

Genericity through Dependent Types

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Abstract

We present general framework for *generic constrained types* that captures the notion of value-dependent and type-dependent (generic) type systems for object-oriented languages. Constraint systems formalize systems of partial information. Constrained types are formulas $C\{c\}$ where C is the name of a class or an interface and c is a constraint on the immutable state of an instance of C (the *properties*).

The basic idea is to formalize the essence of nominal object-oriented types as a constraint system, and to permit both value and type properties and parameters. Type-generic dependence is now expressed through constraints on these properties and parameters. Type-valued properties are required to have a run-time representation—the run-time semantics is not defined through erasure.

Many type systems for object-oriented languages developed over the last decade can be thought of as constrained type systems in this formulation. This framework is parametrized by an arbitrary constraint system C of interest. It permits the development of languages with pluggable type systems, and supports dynamic code generation to check casts at run-time.

The paper makes the following contributions: (1) We show how to accommodate generic object-oriented types within the framework of constrained types. (2) We illustrate the type system with the development of a formal calculus GenX10 and establish type soundness. (3) We discuss the design and implementation of the type system for X10, a modern object-oriented language, based on constrained types. The type system integrates and extends the features of nominal types, virtual types, and Scala’s path-dependent types, as well as representing generic types.

1. Introduction

todo: Awkward, repetitive

todo: More positioning, relative to: DML, HM(X), constrained types (Trifonov, Smith) and subtyping constraints, Java generics, GJ, PolyJ, C# generics, virtual types, liquid types

todo: Possible claim: first type system that combines genericity and dep types in some vague general way.

todo: Incorporate some text from OOPSLA paper on deptypes.

todo: Cite liquid types and whatever it cites

Modern object-oriented type systems provide many features to improve productivity by allowing programmers to express strong program invariants as types that are checked by the compiler, without sacrificing the ability to reuse code.

Examples include generics, self types, dependent types. We present a dependent type system that extends a class-based language with statically-enforced constraints on types and values. This type system supports several features of modern object-oriented

language through natural extensions of the core dependent type system: generic types, virtual types, and self types among them.

We have formalized the type system in an extension of Featherweight Java [12] and provide proof of soundness. The type system is parameterized on the constraint system. By augmenting the default constraint system, the type system can serve as a core calculus for formalizing extensions of a core object-oriented language.

This work is done in the context of the X10 programming language [22]. The key idea is to define *constrained types*, a form of dependent type defined on predicates over types and over the immutable state of the program.

In X10, objects may have both value members (fields) and type members. The immutable state of an object is captured by its *value properties*: public final fields of the object. For instance, the following class declares a two-dimensional point with properties x and y of type float:

```
class Point(x: float, y: float) { }
```

A constrained type is a type $C\{e\}$, where C is a class and e is a boolean predicate on the properties of C and the final variables in scope at the type. For example, given the above class definition, the type $\text{Point}\{x^2+y^2 < 1\}$ is the type of all points within the unit circle.

In previous work [22, 21], we considered only value properties. To support genericity these types are generalized to allow *type properties* and constraints on these properties. Like a value property, a type property is an instance member. The type properties of an object are bound to concrete types when the object is created. Types may be defined by constraining the type properties as well as the value properties of a class.

The following code declares a class `Cell` with a type property named `T`.

```
class Cell[T] {  
  var x: T;  
  def get(): T = x;  
  def set(x: T) = { this.x = x; }  
}
```

The class has a mutable field `x` of type `T`, and has `get` and `set` methods for accessing the field.

This example shows that type properties are in many ways similar to type parameters as provided in object-oriented languages such as Java and Scala and in functional languages such as ML and Haskell.

As the example illustrates, type properties are types in their own right: they may be used in any context a type may be used, including in `instanceof` and cast expressions. However, the key distinction between type properties and type parameters is that type properties are instance members. Thus, for an expression `e` of type `Cell`, `e.T` is a type, equivalent to the concrete type to which `T` was initialized when the object `e` was instantiated. To ensure soundness, `e` is restricted to final access paths. Within the body of a class, the unqualified property name `T` resolves to `this.T`.

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All properties of an object, both type and value, must be bound at object instantiation and are immutable once bound. Thus, the type property "T" of a given "Cell" instance must be bound to a concrete type by the constructor.

As with value properties, type properties may be constrained by predicates to produce new types. X10 supports equality constraints, written $T_1 = T_2$, and subtyping constraints, written $T_1 <: T_2$. For instance, the type `Cell{T==String}` is the type of all `Cells` that contain a `String`.

In general, the syntax of a constrained type is `C{c}`, where `C` is a base class and `c` is a predicate on the properties of `C`. For brevity, a constraint can be written as a comma-separated list of conjuncts; that is, the constraint `c1 && c2` can be written `c1, c2`.

Constraints on properties induce a natural subtyping relationship: `C{c}` is a subtype of `D{d}` if `C` is a subclass of `D` and `c` entails `d`.

1.1 Generics

Type properties and subtyping constraints may be used to provide genericity. Unlike existing proposals for generic types in Java-like languages [11, 25, 4, 19, 3, 23, 1, 2, 8, 9, 18], which are implemented via type erasure, our design supports run-time introspection of generic types.

Another problem with many of these proposals is inadequate support for primitive types, especially arrays. The performance of primitive arrays is critical for the high-performance applications for which X10 is intended. These proposals introduce unnecessary boxing and unboxing of primitives. Our design does not require primitives be boxed.

The `Cell` defined class above is a generic class. The type property `T` may be constrained with equality and subtyping constraints.

The type `Cell{T==float}` binds the type property `T` to `float`. Assuming `c` has this type, the following code is legal:

```
val x: float = c.get();
c.set(1.0);
```

The type of `c.get()` is `c.T`, which is equivalent to `float`. Similarly, the `set` method takes a `float` as argument.

Subtyping constraints enable use-site variance: The type `Cell{T<:int}` constrains `T` to be a subtype of `int`. All instances with this type must bind `T` to a subtype of `int`. The following expressions have this type:

```
new Cell[int](1);
new Cell[int{self==3}](3);
```

The cell in the first expression may contain any `int`. The cell in the second expression may contain only 3. If `c` has the type `Cell{T<:int}`, then `c.get()` has the abstract type `c.T`, which is an unknown but fixed subtype of `int`. The `set` method of `c` can only be called with an object of type `c.T`.

Types properties may also be constrained contravariantly.

Subtyping constraints provide similar expressive power as Java wildcards. We describe an encoding of wildcards in Section ??.

todo: move this para Type and value constraints are also used to support generalized constraints [9], optional methods [?] and optional interfaces.

1.2 Self types

Type properties can also be used to support self types [5, 6].

We introduce a type property `class` to the root of the class hierarchy, `Object`:

```
class Object[class] { ... }
```

The class invariant for all classes `C` implies `this.class<: C`.

1.3 Virtual types

Type properties share many similarities with virtual types [15, 14], particularly with sound formulations of virtual types using path-dependent types, as found in `gbeta` [10], `Scala` [18], and `J&` [17]. Constrained types are more expressive than virtual types since they can be constrained at the use-site, can be refined on a per-object basis without explicit subclassing, and can be refined contravariantly as well as covariantly.

Thorup [23] proposed adding genericity to Java using virtual types. For example, a generic `List` class can be written as follows:

```
abstract class List {
  abstract typedef T;
  void add(T element) { ... }
  T get(int i) { ... }
}
```

This class can be refined by bounding the virtual type `T` above:

```
abstract class NumberList extends List {
  abstract typedef T as Number;
}
```

And this abstract class can be further refined to *final bind* `T` to a particular type:

```
class IntList extends NumberList {
  final typedef T as Integer;
}
```

These classes are related by subtyping: `IntList<: NumberList<: List`.

