Projectagon-Based Reachability Analysis for Circuit-Level Formal Verification

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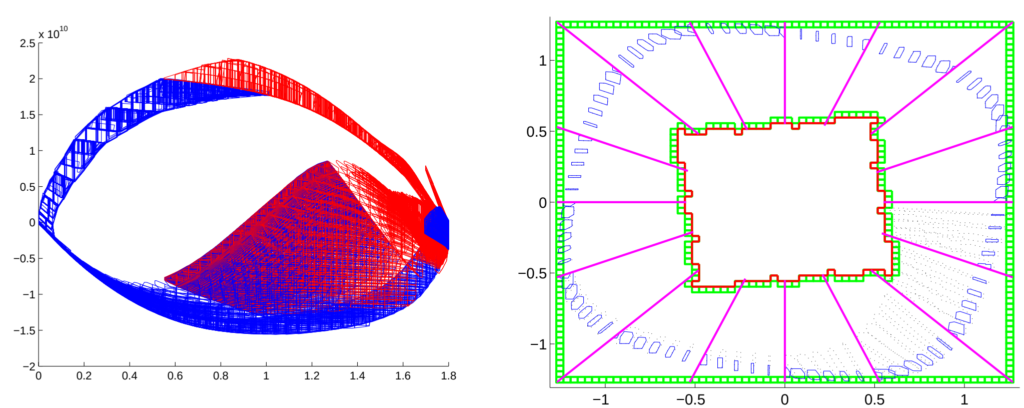


Motivation

- Circuit-Level Verification: verifying circuits using non-linear ODE models
  - Analog and mixed signal (AMS) circuits
  - Deep-submicron circuit effects undermine digital abstractions
- Analog Bugs
  - Account for large percentage of critical bugs
    - Analog design lacks systematic design flow and test methodologies
    - Analog design relies on designer intuition and expertise
  - Intel Sandy Bridge Chipset.
    - Lost: one billion $!
    - Passed all Intel's internal tests and all of OEM tests.
- Simulations cannot find all critical bugs before fabrication.
  - Incomplete coverage: difficult to cover all corner cases.
  - Inaccurate models: ideal working conditions and input signals.
- Problem Statement

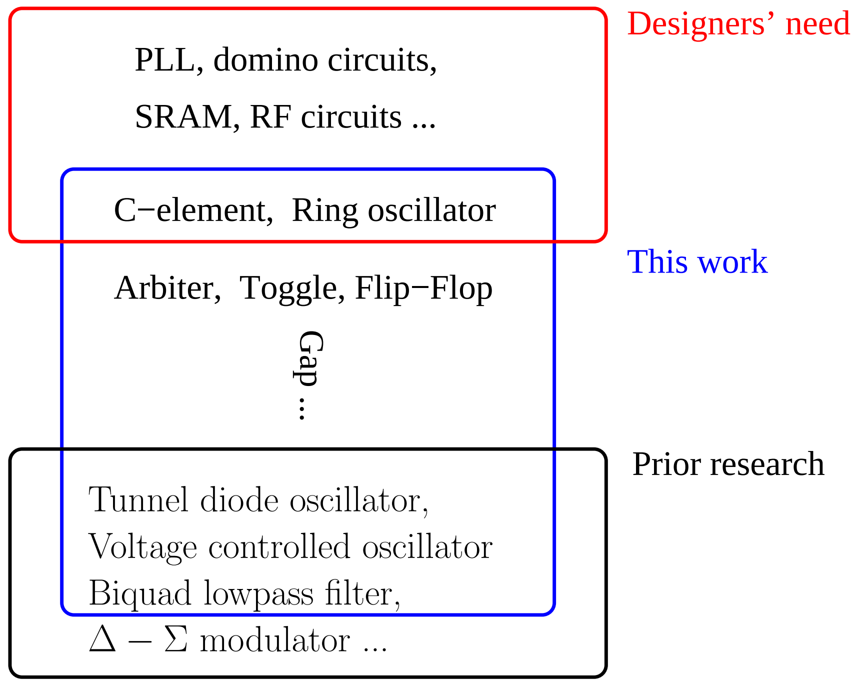
We need formal methods to circuit-level design errors.

Contributions

This thesis demonstrates the feasibility of formally verifying behaviors for circuits modeled by non-linear, ordinary differential equations.

- Verification as Reachability
  - Methodology for specifying circuit properties
  - Table-based methods for modeling circuits
  - Circuit verification $\rightarrow$ reachability analysis
- Reachability Analysis
  - Projectagon-based reachability analysis
  - Solve nonlinear ODEs by maximum principle
  - Improvements: robustness, efficiency, accuracy, usability
- Practical Circuit Verification Examples
  - Yuan-Svensson toggle circuit,
  - A flip-flop circuit
  - An arbiter circuit
  - The Rambus ring oscillator

Circuit-Level Formal Verification



- Limitations of Prior Work
  - Small circuits: *e.g.,* 2-3 variables
  - Simplistic dynamics: *e.g.,* (piecewise) linear
  - Verify simple properties
- Our Goal
  - Real circuits ($\sim$ 10-dim)
  - Nonlinear models
  - Verification need of designers

