Solving Large, Irregular Graph Problems in X10

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Abstract

Graph problems are finding increasing applications in high performance computing disciplines. Obtaining efficient implementations for large, irregular graph instances remains a challenge. There exists a large body of theoretically fast parallel graph algorithms. However, experimental studies show that they often fail to achieve good parallel speedups in practice since fine-grained parallelism is not well supported on current systems.

This paper shows how *work-stealing* ideas for implementing fine-grained parallelism may be extended to apply to graph problems, and implemented efficiently on existing multicore systems. Work-stealing is a provably optimal technique for scheduling certain kinds of series-parallel dependency graphs (implemented in systems such as Cilk) in which a collection of worker threads generate parallel work separately and steal from each other when their generated work is exhausted. In order to express graph algorithms, we extend Cilk's work-stealing scheduler in three ways. (1) We extend the scheduler to correctly implement *improperly nested* computations (computations in which a procedure may return without checking that all asynchronous tasks spawned during its execution have terminated). (2) We introduce the notion of *global termination detection* – to determine the computation has terminated we track the distributed property "For every worker, the work queue is empty". This is an improvement when the number of workers (typically dozens) is much smaller than the number of tasks (typically millions). (3) We generalize (2) to *repeated global quiescence detection* by integrating a notion of barrier-based computation into work-stealing. This is necessary to express phased computations such as breadth-first search in which one phase (consisting of the execution of data-dependent number of tasks) must terminate before the next phase is started.

We have implemented these ideas in the X10 Work-stealing system (XWS), a runtime library for the X10 programming language. We show that several graph algorithms can naturally and elegantly be expressed in X10 and translated into calls to XWS. We measure performance on an Opteron-based system and a Niagara system. In those cases in which corresponding programs can be written in Cilk, we show that performance of the XWS program is better than the Cilk program. In all cases we show that performance is comparable to that obtained by hand-written C code, and that performance scales well with the number of processors.

1 Introduction

Graph theoretic problems arise in traditional and emerging scientific disciplines such as VLSI design, optimization, databases, and computational biology. There are plenty of theoretically fast parallel algorithms, for example, work-time optimal PRAM algorithms, for graph problems; however, in practice few parallel implementations beat the best sequential implementations for arbitrary, sparse graphs. The mismatch between theory and practice suggests a large gap between algorithmic model and the actual architecture. We observe that the gap is increasing as new diversified architectures emerge. Elegant solutions with high performance seem hard to come by even from combined efforts of algorithmic and architectural improvement.

Due to their irregular and combinatorial nature, large-scale graph problems are challenging to solve in parallel. Many important real world graph instances, for example, the Internet, social interaction networks, transportation networks, and protein-protein interaction networks, are irregular. These graphs can be modeled as 'scale-free" graphs [3]. For random and scale-free graphs no known efficient partitioning technique exists, which makes them extremely hard to solve on distributed-memory systems. Moreover, the irregular memory access pattern dictated by the input instances is not cache-friendly. Obtaining high performance on shared-memory systems is challenging. Compared with their numerical counterparts, parallel graph algorithms take drastically different approaches than the sequential algorithms, and usually employ fine-grained parallelism. For example, depth-first search (DFS) or breadth-first search (BFS) are two popular sequential algorithms for the spanning tree problem. Many parallel spanning tree algorithms, represented by the Shiloach-Vishkin algorithm [13], take the "graft-and-shortcut" approach, and provide massive amount of fine-grained parallelism in the order of $O(n)$. In the absence of efficient scheduling support of parallel activities, fine-grained parallelism incurs large overhead on current systems, and oftentimes the algorithms do not show practical performance advantage. Graph algorithms also tend to be load/store intensive [5], and they lay great pressure on the memory subsystem. It gets even worse on distributed-memory architectures if necessary task management and memory affinity scheduling are not provided.

Features of X10 such as shared-memory interface, asynchronous parallelism, and runtime task scheduling make it ideal for solving large-scale graph problems. The shared-memory address space obviates the need to partition a graph and issue requests explicitly to access remote data. Without this, writing code for irregular graph algorithms is daunting. In fact, none of the SSCA [10] graph benchmarks has implementations on distributed-memory systems. X10 in addition allows specifying the location of a parallel activity so that the affinity between tasks and data is exploited. Efficient mapping of fine-grained parallelism to target architectures with high performance is the highlight of the X10 programming model. X10 provides a rich collection of programming constructs that may be used to express various levels of parallelism and synchronization scheme. Depending on the input and the target systems, different algorithms can be easily implemented to fit with the architecture. X10 runtime manages parallel activities effectively with low cost. X10 effectively helps reduce the gap between theory and practice in solving large-scale graph problems. With X10 fine-grained parallelism is elegantly expressed, and compared with the best native C implementation, X10 program achieves comparable, and sometimes even better performance.

