

## Abstract

Notes on a formal semantics for dependently typed X10, and generically typed X10.

## 1. Judgements

Here is the syntax for CFJ, converted to the new X10 1.7 style syntax. (Note: we should convert the OOPSLA paper to this syntax as well.)

*Need to answer reviewer's question: Does the invariant associated with a class C change when that associated with the class D that it extends, changes? (Yes.) Need to figure out what this means for separate compilation. Should still be able to compile separately. It is just the case now that when you link a new class in, the invariant for this class will depend on the invariant that you have loaded for its superclasses. Java and X10 do nothing to keep compile-time types consistent with link-time types.*

(C Term)	$t ::= x \mid \text{self} \mid t.f$
	$\mid \text{new } C(\bar{t}) \mid g(\bar{t})$
(Const.)	$c, d ::= \text{true} \mid p(\bar{t}) \mid t = t$
	$\mid c, c \mid \text{val } x : T; c$
(Class)	$L ::= \text{class } C(\bar{f} : \bar{T})\{c\} \text{ extends } T \{ \bar{M} \}$
(Method)	$M ::= \text{def } m(\bar{x} : \bar{T})\{c\} : T = e;$
(Exp.)	$e ::= x \mid e.f \mid e.m(\bar{e}) \mid \text{new } C(\bar{e})$
	$\mid e \text{ as } T$
(Type)	$S, T,$
	$U, Z ::= C\{d\}$

We will use the abbreviation  $\text{val } x : S; \text{val } y_1 : T_1; \dots; \text{val } y_n : T_n; T$  for  $\text{val } x : S; \text{val } y_1 : T_1; \dots; \text{val } y_n : T_n; T$ .

In the rest of this paper we will assume give some fixed but unknown constraint system  $\mathcal{D}$ . We will assume that the program  $P$  is written using constraints from  $\mathcal{D}$ , and further that classes defined in  $P$  do not have a cyclic inheritance struture.

### 1.1 The Object constraint system, $\mathcal{O}$

From  $P$  and  $\mathcal{D}$  we also generate a new constraint system  $\mathcal{O}$ , the constraint system of *objects* over  $P$  and  $\mathcal{D}$  as follows. Let  $C, D$  range over names of classes in  $P$ ,  $f$  over field names,  $m$  over method names,  $T$  over types,  $c$  over constraints in the underlying data constraint system  $\mathcal{D}$ . The constraints of  $\mathcal{O}$  are given by:

(Const.)	$c, d ::= \text{class}(C) \mid C \text{ extends } D$
	$\mid \text{fields}(x, \bar{f} : \bar{T})$
	$\mid \text{mtype}(x, m, (\bar{x} : \bar{T}, c \rightarrow T))$

In the method signature formula  $(\bar{x} : \bar{T}, c \rightarrow T)$ , the variables  $\bar{x}$  are considered bound; formulas with bound variables are considered equivalent upto  $\alpha$ -renaming.

The constraint system satisfies the following axioms and inference rules.

For every class  $C$  defined in  $P$ , we have:  $\vdash_{\mathcal{O}} \text{class}(C)$ .

The *extends* relation is transitive and  $C$  *extends*  $D$  whenever  $C$  *extends*  $D$  in the given program.

Let  $\bar{f} : \bar{T}$  be the fields defined or inherited at class  $C$ . The  $\vdash_{\mathcal{O}}$  also satisfies:

$$\Gamma, z : D\{d\} \vdash_{\mathcal{O}} \text{fields}(z, \bar{f} : \bar{T}) \quad (\text{FIELDS})$$

where for any constraint  $e$ ,  $e'$  is the constraint  $\text{val this; this} = z, d, e$ , and for any type  $F\{f\}$ , the type  $F'$  is  $F\{f'\}$ .