Outline

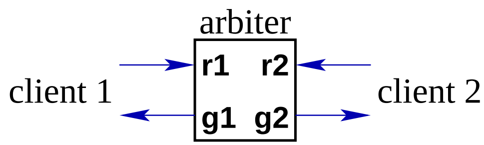
- Motivation and Contributions
- Verification as Reachability
- COHO: Reachability Analysis
- Circuit Verification Examples
- Conclusion

Formal Verification

- Goal: show Model $\models$ Specification
- Model: nonlinear ordinary differential inclusion
  - Nonlinear
    - Real circuits are nonlinear
    - Linear dynamics can be analyzed by simulations or other methods
  - Inclusion
    - Deterministic models are approximations of reality
    - Non-deterministic behaviors: *e.g.,* input uncertainty, PVT variation, noise
- Specification
  - Continuous extension of LTL
    - Continuous variables
    - Dense time
  - Brockett's annulus
    - Specifying analog signals
    - Providing propositions over continuous spaces
  - Probability
    - Specify analog properties
    - Metastability behaviors are common

Example: 2-input Arbiter

- A Black-Box View



  - Request signals: r_1, r_2; grant signals: g_1, g_2
  - Mutually exclusive access
  - Handshake protocol
  - Liveness

Initially:

$$\forall i \in \{1,2\}.\ \neg \mathsf{r}_i \wedge \neg \mathsf{g}_i$$

Assume (environment controls r_1 and r_2):

$$\forall i \in \{1,2\}. \quad \Box(\mathsf{r}_i \overset{\mathbf{W}}{\Rightarrow} \mathsf{g}_i) \wedge \Box(\neg \mathsf{r}_i \overset{\mathbf{W}}{\Rightarrow} \neg \mathsf{g}_i) \wedge \Box(\mathsf{g}_i \overset{\mathbf{U}}{\Rightarrow} \neg \mathsf{r}_i)$$

Guarantee (arbiter controls g_1 and g_2):

Handshake:

$$\forall i \in \{1,2\}.\ \Box(\neg \mathsf{g}_i \overset{\mathbf{W}}{\Rightarrow} \mathsf{r}_i) \wedge \Box(\mathsf{g}_i \overset{\mathbf{W}}{\Rightarrow} \neg \mathsf{r}_i)$$

Mutual Exclusion:

$$\Box \neg(\mathsf{g}_1 \wedge \mathsf{g}_2)$$

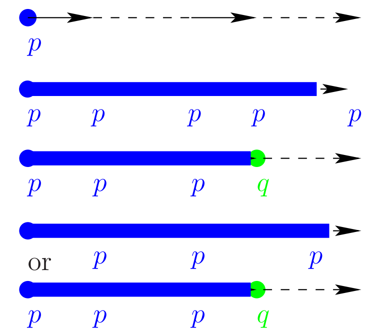
Liveness:

$$\forall i \in \{1,2\}.\ (\Box(\mathsf{r}_i \overset{\mathbf{U}}{\Rightarrow} \mathsf{g}_i)) \wedge (\Box(\neg \mathsf{r}_i \overset{\mathbf{U}}{\Rightarrow} \neg \mathsf{g}_i))$$

Specification

- Continuous LTL
  - Continuous variables & trajectories
  - Dense time

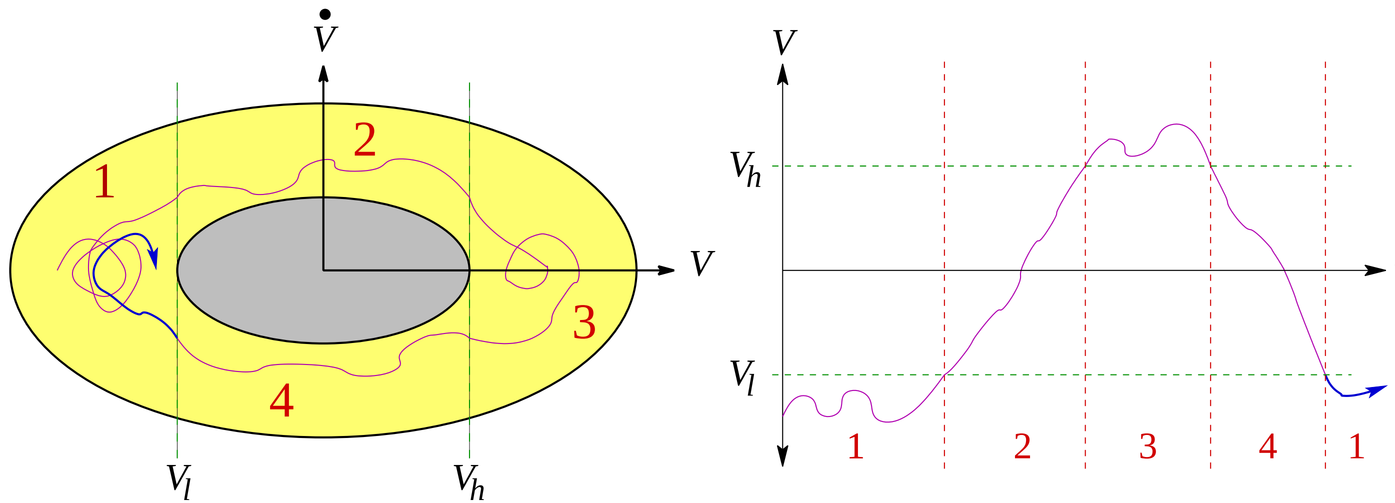
p:	p holds for the current time.
$\Box p$:	p holds for this and all subsequent time.
$p\ \mathbf{U}\ q$:	p holds until a time for which q holds.
$p\ \mathbf{W}\ q$:	p holds forever or until a time for which q holds.