Only classes where `T` is final bound can be non-abstract.

In X10, an analogous `List` class would be written as follows:

```
class List[T] {
  def add(element: T) = { ... }
  def get(i: int): T = { ... }
}
```

`NumberList` and `IntList` can be written as follows:

```
class NumberList extends List{T<:Number} { }
class IntList extends NumberList{T==Integer} { }
```

However, note that X10's `List` is not abstract. Instances of `List` can instantiate `T` with a particular type and there is no need to declared classes for `NumberList` and `IntList`. Instead, one can use the types `List[+Number]` and `List[Integer]`.

Unlike virtual types, type properties can be refined contravariantly. For instance, one can write the type `List[-Integer]`, and even `List{Integer<:T, T<:Number}`.

1.4 Structural constraints

Type constraints need not be limited to subtyping constraints. By introducing structural constraints on types, GenX10 allows type properties to be instantiated on any type with a given set of methods and fields. This feature is useful for reusing code in separate libraries since it does not require code of one library to implement an interface to satisfy a constraint of another library.

1.5 Outline

The rest of the paper...

2. X10

Before describing the generic type system, we present an overview of X10's syntax and semantics. X10 is a class-based language similar to Java or Scala. Superficially, the language may be thought of as sequential Java with some elements of Scala syntax and

with new constructs for concurrency and distribution. Like Java, the language provides both classes and interfaces; it does not yet support traits, as found in Scala.

Both classes and interfaces may define properties. Value properties may be considered to be public final fields. Whereas Java supports only static fields in interfaces, X10 allows interfaces to define value properties. Any class implementing an interface must declare, and initialize in its constructor, the properties inherited from the interface.

Classes may define fields, methods, and constructors. The declaration syntax, illustrated in the `Cell` example above, is similar to Scala's. Fields may be declared either `val` or `var`. A `val` is *final* and must be assigned exactly once. Methods are declared with a `def` keyword. As in Java, methods may be declared `static`, however fields cannot. Constructor syntax is similar to method syntax and X10 adopts Scala's convention of using the name `this` for constructors. In X10, constructors have a return type, which constrains the properties of the new object.

3. Generics

Type properties and subtyping constraints may be used to provide genericity. Unlike existing proposals for generic types in Java-like languages [11, 25, 4, 19, 3, 23, 1, 2, 8, 9, 18], which are implemented via type erasure, our design supports run-time introspection of generic types.

Another problem with many of these proposals is inadequate support for primitive types, especially arrays. The performance of primitive arrays is critical for the high-performance applications for which X10 is intended. These proposals introduce unnecessary boxing and unboxing of primitives. Our design does not require primitives be boxed.

The `Cell` defined class above is a generic class. The following are legal types:

- `Cell`. This type has no constraints on `T`. Any type that constrains `T`, those below, is a subtype of `Cell`. The type `Cell` is equivalent to `Cell{true}`. For a `Cell c`, the return type of the `get` method is `c.T`. Since the property `T` is unconstrained, the caller can only assign the return value of `get` to a variable of type `c.T` or of type `Object`. In the following code, `y` cannot be passed to the `set` method because it is not known if `Object` is a subtype of `c.T`.

```
val x: c.T = c.get();
val y: Object = c.get();
c.set(x); // legal
c.set(y); // illegal
```

- `Cell{T==float}`. The type property `T` is bound to `float`. Assuming `c` has this type, the following code is legal:

```
val x: float = c.get();
c.set(1.0);
```

The type of `c.get()` is `c.T`, which is equivalent to `float`. Similarly, the `set` method takes a `float` as argument.

- `Cell{T<:int}`. This type constrains `T` to be a subtype of `int`. All instances of this type must bind `T` to a subtype of `int`. The following expressions have this type:

```
new Cell[int](1);
new Cell[int{self==3}](3);
```

The cell in the first expression may contain any `int`. The cell in the second expression may contain only 3. If `c` has the type `Cell{T<:int}`, then `c.get()` has type `c.T`, which is an

unknown but fixed subtype of `int`. The `set` method of `c` can only be called with an object of type `c.T`.

- `Cell{T:>String}`. This type bounds the type property `T` from below. The `set` method may be called with any supertype of `String`; the return type of the `get` method is known to be a supertype of `String` (and implicitly a subtype of `Object`).

For brevity, the constraint may be omitted and interpreted as `true`. The syntax `C[T1,...,Tm](e1,...,en)` is sugar for `C{X1==T1,...,Xm==Tm,x1==e1,...,xn==en}` where `Xi` are the type properties and `xi` are the value properties of `C`. If either list of properties is empty, it may be omitted.

In this shortened syntax, a type argument `T` used may also be annotated with a *use-site variance tag*, either `+` or `-`: if `X` is a type property, then the syntax `C[+T]` is sugar `C{X<:T}` and `C[-T]` is sugar `C{X>T}`; of course, `C[T]` is sugar `C{X==T}`.

3.1 Subtyping

`C{c}` is a subtype of `D{d}` if `C` is a subclass of `D` and `c` entails `d`.

3.2 Use-site variance

3.3 Class declarations

Classes may be declared with any number of type properties and value properties. These properties can be constrained with a *class invariant*, specified by a `where` clause, a predicate on the properties of any instance of the class. The general form of a class definition is:

```
class C[X1, ..., Xp](x1: T1, ..., xk: Tk)
  where c
  extends B{c0}
  implements I1{c1}, ..., In{cn} {...}
```

3.4 Definition-site variance

In a class definition, a type property may be declared with a *definition-site variance tag*, either `+` or `-`. A `+` tag indicates that the class is covariant on the property; that is, given a definition `Cell[+T]`, if `A <: B`, then `Cell[A] <: Cell[B]`. Similarly, `Cell[-T]` indicates that `T` is contravariant in `Cell`; that is, if `A <: B`, then `Cell[B] <: Cell[A]`.

A definition-site variance tag changes the meaning of the syntactic sugar for the type `Cell[A]`. If the property is covariant (i.e., is declared as `+T`), `Cell[A]` is sugar for `Cell{T<:A}`. If the property is contravariant (`-T`), then `Cell[A]` is sugar for `Cell{T>A}`. Otherwise, the property is invariant and `Cell[A]` is sugar for `Cell{T==A}`.

The compiler should issue a warning if a covariant property is used in a negative position (e.g., in a method parameter type) in its class definition, or if a contravariant property is used in a positive position (e.g., in a method return type). Without these restrictions, methods or fields with types dependent on the property would be safe, but not be accessible using the default instantiation (e.g., `Cell[int]`).

3.5 Class invariant

3.6 Method parameters

Methods and constructors may have type parameters. For instance, the `List` class below defines a `map` method that maps each element of a list of `T` to a value of another type `S`, constructing a new list of `S`.

```
class List[T] {
  val array: Array[T];
  def map[S](f: T => S): List[S] {
```

```

    val newArray = new Array[S](array.length);
    for (i in [0:array.length-1]) {
        newArray(i) = f(array(i));
    }
    return new List(newArray);
}
}

```

A parameterized method can be invoked by giving type arguments before the expression arguments. For example, the following code takes a list of Strings and returns a list of string lengths of type `int`

```

xs: List[String] = ...;
ys: List[int] = xs.map[int](
    (x: String) => x.length());

```

3.7 Method where clauses

Method and constructor parameters, both value parameters and type parameters, can be constrained with a `where` clause on the method. For type parameters, this feature is similar to generalized constraints proposed for C# [9]. In the following code, the `T` parameter is covariant and so the `append` methods below are illegal:

```

class List[+T] {
    def append(other: T): List[T] = { ... }
    // illegal
    def append(other: List[T]): List[T] = { ... }
    // illegal
}

```

However, one can introduce a method parameter and then constrain the parameter from below by the class's parameter: For example, in the following code,

```

class List[+T] {
    def append[U](other: U)
        {T <: U}: List[U] = { ... }
    def append[U](other: List[U])
        {T <: U}: List[U] = { ... }
}

```

The constraints must be satisfied by the callers of `append`. For example, in the following code:

```

xs: List[Number];
ys: List[Integer];
xs = ys; // ok
xs.append(1.0); // legal
ys.append(1.0); // illegal

```

the call to `xs.append` is allowed and the result type is `List[Number]`, but the call to `ys.append` is not allowed because the caller cannot show that `Number <: Double`.