For every method  $m(\bar{x} : \bar{E})\{c\} : F = \{\dots\}$  defined or inherited at class  $D$ ,  $\vdash_{\mathcal{O}}$  satisfies:

$$\Gamma, z : D\{d\} \vdash_{\mathcal{O}} \text{mtype}(z, m, \bar{x} : \bar{E}', c' \rightarrow F') \quad (\text{MTYPE})$$

where  $E', c', F'$  are as above.

The constraint system  $\mathcal{C}$  is the (disjoint) conjunction  $\mathcal{D}, \mathcal{O}$  of the constraint systems  $\mathcal{D}$  and  $\mathcal{O}$ . (This requires the assumption that  $\mathcal{D}$  does not have any constraints in common with  $\mathcal{O}$ .)

*Note: Figure out whether consistency checks need to be added.*

## 2. Rules

### 2.1 Judgements

The following judgements will be defined. In all of them  $\Gamma$  is a well-typed context.

- Given  $\Gamma$ , the type  $T$  is well-formed:  $\Gamma \vdash T \text{ type}$
- Given  $\Gamma$ , the type  $S$  is a subtype of  $T$ :  $\Gamma \vdash S <: T$
- Given  $\Gamma$ , the expression  $e$  is of type  $T$ :  $\Gamma \vdash e : T$
- Given  $\Gamma$ , the method  $M$  is well-defined for the class  $C$ :  $\Gamma \vdash M \text{ OK in } C$
- Given  $\Gamma$ , the field  $M$  is well-defined for the class  $C$ :  $\Gamma \vdash f : T \text{ OK in } C$
- Given  $\Gamma$ , the class definition  $Cl$  is well-defined:  $\Gamma \vdash Cl \text{ OK}$

In what follows we will sometimes think of the family of five judgements as a single judgement  $\Gamma \vdash \phi$ , where  $\phi$  ranges over the formulas  $T \text{ type}$ ,  $e : T$ ,  $S <: T$ ,  $f : T \text{ OK in } C$ ,  $M \text{ OK in } C$ , and  $C \text{ OK}$ .

In defining these judgements we will use  $\Gamma \vdash_{\mathcal{C}} c$ , the judgement corresponding to the underlying constraint system. For simplicity, we define  $\Gamma \vdash c$  to mean  $\sigma(\Gamma) \vdash_{\mathcal{C}} c$ , where the *constraint projection*,  $\sigma(\Gamma)$  is defined thus:

$$\begin{aligned} \sigma(\epsilon) &= \text{true} \\ \sigma(x : C\{c\}, \Gamma) &= (\text{val } x : C; (x = \text{self}.c)), \sigma(\Gamma) \\ \sigma(c, \Gamma) &= c, \sigma(\Gamma) \end{aligned}$$

Now, these judgements need to satisfy certain properties:

- $\Gamma \vdash T \text{ type}$  whenever  $\Gamma \vdash e : T$ .
- That is, if we can conclude that  $e$  has type  $T$  (under certain assumptions), then under those assumptions we must be able to conclude that  $T$  is well-defined.
- $\Gamma \vdash S \text{ type}$  and  $\Gamma \vdash T \text{ type}$  whenever  $\Gamma \vdash S <: T$ .
- If  $\Gamma \vdash e : T$  and  $x$  is a variable occurring free in  $e$  then for some type  $U$ ,  $\Gamma \vdash x : U$ . That is, all free variables on the RHS are actually defined by the LHS.

Keeping in mind these requirements, the rules are as follows. Below wherever we use the assertion “ $x$  free” in the antecedent of a rule we mean that  $x$  is not free in the consequent of the rule.

### 2.2 Structural and Logical Rules

All the judgements are intuitionistic. In particular this means that all constraint systems satisfy the rules and axioms of inference below.

### 2.3 Well formedness rules

We use the judgement for well-typedness for expressions to represent well-typedness for constraints. That is, we posit a special type  $\text{o}$  (traditionally the type of propositions), and regard constraints as expressions of type  $\text{o}$ . Further, we change the formulation slightly so that there are no constraints of the form  $p(t_1, \dots, t_n)$ ; rather instance method invocation syntax is used to express invocation of pre-defined constraints. This logically leads to the step of simply marking certain classes as “predicate” classes – all the (instance) methods of these classes whose return type is  $\text{o}$  then correspond to “primitive constraints.”