- Brockett's Annulus
- Continuous Specification

Specification

- Continuous LTL
- Brockett's Annulus



  - Brockett's annulus allows entire families of signals to be specified.
  - Region 1 represents a logical low signal.
  - Region 2 represents a monotonically rising signal.
  - Region 3 represents a logical high signal.
  - Region 4 represents a monotonically falling signal.
  - Map continuous trajectories to discrete sequences.
- Continuous Specification

Specification

- Continuous LTL
- Brockett's Annulus
- Continuous Specification

Initially:

$$\forall i \in \{1,2\}.\ B_1(r_i) \wedge B_1(g_i)$$

Assume (environment controls r_1 and r_2):

$$\forall i \in \{1,2\}.\quad \Box(B_3(r_i) \overset{\mathbf{W}}{\Rightarrow} B_{2,3}(g_i)) \wedge \Box(B_1(r_i) \overset{\mathbf{W}}{\Rightarrow} B_{4,1}(g_i)) \wedge$$
$$\Box(B_3(g_i) \overset{\mathbf{U}}{\Rightarrow} B_{4,1}(r_i))$$

Guarantee (arbiter controls g_1 and g_2):

Handshake:

$$\forall i \in \{1,2\}.\ \Box(B_1(g_i) \overset{\mathbf{W}}{\Rightarrow} B_{2,3}(r_i)) \wedge \Box(B_3(g_i) \overset{\mathbf{W}}{\Rightarrow} B_{4,1}(r_i))$$

Mutual Exclusion:

$$\Box\neg(B_{2,3}(g_1) \wedge B_{2,3}(g_2))$$

Liveness:

$$\forall i \in \{1,2\}.\ (\Box(B_3(r_i) \overset{\mathbf{U}}{\Rightarrow} B_{2,3}(g_i))) \wedge (\Box(B_1(r_i) \overset{\mathbf{U}}{\Rightarrow} B_{4,1}(g_i)))$$

Liveness and Probability

- Metastability
  - There must exist trajectories where contested request are never granted
  - Liveness is not “always” satisfied
  - No ideal arbiter
- Almost Surely
  - Liveness is not always satisfied, but it may be satisfied with probability 1.
  - Solution: introduce probability theory to logic
  - *Almost-surely* version of LTL “always” operator
    - A trajectory ϕ satisfies $\Box_Z S$ iff S holds everywhere along ϕ, or if ϕ is in a negligible set, Z.
    - The probability of S holding everywhere along ϕ is equal to 1.
    - Z is the same for all trajectories.
- Continuous Specification

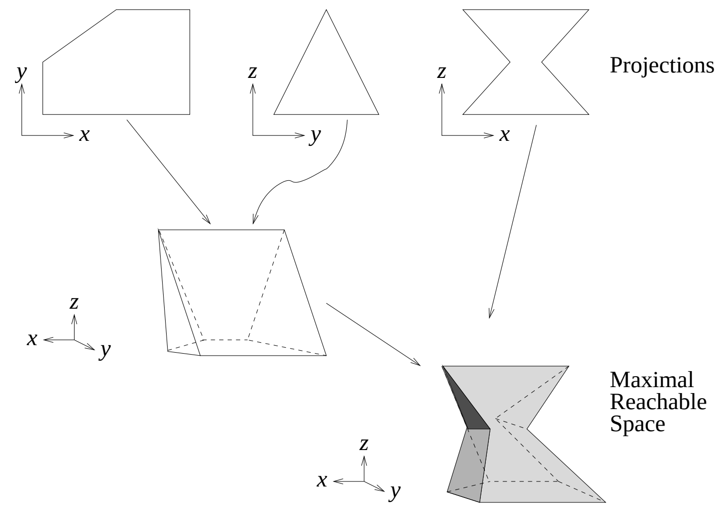
$$\begin{array}{lll} \forall i \in \{1,2\}. & & \alpha\text{-ins} \Rightarrow (\Box_Z (B_3(r_i) \overset{\mathbf{U}}{\Rightarrow} B_{2,3}(g_i))) \\ & \wedge & (B_3(r_i) \overset{\mathbf{U}}{\Rightarrow} (B_{2,3}(g_i) \vee B_3(r_{\sim i}))) \wedge (\Box (B_1(r_i) \overset{\mathbf{U}}{\Rightarrow} B_4(g_i))) \end{array}$$

Outline

- Motivation and Contributions
- Verification as Reachability
- COHO: Reachability Analysis
- Circuit Verification Examples
- Conclusion

