Finishing the Inverse Function Theorem

Thomas Cohn

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Lemma: $f, g \in C^r$, $f \circ g$ defined $\Rightarrow f \circ g \in C^r$.
$r = 1$: use the chain rule.
$D(f \circ g) = \mu(Dg, Df \circ g) \Rightarrow C^\infty \circ C^{r-1} \circ C^r \Rightarrow f \circ g \in C^r$. $\square$

The inverse function theorem also holds for $f \in C^r(A^{\text{osso}\ V}, W)$ if V, W are complete normed vector spaces (Banach spaces), and $Df(\vec{a})$ is bijective and bounded.

Ex: $V = C([-1,1], \mathbb{R})$, $\begin{array}{rcl} T : V & \to & \mathbb{R} \\ f & \mapsto & f(0) \end{array}$ linear. $||f||_1 = \int_{-1}^{1} |f|$, and $||f||_\infty = \max |f(x)| : -1 \le x \le 1$.

Ex: $||\cdot||_1$ and $||\cdot||_2$ are norms. $||f||_p = \sqrt[p]{\left(\int_{-1}^{1} |f|\right)^p}$.
$||f||_1 < 2\,||f||_\infty$, but $||f||_\infty$ not bounded by $C\,||f||_1$ $\forall C \in \mathbb{R}$.

We then show that not all norms are equivalent in an infinite-dimensional. This proof doesn't really make sense without the picture (and I don't know how to TeX graphs), so just check your notes.