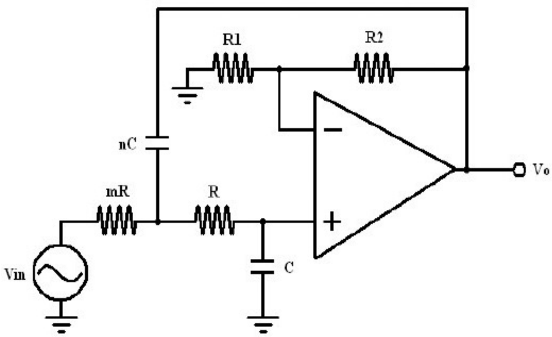
Lab3 Filter Theory and Laboratory 0811562 何祁恩

* Phase 1: Second order active low-pass filter

Spec: $\begin{cases} \text{Magnifying power} = A_{dc} = 2\,(V/V) \\ \text{Cut-off frequency} = f_0 = 200\,Hz \\ \text{Input Signal}: V_{in} = 2\,V_{pp} \end{cases}$



Second-order Sallen Key LPF

* Determine the resistors & capacitor to use to meet the Spec.

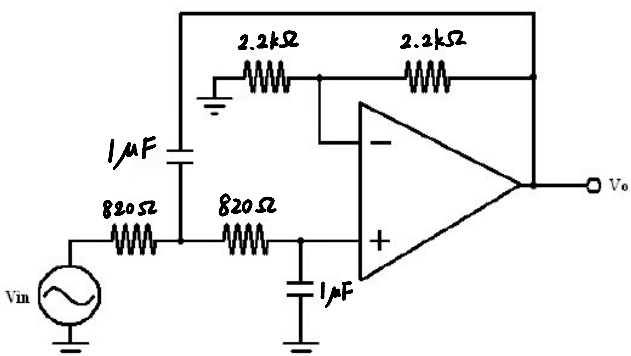
1° To simply the question, let's set $m = n = 1$

2° $A_{dc} = 2 = \frac{R_1 + R_2}{R_1}$, WLOG choose $R_1 = R_2 = 2.2k\Omega$

3° $f_o = 200Hz = \frac{1}{2\pi RC} \Rightarrow RC = \frac{1}{400\pi}$

Since we got $C = 1\mu F$, we can find the best matched $R = 820\Omega$.

Our Schematic of Second-order LPF



$f_{o\ expected} = \frac{1}{2\pi(820)(10^{-6})} = 194\,Hz$

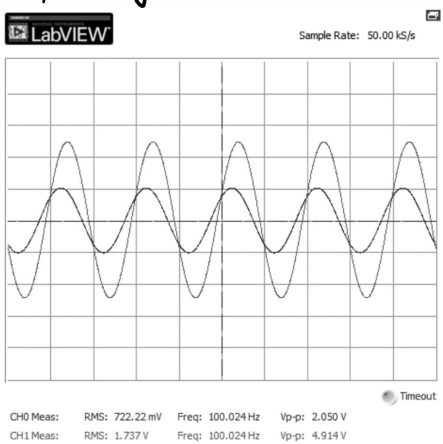
↳ Expected f_o in our filter design.

$A_{dc} = \frac{2.2k + 2.2k}{2.2k} = 2$

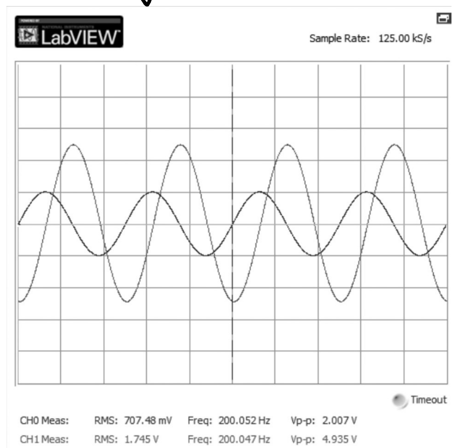
* Note that we should avoid using resister under 100Ω, which will draw a lot of current through it, and behave unpredictable!

Physical circuit result :

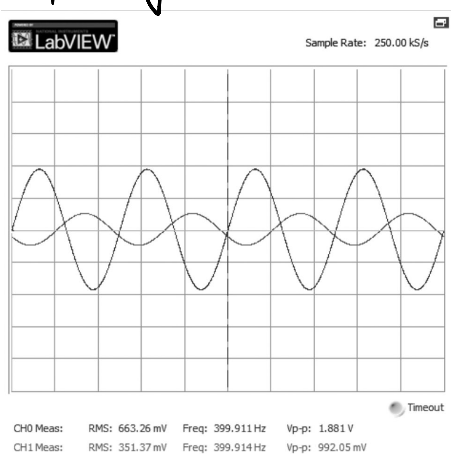
Input Signal with 100 Hz



Input Signal with 200 Hz



Input Signal with 400 Hz



Input f (Hz)	Vin Vpp (V)	Vout Vpp (V)	Gain (V/V)
100	2.050	4.914	2.397
200	2.007	4.935	2.458
400	1.881	0.992	0.527

