Very Brief Notes

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Defn: $f \in C^\infty \leftrightarrow f \in C^r \ \forall r \in \mathbb{N} \cup \{0\}$.

$f \in C^1(A^{\text{osso}V}, W)$
$f \in C^2 \leftrightarrow Df \in C^1$
$f \in C^r \leftrightarrow Df \in C^{r-1}$ (for $r = 2, 3, 4 \ldots$)
$f \in C^r \leftrightarrow D_{\vec{u_1}} D_{\vec{u_2}} \cdots D_{\vec{u_k}} f$ exists and is continuous $\forall \vec{u_1}, \ldots, \vec{u_k} \in V$ with $k \leq r$.

From Wednesday, $f \in C^2(A^{\text{osso}V}, \mathbb{R}^n) \to D_{\vec{u_1}} D_{\vec{u_2}} f = D_{\vec{u_2}} D_{\vec{u_1}} f$.
This also works for $f \in C^2(A, W$ with $\dim W < +\infty$ (see study exercise 7).

Suppose $f \in C^r$. Consider $D_{\vec{u_1}} D_{\vec{u_2}} \cdots D_{\vec{u_r}} f$. We can interchange $\vec{u_j}$ with $\vec{u_{j+1}}$, so we can arbitrarily permute $\vec{u_j}$.

Ex: Suppose $f \in C^r(\mathbb{R}^n, \mathbb{R})$. How many distinct $D_{j_1} \cdots D_{j_r} f$ are there?