Error detection

(1) Compute $c(x)$ and $T(x)$, the CRC bits and transmitted string respectively.

$S(x) = 110101$ $(k=5)$, $g(x) = 1001$ $(L=3)$

Am: $S(x) = x^5+x^4+x^2+1$, $g(x) = x^3+1$

$S(x)\,x^L = (x^5+x^4+x^2+1)x^3 = x^8+x^7+x^5+x^3$

$\frac{S(x)\,x^L}{g(x)}$.

$1+x^3 \;)\; x^8+x^7+x^5+x^3 \;(\; x^5+x^4+x+1$

x^8+x^5

x^7+x^3

x^7+x^4

x^4+x^3

x^4+x

x^3+x

x^3+1

$x+1$ → $c(x)$

$T(x) = S(x)\,x^L + c(x)$

$= x^8+x^7+x^5+x^3+x^2+1$

$= (x^8+x^7+x^5+x^3+x+1)$

Am:

$g(x)$ = 1001, $S(x).x^L$ = 1101010000

1001) 1101010000 (1100 11

1001

1000

1001

0011

0000

0110

0000

1100

1001

1010

1001

011 → $1+x$ → $c(x)$

* Append L zeros to $S(x)$ and then divide by $g(x)$.

$T(x) = S(x).x^L + c(x)$

$= 110101000 + 011$

$= (110101011) \rightarrow x^8+x^7+x^5+x^3+x+1$

(2) Suppose $g(x) = 1001$ and $T(x) = 1010101$, did any transmission error occur?

1001) 1010101 (1011

1001

0111

0000

1110

1001

1111

1001

110

Since remainder is not zero, there must have been errors in transmission.

(3) Distance between two Codewords: (Hamming) distance between two n-bit codewords w_1 and w_2 is the number of bit positions in which they differ.

Thus $0 \le HD(w_1, w_2) \le n$.

minimum Hamming distance of a code: minimum Hamming distance between any two codewords in the code: That is, $\min\{HD(w_1, w_2) : \forall w_1, w_2 \in C\}$.

A code is a set of codewords.

A code with minimum Hamming distance of D can detect any error pattern of D-1 or fewer errors. Moreover, there is at least one error pattern with D errors that cannot be detected.

A <u>linear code</u> produces codewords from message bits by restricting the algebraic operations to linear functions. By linear we mean that any check bit in a codeword is computed as the weighted sum of one or more message bits.

Binary Linear Code: uses only arithmetic modulo 2.

A code is linear if and only if the sum of any two codeword is another codeword. This implies that all-zeros codeword has to be in any linear code, because it results from adding a codeword to itself.

The weight of a codeword is the number of 1's in the codeword. The minimum Hamming distance of a linear code is equal to the minimum weight of the non-zero codewords: $\min\{\text{weight}(w) : w \text{ is non-zero and } \in C\}$.

(n, k) code → n is the length of the codeword.
k is the length of the message bits

So (n-k) is the number of check bits.

(2) Consider (3,2) even parity code and (4,3) odd parity code.

i) Are they linear codes? why?

~~ii) what are the parity check equations for these~~

The set of codewords for (3,2) even parity is.

	0	0	0
a	0	1	1
b	1	0	1
c	1	1	0

* 000 is present 000 + w = w

~~000 + 011 → 011 ∈ C~~
~~+ 101 → 101 ∈ C~~

+

ab * 011 ⊕ 101 = 110 ∈ C
ac 011 ⊕ 110 = 101 ∈ C
bc 101 ⊕ 110 = 011 ∈ C.

Sum of any two codewords is another codeword. So (3,2) is linear.

The set of codewords for (4,3) odd parity code is

0001
0010
0100
0111
1000
1011
1101
1110

Since $0001 \oplus 0010 = 0011$ which is not a codeword.

Also, $0001 \oplus 0001 = 0000$ which is not present.

So, (4,3) code is not linear.

(b) ii) what are the set of parity check equations for (3,2) even and (4,3) odd parity code?

for (3,2) even parity:

$u_2 u_1 \; p$

$p = u_1 \oplus u_2$

for (4,3) odd parity:

$u_3 u_2 u_1 \; p$

$p = u_1 \oplus u_2 \oplus u_3 \oplus 1$

If odd number of 1 in $u_1 u_2 u_3$ then $u_1 \oplus u_2 \oplus u_3 = 1$, so $p = 0$. This implies $u_1 u_2 u_3 p$ has odd number of 1s.

If even number of 1 in $u_1 u_2 u_3$ then $u_1 \oplus u_2 \oplus u_3 = 0$, so $p = 1$. This implies $u_1 u_2 u_3 p$ has odd number of 1s.

if odd number of 1s in $u_1 u_2$ then $u_1 \oplus u_2 = 1$. So $p = 1$. This implies $u_1 u_2 p$ has even number of 1s.

If even number of 1s in $u_1 u_2$ then $u_1 \oplus u_2 = 0$, so, $p = 0$. This implies $u_1 u_2 p$ has even no. of 1s.

iii) find minimum Hamming distance and error detecting capability of (3,2) and (4,3):

For (3,2) even:

000 →	weight = 0
011 →	" = 2
101 →	" = 2
110 →	" = 2

minimum non-zero weight = 2.

So error detecting capability = 2-1 = 1.

Since (3,2) is linear

For (4,3) Code

Since (4,3) is not linear.

* minimum Hamming distance is not necessarily equal to minimum Hamming weight.
* So we need to compute the distance between each pair of codewords.
* Alternatively, we can argue that two distinct codewords must differ by at least 2 otherwise the odd parity will not be maintain. Hence $d_{min} = 2$ implying error detecting capability $= 1$.

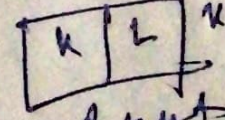
(4) Consider Hamming code with $g(x) = 1+x+x^3$. Determine if the codewords described by $c_1(x) = 1+x+x^3+x^7$ and $c_2(x) = 1+x^3+x^5+x^6$ are valid codewords for this generator polynomial.

Ans $c_1(x) = g(x)(1+x^3)$.

* If we divide $c_1(x)$ by $g(x)$, we get remainder zero. So valid codeword.
* If we divide $c_2(x)$ by $g(x)$, we get non-zero remainder. So $c_2(x)$ is not valid.

(5) Suppose an error occurs only in data bits and not in CRC bits. Argue that this error may not be detected.

Ans: Add $e(x) = g(x)x^L$ to $T(x)$ to get $T'(x)$.
Such $T'(x)$ is divisible by $g(x)$.



Suppose an error occurs only in CRC bit and not in data bits. Argue that such error must be detected.

Ans: Let $T(x)$ is transmitted and $T'(x)$ is received. Since error occurs only in CRC bit $e(x) = T(x) - T'(x)$. is of degree $L-1$. But $g(x)$ has degree L. So $e(x)$ is not divisible by $g(x)$. Hence errors are detected.