In this paper we present solving irregular graph problems in X10 on a cluster of SMPs. As most current and emerging supercomputers are clusters of SMPs, it is important to solve the problems efficiently on these platforms. As X10 is an on-going project, we present performance results on SMP nodes as across-node runtime support is still in development. The algorithms we consider include both PRAM algorithms and efficient algorithms that based on more realistic models such as the SMP model [8]. PRAM algorithms are synchronous and provide massive amount of parallelism. The other types of algorithms are either asynchronous or bulk-synchronous with limited amount of parallelism that maps well to architectures with a moderate number of processors. Both classes of algorithms can be expressed and implemented efficiently in X10.

As an example, we consider the spanning tree problem. Despite dozens of parallel spanning tree algorithms, it is notoriously hard to achieve good parallel performance [1]. We design and/or implement in X10 three parallel algorithms that are representative of different algorithmic approaches. The performance is comparable to or better than the best known prior implementations. Moreover, the algorithm expressed in X10 code is concise and elegant.

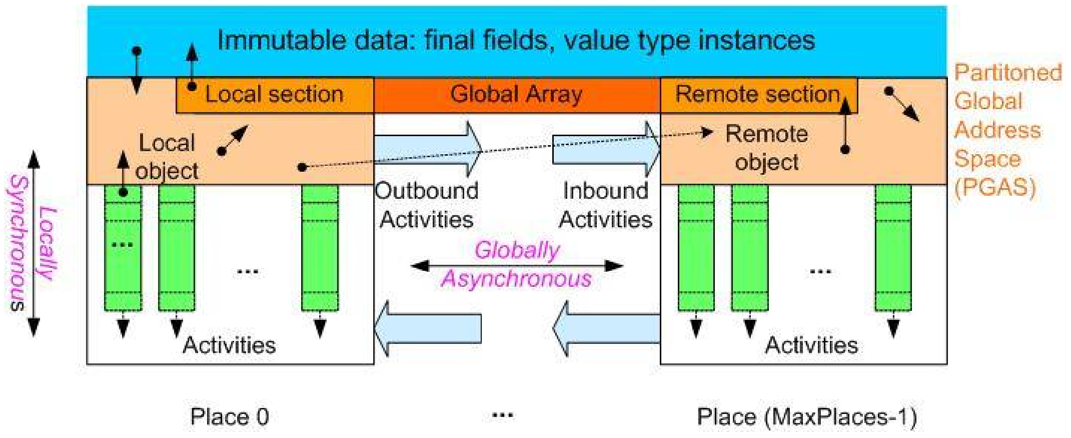
The rest of the paper is organized as follows. Section 2 describes the language features of X10. Section 3 presents spanning tree algorithms in X10. Section 4 presents the runtime support for X10, especially for activity scheduling, with comparison to other runtime systems. Section 5 gives our experimental results. In Section 6 we conclude and describe future work.

2 X10: Designed for High Productivity

X10 is a new Partitioned Global Address Space (PGAS) language being developed at IBM as part of the DARPA HPCS project [4]. It is designed to address both programmer productivity and parallel performance for modern architectures from the multicores, to the heterogeneous accelerators (as in the Cell processor), and to the scale-out clusters of SMPs such as Blue Gene. The language is based on sequential Java with extensions for programming fine-grained and massive parallelism. Unlike other PGAS languages such as Co-Array Fortran, Titanium, and UPC whose model of parallelism is Single Program Multiple Data (SPMD), X10 supports dynamic and structured concurrency with SPMD as a special case. In this section, we provide a brief introduction to the basic concepts in the X10 programming model and the language constructs used to implement the graph algorithms of interest. For more details and other features of X10, readers please refer to [4].

1. **Activities** – All concurrency in X10 is expressed as asynchronous *activities*. An activity is a lightweight thread of execution, which can be spawned recursively in a fork-join manner. The syntax of spawning an activity is `async S`, where a new child activity is created executing statement `S`. The granularity of an activity is arbitrary – `S` can be a single statement reading a remote variable or a sequence of statements performing a stencil operation on a grid. Our experience has been that this single notion of an asynchronous activity can subsume many levels of parallelism that a programmer may encounter such as threads, structured parallelism (including OpenMP), messaging (including MPI), and DMA transfers.

2. **Places** – The main program starts as single activity at *place* 0. Place is an X10 concept which can be considered as a virtual SMP, but multiple places can be mapped to one physical SMP node. The global address space is partitioned across places. Data and activities have affinity with and only with one place. Activities can only operate on data local to them, that is, within the same place. To access data at another place, a new activity has to be spawned



Dynamic parallelism with a Partitioned Global Address Space. All concurrency is expressed as asynchronous activities. Each vertical green rectangle above represents the stack for a single activity. An activity may hold references to remote objects, that is, at a different place. However, if it attempts to operate on a remote object, then it has to spawn a new activity at the remote place to perform the operation. Immutable (read only) data is special which can be accessed freely from any place providing opportunity for single-assignment parallelism.

Figure 1: X10 Programming Model

there to perform the operation. The syntax for spawning an activity at place p is `async (p) S`. The diagram in Figure 1 describes the X0 programming model.

3. **X10 arrays** – X10 supports a rich set of multidimensional array abstractions and domain calculus as in Titanium [6]. The array index space is global where each index is an integer vector named *point*, and a *domain* is a set of points which can be either rectangular or not. The distribution of an array across places is specified by a *distribution* which can be defined by users. Each distribution maps a set of points in a region to a set of places.

