Stats 426 Lecture 2

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1/25/21

Defn: The sample space, denoted Ω, is the collection of all outcomes of an experiment or process.

Ex:

- Tossing a coin: $\Omega = \{H, T\}$
- Tossing 2 coins: $\Omega = \{(H, H), (H, T), (T, H), (T, T)\}$
- Rolling 2 dice and summing: $\Omega = \{2, \dots, 12\}$

Defn: An event is a subset of Ω. A simple event is a single outcome, whereas a compound event consists of more than one outcome.

Defn: An event A is said to occur if the actual outcome of the experiment is contained in A.

Defn: A discrete sample space has a finite or countably infinite number of elements. A continuous sample space has an uncountable number of elements.

Defn: Two events A and B are said to be mutually exclusive if they're disjoint, i.e., $A \cap B = \emptyset$.

The probability axioms:

- $P(\Omega) = 1$
- $P(A) \geq 0$, $\forall A \subseteq \Omega$
- For a finite family of mutually exclusive events $A_1, \dots, A_k$, $P(\bigcup_{i=1}^k A_i) = \sum_{i=1}^k P(A_i)$.
- For a countably infinite family of mutually exclusive events $A_1, \dots$, $P(\bigcup_{i=1}^\infty A_i) = \sum_{i=1}^\infty P(A_i)$.

In general, a compound event is the (disjoint) union of simple events.

If Ω consists of N equally likely simple events, then $\forall A \subseteq \Omega$, $P(A) = \frac{|A|}{N}$.

Properties of probability:

- $P(A^\complement) = 1 - P(A)$
- $P(\emptyset) = 0$
- $A \subseteq B \Rightarrow P(A) \leq P(B)$
- $\forall A, B$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Defn: The probability of an event A, given another event B, is called the conditional probability of A given B, and is denoted $P(A|B)$. It's defined by $P(A|B)P(B) = P(A \cap B)$.

Defn: Two events A and B are independent if $P(A|B) = P(A)$; $P(B|A) = P(B)$.

If A and B are independent, then $P(A \cap B) = P(A)P(B)$.

Consider selecting r (distinguishable) items from a set of n.

- When sampling with replacement, there are n^r possibilities.
- When sampling without replacement (and consider order), there are $\frac{n!}{(n-r)!}$ possibilities.
- When sampling without replacement (and not considering order), there are $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ possibilities.