Syntactically, we will continue to use the symmetric syntax  $p(t_1, \dots, t_n)$  rather than  $t_1.p(t_2, \dots, t_n)$ .

$$\begin{array}{c}
\frac{}{\Gamma, c \vdash c} \text{ (ID)} \qquad \frac{\Gamma \vdash c \quad \Gamma, c \vdash d}{\Gamma \vdash d} \text{ (CUT)} \\
\\
\frac{\Gamma \vdash \phi \quad \Gamma \vdash S \text{ type} \quad x \notin \text{var}(\Gamma)}{\Gamma, x : S \vdash \phi} \text{ (WEAK-1)} \qquad \frac{\Gamma \vdash \phi \quad \Gamma \vdash c : o}{\Gamma, c \vdash \phi} \text{ (WEAK-2)} \qquad \frac{\Gamma, \psi_0, \psi_1 \vdash \phi}{\Gamma, (\psi_0, \psi_1) \vdash \phi} \text{ (AND-L)} \\
\\
\frac{\Gamma \vdash \psi_0 \quad \Gamma \vdash \psi_1}{\Gamma \vdash \psi_0, \psi_1} \text{ (AND-R)} \qquad \frac{\Gamma \vdash \phi[t/x] \quad \Gamma \vdash t : S}{\Gamma \vdash \text{val } x; \phi} \text{ (EXISTS-R)} \qquad \frac{\Gamma, x : S, \psi \vdash \phi \quad (x \text{ fresh})}{\Gamma, \text{val } x : S; \psi \vdash \phi} \text{ (EXISTS-L)}
\end{array}$$


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**Figure 1.** Logical rules

(The alternative is to introduce static methods, and static method invocations in the expression language. This is not too difficult, but it is annoying to have to repeat most of the formulation of instance methods, and to have one more case to prove.)

So this means that the only cases left to handle are all the simple ones, expressing the availability of certain constants and operators at type  $o$  (see Figure 2).

### 3. Type inference rules

#### 3.1 Subtype judgement

Only one inference rule needs to be specified:

$$\frac{\Gamma \vdash C \text{ extends } D \quad \Gamma \vdash C\{c\} \text{ type}, D\{d\} \text{ type} \quad \Gamma, \text{self} : C, c \vdash d}{\Gamma \vdash C\{c\} <: D\{d\}} \text{ (DEP TYPE)}$$

#### 3.2 Expression typing judgement

Now we consider the rule for method invocation. Assume that in a type environment  $\Gamma$  the expressions  $\bar{e}$  have the types  $\bar{T}$ . Since the actual values of these expressions are not known, we shall assume that they take on some fixed but unknown values  $\bar{z}$  of type  $\bar{T}$ . Now for  $z$  as receiver, let us assume that the type  $T \equiv C\{d\}$  has a method named  $m$  with signature  $\bar{z} : \bar{Z}, c \rightarrow U$ . If there is no method named  $m$  for the class  $C$  then this method invocation cannot be type-checked. Without loss of generality we may assume that the parameters of this method are named  $\bar{z}$ , since we are free to choose variable names as we wish because of  $\alpha$ -equivalence. Now in order for the method to be invocable, it must be the case that the types  $\bar{T}$  are subtypes of  $\bar{Z}$ . (Note that there are no occurrences of `this` in  $\bar{Z}$ ; they have been replaced by  $z$  – see Section 1.1) Further, it must be the case that for these parameter values, the constraint  $c$  is entailed. Given all these assumptions it must be the case that the return type is  $U$  — with all the parameters  $\bar{z}$  existentially quantified.

