Coho-Reach: Performance Analysis

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1 Introduction

For long time, I want to know the complexity of the reachability analysis algorith in COHO-REACH. By theory analysis, the complexity should be between $O(n^2)$ and $O(n^3)$. However, it is not proved by real applications, for several reasons. First, reachability analysis is usually too expensive for systems with more than four variables. Second, it lacks meanful examples with similar dynamics but different number of variables. So many algorithms have been presented and claimed to be efficient for high-dimensional systems. But the claim can not be proved.

In this document, we use n-stage ring oscillators an the example to analyze the performance of COHO-REACH and show the efficiency of our algorithms.

2 Examples

In this document, we use n-stage inverter oscillator as the example, where n is an odd number. What is more, we also use $2n$-stage Rambus ring oscillator as an example, where n is a even number. These circuits are well described in many places, so we igore it here.

Instead of using real transistor models like BSIM3 models or Hspice simulation data, we use a very simple model for the inverter circuit from [1].

$$\dot{V_{out}} = -\frac{V_{out} + V_{sat} tanh(\frac{V_{in}}{V_s})}{C} \tag{1}$$

where V_{in}, V_{out} are the voltage of inverter input and output nodes; C is the capacitance, V_{sat} is the saturation voltage and V_s is used to adjust the internal slope. The model provides a simple abstraction of inverter circuit. Meanwhile, it is easy to use and analyze.

By using this inverter model, it is simple to get the ODE model for the inverter ring oscillator and Rambus ring oscillator. In this document, we use $C = 1, V_{sat} = 1, V_s = 0.25$ to make the circuit oscillates.

3 Result

The machine we used is *mala.cs.ubc.ca*. The sever has 16-core Intel Xeon E5520 CPU with 2.27GHZ and 32G memory. COHO-REACH uses only one core and around 4G memory.

3.1 Inverter Ring Oscillator

We use the n-stage inverter oscillator as an example, where n is an odd number in the range $[3, 5, \ldots, 21]$. We perform reachability analysis from initial regions $HLHL \ldots H$ for 50 steps. As we use the *guess-verify* strategy for calculating step size, the forward time is not the same for each runs, but very close to 2.65. We have analyzed the performance for three typical configurations of COHO-REACH:

1. A: Convex projectagon, calculate one model for the projectagon
2. B: Convex projectagon, calculate models and advance projectagon faces individually.
3. C: Non-convex projectagon, calculate models and advance projectagon faces individually.

The running time is shown in figures 1 2 3.

The complexity is around $O(n^3)$, as shown in figures 4 5 6 ($T^{1/3}$ v.s. N).

The relative performance of configurations B and C to configuration A is shown in figure 7. The relative performance of configurations C to configuration B is shown in figure 8. We can see that configuration C is around 1.5 slower than configuration B. Configuration A is much faster than others the number of faces for configuration B and C increase linearly with the system dimensions.

The accuracy is not compared as I did figure out a easy way to measure the over-approximation error.