4. **Parallel loops** – There are two kinds of parallel loop in X10: `foreach` and `ateach`, for looping over a region and a distribution respectively. Their difference is that the `for` loop spawns activities locally, whereas the `ateach` loop spawns activities at the places specified by the distribution.

5. **Finish and clock** – The statement `async S` returns immediately when it is executed even if the statement `S` is not finished, which may also spawn other activities. The `finish S` construct is used to specify that the next instruction is executed only when the statement `S` has finished globally, that is, all transitively spawned child activities have finished. While `finish` permits the detection of global termination, there are many cases in which a barrier-like coordination is needed for a set of activities during the middle of their computation. X10 uses *clock* to coordinate such a set of activities. A clock has phases, and the activities registered with this clock can be synchronized by waiting for their finish of the current clock phase. An activity can be registered with multiple clocks and it can drop any of them at any time.

6. **Atomic blocks** – X10 uses *atomic* blocks for mutual exclusion. An atomic statement/method is conceptually executed in a single step, while other activities are suspended. An atomic block must be nonblocking, sequential (without spawning activities), and local (no remote data access). *Conditional* atomic block is another parallel language construct of X10 which can be used, for example, to implement point-to-point synchronization. The syntax for a conditional atomic block is `when (E) S`, where the executing activity suspends until the boolean expression `E` is true, then `S` is executed atomically.

3 Designing Parallel Graph Algorithms in X10

Large scale graph problems with irregular instances are challenging to solve on current and emerging parallel systems [2]. Despite the large existing body of theoretically-fast PRAM algorithms and communication-optimal BSP algorithms, there are few implementations that achieve good parallel performance. PRAM is highly idealistic and conducive to exploring the inherent parallelism of a problem. For practical purposes, PRAM does not reflect features of main-stream architectures that are critical to performance. Significant effort is necessary to produce efficient implementations on even shared-memory systems, for example, SMPs. Few PRAM algorithms have been adapted to the distributed-memory environment with significant overhaul of major algorithm steps. More practical models such as BSP and LogP, on the other hand, parameterize communication, synchronization, and even memory access costs. As a result, algorithms may be mapped onto real machines with reasonable ease and performance, yet design choices are severely limited as the dimension of fine-grained parallelism is virtually excluded. After decades of effort, designing and implementing parallel graph algorithms for large, irregular inputs on either class of models remains challenging.

One of the biggest challenges of efficiently simulating a PRAM algorithm on modern architectures is load balancing. Load-balancing is not an issue with PRAM as there are always enough processors. Consider a graph algorithm for sparse, irregular instances with adjacency list as the input. A PRAM algorithm may assign for each edge (in this case each neighbor v in the adjacency list of vertex u) a processor, and takes $O(n)$ processors. When simulating on p ($p \ll n$) processors, $\frac{n}{p}$ vertices are usually assigned to one processor. As the graph is irregular with potentially huge difference among the number of neighbors for each processor, load-imbalance constitutes a serious performance problem. Keeping track of workload distribution becomes even more cumbersome when the algorithms compact the input graphs. The load-balancing challenge with popular distributed-memory models such as BSP and LogP is that no known practical technique exists that partitions a graph evenly.

The programming model of X10 enables an algorithm designer to focus on expressing the appropriate parallelism, and leaves to the runtime system mapping and scheduling activities on to the processors in a load-balanced fashion. X10 makes a big step in terms of productivity and performance in solving large-scale, irregular graph problems on current supercomputers.

We next present two algorithms that are programmed in X10 and showcase its expressiveness in designing graph algorithms. Among the many productivity-improving features, we here choose to show those that are related to activity scheduling as this is the foundation of high performance guarantee from X10. We take the spanning tree problem as our example. Finding a spanning tree of a graph is an important building block for many graph algorithms, for example, biconnected components and ear decomposition [12], and can be used in graph planarity testing [11]. Spanning

```
void traverse(int u) {
  finish for(int k=0;k<G[u].degree;k++) {
    final int v=G[u].neighbors[k];
    if(G[v].color.compareAndSet(0,1)) {
      G[v].parent=u;
      async traverse(v);
    }
  }
}
```

Single-place code

```
void traverse(int u) {
  finish for(int k=0;k<G[u].degree;k++) {
    final int v=G[u].neighbors[k];
    async (G.distribution[v]) {
      if(G[v].color.compareAndSet(0,1)) {
        G[V].parent=U;
        traverse(v);
      }
    }
  }
}
```

Multi-place code

Table 1: Single- and Multi-place spanning tree algorithm core

tree represents a wide range of graph problems that have fast theoretic parallel algorithms but no known efficient parallel implementations that achieve speedup without serious restricting assumptions about the inputs. Section 3.1 describes a new algorithm on SMP, whose performance is further studied in subsequent sections. As an illustration of the ease to extend an algorithm designed for an SMP to run on a cluster of SMPs, section 3.1 is the algorithm for distributed environment.