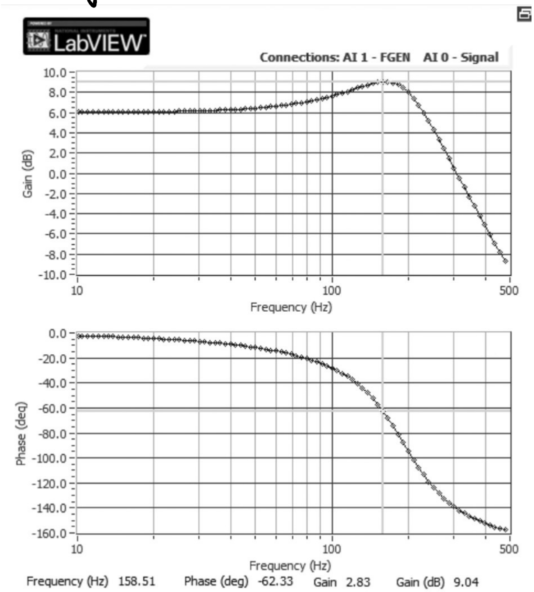
* We should select another frequency smaller than 100 Hz, so that we can clearly observe the behavior of LPF.

1. $f = 100$ Hz, the gain is almost 2 V/V, and exists a little phase shift
2. $f = 200$ Hz, the gain reached its maximum, and exists $60°$ phase shift
3. $f = 400$ Hz $> f_0$, the gain is decreased to 0.527, and it's almost out of phase.

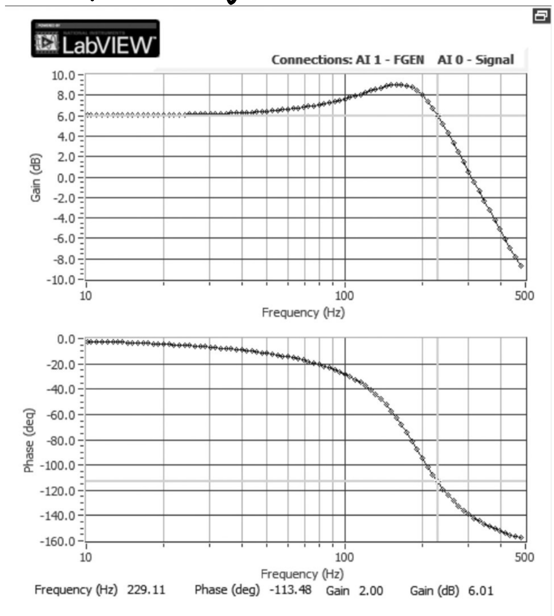
Physical circuit Bode plot :

$-3dB$

Peak gain at $f = 158Hz$ with $9.04dB$



$-3dB$ point at $f = 229Hz$, with $6.01dB$



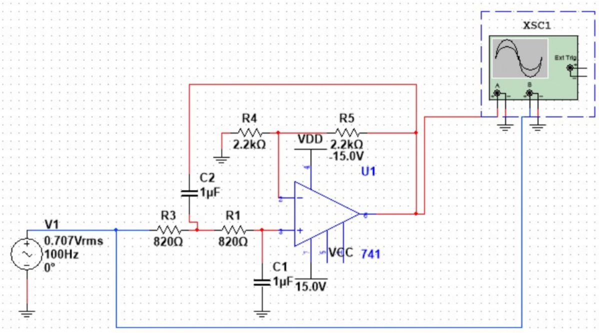
1° $f < 40Hz$, LPF act very ideal with $gain = 6dB = 20\log 2 \approx A_{dc}$ with very small phase shift $\approx V_{out} = 2V_{in} \angle 0°$

2° $150Hz < f < 200Hz$, LPF has the max gain in this range $\approx 9dB$ and exists $60° \sim 90°$ phase shift $\approx V_{out} = 2.8 V_{in} \angle \frac{\pi}{3} \sim \frac{\pi}{2}$

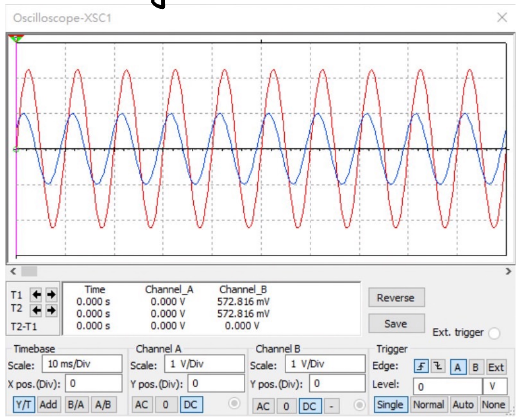
3° $f > 200Hz = f_0$, the magnitude response decrease very fast, and therefore the output signal became smaller in this range, and phase will close to out of phase! It's a LPF, filter out the high frequency input signal.

