Tutorial

Saturday, 20 November 2021 2:21 PM

$\left(r + \frac{1}{r}\right)$ is an integer $\qquad r^i + \frac{1}{r^i}$

$i \le n$

To prove: $r^n + \frac{1}{r^n}$ is an integer.

$n=0$, $r^0 + \frac{1}{r^0}$ is an integer

$n=1$, $r + \frac{1}{r}$ is an integer.

$$\left(r^{n-1} + \frac{1}{r^{n-1}}\right)\left(r + \frac{1}{r}\right) = r^n + \frac{1}{r^n} + \frac{1}{r^{n-2}} + r^{n-2}$$

$$r^n + \frac{1}{r^n} = \left(r^{n-1} + \frac{1}{r^{n-1}}\right)\left(r + \frac{1}{r}\right) - \left(r^{n-2} + \frac{1}{r^{n-2}}\right)$$

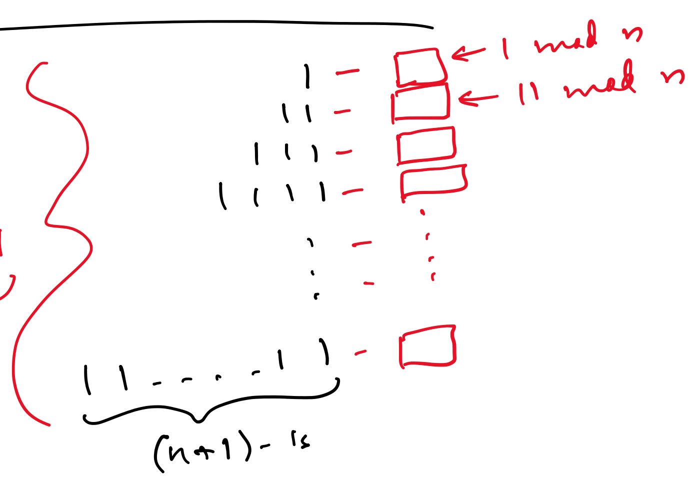
$\to$ integer.

$$r^2 + \frac{1}{r^2} = \left(r + \frac{1}{r}\right)\left(r + \frac{1}{r}\right) - \left(r^0 + \frac{1}{r^0}\right)$$

$$r^3 + \frac{1}{r^3} = \left(r^2 + \frac{1}{r^2}\right)\left(r + \frac{1}{r}\right) - \left(r + \frac{1}{r}\right)$$

$\boxed{n}$

Different remainders possible: $\underbrace{0, 1, 2, \ldots, n-1}_{n}$



$(j-i)$ will be divisible by n //

1111111111 − j

1111111 − i

1100000000

---

① $\quad$ 2, 3, . . . , n $\quad$ {①, 2, 3} $\quad$ {① ◯ }

2^{n-1} possible subsets $\qquad$ every subset of 2, 3, .., n

{1} ∪ S

$\{1, 2, 3, \ldots, n, \underbrace{n+1, n+2, \ldots, 2n}\}$

for every $i \in \{1, 2, \ldots, 2n\}$

$\exists\, t$ s.t. $2^t i \in \{n+1, n+2, \ldots, 2n\}$

$i \to f(i) \qquad f(i) \in \underbrace{\{n+1, n+2, \ldots, 2n\}}_{n \text{ diff. no.s.}}$

$S \subseteq \{1, 2, \ldots, 2n\}$

$|S| \geq n+1 \Rightarrow \exists\, i, j \in S$ s.t. $f(i) = f(j)$

$f(i) = 2^{t_1} i, \quad f(j) = 2^{t_2} j$

$i < j, \quad 2^{t_1} i = 2^{t_2} j \Rightarrow t_1 > t_2$

$2^{t_1 - t_2} i = j \Rightarrow i$ divides j

---

52 numbers

$n_1 - \boxed{\ } \; n_1 \bmod 100$

$n_2 - \boxed{\ } \; n_2 \bmod 100$

$\vdots$

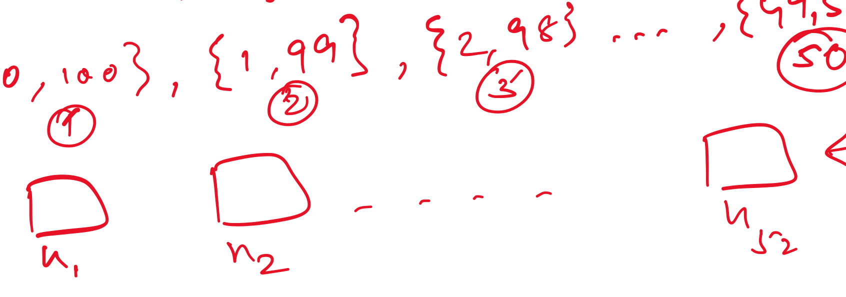
$n_{52} - \boxed{\ } \; n_{52} \bmod 100$

