtt{x}}:\overline{\mathtt{T}}, \overline{\mathtt{X}}==\overline{\mathtt{V}} \\ \sigma(\Gamma') \vdash_{\mathcal{C}} \mathtt{c} \\ \sigma(\Gamma') \vdash_{\mathcal{C}} \overline{\mathtt{T}} <: \overline{\mathtt{S}}\end{array}}{\Gamma \vdash \mathtt{e}_0\mathtt{.m[}\overline{\mathtt{V}}\mathtt{](}\overline{\mathtt{e}}\mathtt{)} : \exists \overline{\mathtt{X}}:\mathtt{type}, \mathtt{this}:\mathtt{T}_0, \overline{\mathtt{x}}:\overline{\mathtt{T}}.\ \mathtt{U}} \quad \text{(T-INVK)}$$

$$\frac{\begin{array}{c}\Gamma \vdash \overline{\mathtt{e}}:\overline{\mathtt{T}} \\ \mathtt{C\ has\ def\ this[}\overline{\mathtt{X}}\mathtt{](}\overline{\mathtt{x}}:\overline{\mathtt{S}}\mathtt{)\{c\}:U = \ldots} \\ \Gamma' = \Gamma, \overline{\mathtt{X}}:\mathtt{type}, \mathtt{this}:\mathtt{C}, \overline{\mathtt{x}}:\overline{\mathtt{T}}, \overline{\mathtt{V}}==\overline{\mathtt{X}} \\ \Gamma'' = \Gamma, \overline{\mathtt{X}}:\mathtt{type}, \mathtt{this}:\mathtt{U}, \overline{\mathtt{x}}:\overline{\mathtt{T}}, \overline{\mathtt{V}}==\overline{\mathtt{X}} \\ \sigma(\Gamma') \vdash_{\mathcal{C}} \mathtt{c} \\ \sigma(\Gamma') \vdash_{\mathcal{C}} \overline{\mathtt{T}} <: \overline{\mathtt{S}} \\ \sigma(\Gamma'') \vdash_{\mathcal{C}} \mathit{inv}(\mathtt{C}),\end{array}}{\Gamma \vdash \mathtt{new\ C[}\overline{\mathtt{V}}\mathtt{](}\overline{\mathtt{e}}\mathtt{)} : \exists \overline{\mathtt{X}}:\mathtt{type}, \mathtt{this}:\mathtt{C}, \overline{\mathtt{x}}:\overline{\mathtt{T}}.\ \mathtt{U}} \quad \text{(T-NEW)}$$

Figure 12. Typing rules

3.7.3 Class OK judgment

The following rule is modified from what we had in the paper to ensure that all the types are well-formed (under the assumption $\mathtt{this}:\mathtt{C}$). Note that the variables $\overline{\mathtt{x}}$ are permitted to occur in the types $\mathtt{T}_0, \overline{\mathtt{T}}$, hence their typing assertions must be added to Γ.

$$\frac{\begin{array}{c}\Gamma = \mathtt{this}:\mathtt{C\{self==this},\mathit{inv}(\mathtt{C})\},\overline{\mathtt{x}}:\overline{\mathtt{T}}\mathtt{\{self==\}}\overline{\mathtt{x}}\mathtt{\},c} \\ \Gamma \vdash \mathtt{e}:\mathtt{U} \\ \sigma(\Gamma) \vdash_{\mathcal{C}} \mathtt{U} <: \mathtt{T}\end{array}}{\mathtt{def\ m[}\overline{\mathtt{X}}\mathtt{](}\overline{\mathtt{x}}:\overline{\mathtt{T}}\mathtt{)\{c\}:T = e\ OK\ in\ C}} \quad \text{(METHOD OK)}$$

This rule did not exist in our submission. This is necessary to ensure that the types of fields are well-formed.

$$\frac{\mathtt{this}:\mathtt{C}, \mathtt{c} \vdash \mathtt{T}:\mathtt{type}}{\mathtt{val\ f\{c\}:T\ OK\ in\ C}} \quad \text{(FIELD OK)}$$

This rule is now modified to ensure that all the types and methods in the body of the class are well-formed.

$$\frac{\begin{array}{c}K\ \text{OK in C} \\ \overline{\mathtt{M}}\ \text{OK in C} \\ \overline{\mathtt{F}}\ \text{OK in C} \\ \mathtt{this}:\mathtt{C} \vdash \mathtt{T}:\mathtt{type}\end{array}}{\mathtt{C[}\overline{\mathtt{X}}\mathtt{](}\overline{\mathtt{x}}:\overline{\mathtt{T}}\mathtt{)\{c\}\ ext\ T\ \{\ K\ \overline{M}\ \overline{F}\ \}\ OK}} \quad \text{(CLASS OK)}$$