Multisim Simulation Result:

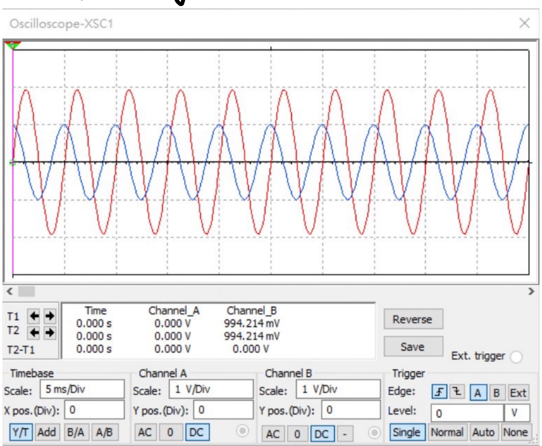
Circuit schematic on multisim (flip OP)



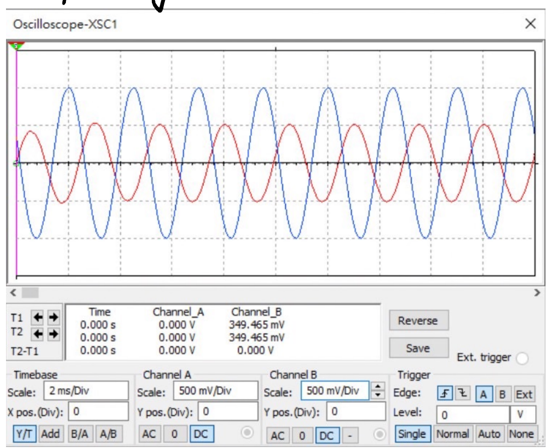
Input Signal with 100Hz



Input Signal with 200Hz



Input Signal with 400Hz

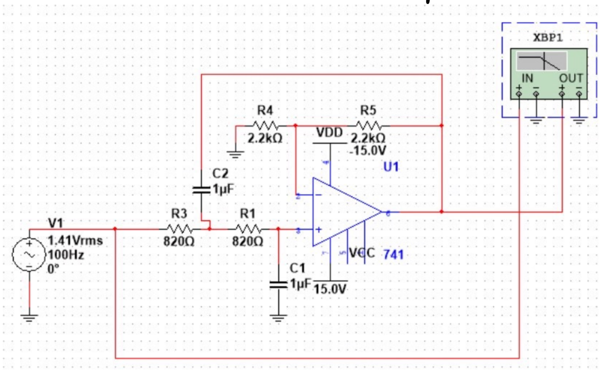


* We choose the same 3 frequencies to simulate on multisim so that we can compare the result with our physical circuit!

* Both magnitude and phase are almost the same as our result on physical circuit.

Multisim simulation Bode plot.

Multisim schematic (flip OP)

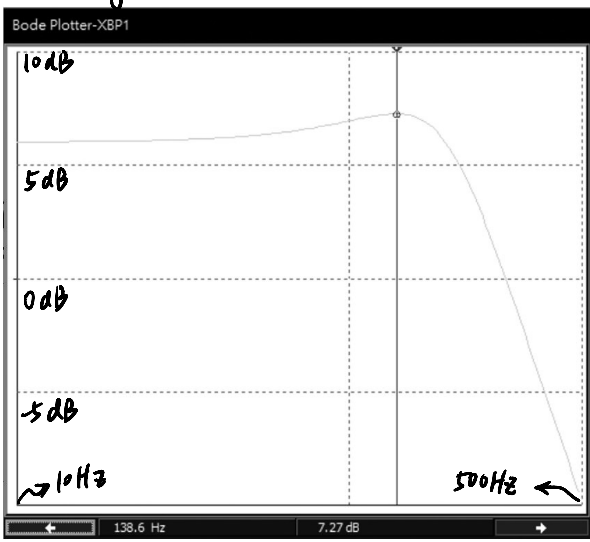


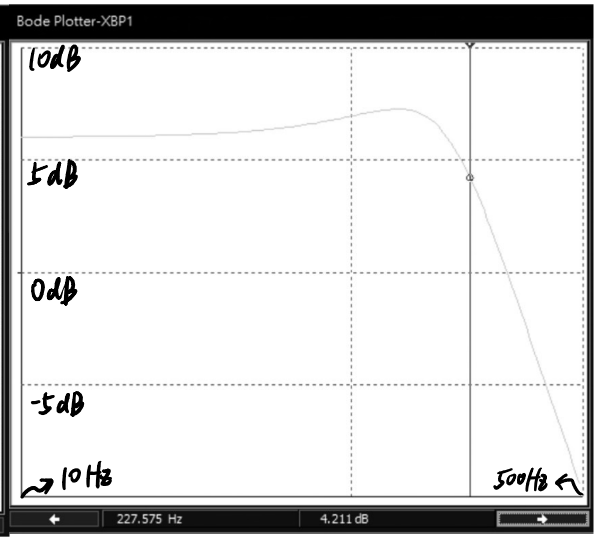
It's quite strange when simulating the bode plot, we need to add AC source to both our circuit input and bode plot input. I think it should be just be setting the start/stop frequency, and bode plot machine will automatically AC sweep the signal to our circuit.

