More Planar Graphs

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We want to come up with a charecterization of planar graphs. We proved K_5 and $K_{3,3}$ are nonplanar.

Defn: H is a minor of G if it can be obtained from G by deleting vertexes, deleting edges, and contracting edges.

Defn: H is a subdivision of G if it can be obtained by adding vertexes to split edges, removing edges, and removing vertexes.

Thm: (Wagner) A graph G is nonplanar iff G contains K_5 or $K_{3,3}$ as a minor.

Thm: (Kuratowski) A graph G is nonplanar iff G containes K_5 or $K_{3,3}$ as a subdivision.
Proof:
Claim 1: If G has no subdivision of K_5 or $K_{3,3}$ and $e \in G$, then $G/e = H$ has no subdivision of K_5 or $K_{3,3}$. If $H' \subset H$ has a subdivision of K_5 or $K_{3,3}$, and $z \in H'$, then H' is a subdivision of G. If $z \in H'$, and $\deg_{H'}(z) = 2$ (then just do every case) **I'm not sure what he means here**
Claim 2: (Inductive-ness) let $G' = G/e$. Then G' has fewer edges than G. By induction, G' is planar. $\square$

A torus is a doughnut. We can draw graphs onto a torus.

Ex: K_7 can be drawn on a torus.

Defn: T_g is the sphere with g handles.

Defn: Given G and T_g, an embedding of G on T_g is a drawing of G on T_g s.t. no 2 edges cross.

Defn: Given G and T_g, a 2-cell embedding of G on T_g is a drawing such that no 2 edges cross and $T_g \setminus (V \cup E) = \sqcup D_i$ where each D_i is a "disc".

Thm: If G has an embedding on T_g, then $v - e + f \geq 2 - 2g$.
If G has a 2-cell embedding on T_g, then $v - e + f = 2 - 2g$.