#### 3.3 Class OK judgement

Note that the variables  $\bar{x}$  are permitted to occur in the types  $T, \bar{T}$ , hence their typing assertions must be added to  $\Gamma$ .

$$\begin{array}{c}
\Gamma \vdash \mathbf{true} : o \quad (\mathbf{true}) \quad \frac{\Gamma \vdash t_0 : T_0 \quad \Gamma \vdash t_1 : T_1 \quad (\Gamma \vdash T_0 <: T_1 \vee \Gamma \vdash T_1 <: T_0)}{\Gamma \vdash t_0 = t_1 : o} \text{ (EQUALS)} \quad \frac{\Gamma \vdash c_0 : o \quad \Gamma \vdash c_1 : o}{\Gamma \vdash (c_0, c_1) : o} \text{ (AND)} \\
\\
\frac{\Gamma \vdash t : T \quad \Gamma \vdash c[t/x] : o}{\Gamma \vdash \mathbf{val } x : T; c : o} \text{ (SOME)} \quad \frac{\Gamma \vdash \mathbf{class}(C) \quad \Gamma, \mathbf{self} : C \vdash c : o}{\Gamma \vdash C\{c\} \text{ type}} \text{ (TYPE)}
\end{array}$$


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**Figure 2.** Well formedness rules

$$\begin{array}{c}
\Gamma, x : T \vdash x : T \quad (\mathbf{T-VAR}) \quad \frac{\Gamma \vdash e : U \quad \Gamma \vdash T \text{ type}}{\Gamma \vdash e \text{ as } T : T} \text{ (T-CAST)} \quad \frac{\Gamma \vdash e : S \quad \Gamma, \mathbf{this} : S \vdash \mathbf{fields}(\mathbf{this}, \bar{f} : \bar{U})}{\Gamma \vdash e.f_i : \mathbf{val this} : S; U_i} \text{ (T-FIELD)} \\
\\
\frac{\Gamma \vdash e : T, \bar{e} : \bar{T} \quad \Gamma, z : T \vdash \mathbf{mtype}(z, m, \bar{z} : \bar{Z}, c \rightarrow U) \quad \Delta \equiv \Gamma, z : T, \bar{z} : \bar{T} \quad \Delta \vdash \bar{T} <: \bar{Z}, c \quad z, \bar{z} \text{ fresh}}{\Gamma \vdash e.m(\bar{e}) : \mathbf{val } z : T, \bar{z} : \bar{T}; U} \text{ (T-INVK)} \quad \frac{\Gamma \vdash \bar{e} : \bar{T} \quad \Gamma, z : C \vdash \mathbf{fields}(z, \bar{f} : \bar{Z}) \quad \Delta \equiv \Gamma, z : C, \bar{z} : \bar{T}, z.\bar{f} = \bar{z} \quad \Delta \vdash \bar{T} <: \bar{Z}, \mathbf{inv}(C, z) \quad z, \bar{z} \text{ fresh}}{\Gamma \vdash \mathbf{new } C(\bar{e}) : C\{\mathbf{val } z : C, \bar{z} : \bar{T}; z.\bar{f} = \bar{z}, \mathbf{self} = z, \mathbf{inv}(C, z)\}} \text{ (T-NEW)} \\
\\
\frac{\Delta \equiv \mathbf{this} : C, \bar{x} : \bar{T}, c \quad \Delta \vdash T \text{ type}, \bar{T} \text{ type}, e : U, U <: T}{\mathbf{def } m(\bar{x} : \bar{T})\{c\} : T = e; \text{ OK in } C} \text{ (METHOD OK)} \quad \frac{\bar{M} \text{ OK in } C \quad \mathbf{this} : C, c \vdash \bar{T} \text{ type}, T \text{ type}}{\mathbf{class } C(\bar{f} : \bar{T})\{c\} \text{ extends } T\{\bar{M}\} \text{ OK in } C} \text{ (CLASS OK)}
\end{array}$$


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**Figure 3.** Type judgement