Lecture 1

Monday, 4 October 2021 12:37 PM

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80 % - Exams (mid-sem, end-sem)
20 % - Assignments

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Mathematical Proofs

"Facts" Axioms

Natural numbers ($\mathbb{N}$)

0, 1, 2, 3, ...

Peano's axioms define what $\mathbb{N}$ is.

(1) Statement: There is at least one natural number that is divisible by only 1 and itself.

$\boxed{2}$ ✓ "witness" (True)

(2) Statement: Every prime number is odd. (False)

2 is a prime number that is not odd.

"2" is a "counterexample".

"Negation" of (1) is: $\neg(1)$ =
"There are no natural numbers that are divisible by only 1 and itself."

$\neg(2)$: There is a prime number that is not odd.

"Law of the excluded middle". ←

"Foundations of mathematics"
↳ Logic, set theory.

$[S_1 \wedge S_2]$ — F $\neg[S_1 \wedge S_2]$
T and F $[\neg S_1 \vee \neg S_2]$

S_1 - this shirt is white
S_2 - this shirt is sleeveless
$S_1 \wedge S_2$ - this shirt is both white and sleeveless

$\neg(S_1 \wedge S_2)$ - $\neg S_1 \vee \neg S_2$
De Morgan's Law

(3) "Every number >1 is either prime or divisible by 2 or 3." - False
25 is a counterexample.

(4) "Prime numbers are infinite."

Proof: Suppose there are only a finite number of prime numbers.

$\underbrace{\{p_1, p_2, \ldots, p_k\}}_{\text{set of all prime numbers}}$ → Can this be an empty set? No. from (1).

$p_1 < p_2 < \ldots < p_k$

$n = p_1 \cdot p_2 \cdot p_3 \cdots p_k$

n is divisible by each of $p_1, p_2, \ldots, p_k$

$n+1$

$\boxed{\text{None of } p_1, p_2, \ldots, p_k \text{ divides } (n+1).}$

$(n+1)$ is greater than each one of $p_1, p_2, \ldots, p_k$.

$(n+1)$ is not one of $p_1, p_2, \ldots, p_k$.

So $(n+1)$ is composite.

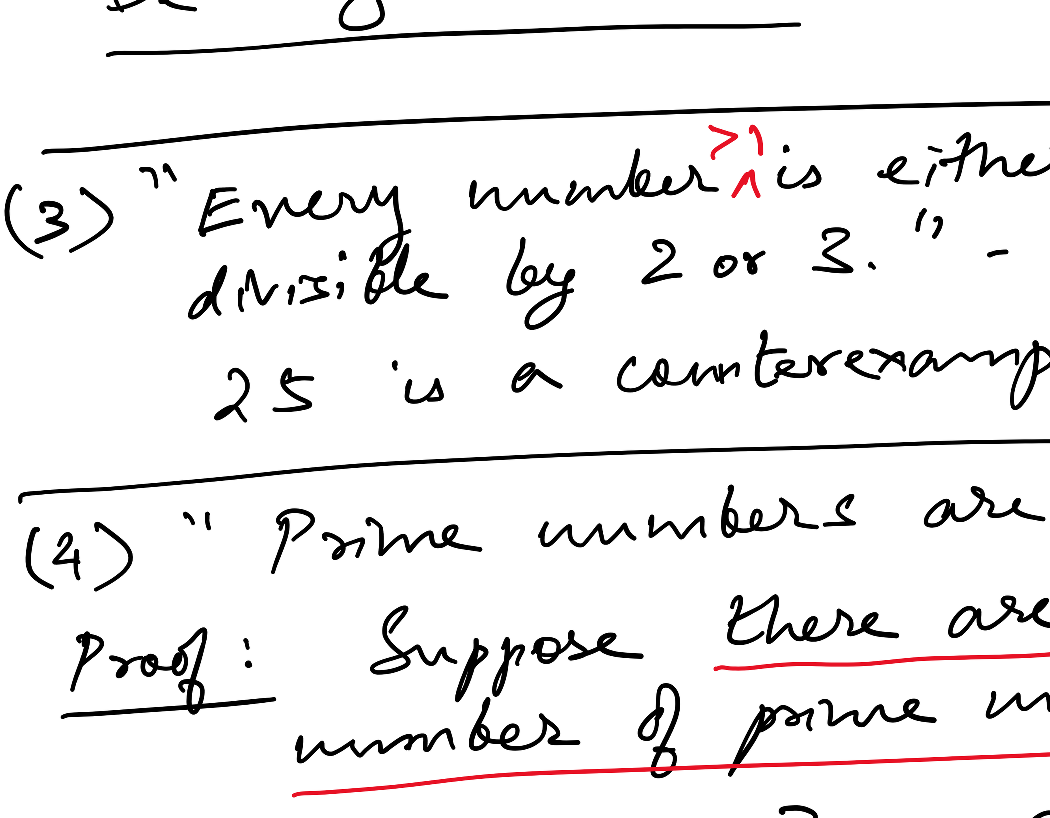
So there is a prime number that divides $n+1$.

