Discrete Mathematics

Assignment 2

Give justifications for all your answers. All graphs are simple and undirected unless otherwise stated. Each question carries 10 marks.

1. A graph G is said to be *self complementary* if G is isomorphic to $\overline{G}$. List all the trees that are self-complementary.
2. Derive a necessary and sufficient condition for a sequence of integers to be the degree sequence of a tree.
3. A graph is *k-regular* if every vertex has degree k. Show that for $k \geq 2$, a bipartite k-regular graph contains no bridge.
4. Show that every 3-regular 3-edge-connected graph is 3-connected.
5. Let d be the maximum degree of a vertex in a tree T. Prove that T has at least d leaves.
6. A directed graph that contains no directed cycles is called a *directed acyclic graph*, or *DAG* for short. Prove that the vertices of any DAG can be arranged in a sequence such that for every edge (u, v) in the DAG, u appears before v in the sequence.
7. Let C be a cycle in a graph G. A *chord* of C is an edge in G between two vertices of C that are not consecutive on C. A cycle without a chord is called a *chordless* cycle. Show that every graph that contains at least one cycle is either bipartite or contains a chordless odd cycle.