Peak gain at f = 138.6 Hz with 7.27 dB

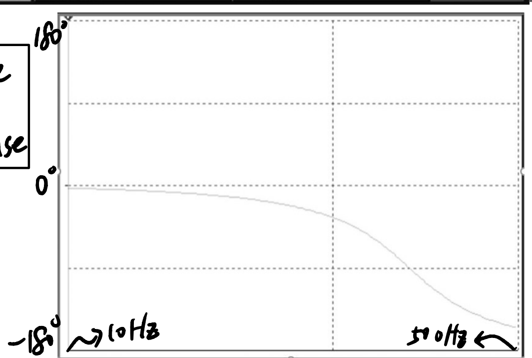
-3dB point at f = 227 Hz with 4.21 dB

-3dB





Phase Response



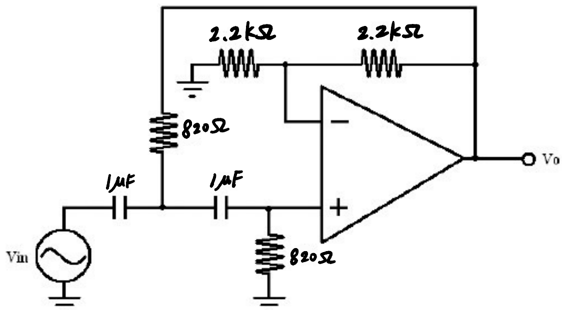
* Both peak gain and -3dB gain are smaller than our physical circuit result.

* -3dB frequency are matched on both multisim and physical circuits.

* Phase 2: Second order active high-pass filter

Spec: $\begin{cases} \text{Magnifying power} = A_{dc} = 2\,(V/V) \\ \text{Cut-off frequency} = f_0 = 200\text{ Hz} \\ \text{Input Signal}: V_{in} = 2\,V_{pp} \end{cases}$

Our schematic of second order HPF



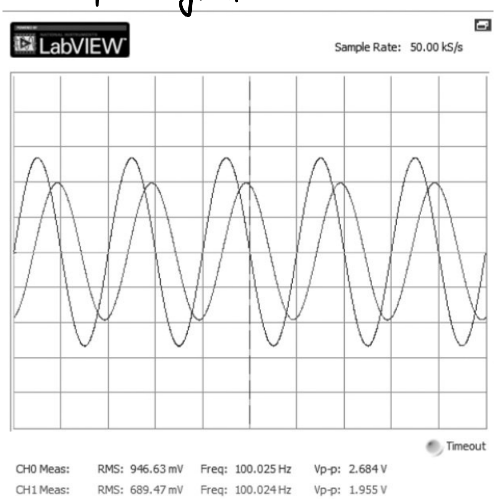
$$f_0 \text{ expected} = \frac{1}{2\pi(820)(10^{-6})} = 194\text{ Hz}$$

↳ Expected f_0 in our filter design

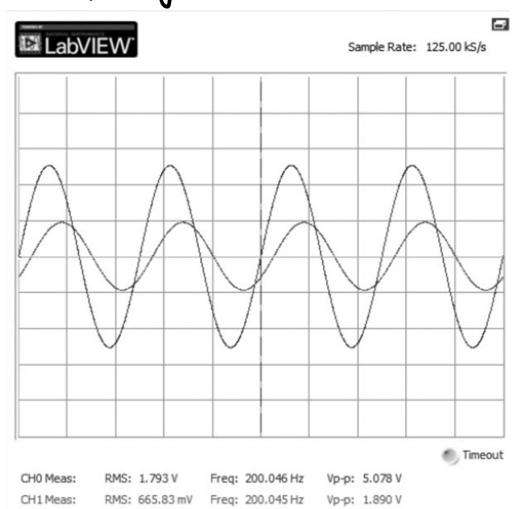
$$A = \frac{2.2k + 2.2k}{2.2k} = 2\,(V/V)$$

Physical circuit result

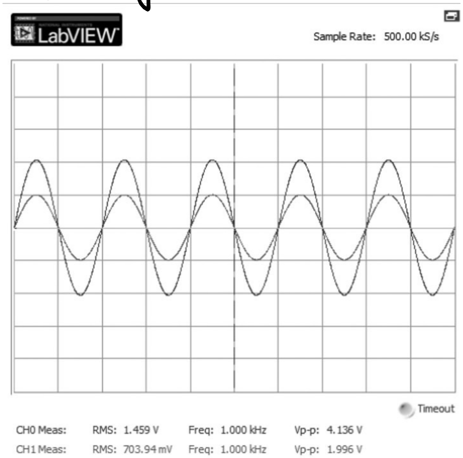
Input Signal with 100Hz



Input Signal with 200Hz



Input Signal with 1kHz



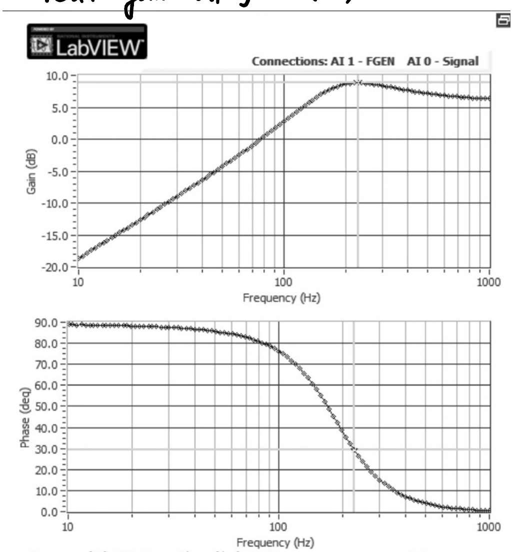
Input f (Hz)	Vin Vpp (V)	Vout Vpp (V)	Gain (V/V)
100	1.955	2.684	1.372
200	1.890	5.078	2.686
1k	1.996	4.136	2.072

* At $f = 100Hz < f_0$, gain is smaller than 2V/V, and exist phase shift.
* At $f = 200Hz$, gain reach its maximum, and exist 40° phase shift.
* At $f = 1kHz$, gain = 2 as the spec request, and there exist no phase shift between input and output.