3.8 Optional methods and interfaces

Method where clauses also provide support for optional methods.

```

class List[T] {
    ...
    def print(){T <: Printable} = {
        for (x: T in this)
            x.print();
    }
}

```

`List.print` may only be called on lists instantiated on subtypes of the `Printable` interface.

Optional methods generalize to optional interfaces.

```

class List[T] implements {T <: Printable} Printable {
    ...
    def print(){T <: Printable} = {
        for (x: T in this)
            x.print();
    }
}

```

In this case `List` implements the `Printable` interface only if `List.T` implements `Printable`. Thus `List{T==String}` and `List{T==List[String]}` are subtypes of `Printable`, but `List{T==DirtyWord}` is not.

3.9 Method overriding

Legal if any call to super method can call sub method.

covariant return contravariant args weaker where clause

3.10 Constructor definitions

Constructors are defined using the syntax `def this`, as shown in Figure 1. Constructors must ensure that all properties of the new object are initialized and that the class invariants of the object's class and its superclasses and superinterfaces hold.

Properties are initialized with a `property` statement. For instance, the constructor for `Cell` ensures that the type property `T` is bound.

```

def this[T](x: T) =
    { property[T](); this.x = x; }

```

The `property` statement is used to set all the properties of the new object simultaneously; the syntax is similar to a `super` constructor call.

If the `property` statement is omitted, the compiler implicitly initializes the properties from the formal type and value parameters of the constructor. The `property` statement for `Cell`'s constructor, for example, could have been omitted.

Constructors have “return types” that can specify an invariant satisfied by the object being constructed. The compiler verifies that the constructor return type and the class invariant are implied by the `property` statement and any `super` or `this` calls in the constructor body.

Classes that do not declare a constructor have a default constructor with a type parameter for each type property and a value parameter for each value property.

4. Formal semantics

We present a core calculus, `GenX10`, for `X10` with generics. `GenX10` is based on Constrained Featherweight Java [21].

todo: Add method overriding rules: covariant return, contravariant args, weaker constraints

The grammar for `GenX10` is shown in Figure 1. The calculus elides features of the full `X10` language not relevant to this paper.

Figure 2 extends the grammar with syntactic sugar for subtyping constraints and existential types. The subtyping constraint $t_1 <: t_2$ is atomic formula. The existential type $\exists x: T. R\{c\}$ is sugar for $R\{\exists x. \sigma(x:T), c\}$.

We assume a fixed but unknown constraint system \mathcal{D} . A program P is written using constraints from \mathcal{D} . We assume classes defined in P do not have a cyclic inheritance structure.

$$\frac{\text{extends}^+ \text{ acyclic}}{\vdash \bar{L} \text{ ok}} \quad (\text{PROGRAM OK})$$

¹ A list of length 7, of course.

program	P	::=	\bar{L}
classes	L	::=	$\text{class } C[\bar{X}] (\bar{x}:\bar{T}) \{c\}$ $\text{extends } T \{ \bar{M} \}$
base types	R		
classes		::=	C
type variables			X
type members			e.X
type type			type
types	T	::=	R{c}
methods	M	::=	def m[\bar{X}] ($\bar{x}:\bar{T}$) {c}: T = e
expressions	e		
literals		::=	true false null n
variables			x
field access			e.x
call			e ₀ .m[\bar{T}] (\bar{e})
new			new C[\bar{T}] (\bar{e})
cast			e as T
constraint terms	t		
self		::=	self
variables			x
properties			t.x
atoms			g(t ₁ , ..., t _n)
new			new C(t ₁ , ..., t _n)
constraint	c		
true		::=	true
equality			t ₁ == t ₂
existentials			$\exists x. c$
conjunction			\bar{c}
predicates			p(t ₁ , ..., t _n)
environments	Γ	::=	ε Γ, c $\Gamma, x:T$ $\Gamma, X:\text{type}$

Figure 1. GenX10 grammar

types	T	::=	\dots $\exists x:T_0. T$ $\exists X:\text{type}. T$
constraint terms	t	::=	\dots $\text{true} \mid n \mid C$
literals			
type variables			X
type properties			t.X
constraint	c	::=	\dots $t_1 <: t_2$ $\text{cons}(T, z)$

Figure 2. GenX10 grammar with subtyping constraints

4.1 The object constraint system, O

From P and \mathcal{D} we generate an *object constraint system* O , shown in Figure 3, as follows. Let C and D range over names of classes in P , f over field names, m over method names, T over types, and c over constraints in the underlying data constraint system \mathcal{D} .

In the method signature $\bar{X}(\bar{x}:\bar{T})\{c\}$, the type variables \bar{X} and data variables \bar{x} are considered bound; formulas with bound variables are considered equivalent upto α -renaming.

The constraint system satisfies the axioms and inference rules in Figure 3. The *class*, *extends*, *fields*, and *mtype* constraints are given directly from the program P .

constraint	c	::=	$\text{class}(C)$ $C \text{ extends } D$ $\text{fields}(x, \bar{f}:\bar{T})$ $\text{mtype}(x, m, [\bar{X}(\bar{x}:\bar{T})\{c\}])$
<hr/>			
$\frac{\text{class } C[\bar{X}] (\bar{f}:\bar{T})\{c\} \text{ extends } D\{d\} \{ \bar{M} \} \in P}{\begin{array}{l} \vdash_O \text{class}(C) \\ \vdash_O C \text{ extends } D \\ \vdash_O C \text{ extends } D \end{array}}$			
<hr/>			
$\frac{\text{class } C[\bar{X}] (\bar{f}:\bar{T})\{c\} \text{ extends } \text{Object} \{ \bar{M} \} \in P}{\Gamma, z:C\{d\} \vdash_O \text{fields}(z, \bar{f}:\bar{T})} \text{ (FIELDS)}$			
<hr/>			
$\frac{\begin{array}{l} \vdash_O C \text{ extends } D \\ \text{class } C[\bar{X}] (\bar{f}:\bar{T})\{c\} \text{ extends } D\{d\} \{ \bar{M} \} \in P \\ \Gamma, z:D\{d\} \vdash_O \text{fields}(z, \bar{f}_0:\bar{T}_0) \end{array}}{\Gamma, z:D\{d\} \vdash_O \text{fields}(z, \bar{f}_0:\bar{T}_0, \bar{f}:\bar{T})} \text{ (FIELDS-EXTENDS)}$			
<hr/>			
$\frac{\begin{array}{l} \text{class } C[\bar{X}] (\bar{f}:\bar{T})\{c\} \text{ extends } \text{Object} \{ \bar{M} \} \in P \\ M_i = \text{def } m_i[\bar{X}] (\bar{x}:\bar{T})\{c\}: T = e \end{array}}{\Gamma, z:C\{d\} \vdash_O \text{mtype}(z, m_i, [\bar{X}(\bar{x}:\bar{T})\{c\}] \rightarrow T)} \text{ (MTYPE)}$			

Figure 3. The constraint system O

The constraint system C is the disjoint conjunction \mathcal{D} , O of the constraint systems \mathcal{D} and O . (This requires the assumption that \mathcal{D} does not have any constraints in common with O .)