3.1 A Parallel Spanning Tree Algorithm based on graph traversal in X10

Many fast PRAM algorithms have been proposed for the spanning tree problem, and they are drastically different from the sequential depth-first search (DFS) or breadth-first search (BFS) approaches. DFS and BFS are efficient with very low overhead, and for quite a long time no parallel implementations beat the best sequential implementation for irregular inputs [1].

Bader and Cong [1] presented the first fast parallel spanning tree algorithm that achieved good speedups on SMPs. Their algorithm is based on a graph traversal approach, and is similar to DFS or BFS. There are two steps to the algorithm. First a small stub tree of size $O(p)$ is generated by having one processor randomly walking the graph, and the vertices are evenly distributed to each processor. The processors then traverse the graph in a manner similar to sequential DFS or BFS. To achieve good load-balancing, a processor checks for non-empty stack/queue on some other processor when it runs out of work, and takes a portion of the stack/queue as its own.

X10 allows the expression of essential parallelism in the algorithm in a very concise and elegant way. The first program in Table 1 is a recursive algorithm that traverses the graph and computes a spanning tree. It is identical to the recursive sequential DFS, except that commands for creating logical parallel activities (*async*) and atomic operations (*compareAndSet*) are used.

While visiting each neighbor v of vertex u, the algorithm spawns a new traversal activity. Since the algorithm is recursive, for a graph with millions of vertices, massive amount of parallelism is available. It puts great pressure on the runtime system for task management, scheduling, and load-balancing.

The second program shows the change required to the traversal code when the graph is distributed across multiple places (as would be necessary when running the program on a cluster of SMPs). The *async* command takes an extra argument specifying the place at which it is to run.

In both cases, computation is initiated by executing `finish traverse(0);`. The `finish` construct ensures that computation terminates only when all activities spawned during the graph search have terminated.

Note that Bader and Cong's spanning tree algorithm contains over 300 lines of C code.

4 Runtime Support for X10 programs

We have implemented a runtime system for X10. As an initial implementation, we have developed a Java-based runtime system to support shared-memory parallelization. It is distributed as a Java package, `x10.runtime.cws`, under an open-source license [9]. In this section, we discuss the implementation of key features of the runtime system.

The computation is organized as collection of *tasks*. A task is a sequence of instructions that can spawn other tasks and wait for completion of the spawned tasks. The computation begins with a single task and is considered complete when there are no more tasks executing in the system.

4.1 Cilk work-stealing

This section describes Cilk work stealing, as implemented in XWS. It closely follows the description of the implementation of Cilk in [7].

In summary, Cilk Work Stealing (CWS) is organized around a collection of cooperating threads called *workers*. Each worker maintains a double-ended queue (deque) of tasks. During execution as a worker creates more tasks it pushes them at the bottom of the queue. When it needs more tasks it retrieves the current task from the bottom of the deque. When a worker runs out of work (its deque is empty), it randomly chooses another worker (the *victim*) and attempts to steal a task by fetching it from the top of the deque. Program begins execution when the environment submits a task to a central task queue.

One of the workers retrieves the task from the global queue and begins executing it. When a worker does not have tasks to execute it *steals* tasks available at other workers. Assuming the computation contains sufficient parallelism stealing happens infrequently. The design ensures that there are few overheads during normal execution, referred to as *the fast path*. The additional overheads incurred to load-balance the computation are proportional to the number of steals in the execution. The design of the worker for the the task types supported is discussed in subsequent sections.

The normal execution corresponds to the depth-first sequential execution of the tasks spawned. Thus the execution corresponds to a sequential execution when there is only one worker.

We now describe the details.

Every *async* in the X10 program is compiled into a sub-class of `Frame`. An *async* is said to consist of *threads*, where a thread is a non-blocking sequence of instructions terminating in a spawn or waiting on an *async*.

Each class created for an *async* contains as member variables the local variables used in the *async* body, a counter (*PC*) that specifies the next instruction in the body of the *async* after the current thread.

Cilk implements a scheme for handling continuations. Two versions of the *async* body are created – the fast and the slow versions. The fast version of the program is executed during the normal course of the program. The slow version is invoked by the worker to initiate processing of a task. Fig. 2 shows the fast and slow versions for the Fibonacci method shown in Fig. 2(a).

```
int fib(int n) {
  int x, y;
  if(n<2) return n;
  finish {
    async x=fib(n-1);
    async y=fib(n-2);
 }
  return x+y;
}
```

(a)

```
int fib(Worker w, int n)
 throws StealAbort {
  int x, y;
  if (n < 2) return n;

  FibFrame frame = new FibFrame(n);
  frame.PC=LABEL_1;
  w.pushFrame(frame);

  x = fib(w, n-1);
  w.abortOnSteal(x);

  frame.x=x;
  frame.PC=LABEL_2;

  y=fib(w, n-2);
  w.abortOnSteal(y);

  w.popFrame();
  return frame.x+y;
}
```

(b)

```
void compute(Worker w,
 Frame frm) throws StealAbort {
  int x, y;
  FibFrame f=(FibFrame)frm;
  int n = f.n;
  switch (f.PC) {
  case ENTRY:
    if (n < 2) {
      result = n;
      setupReturn();
      return;
   }
    f.PC=LABEL_1;
    x = fib(w, n-1);
    w.abortOnSteal(x);
    f.x=x;
  case LABEL_1:
    f.PC=LABEL_2;
    int y=fib(w,n-2);
    w.abortOnSteal(y);
    f.y=y;
  case LABEL_2:
    f.PC=LABEL_3;
    if (sync(w)) return;
  case LABEL_3:
    result=f.x+f.y;
    setupReturn();
 }
}
```

(c)

Figure 2: (a) X10 program for Fibonacci. (b) Fast version. (c) Slow version

Each worker maintains a deque consisting of stack of Frames. On entering a fast version, a frame object is created and pushed onto the stack (method: `pushFrame()`). This frame contains the up-to-date values of variables whenever any other worker might steal this task. This frame is popped from the stack (method: `popFrame()`) when this method returns.