TODO: method overriding

3.7.4 Subtype judgment

$$\frac{\sigma(\Gamma) \vdash_{\mathcal{C}} \mathtt{T}_1 <: \mathtt{T}_2}{\Gamma \vdash \mathtt{T}_1 <: \mathtt{T}_2}$$

4. Translation

This section describes an implementation approach for generic constrained types on a Java virtual machine. We describe the implementation as a translation to Java.

The design is a hybrid design based on the implementation of parametrized classes in NextGen [2, 3] and the implementation of PolyJ [6]. Generic classes are translated into template classes that are instantiated on demand at run time by binding the type properties to concrete types. To implement run-time type checking (e.g., casts), type properties are represented at run time using *adapter objects*.

This design, extended to handle language features not described in this paper, has been implemented in the X10 compiler. The X10 compiler is built on the Polyglot framework and translates X10 source to Java source[1]

4.1 Classes

Each class is translated into a *template class*. The template class is compiled by a Java compiler (e.g., javac) to produce a class file. At run time, when a constrained type `C{c}` is first referenced, a class loader loads the template class for `C` and then transforms the template class bytecode, specializing it to the constraint `c`.

For example, consider the following classes.

```
class A[T] {
    var a: T;
}
```

[1] There is also a translation from X10 to C++ source, not described here.

```
class C {
    val x: A[Int] = new A[Int]();
    val y: Int = x.a;
}
```

The compiler generates the following code:

```
class A {
    // Dummy class needed to type-check uses of T.
    @TypeProperty(1) static class T { }

    T a;

    // Dummy getter and setter; will be eliminated
    // at run time and replaced with actual gets
    // and sets of the field a.
    @Getter("a") <S> S get$a() { return null; }
    @Setter("a") <S> S set$a(S v) { return null; }
}

class C {
    @ActualType("A$Int")
    final A x = Runtime.<A>alloc("A$Int");
    final int y = x.<Integer>get$a();
}
```

The member class `A.T` is used in place of the type property `T`. The `Runtime.alloc` method is used used in place of a constructor call. This code is compiled to Java bytecode.

Then, at run time, suppose the expression `new C()` is evaluated. This causes `C` to be loaded. The class loader transforms the bytecode as if it had been written as follows:

```
class C {
    final A$Int x = new A$Int();
    final int y = x.a;
}
```

The `ActualType` annotation is used to change the type of the field `x` from `A` to `A$Int`. The call to `Runtime.alloc` is replaced with a constructor call. The call to `x.get$a()` is replaced with a field access.

The implementation cannot generate this code directly because the class `A$Int` does not yet exist; the Java source compiler would fail to compile `C`.

Next, as the `C` object is being constructed, the expression `new A$Int()` is evaluated, causing the class `A$Int` to be loaded. The class loader intercepts this, demangles the name, and loads the bytecode for the template class `A`.

The bytecode is transformed, replacing the type property `T` with the concrete type `int`, the translation of `Int`.

```
class A {
    x10.runtime.Type T;
}

class A$Int extends A {
    int x;
}
```

Type properties are mapped to the Java primitive types and to Object. Only nine possible instantiations per parameter. Instantiations used for representation. Adapter objects used for run time type information.

Could do instantiation eagerly, but quickly gets out of hand without whole-program analysis to limit the number of instantiations: 9 instantiations for one type property, 81 for two type properties, 729 for three.

Value constraints are erased from type references.

Constructors are translated to static methods of their enclosing class. Constructor calls are translated to calls to static methods.

Consider the code in Figure 13. It contains most of the features of generics that have to be translated.

4.2 Eliminating method type parameters

4.3 Translation to Java

4.4 Run-time instantiation

We translate `instanceof` and cast operations to calls to methods of a `Type` because the actual implementation of the operation may require run-time constraint solving or other complex code that cannot be easily substituted in when rewriting the bytecode during instantiation.

5. Discussion

todo: Move some of this to Section 6

5.1 Type properties versus type parameters

Type properties are similar, but not identical to type parameters. The differences may potentially confuse programmers used to Java generics or C++ templates. The key difference is that type properties are instance members and are thus accessible through access paths: `e.T` is a legal type.