4.2 Structural and logical rules

All judgments are intuitionistic. In particular, this means that all constraint systems satisfy the rules and axioms in Figure 4.

4.3 Well-formedness rules

We use the judgment for well-typedness for expressions to represent well-typedness for constraints. That is, we posit a special type \mathbf{o} (traditionally the type of propositions), and regard constraints as expressions of type \mathbf{o} .

Further, we change the formulation slightly so that there are no constraints of the form $p(t_1, \dots, t_n)$; rather instance method invocation syntax is used to express invocation of pre-defined constraints. This logically leads to the step of simply marking certain classes as “predicate” classes—all the (instance) methods of these classes whose return type is \mathbf{o} then correspond to “primitive constraints”. Syntactically, we continue to use the symmetric syntax $p(t_1, \dots, t_n)$ rather than $t_1.p(t_2, \dots, t_n)$. The alternative is to introduce static methods and static method invocations in the expression language. This is not difficult, but is annoying to have to repeat most of the formulation of instance methods.

This means that the only cases left to handle are all the simple ones, expression the availability of certain constraints and operations of type \mathbf{o} .

4.4 Subtyping constraints

4.5 Constraint projection

First, for a type environment Γ , we define the *constraint projection*, $\sigma(\Gamma)$ thus:

$\Gamma, c \vdash c$	(ID)
$\frac{\Gamma \vdash c \quad \Gamma, c \vdash d}{\Gamma \vdash d}$	(CUT)
$\frac{\Gamma \vdash \phi \quad \Gamma \vdash T : \text{type} \quad x \notin \text{var}(\Gamma)}{\Gamma, x : T \vdash \phi}$	(WEAK-1)
$\frac{\Gamma \vdash \phi \quad \Gamma \vdash c : o}{\Gamma, c : \phi}$	(WEAK-2)
$\frac{\Gamma, \psi_0, \psi_1 \vdash \phi}{\Gamma, (\psi_0, \psi_1) \vdash \phi}$	(AND-L)
$\frac{\Gamma \vdash \psi_0 \quad \Gamma \vdash \psi_1}{\Gamma \vdash (\psi_0, \psi_1)}$	(AND-R)
$\frac{\Gamma \vdash \phi[t/x]}{\Gamma \vdash_C \exists x. \phi}$	(EXISTS-R)
$\frac{\Gamma, x : T, \psi \vdash \phi \quad x \text{ fresh}}{\Gamma, \exists x : T. \psi \vdash \phi}$	(EXISTS-L)

Figure 4. Logical rules

$\Gamma \vdash \text{true} : o$	(TRUE)
$\frac{\Gamma \vdash t_0 : T_0 \quad \Gamma \vdash t_1 : T_1 \quad (\Gamma \vdash T_0 <: T_1 \vee \Gamma \vdash T_1 <: T_2)}{\Gamma \vdash t_0 == t_1 : o}$	(EQUALS)
$\frac{\Gamma \vdash c_0 : o \quad \Gamma \vdash c_1 : o}{\Gamma \vdash (c_0, c_1) : o}$	(AND)
$\frac{\Gamma \vdash t : T \quad \Gamma \vdash c[t/x] : o}{\Gamma \vdash \exists x : T. c : o}$	(SOME)
$\frac{\Gamma \vdash \text{class}(C) \quad \Gamma, \text{self} : C \vdash c : o}{\Gamma \vdash C\{c\} : \text{type}}$	(TYPE)

Figure 5. Well-formedness rules

$$\begin{aligned}
\sigma(\varepsilon) &= \text{true} \\
\sigma(\Gamma, x : T) &= \sigma(\Gamma), \text{cons}(T, x) \\
\sigma(\Gamma, c) &= \sigma(\Gamma), c
\end{aligned}$$

The auxiliary function *cons* specifies the constraint for a type *T* with *self* bounds to *x*. The constraint projection uses an atomic formula *cons*, which is equated to the constraint of *T* if *T* is not a type variable.

$\frac{\bar{c} \vdash_C T_1 == T_2}{\bar{c} \vdash_C \text{cons}(T_1, z) == \text{cons}(T_2, z)}$	(CONS-EQ)
$\frac{\bar{c} \vdash_C T_1 <: T_2}{\bar{c}, \text{cons}(T_1, z) \vdash_C \text{cons}(T_2, z)}$	(CONS-SUB)
$\vdash_C \text{cons}(C, z)$	(CONS)
$\frac{\bar{c} \vdash_C \bar{s} == \bar{t}}{\bar{c} \vdash_C f(\bar{s}) == f(\bar{t})}$	(EQ-ATOM)
$\bar{c} \vdash_C t == t$	(EQ-REFL)
$\frac{\bar{c} \vdash_C t_1 == t_2 \quad \bar{c} \vdash_C t_2 == t_3}{\bar{c} \vdash_C t_1 == t_3}$	(EQ-TRANS)
$\frac{\bar{c} \vdash_C t_1 == t_2 \quad \bar{c} \vdash_C t_2 == t_1}{\bar{c} \vdash_C t_1 == t_1}$	(EQ-SYM)
$\frac{\bar{c} \vdash_C T_1 <: T_2 \quad \bar{c} \vdash_C T_2 <: T_1}{\bar{c} \vdash_C T_1 == T_2}$	(EQ-SUB)

$$\frac{\bar{c} \vdash_C C\{c\} : \text{type} \quad \bar{c}, c \vdash_C d}{\bar{c} \vdash_C C\{c\} <: C\{d\}} \quad (\text{SUB-CONS})$$

$$\frac{C[\bar{X}] (\bar{x} : \bar{T}) \{c\} \text{ ext } T \{ K \bar{M} \bar{F} \}}{\vdash_C C <: T} \quad (\text{SUB-SUPER})$$

$$\vdash_C T <: \text{Object} \quad (\text{SUB-OBJECT})$$

$$\frac{\bar{c} \vdash_C T_1 == T_2}{\bar{c} \vdash_C T_1 <: T_2} \quad (\text{SUB-EQ})$$

$$\frac{\bar{c} \vdash_C T_1 <: T_2 \quad \bar{c} \vdash_C T_2 <: T_3}{\bar{c} \vdash_C T_1 <: T_3} \quad (\text{SUB-TRANS})$$

Figure 6. Equality and subtyping rules

$$\begin{aligned}
\text{cons}(C, z) &= \text{cons}(C, z) \\
\text{cons}(C\{c\}, z) &= c[z/\text{self}], \text{cons}(C\{c\}, z) == c[z/\text{self}] \\
\text{cons}(p.X, z) &= \text{cons}(p.X, z) \\
\text{cons}(X, z) &= \text{cons}(X, z)
\end{aligned}$$

Thus, for example, the constraint projection of the environment:

$$b : D, \quad a : C\{\text{self}.X == D\{d\}, \text{self}.Y <: b.Z\}$$

is:

$$a.X == D\{d\}, \quad a.Y <: b.Z$$

$$\frac{C[\bar{X}](\bar{x}:\bar{T})\{c\} \text{ ext } T \{ \bar{M} \bar{F} \}}{\vdash C:\text{type}}$$

$$\frac{\Gamma \vdash T:\text{type} \quad \Gamma, \text{self}:T \vdash c:\text{Boolean} \quad \sigma(\Gamma) \vdash_C c \text{ OK}}{\Gamma \vdash T\{c\}:\text{type}}$$

$$\frac{\Gamma \vdash p:T \quad \Gamma \vdash T \text{ has } X}{\Gamma \vdash p.X:\text{type}}$$

$$\Gamma, X:\text{type} \vdash X:\text{type}$$

Figure 7. Type well-formedness

$$\frac{C[\bar{X}](\bar{x}:\bar{T})\{c\} \text{ ext } T \{ \bar{K} \bar{M} \bar{F} \}}{\begin{array}{l} \vdash C \text{ has } \bar{K} \\ \vdash C \text{ has } \bar{X}_i \\ \vdash C \text{ has } \bar{x}_i:T_i \\ \vdash C \text{ has } \bar{M}_i \\ \vdash C \text{ has } \bar{F}_i \end{array}} \text{ (HAS-CLASS)}$$

$$\frac{Z \neq K \quad \Gamma \vdash T_1 \text{ has } Z \quad \sigma(\Gamma) \vdash T_2 <: T_1}{\Gamma \vdash T_2 \text{ has } Z} \text{ (HAS-SUB)}$$

Figure 8. Structural constraints

4.6 Type well-formedness

4.7 Type inference rules

4.7.1 Constraint rules

4.7.2 Expression typing judgment

The cast rule T-CAST requires that the cast type be well-formed.