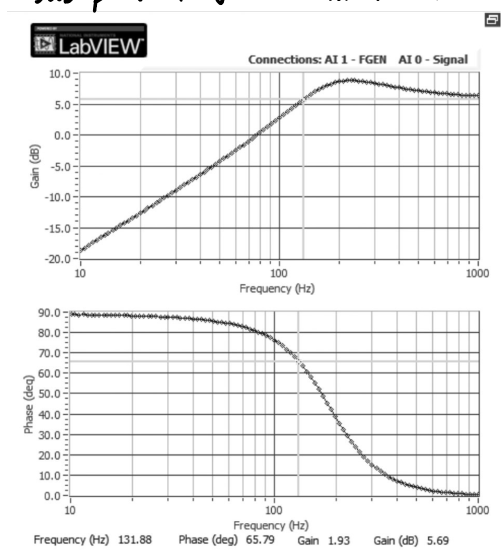
Physical circuit Bode plot.

-3dB

Peak gain at f=229Hz, with 8.8dB



-3dB point at f=131Hz with 5.69dB



From Bode plot, we have:

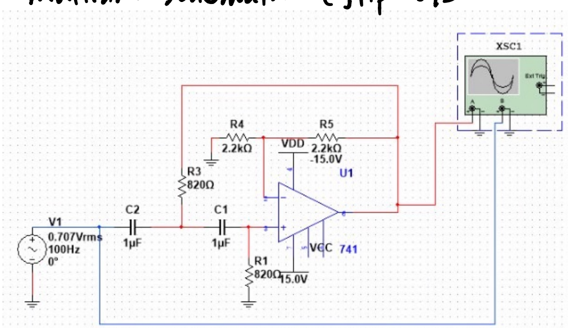
* At $f < 80$ Hz, the gain < 0 dB, which means the output signal is smaller than the input signal. Filtered out the low-frequency input, therefore, we can confirm our design is a HPF.

* At $f \approx 200$ Hz, gain reach its maximum $= 8.8$ dB $\Rightarrow V_{out} \approx 2.75 \cdot V_{in} \angle \frac{\pi}{6}$

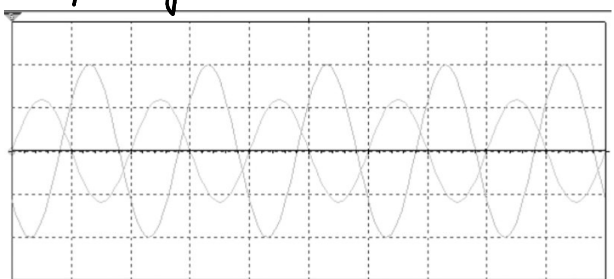
* At $f > 600$ Hz, HPF act very ideal with gain $= 6$dB $= 2$ V/V, with almost 0 phase shift $\Rightarrow V_{out} = 2 V_{in} \angle 0°$

Multisim Simulation Result:

multisim schematic (flip OP)