Type properties, unlike type parameters, are inherited. For example, in the following code, `T` is defined in `List` and inherited into `Cons`. The property need not be declared by the `Cons` class.

```
class List[T] { }
class Cons extends List {
    def head(): T = { ... }
    def tail(): List[T] = { ... }
}
```

The analogous code for `Cons` using type parameters would be:

```
class Cons[T] extends List[T] {
    def head(): T = { ... }
    def tail(): List[T] = { ... }
}
```

We can make the type system behave as if type properties were type parameters very simply. We need only make the syntax `e.T` illegal and permit type properties to be accessible only from within the body of their class definition via the implicit `this` qualifier.

5.2 Wildcards

Wildcards in Java [23, 52] were motivated by the following example (rewritten in X10 syntax) from [52]. Sometimes a class needs a field or method that is a list, but we don't care what the element type is. For methods, one can give the method a type parameter:

```
def aMethod[T](list: List[T]) = { ... }
```

This method can then be called on any `List` object. However, there is no way to do this for fields since they cannot be parametrized. Java introduced wildcards to allow such fields to be typed:

```
List<?> list;
```

In X10, a similar effect is achieved by not constraining the type property of `List`. One can write the following:

```
list: List;
```

Similarly, the method can be written without type parameters by not constraining `List`:

```
class C[T] {
    var x: T;
    def this[T](x: T) { this.x = x; }
    def set(x: T) { this.x = x; }
    def get(): T { return this.x; }
    def map[S](f: T => S): S { return f(this.x); }
    def d() { return new D[T](); }
    def t() { return new T(); }
    def isa(y: Object): boolean { return y instanceof T; }
}

val x : C = new C[String]();
val y : C[int] = new C[int]();
val z : C{T <: Array} = new C[Array[int]]();
x.map[int](f);
new C[int{self==3}]() instanceof C[int{self<4}];
```

Figure 13. Code to translate

```
def aMethod(list: List) = { ... }
```

In X10, `List` is a supertype of `List[T]` for any `T`, just as in Java `List<?>` is a supertype of `List<T>` for any `T`. This follows directly from the definition of the type `List` as `List{true}`, and the type `List[T]` as `List{X==T}`, and the definition of subtyping.

Wildcards in Java can also be bounded. We achieve the same effect in X10 by using type constraints. For instance, the following Java declarations:

```
void aMethod(List<? extends Number> list) { ... }
<T extends Number> void aParameterizedMethod(List<T> list) { ... }
```

may be written as follows in X10:

```
def aMethod(list: List{T <: Number}) = { ... }
def aParameterizedMethod[T{self <: Number}](list: List[T]) = { ... }
```

Wildcard bounds may be covariant, as in the following example:

```
List<? extends Number> list = new ArrayList<Integer>();
Number num = list.get(0);     // legal
list.set(0, new Double(0.0)); // illegal
list.set(0, list.get(1));     // illegal
```

This can also be written in X10, but with an important difference:

```
list: List{T <: Number} = new ArrayList[Integer]();
num: Number = list.get(0);    // legal
list.set(0, new Double(0.0)); // illegal
list.set(0, list.get(1));     // legal! (when list is final)
```

Note because `list.get` has return type `list.T`, the last call in above is well-typed in X10; the analogous call in Java is not well-typed.

Finally, one can also specify lower bounds on types. These are useful for comparators:

```
class TreeSet[T] {
  def this[T](cmp: Comparator{T :> this.T}) { ... }
}
```

Here, the comparator for any supertype of `T` can be used as to compare `TreeSet` elements.

Another use of lower bounds is for list operations. The `map` method below takes a function that maps a supertype of the class parameter `T` to the method type parameter `S`:

```
class List[T] {
  def map[S](fun: Object{self :> T} => S) : List[S] = { ... }
}
```

5.3 Proper abstraction

Consider the following example adapted from [52]:

```
def shuffle[T](list: List[T]) = {
  for (i: int in [0..list.size()-1]) {
    val xi: T = list(i);
    val j: int = Math.random(list.size());
    list(i) = list(j);
    list(j) = xi;
  }
}
```

The method is parametrized on `T` because the method body needs the element type to declare the variable `xi`.

However, the method parameter can be omitted by using the type `list.T` for `xi`. Thus, the method can be declared with the signature:

```
def shuffle(list: List) { ... }
```

This is called *proper abstraction*.

This example illustrates a key difference between type properties and type parameters: A type property is a member of its class, whereas a type parameter is not. The names of type properties are visible outside the body of their class declaration.