The field access rule T-FIELD differs from the rule in the paper in that there is no need to substitute a fresh variable for the receiver. Note that `this` may be free in S —that would be a reference to the current object in the code in which $e.f$ occurs, not a reference to the receiver of the $e.f$ field selection (i.e., the object obtained by evaluating e).

if we allow adding constraints to arbitrary types—do we?

TODO: type parameters!

Now we consider the rule for method invocation. Assume that in a type environment Γ the expressions e_0, \dots, e_n have the types T_0, \dots, T_n . Since the actual values of these expressions are not known, we shall assume that they take on some fixed but unknown values z_0, \dots, z_n of types T_0, \dots, T_n . Now, for z_0 as receiver, let us assume that the type T_0 has a method named m with signature $[\bar{Z}](\bar{z}:\bar{S})\{c\} \rightarrow U$ (Let $T_0 = C\{d\}$. If there is no method named m for the class C then this method invocation cannot be type-checked. Without loss of generality, we may assume that the type parameters of this method are named Z_1, \dots, Z_k , and the value parameters are named z_1, \dots, z_n since we are free to choose variable names as we wish.) Now, for the method to be invocable, it must be the case that the types T_1, \dots, T_n are subtypes of S_1, \dots, S_n . (Note that there may be no occurrences of `this` in S_1, \dots, S_n —they have been replaced by z_0 .) Further, it must be the case that for these parameter values, the constraint c is entailed. Given all these assumptions it must be the case that the return type is U , with all the parameters z_0, \dots, z_n existentially quantified.

$$\frac{\Gamma \vdash e:S \quad \sigma(\Gamma) \vdash_C S <: T \quad \Gamma \vdash T:\text{type}}{\Gamma \vdash e:T} \text{ (T-SUB)}$$

$$\begin{array}{l} \vdash \text{true}:\text{Boolean}\{\text{self}==\text{true}\} \\ \vdash \text{false}:\text{Boolean}\{\text{self}==\text{false}\} \end{array} \text{ (T-BOOL)}$$

$$\vdash n:\text{Int}\{\text{self}==n\} \text{ (T-INT)}$$

$$\frac{\Gamma \vdash e_1:T_1 \quad \Gamma \vdash e_2:T_2}{\Gamma \vdash e_1 == e_2 : \exists z_1:T_1, z_2:T_2. \text{Boolean}\{\text{self}==(z_1==z_2)\}} \text{ (T-EQ)}$$

$$\frac{\Gamma \vdash T_1:\text{type} \quad \Gamma \vdash T_2:\text{type}}{\Gamma \vdash T_1 == T_2 : \text{Boolean}} \text{ (T-TEQ)}$$

$$\frac{\Gamma \vdash T_1:\text{type} \quad \Gamma \vdash T_2:\text{type}}{\Gamma \vdash T_1 <: T_2 : \text{Boolean}} \text{ (T-TSUB)}$$

$$\Gamma, x:T \vdash x:T \text{ (T-VAR)}$$

$$\frac{\Gamma \vdash e:S \quad \Gamma \vdash T:\text{type}}{\Gamma \vdash e \text{ as } T:T} \text{ (T-CAST)}$$

$$\frac{\begin{array}{l} \Gamma \vdash e:T \\ T \text{ has } f\{c\}:U \\ \sigma(\Gamma, \text{this}:T) \vdash_C c \end{array}}{\Gamma \vdash e.f : \exists \text{this}:T. U\{\text{self}==\text{this}.f\}} \text{ (T-FIELD)}$$

$$\frac{\begin{array}{l} \Gamma \vdash e_0:T_0 \quad \Gamma \vdash \bar{e}:\bar{T} \\ T_0 \text{ has def } m[\bar{X}](\bar{x}:\bar{S})\{c\}:U = e \\ \Gamma' = \Gamma, \bar{x}:\text{type}, \text{this}:T_0, \bar{x}:\bar{T}, \bar{x}==\bar{V} \\ \sigma(\Gamma') \vdash_C c \\ \sigma(\Gamma') \vdash_C \bar{T} <: \bar{S} \end{array}}{\Gamma \vdash e_0.m[\bar{V}](\bar{e}) : \exists \bar{x}:\text{type}, \text{this}:T_0, \bar{x}:\bar{T}. U} \text{ (T-INVK)}$$

$$\frac{\begin{array}{l} \Gamma \vdash \bar{e}:\bar{T} \\ C \text{ has def } \text{this}[\bar{X}](\bar{x}:\bar{S})\{c\}:U = \dots \\ \Gamma' = \Gamma, \bar{x}:\text{type}, \text{this}:C, \bar{x}:\bar{T}, \bar{V}==\bar{X} \\ \Gamma'' = \Gamma, \bar{x}:\text{type}, \text{this}:U, \bar{x}:\bar{T}, \bar{V}==\bar{X} \\ \sigma(\Gamma') \vdash_C c \\ \sigma(\Gamma') \vdash_C \bar{T} <: \bar{S} \\ \sigma(\Gamma'') \vdash_C \text{inv}(C), \end{array}}{\Gamma \vdash \text{new } C[\bar{V}](\bar{e}) : \exists \bar{x}:\text{type}, \text{this}:C, \bar{x}:\bar{T}. U} \text{ (T-NEW)}$$

Figure 9. Typing rules

4.7.3 Class OK judgment

The following rule is modified from what we had in the paper to ensure that all the types are well-formed (under the assumption $\text{this } C$). Note that the variables \bar{x} are permitted to occur in the types T_0, \bar{T} , hence their typing assertions must be added to Γ .

$$\frac{\begin{array}{c} \Gamma = \text{this} : C \{ \text{self} == \text{this}, \text{inv}(C) \}, \bar{x} : \bar{T} \{ \text{self} == \} \bar{x} \}, c \\ \Gamma \vdash e : U \\ \sigma(\Gamma) \vdash_C U < : T \end{array}}{\text{def } m[\bar{X}] (\bar{x} : \bar{T}) \{ c \} : T = e \text{ OK in } C} \quad (\text{METHOD OK})$$

This rule did not exist in our submission. This is necessary to ensure that the types of fields are well-formed.

$$\frac{\text{this} : C, c \vdash T : \text{type}}{\text{val } f \{ c \} : T \text{ OK in } C} \quad (\text{FIELD OK})$$

This rule is now modified to ensure that all the types and methods in the body of the class are well-formed.

$$\frac{\begin{array}{c} K \text{ OK in } C \\ \bar{M} \text{ OK in } C \\ \bar{F} \text{ OK in } C \\ \text{this} : C \vdash T : \text{type} \end{array}}{C[\bar{X}] (\bar{x} : \bar{T}) \{ c \} \text{ ext } T \{ K \bar{M} \bar{F} \} \text{ OK}} \quad (\text{CLASS OK})$$

TODO: method overriding

4.7.4 Subtype judgment

$$\frac{\sigma(\Gamma) \vdash_C T_1 < : T_2}{\Gamma \vdash T_1 < : T_2}$$

5. Odds and ends

static methods cannot mention T

interfaces can have static methods; a property can implement I , allowing $T.m()$ static calls

Extensions: structural constraints, optional methods, interfaces enabled flag trick

6. Constraint solver

The goal of the constraint solver is to check an assertion $\bar{c} \vdash_C d$.