Whenever a task is spawned, the worker immediately proceeds to execute the spawned task like a sequential method call. When the spawned task finishes execution, the worker checks whether the current frame was stolen. If so, the result computed by the child task is stored to be passed onto its parent and the execution of the task aborts (method: `abortOnSteal()`). The method call stack is unwound by throwing an exception (`StealAbort`) that is caught in the main routine executed by the worker. The values in the frame of the parent stack are updated before proceeding to execute a spawned child, as the worker now has a non-executing task and hence is target of a steal.

All the descendents are guaranteed to be completed in the fast version when a `finish` is encountered. Hence the finish statements are ignored.

The slow version restores any local variables and uses the *PC* to start execution of the task past the execution point at which it might have been stolen. A task might have been stolen when its descendents are executing. Thus a *finish* statement, translated to the `sync()` method, might cause the invoking task to suspend on completion of non-terminated children. The value of the `result` variable is returned to the parent task on invocation of the `setupReturn()` method.

In a program with sufficient parallelism, the slow version is expected to execute infrequently. The design tries to minimize the overheads in the fast version even at the expense of the slow version (the "work-first" principle).

4.2 Support for Properly Nested Tasks

Cilk requires *fully-strict* programs in which a task waits for all its descendents to complete before returning. Such tasks are also called properly nested tasks.

The X10 runtime system is designed to leverage the Cilk design while supporting a larger class of programs. X10 provides support for *strict* computations, in which a ancestor task need not wait for its descendent tasks to be completed. Such tasks are said to be *improperly nested*.

Each worker contains a *closure*. A closure is an object used to return values from the spawned tasks to their parents in the presence of work stealing.

Each closure maintains a stack of frames. Frames corresponding to spawned tasks are pushed into the stack on entry, and popped on return. In the fast path, return values are propagated as they would be in a sequential program. The thief steals a closure together with the bottom-most available frame from its *victim*.

When a thief steals a task, the descendants of the task in the victim's dequeue continue to execute. In order to return values from the descendents to the parent, a new closure is created that on completion returns the result to the parent closure that was stolen.

Thus the closures form a tree of return value propagation corresponding to the steal operation performed. Termination is detected when the closure corresponding to the task inserted by the driver thread returns.

The procedure executed by the workers to handle properly nested tasks is shown in Fig. 3(a). On completing execution of a closure, a worker first attempts to obtain another closure from its local queue (method:`extractBottom()`). If no local closure is available to execute, the worker attempts to obtain a task either by stealing or from the global queue (method:`getTask()`). It then executes the slow version of the task obtained (method:`execute()`).

4.3 Support for Improperly Nested Tasks

Properly nested tasks t satisfy the property that at the moment when the slow version terminates (method:`compute()`) the frame at the bottom of the worker's dequeue is t. Hence the task can be completed (i.e., removed from the dequeue) by including a `w.popFrame()` call at the end of the compute method. In essence, if a worker is executing only properly nested tasks (this is true when it is executing Cilk code), there is a one-to-one correspondence between the frame stack and the tasks being processed.

X10 permits improperly nested tasks. Such tasks q are used, for instance, to implement the pseudo-depth-first search discussed in this paper. Such a task may add a task r to the deque of its worker (say w) without necessarily transferring control to r. This has two consequences. First, recall that as soon as a worker's dequeue contains more than one task the worker may be the target of a theft. Therefore as soon as q pushes r onto w's dequeue, q is available to be stolen. Therefore q's compute method must record the fact that is computation has begun so that the stealing worker z may do the right thing. For instance, if q's compute method does not contain any internal suspension point then z must immediately terminate execution of q and pop q off its deque. This can be accomplished by defining a volatile int PC field in q, and adding the following code at the beginning of q's compute method

```
if (PC==1) {
  w.popFrame();
```

```
public void run() {
  Executable cl=null; //frame/closure
  int yields = 0;
  while (!done) {
    if (cl == null ) {
      //Extract work from local queue.
      //It will be a closure
      //cl may be null. When non-null
      //cl is typically RETURNING.
      lock(this);
      try {
        cl = extractBottom(this);
      } finally {
        unlock();
      }
    }
  }
  if (cl == null)
    //Steal or get from global queue
    cl = getTask(true);
  if (cl !=null) {
    // Found some work! Execute it.
    Executable cl1 = cl.execute(this);
    cl=cl1;
    cache.reset();
  } else Thread.yield();
}
                (a)
```

```
public void run() {
  Executable cl=null; //frame or closure
  int yields = 0;
  while (!done) {
    if (cl == null ) {
      //Addition for GlobalQuiescence.
      //Keep executing current frame
      //until dequeue becomes empty.
      if (jobMayHaveImproperTask) {
       Cache cache = this.cache;
        for(;;) {
          if(!cache.empty())
            Frame f=cache.currentFrame();
          if (f == null) break;
          Executable cl1=f.execute(this);
          if (cl1 != null) {
            cl=cl1;
            break;
          }
        }
      }
      //Rest of worker code same as for
      //properly nested tasks ...
    }
  }
}
                (b)
```