In Java, Wildcard capture allows the parametrized method to be called with any `List`, regardless of its parameter type. However, the method parameter cannot be omitted: declaring a parameterless version of shuffle requires delegating to a private parametrized version that "opens up" the parameter.

5.4 Conditional methods and generlized constraints

For type parameters, method constraints are similar to generalized constraints proposed for C# [15]. In the following code, the `T` parameter is covariant and so the `append` methods below are illegal:

```
class List[+T] {
  def append(other: T): List[T] = { ... }
      // illegal
  def append(other: List[T]): List[T] = { ... }
      // illegal
}
```

However, one can introduce a method parameter and then constrain the parameter from below by the class's parameter: For example, in the following code,

```
class List[+T] {
  def append[U](other: U)
      {T <: U}: List[U] = { ... }
  def append[U](other: List[U])
      {T <: U}: List[U] = { ... }
}
```

The constraints must be satisfied by the callers of append. For example, in the following code:

```
xs: List[Number];
ys: List[Integer];
xs = ys; // ok
xs.append(1.0); // legal
ys.append(1.0); // illegal
```

the call to `xs.append` is allowed and the result type is `List[Number]`, but the call to `ys.append` is not allowed because the caller cannot show that `Number <: Double`.

5.5 Self types

The generic constrained type system generalizes features of type systems from other object-oriented languages.

Type properties can also be used to support a form of self types [8, 9]. Self types can be implemented by introducing a type property `class` to the root of the class hierarchy, `Object`:

```
class Object[class] { ... }
```

Scala's path-dependent types [38] and J&'s dependent classes [37] take a similar approach.

Self types are achieved by implicitly constraining types so that if an path expression `p` has type `C`, then `p.class <: C`. In particular, `this.class` is guaranteed to be a subtype of the lexically enclosing class; the type `this.class` represents all instances of the fixed, but statically unknown, run-time class referred to by the `this` parameter.

Self types address the binary method problem [8]. In the following example, the class `BitSet` can be written with a `union` method that takes a self type as argument.

```
interface Set {
  def union(s: this.class): void;
}

class BitSet implements Set {
  int bits;
  def union(s: this.class): void {
    this.bits |= s.bits;
  }
}
```

Since `s` has type this `this.class`, and the class invariant of `BitSet` implies `this.class <: BitSet`, the implementation of the method is free to access the `bits` field of `s`.

Callers of `BitSet.union()` must call the method with an argument that has the same run-time class as the receiver. For a receiver `p`, the type of the actual argument of the call must have a constraint that entails `self.class==p.class`.

5.6 Virtual types

Type properties share many similarities with virtual types [33, 32, 16, 17, 13] and similar constructs built on path-dependent types found in languages such as Scala [38], and J& [37]. Constrained types are more expressive than virtual types since they can be constrained at the use-site, can be refined on a per-object basis without explicit subclassing, and can be refined contravariantly as well as covariantly.

constraints	c	::=	...
		|	T has Sig
signatures	Sig	::=	def this[$\overline{X}$]($\overline{x}$: $\overline{T}$){c}: T
		|	def m[$\overline{X}$]($\overline{x}$: $\overline{T}$){c}: T
		|	val x{c}: T

Figure 14. Grammar for structural constraints

Thorup [50] proposed adding genericity to Java using virtual types. For example, a generic `List` class can be written as follows:

```
abstract class List {
    abstract typedef T;
    void add(T element) { ... }
    T get(int i) { ... }
}
```

This class can be refined by bounding the virtual type `T` above:

```
abstract class NumberList extends List {
    abstract typedef T as Number;
}
```

And this abstract class can be further refined to *final bind* `T` to a particular type:

```
class IntList extends NumberList {
    final typedef T as Integer;
}
```

These classes are related by subtyping: `IntList <: NumberList <: List`. Only classes where `T` is final bound can be non-abstract.

The analogous definition of `List` using type properties is as follows:

```
class List[T] {
    def add(element: T) = { ... }
    def get(i: int): T = { ... }
}
```

`NumberList` and `IntList` can be written as follows:

```
class NumberList extends List{T<:Number} { }
class IntList extends NumberList{T==Integer} { }
```

However, note that our version of `List` is not abstract. Instances of `List` can instantiate `T` with a particular type and there is no need to declared classes for `NumberList` and `IntList`. Instead, one can simply use the types `List{T<:Number}` and `List{T==Integer}`.

In addition, unlike virtual types, type properties can be refined contravariantly. For instance, one can write the type `List{T:>Integer}`, and even `List{Integer<:T, T<:Number}`.