We add the following rules to allow type arguments to calls to be omitted.

$$\frac{\begin{array}{c} \bar{Y} \text{ fresh} \\ \Gamma, \bar{Y} : \text{type} \vdash e_0.m[\bar{Y}] (\bar{e}) : T \end{array}}{\Gamma \vdash e_0.m(\bar{e}) : T} \quad (\text{T-INVK-INFERRED})$$

$$\frac{\begin{array}{c} \bar{Y} \text{ fresh} \\ \Gamma, \bar{Y} : \text{type} \vdash \text{new } C[\bar{Y}] (\bar{e}) : T \end{array}}{\Gamma \vdash \text{new } C(\bar{e}) : T} \quad (\text{T-NEW-INFERRED})$$

6.1 Constraint representation

Represent a constraint as a graph G . Each node represents a constraint term for a value or a type. The node for a path p is written v_p ; the node for a type T is written V_T . There are four kinds of edges:

1. undirected equivalence edges, $v_p \sim v_q$ and $V_S \sim V_T$,

2. type edges, $v_p \mapsto_{\text{type}} V_T$,
3. tree edges, $v_p \mapsto_f v_{p.f}$ and $v_p \mapsto_X V_{p.X}$, and
4. flow edges, $V_S \rightarrow V_T$.

First, each constraint term is mapped to a node in the graph as follows. Associate each term t with a node v_t . For each access path $p.x$, add a tree edge $v_p \mapsto_x v_{p.x}$. For each path type $p.X$, add a tree edge $v_p \mapsto_X V_{p.X}$. For each atomic formula $f(\bar{t})$, add the tree edge $v_{f(\bar{t})} \mapsto_i v_{t_i}$ for all i . If term t has type T , add $v_t \mapsto_{\text{type}} V_{T.\text{type}}$ and add $V_T \sim V_{T.\text{type}}$ to G .

Type nodes are sets of classes.

Next, constraints are incorporated into the graph:

- For constraint $p==q$, add $v_p \sim v_q$ to G .
- For constraint $S==T$, add $V_S \sim V_T$ to G .
- For constraint $S<:T$, add $V_S \rightarrow V_T$ to G .

6.2 Solving

A flow-path is a path that follows flow and equivalence edges only.

A type-path is a path that follows type and equivalence edges only.

Now, we saturate: If there is a type-path $v_t \mapsto_{\text{type}}^* V_{C\{c\}}$, add $c[t/\text{self}]$ to the worklist.

Can saturate lazily when doing a lookup. EXCEPT: a type may have an arbitrary constraint $C\{\text{self}.x==3 \ \&\& \ y > 7\}$, so affect is non-local EXCEPT: c is $x.f==...$ with x : Cc need to avoid infinite loop

To check:

- To check constraint $p==q$, check if $v_p \sim^* v_q$.
- To check constraint $S<:T$, check if there is a flow-path from V_S to V_T . This requires checking entailment of the type constraints and adding more edges to the graph. (XXX details!) Add the flow edge to memoize.

7. Translation

This section describes an implementation approach for generic constrained types on a Java virtual machine. We describe the implementation as a translation to Java.

The design is a hybrid design based on the implementation of parameterized classes in NextGen [1, 2] and the implementation of PolyJ [3]. Generic classes are translated into template classes that are instantiated on demand at run time by binding the type properties to concrete types. To implement run-time type checking (e.g., casts), type properties are represented at run time using *adapter objects*.

This design, extended to handle language features not described in this paper, has been implemented in the X10 compiler. The X10 compiler is built on the Polyglot framework and translates X10 source to Java source²

7.1 Method parameters

The first step in translation is to remove method parameters by introducing a generic member class for each generic method. The member class is static iff the method is static. Constructor type parameters are left unchanged. After this step, the code consists only of generic classes. The remaining translation introduces a run-time representation for the type properties of these classes.

7.2 Classes

Each class is translated into a *template class*. The template class is compiled by a Java compiler (e.g., javac) to produce a class file.

²There is also a translation from X10 to C++ source, not described here.

At run time, when a constrained type $C\{c\}$ is first referenced, a class loader loads the template class for C and then transforms the template class bytecode, specializing it to the constraint c .

For example, consider the following classes.

```
class A[T] {
    var a: T;
}
class C {
    val x: A[Int] = new A[Int]();
    val y: Int = x.a;
}
```

The compiler generates the following code:

```
class A {
    // Dummy class needed to type-check uses of T.
    @TypeProperty(1) static class T { }

    T a;

    // Dummy getter and setter; will be eliminated
    // at run time and replaced with actual gets
    // and sets of the field a.
    @Getter("a") <S> S get$a() { return null; }
    @Setter("a") <S> S set$a(S v) { return null; }
}

class C {
    @ActualType("A$Int")
    final A x = Runtime.<A>alloc("A$Int");
    final int y = x.<Integer>get$a();
}
```

The member class $A.T$ is used in place of the type property T . The `Runtime.alloc` method is used in place of a constructor call. This code is compiled to Java bytecode.

Then, at run time, suppose the expression `new C()` is evaluated. This causes C to be loaded. The class loader transforms the bytecode as if it had been written as follows:

```
class C {
    final A$Int x = new A$Int();
    final int y = x.a;
}
```

The `ActualType` annotation is used to change the type of the field x from A to $A\$Int$. The call to `Runtime.alloc` is replaced with a constructor call. The call to `x.get$a()` is replaced with a field access.

The implementation cannot generate this code directly because the class $A\$Int$ does not yet exist; the Java source compiler would fail to compile C .

Next, as the C object is being constructed, the expression `new A$Int()` is evaluated, causing the class $A\$Int$ to be loaded. The class loader intercepts this, demangles the name, and loads the bytecode for the template class A .

The bytecode is transformed, replacing the type property T with the concrete type `int`, the translation of Int .

```
class A {
    x10.runtime.Type T;
}

class A$Int extends A {
    int x;
}
```

expressions	e	$::=$	\dots
		$ $	$T \text{ has Sig}$
signatures	Sig	$::=$	$\text{def this}[\bar{X}](\bar{x}: \bar{T})\{c\}: T$
		$ $	$\text{def } m[\bar{X}](\bar{x}: \bar{T})\{c\}: T$
		$ $	$\text{val } x\{c\}: T$

Figure 12. Grammar for structural constraints

Type properties are mapped to the Java primitive types and to `Object`. Only nine possible instantiations per parameter. Instantiations used for representation. Adapter objects used for run time type information.

Could do instantiation eagerly, but quickly gets out of hand without whole-program analysis to limit the number of instantiations: 9 instantiations for one type property, 81 for two type properties, 729 for three.

Value constraints are erased from type references.

Constructors are translated to static methods of their enclosing class. Constructor calls are translated to calls to static methods.

Consider the code in Figure 10. It contains most of the features of generics that have to be translated.