COHO: Basic Ideas and Algorithms

- Representing and manipulating high-dimensional regions: projectagons



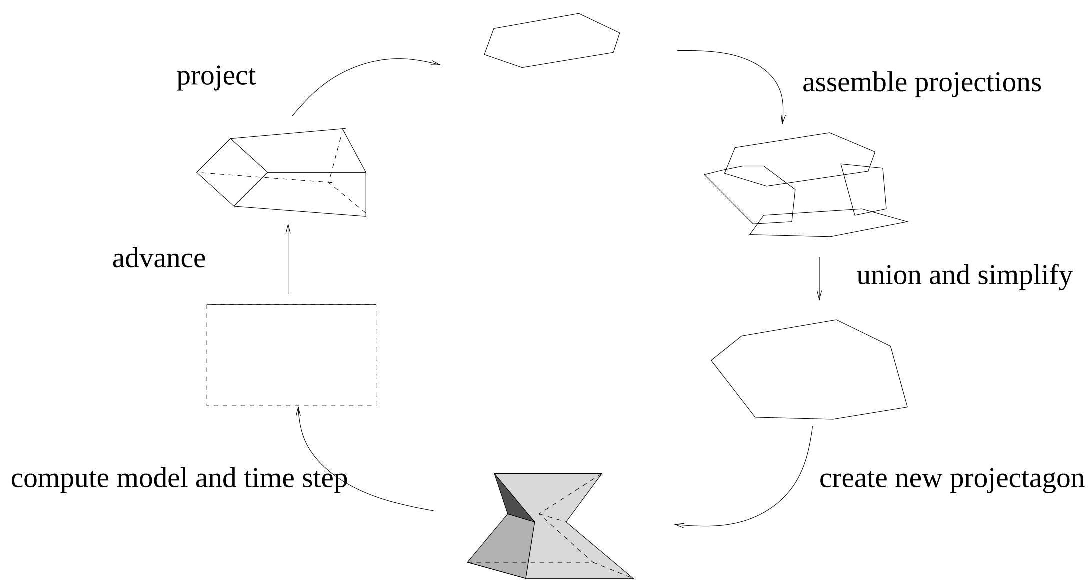
  - COHO projects high-dimensional polyhedra onto two-dimensional subspaces.
  - Projectagons are efficiently manipulated using two-dimensional computational geometry algorithms.
  - COHO uses both geometrical and inequality representations.
  - Projectagon faces correspond to edges of the projection polygons.
- Solving dynamic systems: linear differential inclusions.
- Basic step of COHO

COHO: Basic Ideas and Algorithms

- Representing and manipulating high-dimensional regions: projectagons
- Solving dynamic systems: linear differential inclusions.
  - Approximate the ODEs by linear differential inclusions.
    - least squares method
    - linear programming
  - Compute forward reachable region using the Maximum Principle
  - Efficient: matrix computations
  - Accurate: work on each face rather than the whole projectagon.
- Basic step of COHO

COHO: Basic Ideas and Algorithms

- Representing and manipulating high-dimensional regions: projectagons
- Solving dynamic systems: linear differential inclusions.
- Basic step of COHO



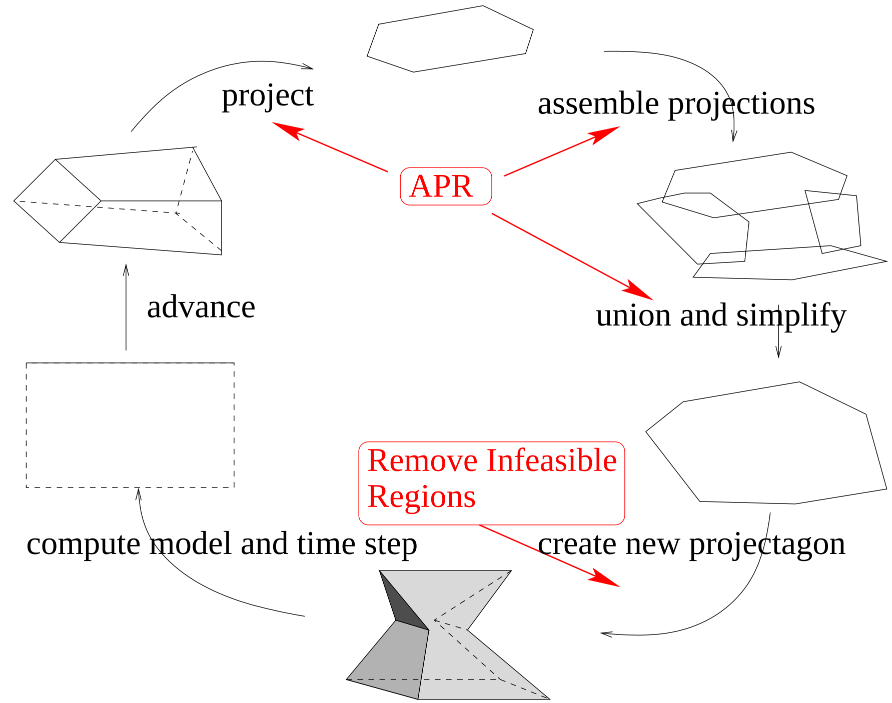
- A bounding projectagon is obtained by moving each face forward in time.
- The advanced face is projected onto two-dimensional subspaces to maintain the structure of projectagon.
- Computation continues until no new reachable region found

COHO: Basic Ideas and Algorithms

- Representing and manipulating high-dimensional regions: projectagons
- Solving dynamic systems: linear differential inclusions.
- Basic step of COHO
- All approximations over-approximate the reachable space: COHO is sound.
- Available at http://coho.sourceforge.net