5.7 Structural constraints

Type constraints need not be limited to subtyping constraints. By introducing structural constraints on types, GFX allows type properties to be instantiated on any type with a given set of methods and fields. This feature is useful for reusing code in separate libraries since it does not require code of one library to implement an interface to satisfy a constraint of another library.

In this section, we consider an extension of the X10 type system to support structural type constraints. The type system need not change except by extending the constraint system. The syntax for structural constraints is shown in Figure 14. A structural constraint of the form `T has Sig` can specify that the type `T` have a constructor, method, or field of the given signature.

Structural constraints on types are found in many languages. Haskell supports type classes [30, 24]. In Modula-3, type equivalence and subtyping are structural rather than nominal as in object-oriented languages of the C family such as C++, Java, Scala, and

X10. The language PolyJ [6] allows type parameters to be bounded using structural where clauses. For example, the sorted list class from Figure 3 could be be written as follows in PolyJ:

```
class SortedList[T] where T { int compareTo(T) } {
    void add(T x) { ... x.compareTo(y) ... }
    ...
}
```

The where clause states that the type parameter `T` must have a method `compareTo` with the given signature.

The analogous code for `SortedList` in the structural extension of X10 is:

```
class SortedList[T]{T has def compareTo(T): int} {
    def add(x: T) = { ... x.compareTo(y) ... }
    ...
}
```

A structural constraint is satisfied if the type has a member of the appropriate name and with a compatible type. The constraint `Z has def m[`$\overline{\texttt{X}}$`](`$\overline{\texttt{x}}$`:`$\overline{\texttt{T}}$`): U` is satisfied by a type `T` if it has a method `m` with signature `def m[`$\overline{\texttt{Y}}$`](`$\overline{\texttt{y}}$`:`$\overline{\texttt{S}}$`): V` and where `([`$\overline{\texttt{Y}}$`](`$\overline{\texttt{y}}$`:`$\overline{\texttt{S}}$`) => V)[T/Z]` is a subtype of `([`$\overline{\texttt{X}}$`](`$\overline{\texttt{x}}$`:`$\overline{\texttt{T}}$`) => U)[T/Z]`. As an example, the constraint `X has def compareTo(X): int` is satisfied by both of the following classes:

```
class C { def compareTo(x: C): int = ...; }
class D { def compareTo(x: Object): int = ...; }
```

todo: The important bit about all this is... don't have to change the type system, just the constraints

6. Related work

Dependent types, constraint-based type systems and generic types have a long history.

Constraint-based type systems. The use of constraints for type inference and subtyping were developed by Mitchell [36] and Reynolds [43]. Trifonov and Smith [53] proposed a type system where types are refined by subtyping constraints, however dependent types are not supported.

HM(X) [47, 41, 42] is a constraint-based framework for Hindley–Milner-style type systems. The framework is parameterized on the specific constraint system X; instantiating X yields extensions of the HM type system. Constraints in HM(X) are over types, not values. Sulzmann and Stuckey [48] showed that the type inference algorithm for HM(X) can be encoded as a constraint logic program parameterized by the constraint system X. Type inference in this framework is thus constraint solving.

Structural constraints on types have been implemented in Haskell type classes [24]. PolyJ [6] also uses structual constraints to bound type parameters.

Dependent types. Several systems have been proposed that refine types in a base type system through constraints on values [22, 1, 31, 25, 18, 19, 44].

With *hybrid type-checking* [18, 19], types can be constrained by arbitrary boolean expressions. While typing is undecidable, dynamic checks are inserted into the program when necessary if the type-checker cannot determine type safety statically. In X10, dynamic type checks, including tests of dependent constraints, are inserted only at explicit casts or `instanceof` expressions.

Logically qualified types, or liquid types [44], permit types in a base HM-style type system to be qualified with dependent constraints. The HM type inference algorithm is extended to infer constraints as well as base types. **todo:** More on this

Concoqtion [21] extends types in OCaml [**?**] with constraints written as Coq [14] rules. While the types are expressive, supporting the full generality of the Coq language, proofs must be provided to satisfy the type checker. X10 supports only constraints that can be checked by a constraint solver during compilation. Concoqtion encodes OCaml types and value to allow reasoning in the Coq formulae; however, there is an impedence mismatch caused by the differing syntax, representation, and behavior of OCaml versus Coq.

Cayenne [5] is a Haskell-like language with fully dependent types. There is no distinction between static and dynamic types. Type-checking is undecidable. There is no notion of datatype refinement as in DML.