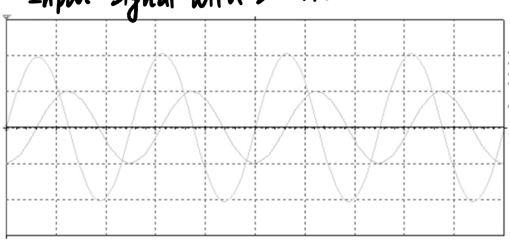


Input Signal with 100Hz



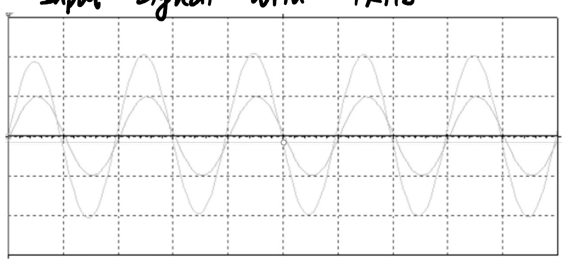
Scale: 500 mV/div, 5ms/div

Input Signal with 200Hz



Scale: 1V/div, 2ms/div

Input Signal with 1kHz

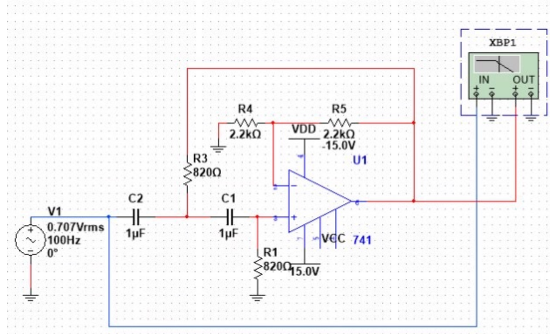


Scale: 1V/div, 500µs/div

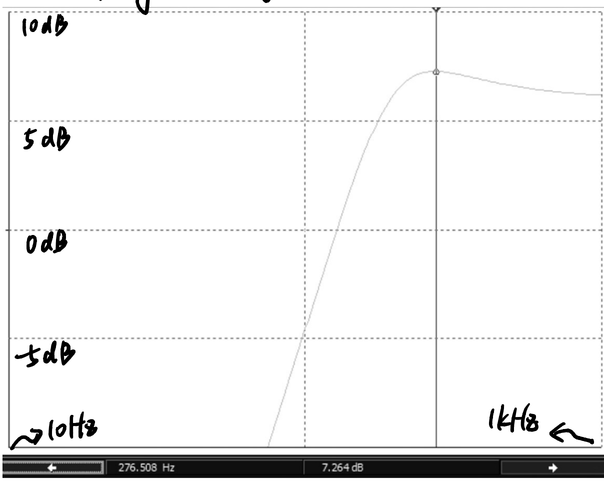
* We got almost same waveform as our physical circuit.

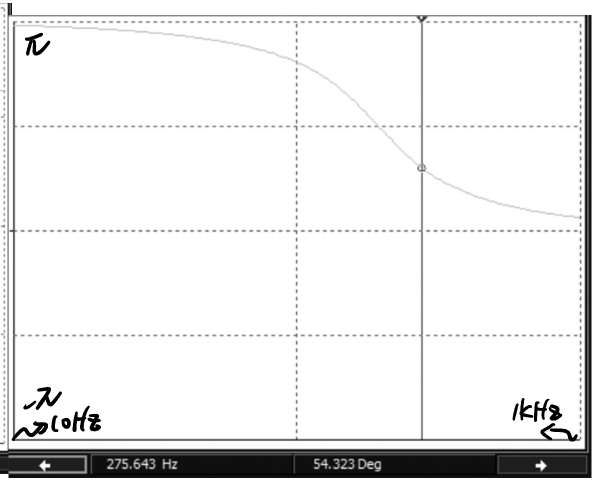
Multisim simulation Bode plot.

Multisim schematic (flip OP)

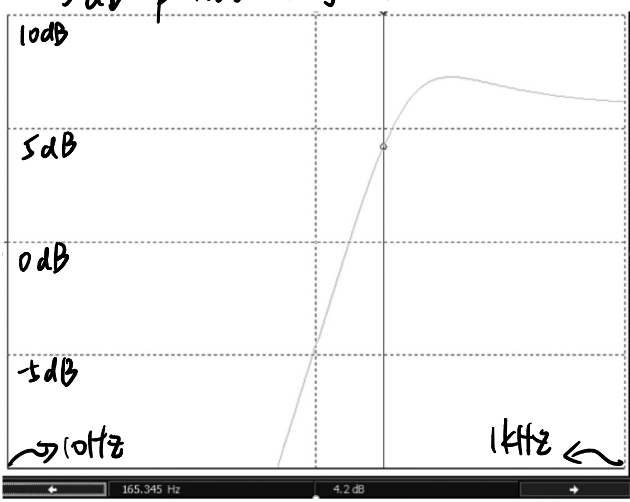


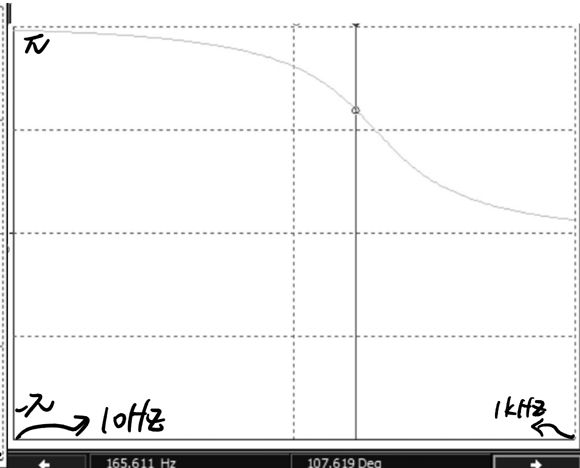
Peak gain at $f=276$ Hz with 7.26 dB and 54.325° phase shift





-3 dB point at $f=165$ Hz with 4.2dB and 107° phase shift





H. Question and Discussion

1. Adjust input signal frequency (輸入訊號頻率) from 100 Hz to 1000Hz and observe the changes of the gain (增益值)

* <Phase 1> LPF

(a) physical circuit

Input f (Hz)	Vin Vpp (V)	Vout Vpp (V)	Gain (V/V)
100	2.050	4.914	2.397
200	2.007	4.935	2.458
400	1.881	0.992	0.527

(b) multisim simulation

Input f (Hz)	Vin Vpp (V)	Vout Vpp (V)	Gain (V/V)
100	2	4.7	2.35
200	2	3.8	1.9
400	2	1	0.5

* <Phase 2> HPF

(a) physical circuit

Input f (Hz)	Vin Vpp (V)	Vout Vpp (V)	Gain (V/V)
100	1.955	2.684	1.372
200	1.890	5.078	2.686
1k	1.996	4.136	2.072

(b) multisim simulation

Input f (Hz)	Vin Vpp (V)	Vout Vpp (V)	Gain (V/V)
100	2	1.4	0.7
200	2	4.2	2.1
1k	2	4.1	2.05

* Gain in multisim are all smaller than physical circuit.