Figure 3: Code executed by workers for (a) only properly nested tasks (b) properly and improperly nested tasks. Note that (b) is an extension of (a)

```
    return;
  }
  PC=1;
```

Second for an improperly nested task when control returns from g's compute method, it may not be the case that the last frame on the dequeue is g. Therefore a call to `popFrame()` at the end of g's compute method would be incorrect. Instead, the compute method returns (without attempting to pop the last frame on the deque). Now whenever the task reaches the bottom of the dequeue, the worker will, as usual, invokes its compute method. However, the code sequence described above will execute, thereby popping the frame from the deque. Thus the code sequence above serves two purposes – it does the cleanup necessary when the task is stolen as well as when it is completed.

The changes to the basic worker code necessary to support improperly nested tasks are shown in Fig. 3(b). With improperly nested tasks, a worker no long enjoys the property that when control returns to it from the invocation of an execute method on the top-level task, the deque is empty. Indeed, control may return to the scheduler leaving several tasks on the deque, including the task whose execute method has just returned. The scheduler must now enter a phase in which it executes the task at the bottom of the deque:

4.4 Global Quiescence

In fully-strict computations, i.e., those involving properly nested tasks, completion of the first task and the return of the corresponding Closure indicates computation termination. Improperly-nested

```
Closure steal(Worker thief) {
  lock(thief); //lock victim deque
  Closure cl = peekTop(thief, victim);
  if (cl==null)
    return null; //nothing to steal
  //cl = Closure that can be stolen
  cl.lock(thief);
  Status status = cl.status();
  if (status == READY) {
    //Closure not processed by victim
    //steal the Closure
  }
  else if (status == RUNNING) {
    //Possible contention with victim
    //Need to steal head frame in Closure
    if (cache.dekker(thief)) {
      //>1 Frame available in Closure
      //Promote child frame to Closure
      //Steal this Closure & head frame
    }
  }
  return null; //No Closure to steal
}
```

(a)

```
Frame stealFrame(Worker thief) {
  Worker victim = this;
  lock(thief);

  //>1 frame in victm's frame stack?
  boolean b=cache.dekker(thief);
  if (b) {
    //Frame available to steal
    Frame frame = cache.headFrame();
    //Mark this frame as stolen
    cache.incHead(); //H=H+1
    return frame;
  }
  return null; //No frame to steal
}
```

(b)

Figure 4: Work stealing algorithm for (a) Properly nested tasks (b) Improperly nested tasks. Both are invoked on victim's Worker object (victim==this). Locks held are freed before returning.

tasks that do not require a return call chain can do away with the closures. We have implemented a mechanism to efficiently identify termination without closures.

The workers share a barrier. The barrier is used to determine when all workers are out of work. Every worker notifies the barrier of its state through two methods. `checkIn()` is used to enter the barrier barrier and notify that the worker is out of work. When such a worker steals work from a victim, it invokes `checkOut()` to leave the barrier. The barrier maintains a `checkoutCount` on the number of workers checked out. It is triggered when all the workers are in it. The action associated with the barrier is triggered and it signals that the computation has terminated.

The algorithm maintains the invariant:

$$(\#\text{workers - checkoutCount}) = \#(\text{workers that know they don't have work })$$

A worker knows it has no work if it stealing. Note that the checkoutCount is not always equal to the number of workers with work to do. In particular, consider a victim that finds the current frame as stolen. The victim cannot identify whether it has work without locking its deque. While it aborts, the thief has the stolen frame and could have invoked `checkOut()`. The barrier identifies both workers as having checked out even though there is one task between them. Note that allowing the victim to `checkIn()` when it identifies a steal would lead to the barrier being incorrectly triggered while the thief still has the stolen task but it yet to invoke `checkOut()`.

4.5 Phased Computations

We also added support for phased computations in which tasks in this phase create tasks to be executed in the next phase. The implementation of the breadth-first search algorithm proceeds one level at a time. The nodes processed at this level are used to determine the nodes to be processed in the next level.

Phased computations are supported as a generalization of global quiescence. Each worker maintains two stacks of frames, referred to as caches. Depending on the phase specified when spawning tasks, a task can be added to the current cache or the next cache, the cache for the next phase.

When global quiescence is detected for this phase, the barrier action invokes `advancePhase()` that steps the computation into the next phase. When a worker runs out of tasks, it checks that the current phase of the worker is the same as the global phase of the computation. If the phase of the computation has advanced further, the workers updates its phase information and swaps the current and next task collections.