Epigram [34, 4] is a dependently typed functional programming language based on a type theory with inductive families. Epigram does not have a phase distinction between values and types.

ESC/Java [20] allow programmers to write object invariants and pre- and post-conditions that are enforced statically by the compiler using an automated theorem prover. Static checking is undecidable and, in the presence of loops, is unsound (but still useful) unless the programmer supplies loop invariants. ESC/Java can enforce invariants on mutable state.

Our work is closely related to DML, [56], an extension of ML with dependent types. DML is also built parametrically on a constraint solver. Types are refinement types; they do not affect the operational semantics and erasing the constraints yields a legal ML program.

At a conceptual level the intuitions behind the development of DML and constrained types are similar. Both are intended for practical programming by mainstream programmers, both introduce a strict separation between compile-time and run-time processing, are parametric on a constraint solver, and deal with mutually recursive data-structures, mutable state, and higher-order functions (encoded as objects in the case of constrained types). Both support existential types.

The most obvious distinction between the two lies in the target domain: DML is designed for functional programming, specifically ML, whereas constrained types are designed for imperative, concurrent OO languages. Hence, technically our development of constrained types takes the route of an extension to FJ. But there are several other crucial differences as well.

First, DML achieves its separation by not permitting program variables to be used in types. Instead, a parallel set of (universally or existentially quantified) "index" variables are introduced. Second, DML permits only variables of basic index sorts known to the constraint solver (e.g., `bool`, `int`, `nat`) to occur in types. In contrast, constrained types permit program variables at any type to occur in constrained types. As with DML only operations specified by the constraint system are permitted in types. However, these operations always include field selection and equality on object references. (As we have seen permitting arbitrary type/property graphs may lead to undecidability.) Note that DML style constraints are easily encoded in constrained types.

Third, DML does not permit any runtime checking of constraints (dynamic casts).

Genericity. Genericity in object-oriented languages is usually suported through type parameterization.

A number of proposals for adding genericity to Java quickly followed the initial release of the language [7, 39, 6, 50, 2]. GJ [7] implements invariant type parameters via type erasure. PolyJ [**?**] supports run-time representation of types via adapter objects, and also permits instantiation of parameters on primitive types and structural parameter bounds. LM [55, 54] also supports a run-time representation of types. NextGen [11, 2] was implemented using run-time instantiation. X10's generics have a hybrid implementation, adopting PolyJ's adapter object approach for dependent types and for

type introspection and using NextGen's run-time instantiation approach for greater efficiency.

C# also supports generics via [49]. Type parameters may be declared with definition-site variance tags. Compilation does not erase types; the CLR performs run-time parameter instantiation. Generalized type constraints were proposed for C# [15]. Methods can be annotated with subtyping constraints that must be satisfied to invoke the method. Generic X10 supports these constraints, as well as constraints on values, with method and constructor where clauses.

GFX does not support *bivariance* [**?**]; a class `C` is bivariant in a type property `X` if `C{self.X==S}` is a subtype of `C{self.X==T}` for any `S` and `T`. Bivariance is useful for writing code in which the property `X` is ignored. One can achieve this effect in GFX simply by leaving `X` unconstrained.

Virtual classes and types. *Virtual classes* [32, 33, 17]. are a language-based extensibility mechanism that where originally introduced in the language BETA [32] as a mechanism for supporting genericity. Virtual classes in BETA are not statically type safe, but this has been remedied in recent formulations [16, 17] and in variants of virtual classes [38, 37, 12, 29] using path-dependent types.

Virtual types, also introduced in BETA [32], are similar to virtual classes. A virtual type is a type binding nested within an enclosing instance. Virtual types may be used to provide genericity; indeed Thorup [50] proposed extending Java with virtual types as a genericity mechanism. Virtual types influenced Java's wildcards [52, 23, 10].

Igarashi and Pierce [27] model the semantics of virtual types and several variants in a typed lambda-calculus with subtyping and dependent types.

Use-site variance based on structural virtual types were proposed by Thorup and Torgerson [51] and extended for parameterized type systems by Igarashi and Viroli [28]. The latter type system lead to the development of wildcards in Java [23, 52, 10].

7. Conclusions

We have presented a preliminary design for supporting genericity in X10 using type properties. This type system generalizes the existing X10 type system. The use of constraints on type properties allows the design to capture many features of generics in languages like Java 5 and C# and then to extend these features with new more expressive power. We expect that the design admits an efficient implementation and intend to implement the design shortly.

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