$\boxed{\text{At least one among } p_1, p_2, \ldots, p_k \text{ divides } (n+1)}$

"Contradiction."

Our assumption that there are only a finite number of prime numbers is false.

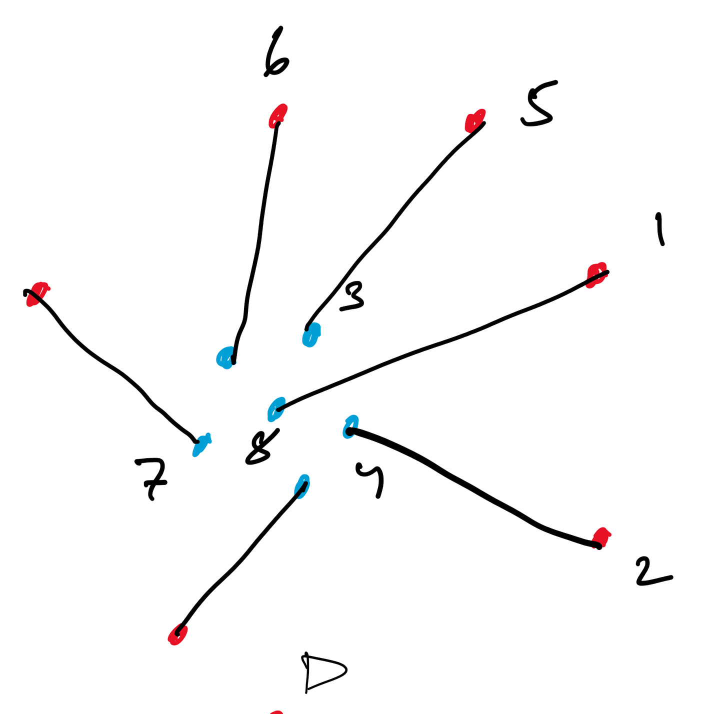
So (4) is true.



Assume: No three points are "collinear"
↓
lie on a straight line

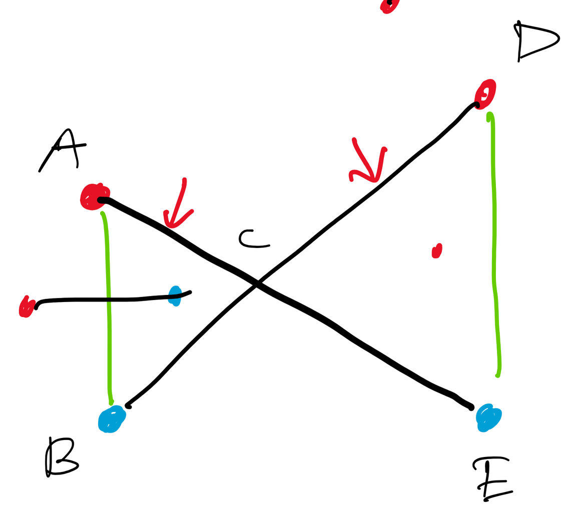
"The points are in general position"

"Given n red points and n blue points in gen. pos. One can pair each red point with a blue point in such a way that if a straight line segment is drawn between the two points in each pair, then no two line segments cross."

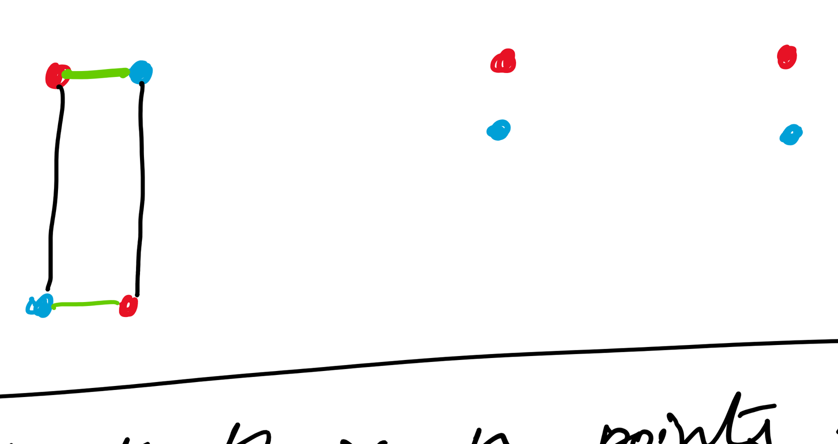


S = Sum of the lengths of all line segments

The value of S strictly decreases when you "swap" two line segments.



AB < AC + BC
DE < CD + CE
AB + DE < (AC + CE) + (BC + CD)
AB + DE < AE + BD



Suppose that there n points in the plane ($n \geq 2$), not all of them are collinear.

Then there is a straight line that passes through exactly two of the n points.