Each worker specifies the number of tasks it has outstanding for the next phase when it invokes `checkIn()`. This information is used to identify if the next phase has any tasks left to be processed.

Note that the global phase could have advanced much further than the phase operated on by this worker. This would happen when this worker has no work for current phase and has checked-in notifying that it has not tasks for the next phase. The other workers could then progress multiple phases before this worker observes the computation progress.

When global quiescence for this phase is triggered, the number of workers with tasks for the next phase is known. The computation is said to have terminated when the current phase has quiesced and no worker has any task for the next phase.

For a given phase, maintaining the invariant mentioned above for global quiescence is more involved with multiple phases. For example, consider a worker advancing its phase to match the global phase of the computation and its next cache is non-empty. Since the worker now has local tasks in this phase, it implicitly checks out of the barrier.

4.6 Explicitly Partitioned Programs

While work-stealing provides a convenient abstraction for load-balancing fine-grained programs, the performance of certain applications can benefit from the explicitly partitioned programs that would exist as a typical parallel global address space program. For example, in the Shiloach-Vishkin algorithm we observe that explicit partitioning of the edges and vertices amongst the workers leads to better performance. This is achieved by having the task, called the job task, submitted by the driver thread spawns tasks, called worker tasks. The number of worker tasks spawned is equal to the number of workers in the system. The job task is improperly nested and returns upon spawning the worker tasks. Each of the spawned tasks now execute one part of the program. Note that these tasks are spawned and available at the worker that executed the job task. This worker now has work and it executes one of the worker tasks. The other workers identify the presence of work in this worker while stealing and steal a worker task. Assuming the worker tasks are sufficiently load-balanced, the rest of the computation proceeds without work-stealing with each worker executing a worker task.

4.7 Performance Analysis

In this section, we discuss the performance implications of the different components of the runtime.

The overheads of the fast version over the sequential execution are:

1. Method's frame needs to be allocated, initialized, pushed onto the deque. Cost: A few assembly instructions.

2. Method's state needs to be saved before each spawn. Cost: writes of local dirty variables and *PC*, and a `StoreLoadBarrier`

3. Method must check if its frame has been stolen after each return from a spawn. Cost: two reads, compare, branch and a `StoreLoadBarrier`

4. On return, frame must be freed. Cost: A few assembly instructions

5. An extra variable is needed to hold the frame pointer. Cost: Increased register pressure.

Note that average cost of allocating and de-allocating memory can be reduced to a couple of statements with an efficient concurrent memory allocation scheme. Thus the overhead in the fast path is very little, as is also demonstrated in the experimental evaluation.

The number of closures created and invocations of the slow version of an *async* is proportional to the number of successful steals. The number of locks requested is proportional to the numbers of attempted steals.

Improperly nested tasks take advantage of the lack of return value to avoid the creation of closures, further reducing overhead.

The computation is identified as terminated, the moment the last worker starts trying to steal. Thus there is no significant delay between the actual termination of the computation and its detection. The mechanism itself incurs the overhead of two atomic updates to a shared counter for every successful steal. The overheads incurred in supported phased execution are similar.

Explicitly partitioned programs begin execution with all the tasks on one worker. There is a delay before which all workers attempt to steal from this worker and obtain a worker task. Assuming a truly random scheme in the choice of the victim to steal from, the worst case in the number of steals before work is found is proportional to the number of workers. Since the worker tasks are load-balanced and there is no work-stealing after this initial delay, the number of steals in the worst case is independent of the program running time and problem size, and hence much smaller than that incurred in a typical work-stealing strategy. We observe this in our experimental evaluation as well.

Thus the common execution path in which all workers are busy involves little overhead. The remaining in other execution paths are proportional to the number of attempted and successful steals performed by the workers. The experimental evaluation section demonstrates that both are far lower than the number of *asyncs* spawned, effectively enabling load-balanced execution of fine-grained parallel programs.

5 Performance Evaluation

The performance numbers for handwritten C application and for the XWS code are given below, for psuedo-depth-first search, breadth-first search and the Shiloach Vishkin algorithm for spanning tree (from top to bottom). Numbers are presented for two machines, altair (containing 4 dual-core Opterons) on the left and moxie, a 32-processor Niagara on the right.