* For HPF at f = 100Hz, there exist big difference! In our physical circuit, the gain still >1, but simulation gain <1.

* The error might come from the propagation delay and parasitic capacitor in solid wire, which may not be concerned in simulation.

2. How well does the measured value (實際量測) of the cutoff frequency (截止頻率) agree with the cutoff frequency run by Multisim software simulation?

* <Phase 1> LPF

Hand-calculation $f_0 = \frac{1}{2\pi RC} = \frac{1}{2\cdot\pi\cdot(820)\cdot 10^{-6}} = 194\,Hz$

Measure from physical circuit: $f_0' = 229\,Hz$

Measure from multisim simulation: $f_0'' = 227\,Hz$

Error between physical circuit & multisim: $\frac{|f_0' - f_0''|}{f_0''} \times 100\% = 0.88\%$

⇒ Our physical circuit cut-off frequency is quite similiar to multisim!

* <Phase 2> HPF

Hand-calculation $f_0 = \frac{1}{2\pi RC} = \frac{1}{2\cdot\pi\cdot(820)\cdot 10^{-6}} = 194\,Hz$

Measure from physical circuit: $f_0' = 131\,Hz$

Measure from multisim simulation: $f_0'' = 165\,Hz$

Error between physical circuit & multisim: $\frac{|f_0' - f_0''|}{f_0''} \times 100\% = 20.6\%$

⇒ Our physical circuit cut-off frequency has 20% error to multisim.

This may comes from:

1. Parasitic capacitance from solid wire.
2. OP741 act as LPF on open-loop gain, therefore, when using OP741 to build the HPF, it might lead to error!

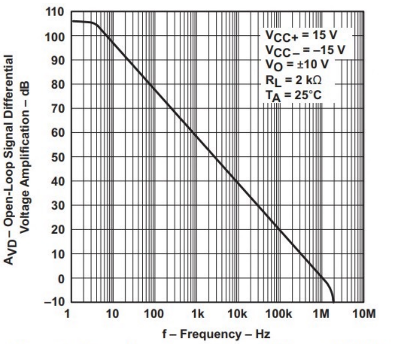
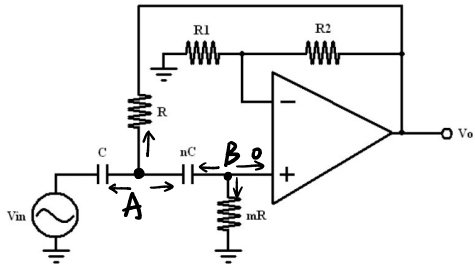


Figure 7. Open-Loop Large-Signal Differential Voltage Amplification vs Frequency

← open-loop gain frequency response from OP741 data sheet. (Rev. G)

3. Derive the transfer function (轉移函數) of the second-order active high-pass filter (二階主動式高通濾波器).

1° Apply KCL on node A/B
(assume ideal op. amp)



$$\begin{cases} A: \dfrac{V_A - V_{in}}{\frac{1}{sC}} + \dfrac{V_A - V_o}{R} + \dfrac{V_A - V_B}{\frac{1}{snC}} = 0 \quad \cdots (1) \\ B: \dfrac{V_B - V_A}{\frac{1}{snC}} + \dfrac{V_B}{mR} = 0 \quad \cdots (2) \end{cases}$$

2° Virtual short on ideal op-amp : $V_B = V_+ = V_- = V_o \cdot \dfrac{R_1}{R_1 + R_2}$
(replace in (1), (2))

3° $$\begin{cases} (V_A - V_{in})\, sC + \dfrac{V_A - V_o}{R} + \left(V_A - \dfrac{V_o R_1}{R_1+R_2}\right) snC = 0 \quad \cdots (3) \\ \left(\dfrac{V_o R_1}{R_1+R_2} - V_A\right) snC + \dfrac{V_o R_1}{(R_1+R_2)\, mR} = 0 \quad \cdots (4) \end{cases}$$

4° From (4), we have $V_A = \dfrac{V_o R_1}{R_1+R_2}\left(1 + \dfrac{1}{smnRC}\right) \quad \cdots (5)$

5° Substitute (5) into (3) and obtain $H(s) = \dfrac{V_o}{V_{in}}$:

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \left(\frac{R_1+R_2}{R_1}\right) \cdot \frac{mnR^2C^2s^2}{mnR^2C^2s^2 + \left[RC + nRC + \frac{R_2}{R_1} mnRC\right]s + 1}$$

$$\left(\times \frac{\cancel{1}/mnR^2C^2}{\cancel{1}/mnR^2C^2}\right) = \left(\frac{R_1+R_2}{R_1}\right) \frac{s^2}{s^2 + \frac{1}{mnRC}\left[1 + n + \frac{R_2}{R_1} mn\right]s + \frac{1}{mnR^2C^2}}$$

