Intro to the Multivariable Calculus Chain Rule

Thomas Cohn

9/21/18

We said earlier that differentiable functions can be locally well-approximated by affine functions.

Loosely, f is differentiable $\rightarrow f(\vec{a}+\vec{h}) \approx f(\vec{a}) + D_f(\vec{a})(\vec{h})$ for $\vec{h} \approx 0$.
$f(\vec{x}) \approx f(\vec{a}) + D_f(\vec{a})(\vec{x}-\vec{a})$.

Suppose we also have $f(\vec{a}) : B \rightarrow Z$, where B is an open subset of W and Z is a normed vector space.
With $\vec{b} \in B$, $g(\vec{y}) \approx g(\vec{b}) + D_g(\vec{b})(\vec{y}-\vec{b})$.

Combine $g(f(\vec{x})) \approx g(\vec{b})D_g(\vec{b})(f(\vec{x}) - f(\vec{a}))$
$\approx g(\vec{b}) + D_g(\vec{b})D_f(\vec{a})(\vec{x}-\vec{a})$
$\approx g(f(\vec{a})) + (D_g(\vec{b}) \circ D_f(\vec{a}))(\vec{x}-\vec{a})$.
This suggests $D_{(g\circ f)}(\vec{a}) = D_g(\vec{b}) \circ D_f(\vec{a})$.

This is the multivariable calculus chain rule.