7.3 Eliminating method type parameters

7.4 Translation to Java

7.5 Run-time instantiation

We translate `instanceof` and cast operations to calls to methods of a `Type` because the actual implementation of the operation may require run-time constraint solving or other complex code that cannot be easily substituted in when rewriting the bytecode during instantiation.

8. Structural constraints

XXX this is an extension of the type system

The type system is general enough to support not only subtyping constraints, but also structural constraints on types. The type system need not change except by extending the constraint system. The syntax for structural constraints is shown in Figure 12.

Structural constraints on types are found in many languages. Haskell [13] supports type classes. In Modula-3, type equivalence and subtyping are structural rather than nominal as in object-oriented languages of the C family such as C++, Java, Scala, and X10. The language PolyJ [3] allows type parameters to be bounded using structural where clauses, a form of F-bounded polymorphism [7]. For example, a sorted list class in PolyJ can be written as follows:

```
class SortedList[T] where T { int compare(T) } {
    void add(T x) { ... x.compare(y) ... }
}
```

The where clause states that the type parameter T must have a method `compare` with the given signatures.

To support this, X10 provides structural constraints on types. The analogous X10 code for `SortedList` is:

```
class SortedList[T] where T has compare(T): int {
    def add(x: T) = { ... x.compare(y) ... }
}
```

A structural constraint is of the form *Type* has *Signature*. A constraint is satisfied if the type has a member of the appropriate name and with a compatible type. The constraint $X \text{ has } f(T1): T2$ is satisfied by a type T if it has a method f whose type is a

```

class C[T] {
  var x: T;
  def this[T](x: T) { this.x = x; }
  def set(x: T) { this.x = x; }
  def get(): T { return this.x; }
  def map[S](f: T => S): S { return f(this.x); }
  def d() { return new D[T](); }
  def t() { return new T(); }
  def isa(y: Object): boolean { return y instanceof T; }
}

val x : C = new C[String]();
val y : C[int] = new C[int]();
val z : C{T <: Array} = new C[Array[int]]();
x.map[int](f);
new C[int{self==3}]() instanceof C[int{self<4}];

```

Figure 10. Code to translate

```

class C[T] where T has T() {
  var x: T;

  def this[T](x: T) { this.x = x; }

  def set(x: T) { this.x = x; }
  def get(): T { return this.x; }

  def d() { return new D[T](); }
  def t() { return new T(); }

  def isa(y: Object): boolean { return y instanceof T; }

  // Translation of map to an inner class
  class map$[T,S] {
    def apply(c: C[T], f: Fun1[T,S]) { return f(c.x); }
  }
}

val x : C = new C[String]();
val y : C[int] = new C[int]();
val z : C{T <: Array} = new C[Array[int]]();
new map$[x.T,int]().apply(x,f);
new C[int{self==3}]() instanceof C[int{self<4}];

```

Figure 11. After removing method parameters

subtype of $(T1 \Rightarrow T2)[T/X]$. As an example, the constraint X has $\text{equals}(X): \text{boolean}$ is satisfied by all three of the following classes:

```

class C { def equals(x: C): boolean; }
class D extends C { }
class E { def equals(x: Object): boolean; }

```

By using function types and where clauses on constructors, X10 can go further than PolyJ. Unlike in PolyJ, where the `compare` method must be provided by `T`, in X10 the `compare` function can be external to `T`. This is achieved as follows:

```

class SortedList[T] {
  val compare: (T,T) => int;
  def this(cmp: (T,T) => int) = { compare = cmp; }
  def add(x: T) = { ... compare(x,y) ... }
}

```

This permits `SortedList` to be instantiated using different `compare` functions:

```

val unixFiles    = new SortedList[String]
                  (String.compareTo.(String));
val windowsFiles = new SortedList[String]
                  (String.compareToIgnoreCase.(String));

```

But, a problem with this approach is that the `compare` function must be provided to the constructor at each instantiation of `SortedList`. The problem can be resolved by using constructors with different structural constraints:

```

class SortedList[T] {
  val compare: (T,T) => int;
  def this[T]() where T has compareTo(T): int = {
    this[T](T.compareTo.(S));
  }
}

```

```
def this[T](cmp: (T,T) => int) = { compare = cmp
def add(x: T) = { ... compare(x,y) ... }
}
```

Now, `SortedList` can be instantiated with any type that has a `compareTo` method without explicitly specifying the method at each constructor call.

9. Discussion

9.1 Type properties versus type parameters

Type properties are similar, but not identical to type parameters. The differences may potentially confuse programmers used to Java generics or C++ templates. The key difference is that type properties are instance members and are thus accessible through access paths: `e.T` is a legal type.

Type properties, unlike type parameters, are inherited. For example, in the following code, `T` is defined in `List` and inherited into `Cons`. The property need not be declared by the `Cons` class.

```
class List[T] {
class Cons extends List {
  def head(): T = { ... }
  def tail(): List[T] = { ... }
}
```

The analogous code for `Cons` using type parameters would be:

```
class Cons[T] extends List[T] {
  def head(): T = { ... }
  def tail(): List[T] = { ... }
}
```

We can make the type system behave as if type properties were type parameters very simply. We need only make the syntax `e.T` illegal and permit type properties to be accessible only from within the body of their class definition via the implicit `this` qualifier.

9.2 Wildcards

Wildcards in Java [11, 25] were motivated by the following example (rewritten in X10 syntax) from [25]. Sometimes a class needs a field or method that is a list, but we don't care what the element type is. For methods, one can give the method a type parameter:

```
def aMethod[T](list: List[T]) = { ... }
```

This method can then be called on any `List` object. However, there is no way to do this for fields since they cannot be parameterized. Java introduced wildcards to allow such fields to be typed:

```
List<?> list;
```

In X10, a similar effect is achieved by not constraining the type property of `List`. One can write the following:

```
list: List;
```

Similarly, the method can be written without type parameters by not constraining `List`:

```
def aMethod(list: List) = { ... }
```

In X10, `List` is a supertype of `List[T]` for any `T`, just as in Java `List<?>` is a supertype of `List<T>` for any `T`. This follows directly from the definition of the type `List` as `List{true}`, and the type `List[T]` as `List{X==T}`, and the definition of subtyping.

Wildcards in Java can also be bounded. We achieve the same effect in X10 by using type constraints. For instance, the following Java declarations:

```
void aMethod(List<? extends Number> list) { ... }
<T extends Number> void aParameterizedMethod(List<T> list) { ... }
```

may be written as follows in X10:

```
def aMethod(list: List{T <: Number}) = { ... }
def aParameterizedMethod[T{self <: Number}](list: List[T])
```

Wildcard bounds may be covariant, as in the following example:

```
List<? extends Number> list = new ArrayList<Integer>();
Number num = list.get(0); // legal
list.set(0, new Double(0.0)); // illegal
list.set(0, list.get(1)); // illegal
```

This can also be written in X10, but with an important difference:

```
list: List{T <: Number} = new ArrayList[Integer]();
num: Number = list.get(0); // legal
list.set(0, new Double(0.0)); // illegal
list.set(0, list.get(1)); // legal! (when list is final)
```

Note because `list.get` has return type `list.T`, the last call in above is well-typed in X10; the analogous call in Java is not well-typed.

Finally, one can also specify lower bounds on types. These are useful for comparators:

```
class TreeSet[T] {
  def this[T](cmp: Comparator{T >: this.T}) { ... }
}
```

Here, the comparator for any supertype of `T` can be used as to compare `TreeSet` elements.