#

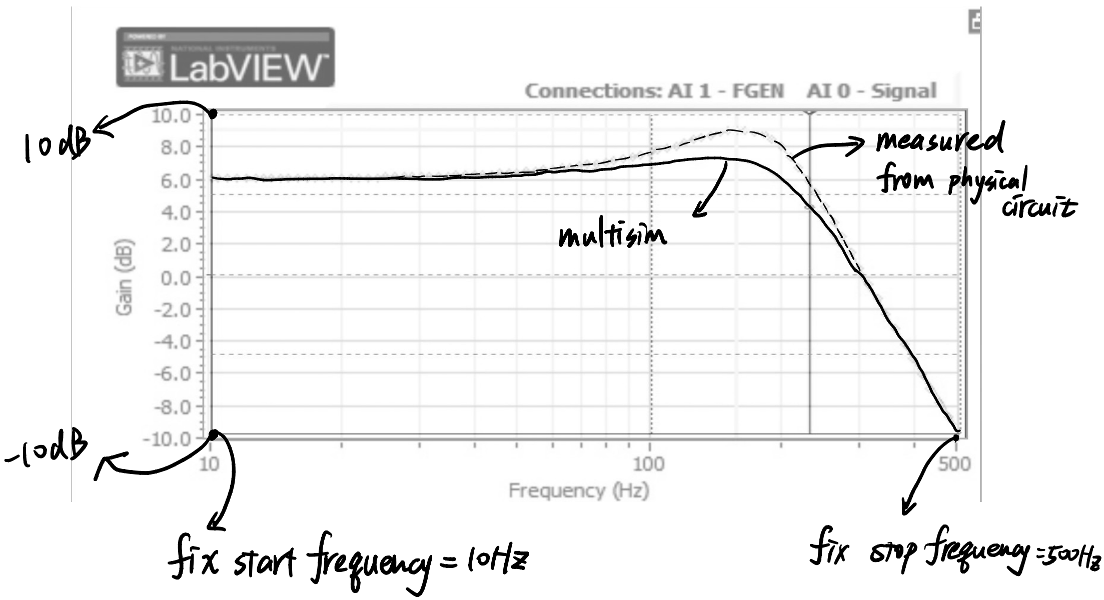
* if $m = n = 1 \Rightarrow H(s) = \left(\dfrac{R_1+R_2}{R_1}\right) \dfrac{s^2}{s^2 + \frac{1}{RC}\left[2 + \frac{R_2}{R_1}\right]s + \frac{1}{R^2C^2}}$

#

4. How well does the measured value of the attenuation level (衰減度) agree with the attenuation level run by MultiSim software simulation?

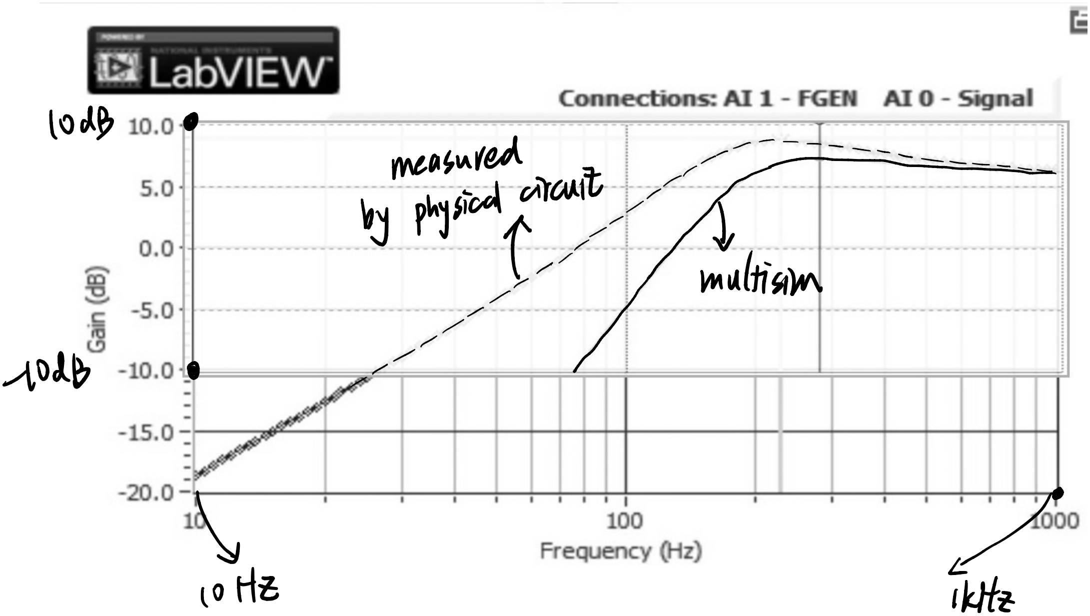
I think it's quite hard for us to use numerical value (slope) to compare with two bode plots, therefore, I try to change the transparency of my bode plots, expand them on the same size, and overlap 2 graph to see whether the attenuation level are similar. Here is my result:

< Phase 1 > LPF



* When $f < 100Hz$, both measured value and multisim are overlapped quite well.
* When $100 Hz < f < 300 Hz$, measured value has a bigger peak than multisim.
* When $f > 300Hz$, the attenuation level are quite the same, overlap quite well!

< Phase 2 >



* When $f > 500$ Hz, both measured value and multisim are overlapped quite well.
* When $f \approx 200$ Hz, physical circuit has bigger peak gain than multisim simulation result.
* When $f < 300$ Hz, multisim attenuation is more obviously than our measured value from physical circuit. It decays faster than measured value!

* Feedback: I have learned these filter theory in signal and system last year, and it is quite insteresting for me to implement the physical circuit and perform some simulation on multisim.