Lecture 21

Monday, 27 December 2021 2:20 PM

A graph G is l-edge-connected

$\iff \lambda(G) \geq l$

"edge connectivity" (vertex)

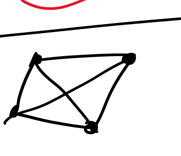
A graph G is k-connected

$\iff \kappa(G) \geq k$

"(vertex)-connectivity"

Reinhard Diestel, "Graph Theory".

How large can $\kappa(G)$ be in terms of n? → no. of vertices

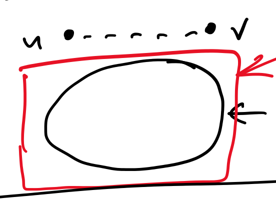


$\kappa(K_n) = n-1$

complete graph on n vertices

$\kappa(G) = n-1 \implies G$ is K_n

Suppose that G is not K_n.

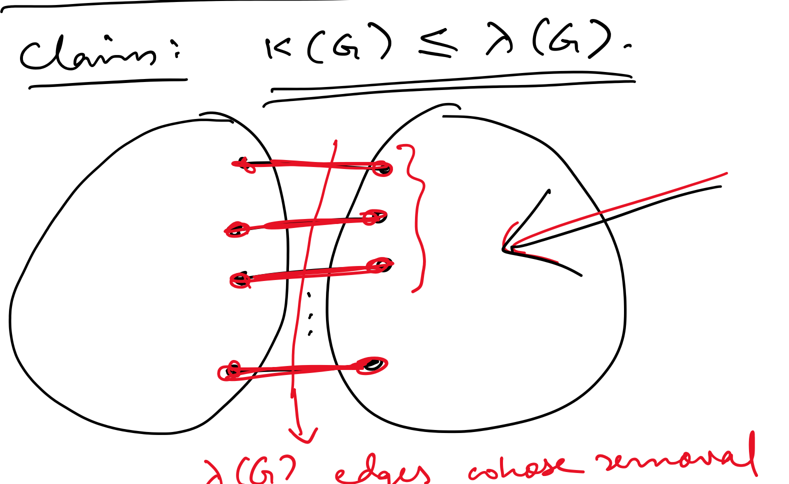


$\leq n-2 \implies \kappa(G) \leq n-2$ [contradiction]

$\leftarrow V(G) - \{u, v\}$

$\lambda(G)$ $\kappa(G)$

Claim: $\kappa(G) \leq \lambda(G)$.

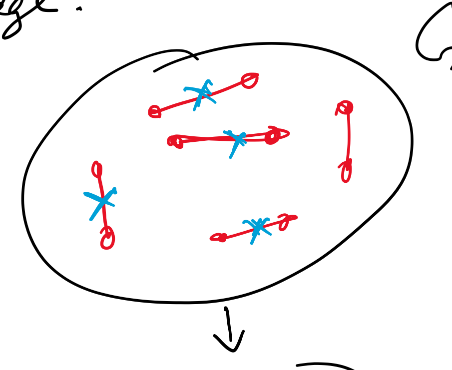


$\lambda(G)$ edges whose removal disconnects the graph.

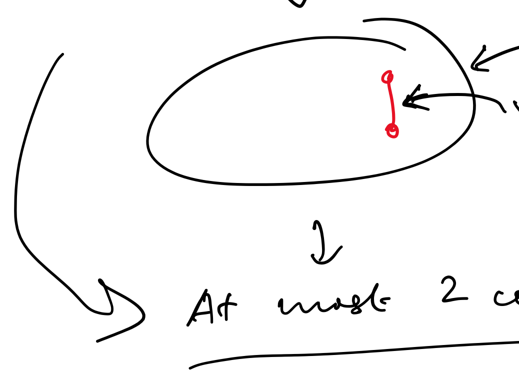
Can G be disconnected by the removal of $\lambda(G)-1$ edges? NO!

No proper subset of the red edges can disconnect the graph.

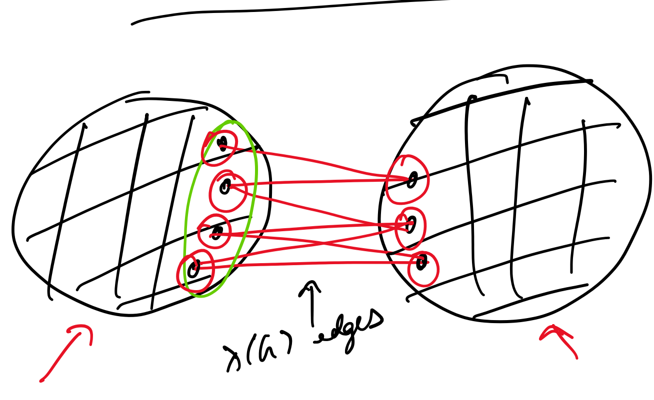
{A connected graph cannot be disconnected into more than two components by removing a single edge.