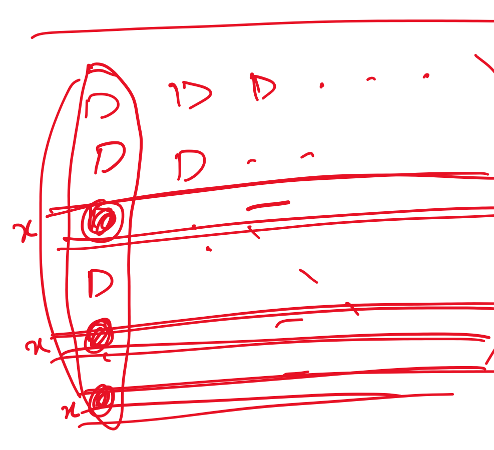
n_i and n_j have the same remainder mod 100, then $n_i - n_j$ is divisible by 100.

If $(n_i \bmod 100) + (n_j \bmod 100) = 100$, then $n_i + n_j$ is divisible by 100.

{0, 100}, {1, 99}, {2, 98} . . . , {49, 51}, {50}

① ② ③ ㊿ (51)



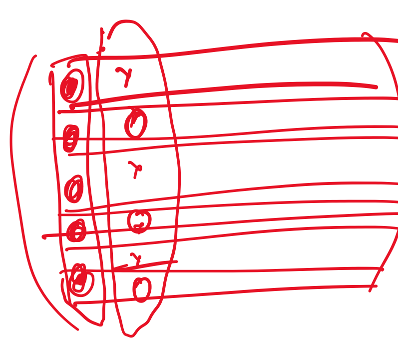


$$\left((s-1)\, n^{(t-1)n+1} + 1\right)$$

1, 2, . . , n

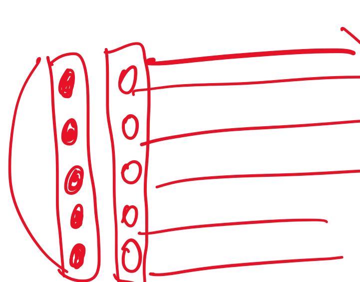
$$\geq \frac{(s-1)\, n^{(t-1)n+1} + 1}{n}$$

$$\geq (s-1)\, n^{(t-1)n} + \frac{1}{n}$$



$$\geq (s-1)\, n^{(t-1)n} + 1$$

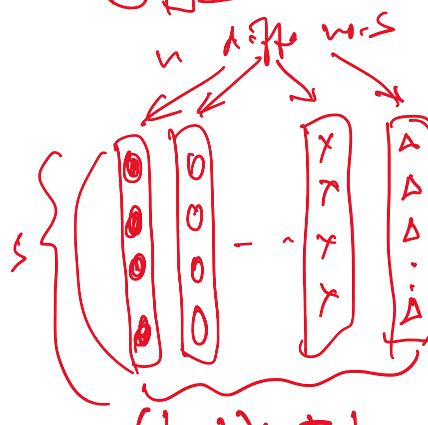
$$\geq \frac{(s-1)\, n^{(t-1)n} + 1}{n}$$