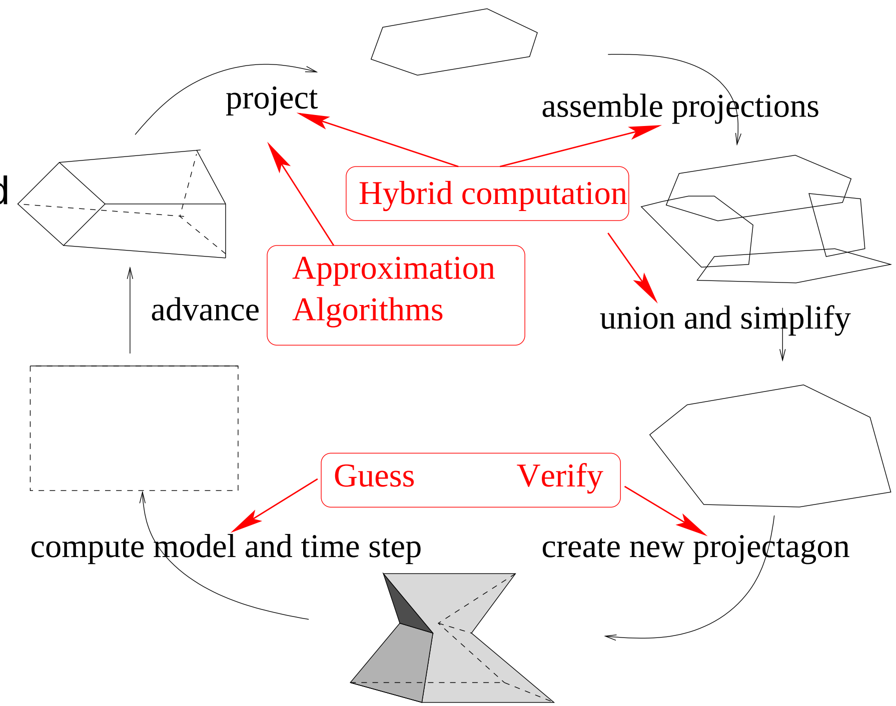
COHO: Improvements

- Robustness
  - Arbitrary precision rational(APR)
    - ill-conditioned problems
    - simple implementation
  - Remove infeasible regions
    - incomplete boundary
    - guarantee non-empty projectagon faces
- Efficiency
- Accuracy
- Usability



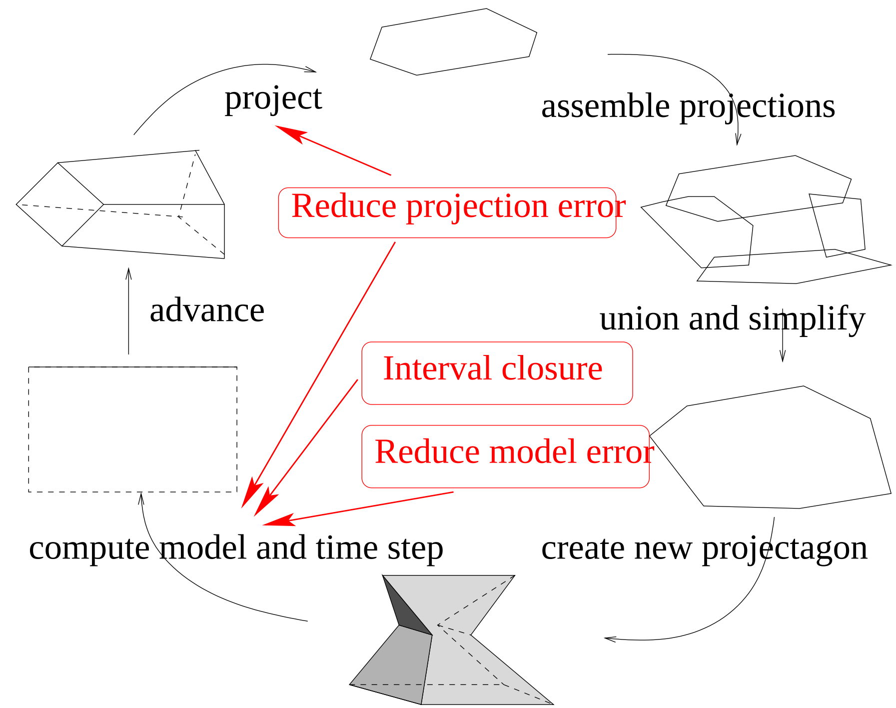
COHO: Improvements

- Robustness
- Efficiency
  - Guess-verify strategy
    - step-size is much smaller than necessary
    - guess a larger step-size based on previous step
    - verify the step-size and model is correct
  - Approximation algorithms
    - linear programming
    - projection algorithm
  - Hybrid computation
    - APR is expensive
    - floating point, interval, APR
- Accuracy
- Usability