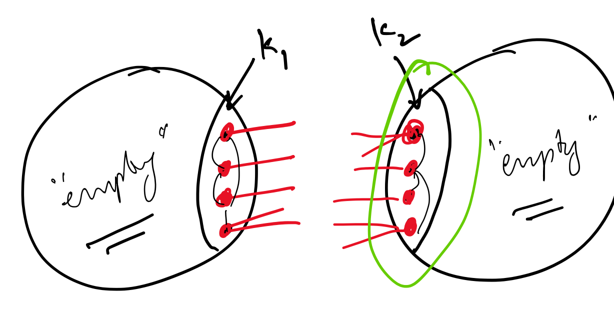


$\lambda(G) = 5$



At most 2 components

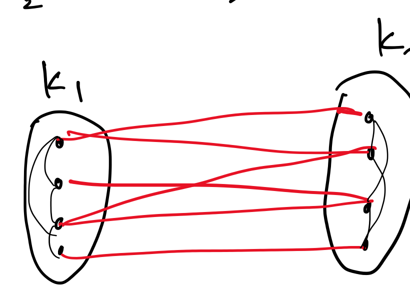


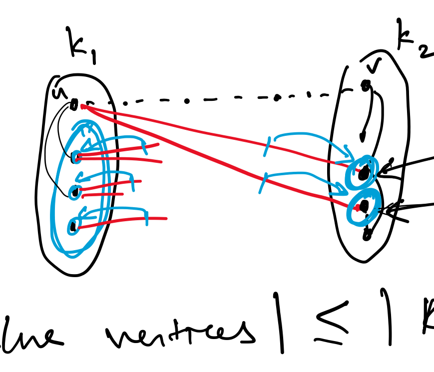


$k_1 > \lambda(G)$? NO

$k_1 \leq \lambda(G)$

$k_2 \leq \lambda(G)$



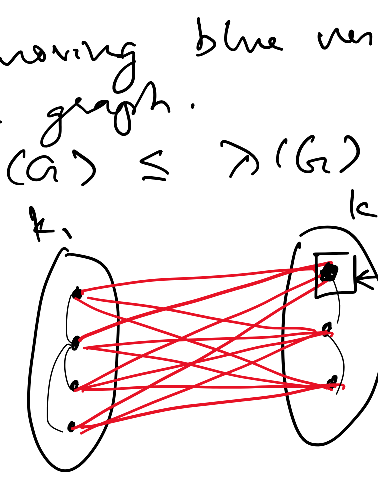


|Blue vertices| $\leq$ |Red edges|

$\leq \lambda(G)$

Removing blue vertices disconnects the graph.

$\kappa(G) \leq \lambda(G)$ [if $u \cdots v$]



$\lambda(G) = k_1 k_2$

Degree of a vertex $\leq k_1 + k_2 - 1$

degree of a vertex $\geq \lambda(G) = k_1 k_2$

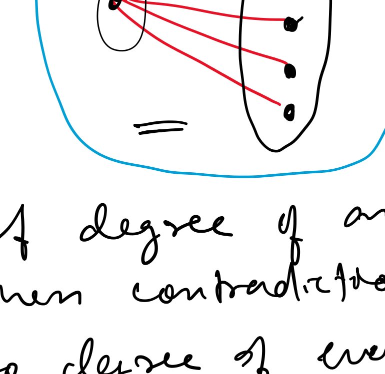
$k_1 k_2 \leq k_1 + k_2 - 1$

$k_1(k_2 - 1) + k_1 \leq k_1 + k_2 - 1$

$k_1(k_2 - 1) \leq k_2 - 1$

Either $k_2 = 1$, or $k_1 \leq 1$

At least one of k_1 or k_2 is 1.



Assume $k_1 = 1$

$\lambda(G) = k_2$

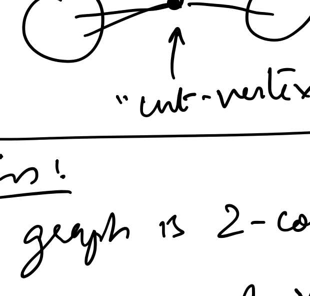
If degree of any vertex $< k_2$, then contradiction to $\lambda(G) = k_2$.

So degree of every vertex $\geq k_2$

Total no. of vertices $= k_2 + 1$

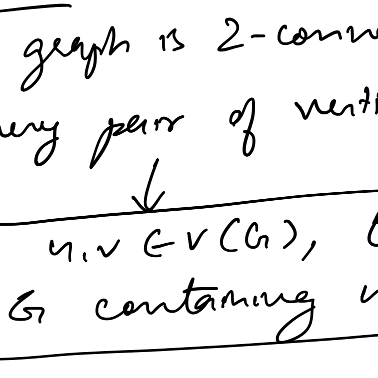
The G is a complete graph.

$\implies \underset{n-1}{\kappa(G)} \leq \underset{n-1}{\lambda(G)}$?



$k_1 k_2 \geq k_1 + k_2 - 1$

Structure of 2-connected graphs

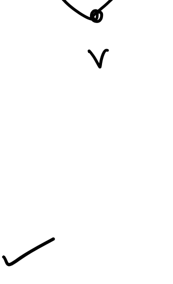


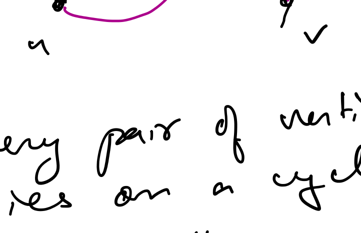
Claim:

A graph is 2-connected if and only if

Every pair of vertices lie on a cycle.

For $u, v \in V(G)$, there is a cycle in G containing u and v.





Every pair of vertices lies on a cycle ✓

⇓

the graph is 2-connected