Signals and Systems
Homework #1

Due Wednesday 9:50am in class, Mar 9, 2022.
100%

1. (10%)

The sinusoidal signal

$$x(\mathrm{t}) = 20\cos(50\pi t - \pi/6)$$

is passed through a square-law device defined by the input–output relation

$$y(t) = x^2(t).$$

Using the trigonometric identity

$$\cos^2\theta = \tfrac{1}{2}(\cos 2\theta + 1),$$

show that the output $y(t)$ consists of a dc component and a sinusoidal component.

(a) Specify the dc component.

(b) Specify the amplitude and fundamental frequency of the sinusoidal component in the output $y(t)$.

2. (10%) Same as 1.
   (a) Plot x(t) and y(t) using MATLAB. Specify the time from 0 to 0.099 second with a step of 1 millisecond.
   (b) Write the code to calculate the mean of y(t). Is it associated with the dc component?
3. (10%)

1.46 The raised-cosine pulse $x(t)$ shown in Fig. P1.46 is defined as

$$x(t) = \begin{cases} \frac{1}{2}[\cos(\omega t) + 1], & -\pi/\omega \le t \le \pi/\omega \\ 0, & \text{otherwise} \end{cases}.$$

Determine the total energy of $x(t)$.

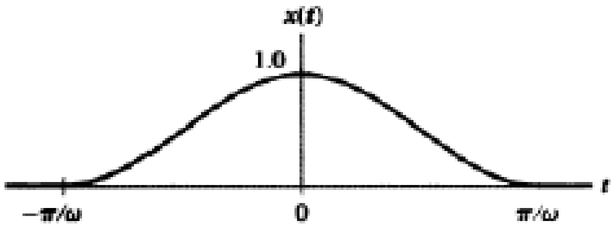

Figure P1.46

4. (10%)

1.51 Sketch the trapezoidal pulse $y(t)$ related to that of Fig. P1.47 as follows:

$$y(t) = x(10t - 5)$$

$$x(t) = \begin{cases} 5 - t, & 4 \le t \le 5 \\ 1, & -4 \le t \le 4 \\ t + 5 & -5 \le t \le -4 \\ 0, & \text{otherwise} \end{cases}$$

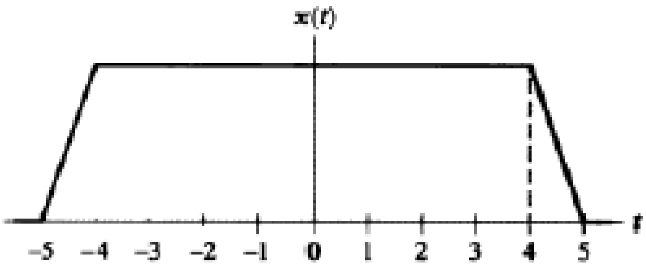

Figure P1.47

5. (10%)

1.55 Figure P1.55(a) shows a pulse $x(t)$ that may be viewed as the superposition of three rectangular pulses. Starting with the rectangular pulse $g(t)$ of Fig. P1.55(b), construct the waveform of Fig. P1.55, and express $x(t)$ in terms of $g(t)$.

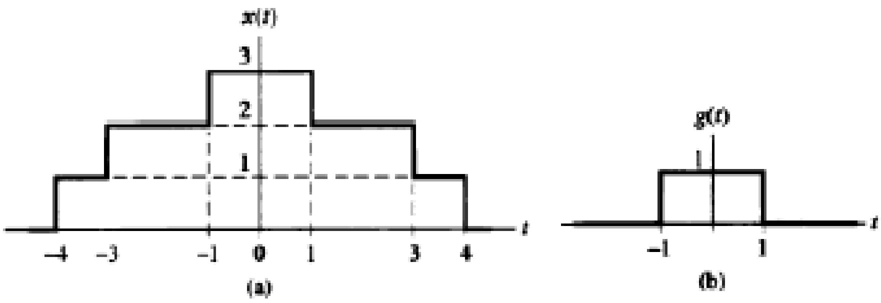

Figure P1.55

6. (20%) 需寫理由

1.64 The systems that follow have input $x(t)$ or $x[n]$ and output $y(t)$ or $y[n]$. For each system, determine whether it is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant.

(a) $y(t) = \cos(x(t))$

(b) $y[n] = 2x[n]u[n]$

(f) $y(t) = \frac{d}{dt}x(t)$

(g) $y[n] = \cos(2\pi x[n + 1]) + x[n]$

7. (15 %)

(a) The output of a discrete-time system is related to its input $x[n]$ as follows:

$$y[n] = a_0x[n] + a_1x[n-1] + a_2x[n-2] + a_3x[n-3].$$

Let the operator S^k denote a system that shifts the input $x[n]$ by k time units to produce $x[n-k]$. Formulate the operator H for the system relating $y[n]$ to $x[n]$. Then develop a block diagram representation for H, using (a) cascade implementation and (b) parallel implementation.

(b) Show that the system described in Problem 1.65 is BIBO stable for all $a_0, a_1, a_2,$ and a_3.

8. (15%) Investigation of SNR (Signal-to-Noise Ratio)

(a) Generate a rectangular pulse x(t) defined by

$$x(t) = 10, \quad 0 \le t \le 3 \text{ seconds}$$
$$= 0, \quad \text{otherwise.}$$

Draw the waveform from -5 to 10 seconds with a time increment of 0.01 seconds.

(b) Generate a noise where its mean is zero and its standard deviation is 1. Then add to the rectangular pulse x(t). The final noise-corrupted result (denoted as y(t)) is plotted together with (a)

(Hint: Use MATLAB function "randn" which is a normal distribution)

(c) Calculate the SNR of y(t) in dB (defined by the peak signal divided by the standard deviation of noise. Then take 20*log ()).

(Hint: The peak signal is the maximum of y(t) using MATLAB function "max". You can use "std" to calculate standard deviation. Be aware that you need to identify the "noise" region of y(t) before calculating noise standard deviation).