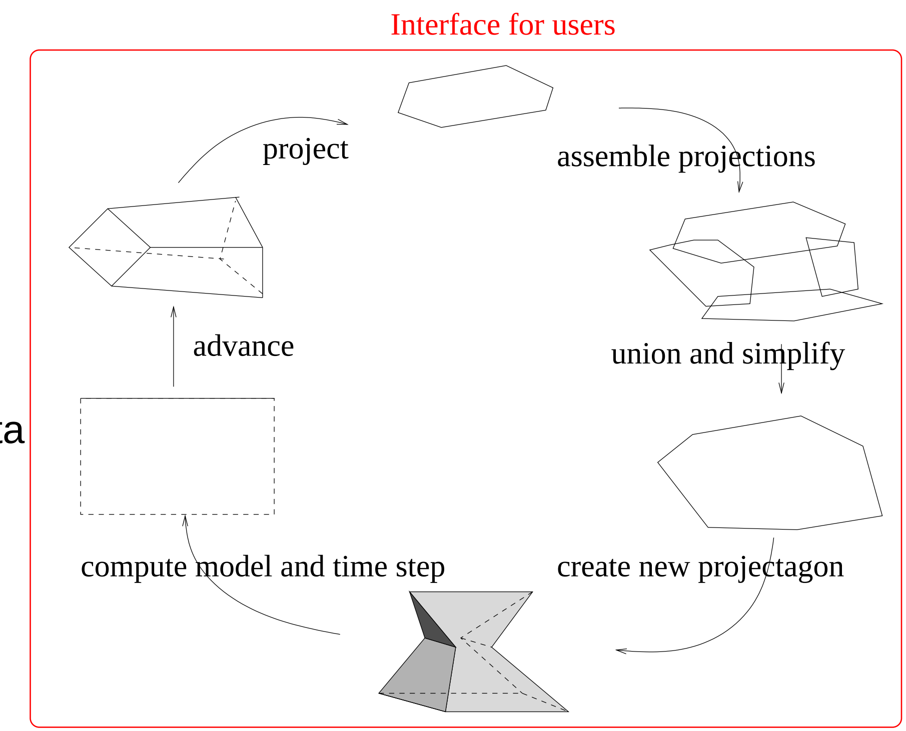
COHO: Improvements

- Robustness
- Efficiency
- Accuracy
  - Interval closure
    - over-approximation by using convex hull
    - non-convex polygons are constraints of variables
    - interval propagation
  - Reduce model error
    - asymmetric bloating
    - anisotropic bloating
    - multiple models
  - Reduce projection error
- Usability



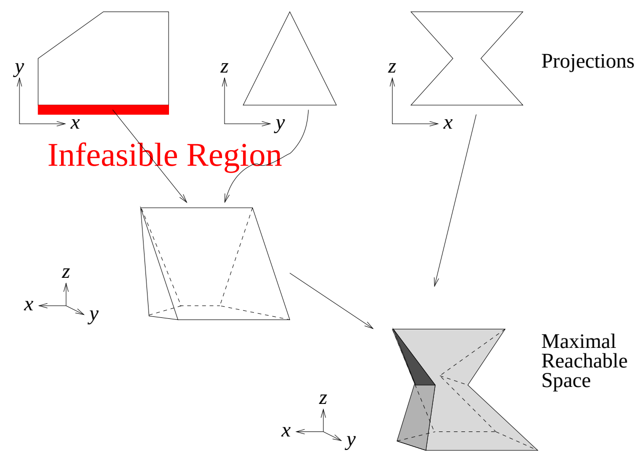
COHO: Improvements

- Robustness
- Efficiency
- Accuracy
- Usability
  - Interface based on hybrid automata
  - Templates for reachability computations
  - Options for trade-off of performance and accuracy



Infeasible Regions

- Infeasible regions
  - Projection polygons are computed independently
  - Infeasible regions: the prism from this region does not intersect with other prisms
  - Leads to incomplete boundaries in the next step



- Clipping infeasible regions
  - The problem of determinating if a projectagon is non-empty is NP-complete
  - Approximation techniques must be applied
  - Make a projectagon feasible to its convex hull

Infeasible Regions

- Algorithm
  - Construct the convex hull of the projectagon
  - Project onto all planes to obtain an updated convex hull
  - Compute the intersection of the projectagon and the updated convex hull
  - Repeat until the result converges
- Projectagon Faces
  - Use interval closure to find an accurate projectagon face if it is feasible

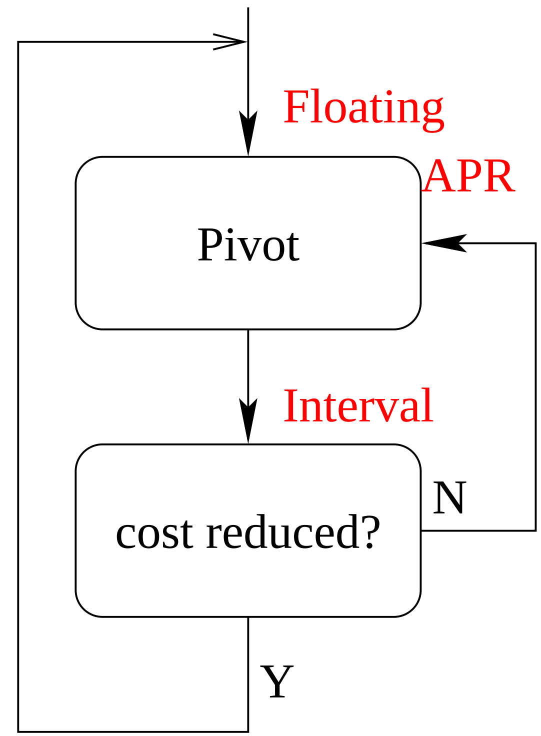
$$prism(e) \bigcap convexhull(P) \bigcap intervalClosure(e, P)$$

  - Use convex hull to find an over-approximated projectagon face otherwise

$$prism(e) \bigcap convexhull(P)$$

Hybrid Computation

- Arbitrary precision rational numbers (APR)
  - Expensive
  - Only necessary for ill-conditioned problems
- Hybrid computation
  - Use double-precision arithmetic for general computation
  - Use interval computation to validate the result
  - Use APR to repeat the computation if failed
- Applications
  - Linear programming
  - Geometric computations
  - Projection algorithms