$$= (s-1)\, n^{(t-1)(n-1)} + \frac{1}{n}$$

$$\geq (s-1)\, n^{(t-1)(n-1)} + 1$$

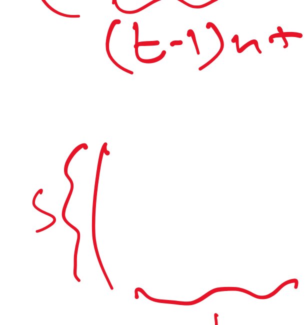
n diff. no.s



$(s-1) + 1$

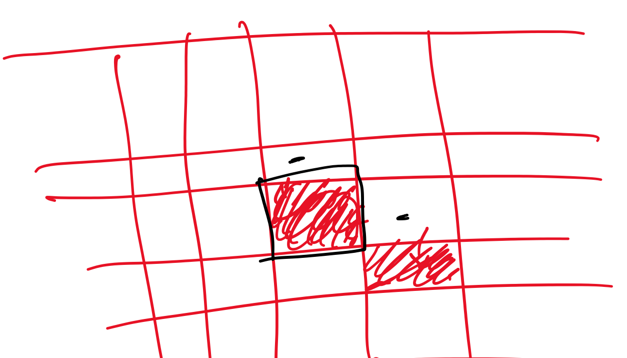
$= s$

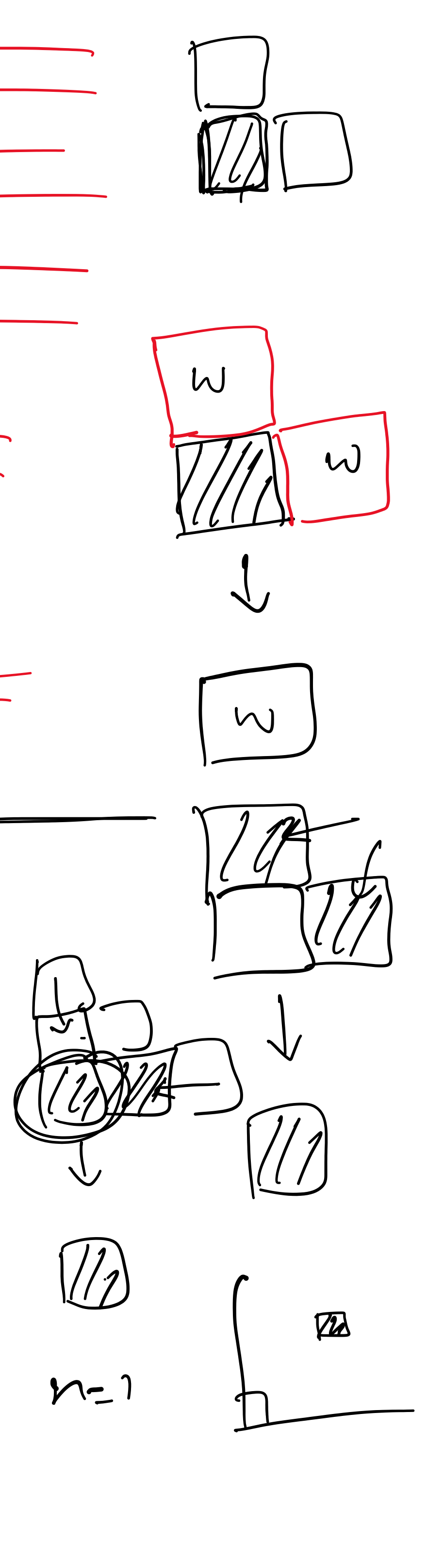
$\geq \frac{(t-1)n+1}{n}$ columns contain the same number.

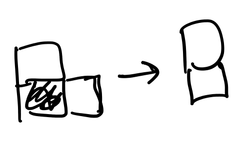


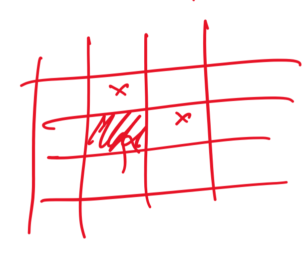
$\geq (t-1) + \frac{1}{n}$

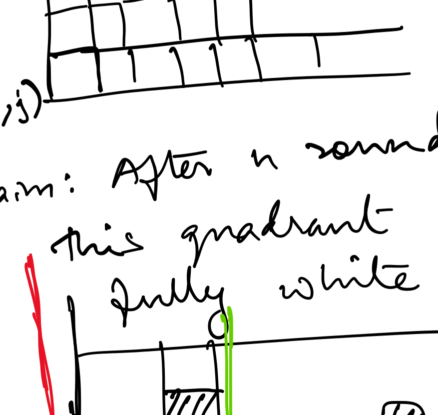
$\geq t$





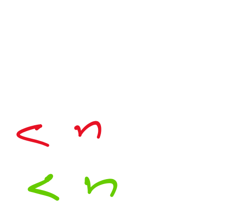


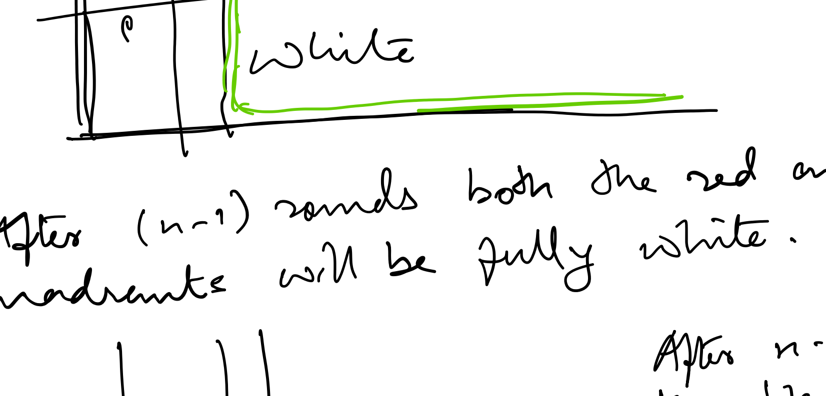




Claim: After n rounds, this quadrant is fully white

$n=1$

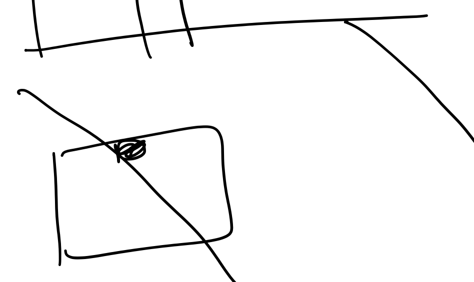




$< n$

$< n$

After $(n-1)$ rounds both the red and green quadrants will be fully white.



After n-rounds the black quad. is fully white.