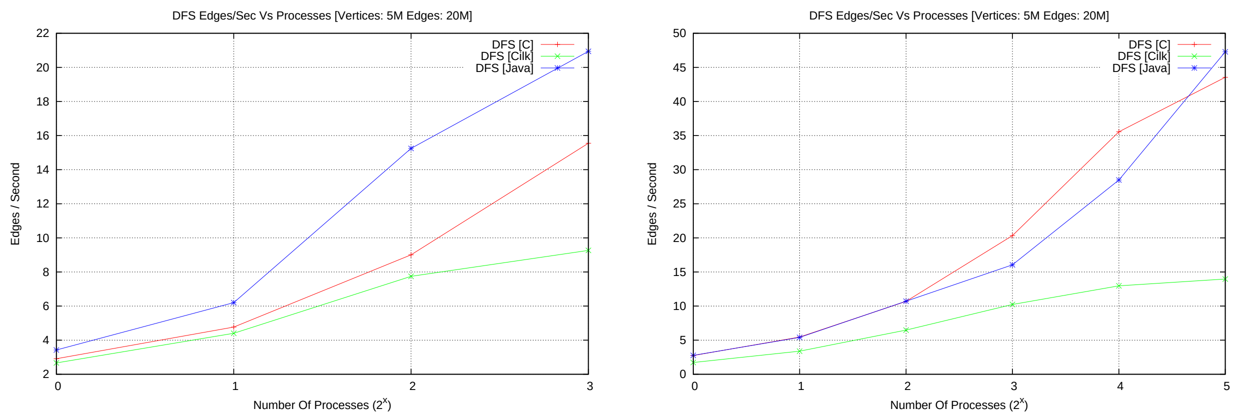


Figure 5: Psuedo-DFS for altair and moxie

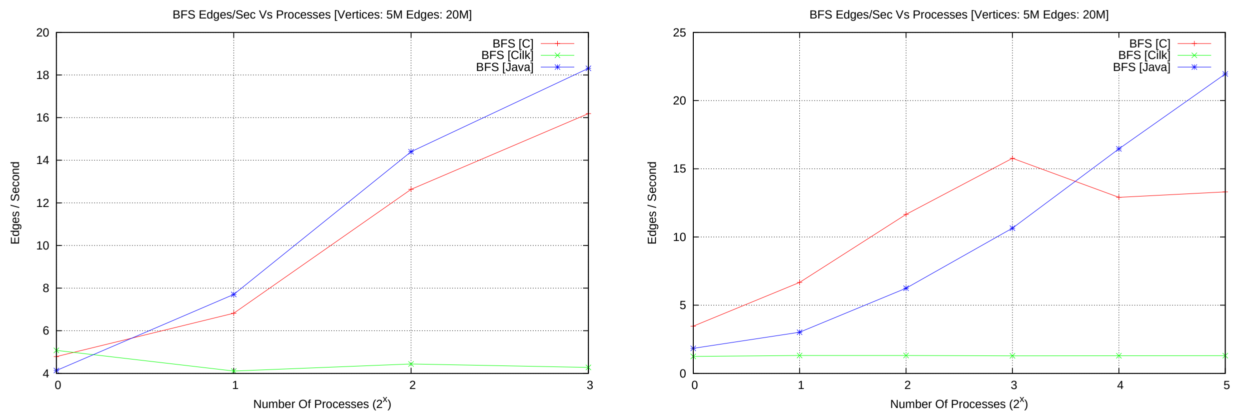


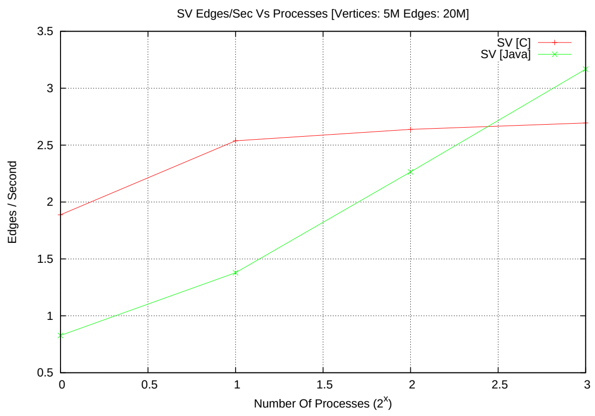
Figure 6: BFS for altair and moxie

The graphs show good scaling for the XWS version of applications on both machines. In all cases the numbers for XWS are better than that for the C code at higher processor counts.

Cilk does not perform well for BFS. (We were unable to get a Cilk version of SV running in time for the submission.)

6 Conclusion

In this paper we have shown how several graph algorithms can be expressed concisely and elegantly in X10. These algorithms rely heavily on support for fine-grained concurrency. The X10 runtime (XWS) implements fine-grained concurrency through an ehanced work-stealing scheduler. Specifically the scheduler supports improperly nested tasks, detection of global termination, and phased work-stealing. We measure the performance of spanning tree algorithms implemented with pseudo-depth-first search (DFS), breadth-first search (BFS) and a Shiloach-Vishkin algorithm on two multicore systems. We show that the XWS programs scale and exhibit performance comparable with hand-written C programs.



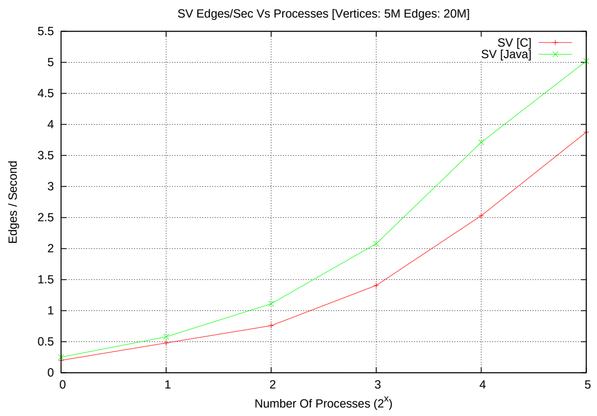


Figure 7: SV for altair and moxie

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