Simplex

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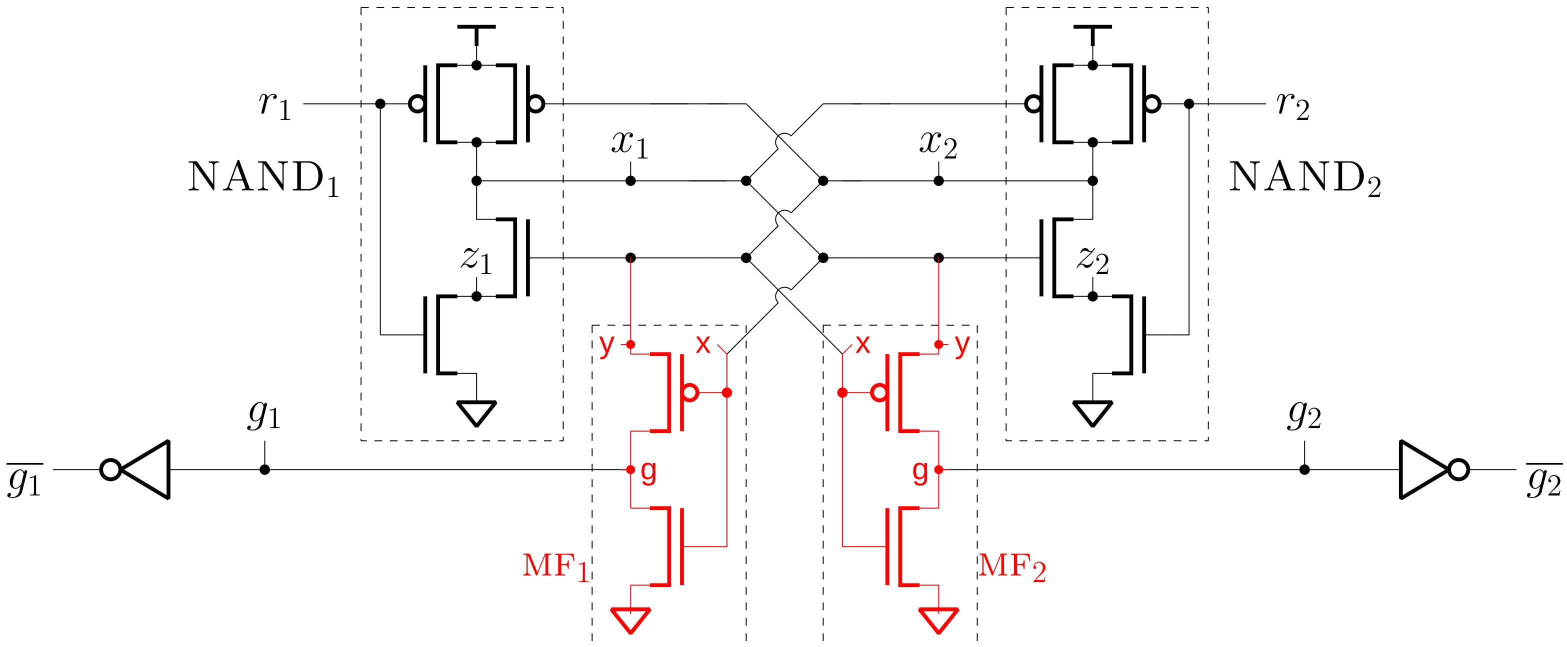
Example: Arbiter

- Arbiter



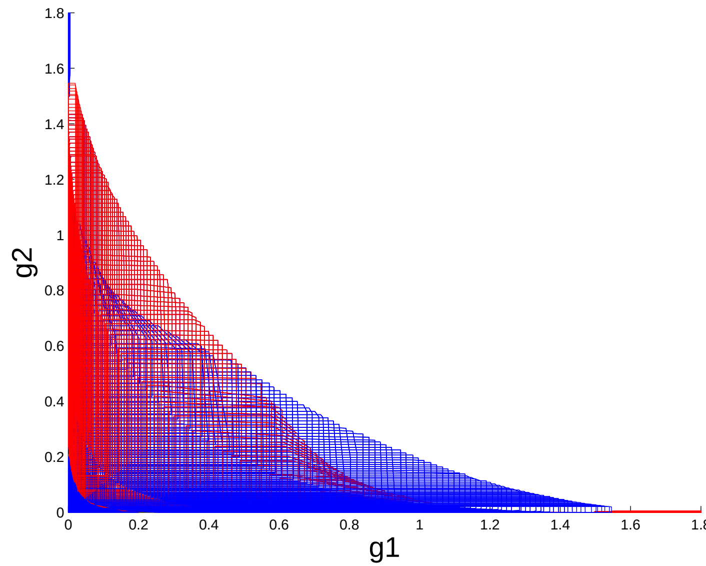
- Based on cross-couple NANDs
- The *metastability filters* ensure that no grant is asserted until metastability has resolved.