3.2 Rambus Ring Oscillator

Will get the data later

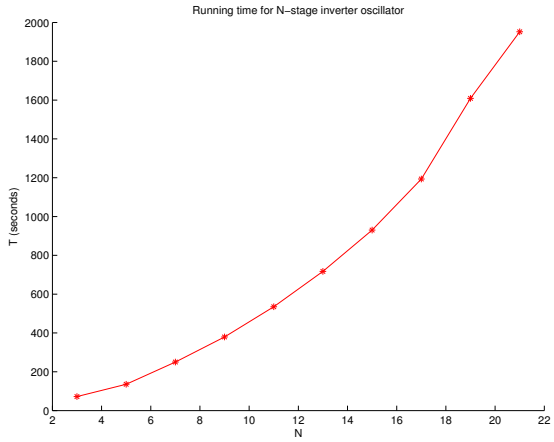

Figure 1: Running time for config A

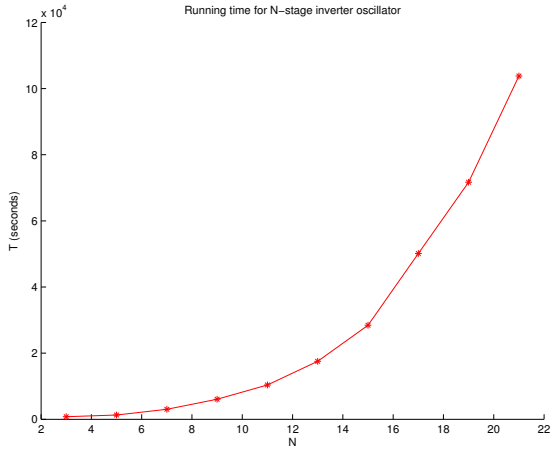

Figure 2: Running time for config B

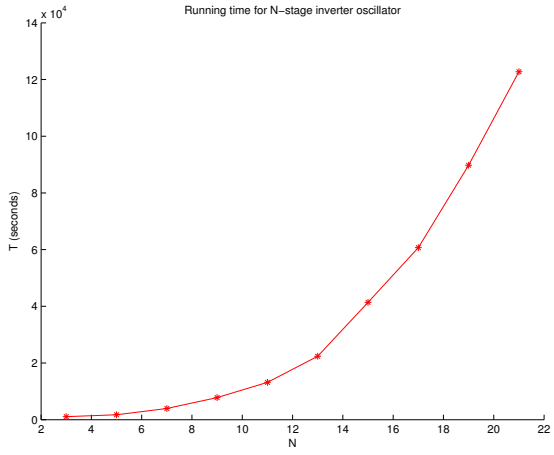

Figure 3: Running time for config C

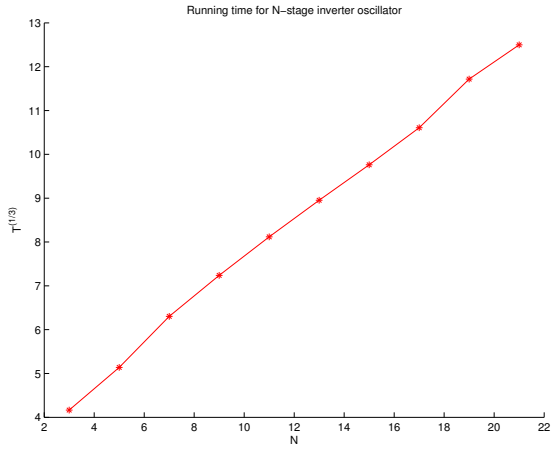

Figure 4: Running time for config A

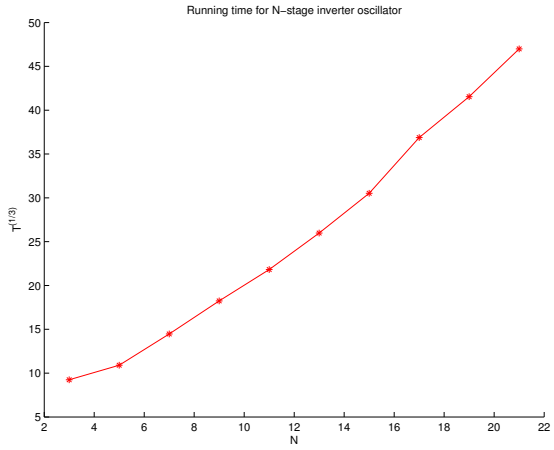

Figure 5: Running time for config B

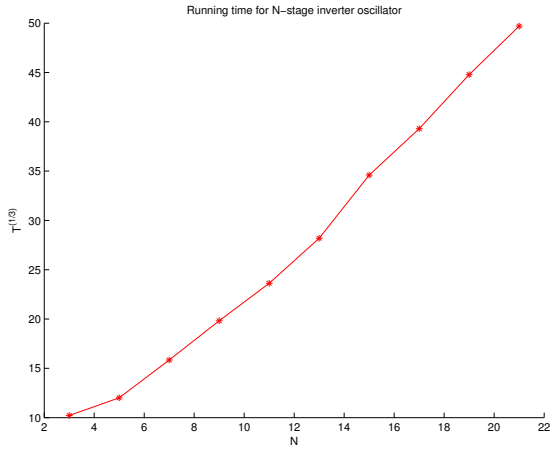

Figure 6: Running time for config C

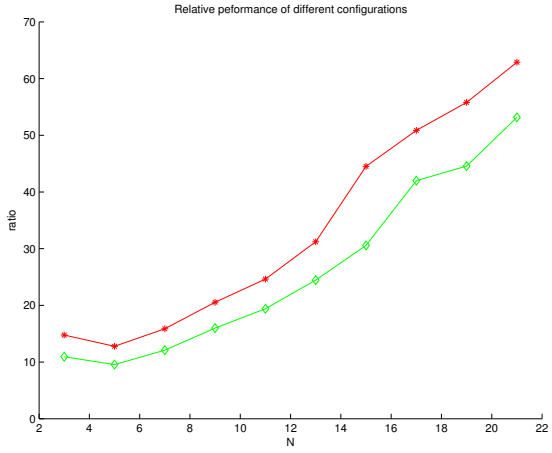

Figure 7: Compare performance of configs

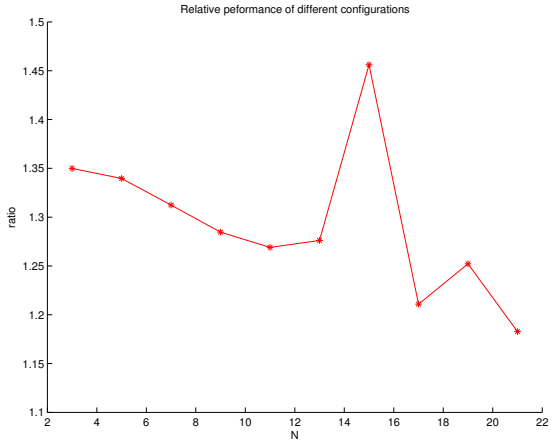

Figure 8: Compare performance of configs

References

[1] Xiaoqing Ge, Murat Arcak, and Khaled N Salama. Nonlinear analysis of ring oscillator and cross-coupled oscillator circuits. *Dynamics of Continuous, Discrete and Impulsive Systems*, December 2010. 3