Another use of lower bounds is for list operations. The `map` method below takes a function that maps a supertype of the class parameter `T` to the method type parameter `S`:

```
class List[T] {
  def map[S](fun: Object{self >: T} => S) : List[S] = { ... }
}
```

9.3 Proper abstraction

Consider the following example adapted from [25]:

```
def shuffle[T](list: List[T]) = {
  for (i: int in [0..list.size()-1]) {
    val xi: T = list(i);
    val j: int = Math.random(list.size());
    list(i) = list(j);
    list(j) = xi;
  }
}
```

The method is parameterized on `T` because the method body needs the element type to declare the variable `xi`.

However, the method parameter can be omitted by using the type `list.T` for `xi`. Thus, the method can be declared with the signature:

```
def shuffle(list: List) { ... }
```

This is called *proper abstraction*.

This example illustrates a key difference between type properties and type parameters: A type property is a member of its class, whereas a type parameter is not. The names of type properties are visible outside the body of their class declaration.

In Java, Wildcard capture allows the parameterized method to be called with any `List`, regardless of its parameter type. However, the method parameter cannot be omitted: declaring a parameterless version of `shuffle` requires delegating to a private parameterized version that “opens up” the parameter.

9.4 Virtual types

10. Extensions

10.1 Self type

10.2 Structural constraints

10.3 Ownership

Consider the following example of generic ownership derived from Potanin et al. [20].

```
class Object(owner: Object) { }

// Map inherits Object.owner
// No need to add explicit vOwner and kOwner properties for Key, Value
class Map[Key, Value]{Key <: Comparable, Value <: Object}
{
    private nodes: Vector[Node[Key, Value](this)](this)

    public def put(key: Key, value: Value): Void = {
        nodes.add(new Node[Key, Value](key, value, this)());
    }

    public def get(key: Key): Value = {
        for (mn: Node[Key, Value](this) in nodes) {
            if (mn.key.equals(key))
                return mn.value;
        }
        return null;
    }

    // OGJ will prevent this from being called, since caller
    // can only assign the result to a supertype of Vector[Node]
    // which would be only Vector(this) or Object(this)
    // BUT: we have Vector :=> Vector(this)
    // Need to require that all class types have an equals constraint
    // on the owner property
    public def exposeVector(): Vector(this) { return nodes; }
}

class Node[Key, Value]
{Key <: Comparable, Value <: Object}
{
    val key: Key;
    val value: Value;

    public def this[K, V](k: Key, v: Value, o: Object): Node[K, V](o) {
        super(o); // set the owner
        property[K, V]; // set the type properties
        this.key = k;
        this.value = v;
    }
}
```

Restrictions:

- owner property must be constrained (define this!)
- owner is always equal to or inside the owner of all other type properties
- types with an actual owner == this, can only be accessed via this

11. Related work

[24] [25] [9] [16] [3] [2] [1] [14] [15] [23]

12. Conclusions

We have presented a preliminary design for supporting genericity in X10 using type properties. This type system generalizes the existing X10 type system. The use of constraints on type properties allows the design to capture many features of generics in languages like Java 5 and C# and then to extend these features with new more expressive power. We expect that the design admits an efficient implementation and intend to implement the design shortly.

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References

- [1] Eric Allen, Jonathan Bannet, and Robert Cartwright. A first-class approach to genericity. In *OOPSLA*, pages 96–114, October 2003.
- [2] Eric E. Allen and Robert Cartwright. Safe instantiation in Generic Java. Technical report, March 2004.
- [3] Joseph A. Bank, Barbara Liskov, and Andrew C. Myers. Parameterized types and Java. In *Proceedings of the 24th Annual ACM Symposium on Principles of Programming Languages (POPL'97)*, pages 132–145, 1997.
- [4] Gilad Bracha, Martin Odersky, David Stoutamire, and Philip Wadler. Making the future safe for the past: Adding Genericity to the Java Programming Language. In *OOPSLA*, 1998.
- [5] Kim B. Bruce, Luca Cardelli, Giuseppe Castagna, Jonathan Eifrig, Scott F. Smith, Valery Trifonov, Gary T. Leavens, and Benjamin C. Pierce. On binary methods. *Theory and Practice of Object Systems*, 1(3):221–242, Fall 1995.
- [6] Kim B. Bruce, Angela Schuett, and Robert van Gent. PolyTOIL: A type-safe polymorphic object-oriented language. In *European Conference on Object-Oriented Programming (ECOOP)*, volume 952 of *LNCS*, pages 27–51. Springer-Verlag, 1995.
- [7] Peter Canning, William Cook, Walter Hill, John Mitchell, and Walter Olthoff. F-bounded polymorphism for object-oriented programming. In *Proceedings of the Conference on Functional Programming Languages and Computer Architecture*, pages 273–280, 1989.
- [8] ECMA-334: C# language specification, June 2006. <http://www.ecma-international.org/publications/files/ecma-st/ECMA-334.pdf>.
- [9] Burak Emir, Andrew Kennedy, Claudio Russo, and Dachuan Yu. Variance and generalized constraints for c# generics. In *ECOOP*, 2006.
- [10] Erik Ernst. *gbeta: A Language with Virtual Attributes, Block Structure, and Propagating, Dynamic Inheritance*. PhD thesis, Department of Computer Science, University of Aarhus, Århus, Denmark, 1999.
- [11] J. Gosling, W. Joy, G. Steele, and G. Bracha. *The Java Language Specification, Third Edition*. Addison Wesley, 2006.
- [12] A. Igarashi, B. Pierce, and P. Wadler. Featherweight Java: A minimal core calculus for Java and GJ. In *ACM Symposium on Object-Oriented Programming: Systems, Languages and Applications*, 1999.
- [13] Haskell 98: A non-strict, purely functional language, February 1999. Available at <http://www.haskell.org/onlinereport/>.
- [14] O. Lehrmann Madsen, B. Møller-Pedersen, and K. Nygaard. *Object Oriented Programming in the BETA Programming Language*. Addison-Wesley, June 1993.

- [15] Ole Lehrmann Madsen and Birger Møller-Pedersen. Virtual classes: A powerful mechanism for object-oriented programming. pages 397–406, October 1989.
- [16] Andrew Myers and Barbara Liskov. Efficient implementation of parameterized types despite subtyping. Technical Report Thor Note 9, MIT LCS, June 1994.
- [17] Nathaniel Nystrom, Xin Qi, and Andrew C. Myers. J&: Nested intersection for scalable software extension. In *Proceedings of the 2006 ACM Conference on Object Oriented Programming Systems, Languages, and Applications (OOPSLA)*, pages 21–36, Portland, OR, October 2006.
- [18] Martin Odersky. Report on the programming language Scala. Technical report, EPFL, 2006.
- [19] Martin Odersky and Philip Wadler. Pizza into Java: Translating theory into practice. pages 146–159, Paris, France, January 1997.
- [20] Alex Potanin, James Noble, Dave Clarke, and Robert Biddle. Generic ownership for generic Java. In *Object-oriented Programming Systems, Languages, and Applications (OOPSLA 2006)*.
- [21] Nathaniel Nystrom Vijay Saraswat, Jens Palsberg, and Christian Grothoff. Constrained types for object-oriented languages. In *Proceedings of the 2008 ACM Conference on Object-Oriented Programming Systems, Languages, and Applications (OOPSLA)*, October 2008.
- [22] V. Saraswat et al. Report on the programming language X10. Technical report, IBM T.J. Watson Research Center, 2006.
- [23] Kresten Krab Thorup. Genericity in Java with virtual types. Number 1241, pages 444–471, 1997.
- [24] Kresten Krab Thorup and Mads Torgersen. Unifying genericity: Combining virtual types and parameterized classes. In *ECOOP*, 1998.
- [25] Mads Torgersen, Christian Plesner Hansen, Erik Ernst, Peter von der Ahé, Gilad Bracha, and Neal Gafter. Adding wildcards to the Java programming language. In *SAC*, March 2004.