Reachability Computation

- Stiffness
  - Vastly different node capacitances: z_1, z_2
  - Ill-conditioned Jacobian matrix for ODE $\dot{x} = f(x)$
  - Difficult to find a good time-step
    - Large: large model error
    - Small: large projection and simplification errors
- Solution
  - Changing variables: converge much more rapidly
  - Additional invariants: reduce over-approximation
- Performance
  - 6-dimensional non-convex regions
  - $\sim$ 90 hours
    - large number of steps
    - circuit modeling, projection, linear programming
  - 1 $\sim$ 2G memory
    - large tables for circuit models

Results

- Safety Properties
  - ⇒ Mutual Exclusion
  - Handshake Protocol
  - Brockett Annuli

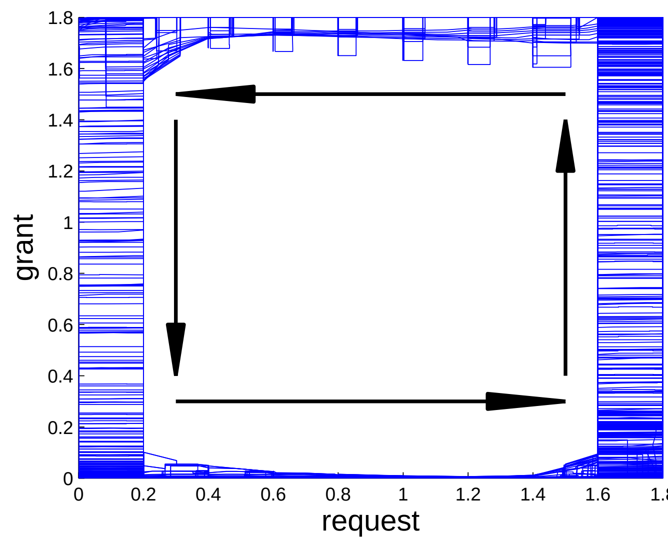


Mutual Exclusion

- Liveness Properties

Results

- Safety Properties
  - Mutual Exclusion
  - ⇒ Handshake Protocol
  - Brockett Annuli

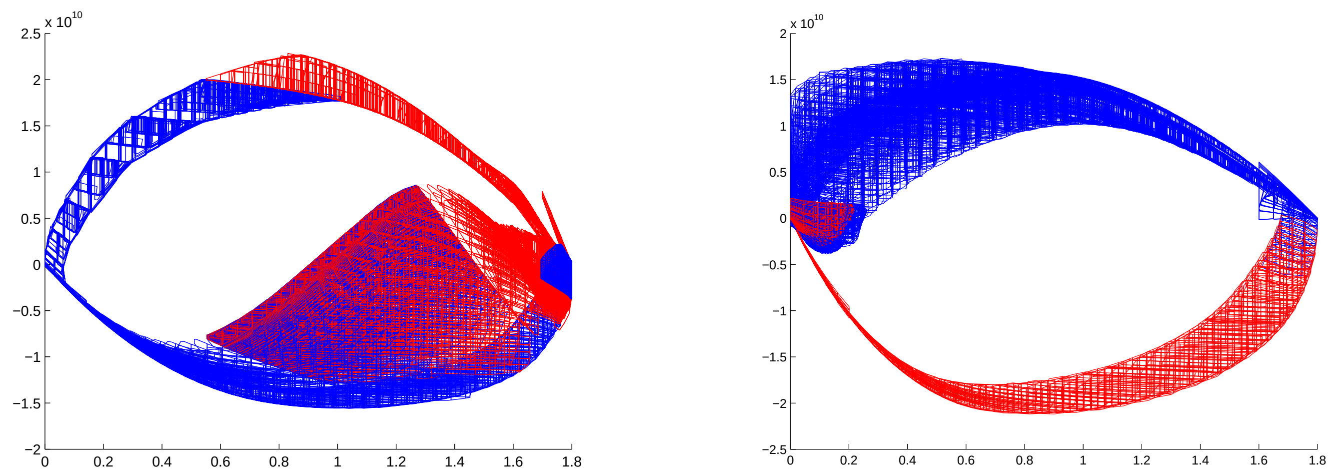


Handshake

- Liveness Properties

Results

- Safety Properties
  - Mutual Exclusion
  - Handshake Protocol
  - ⇒ Brockett Annuli



$\dot{x}$ vs. x $\dot{g}$ vs. g.

Brockett Annuli

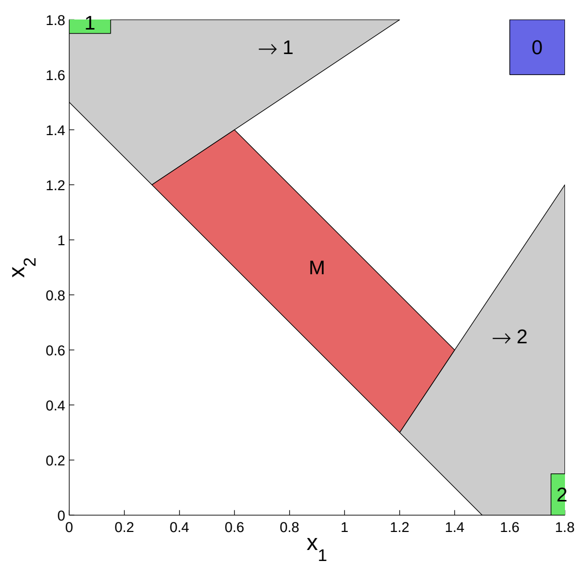
- Liveness Properties

Results

- Safety Properties
  - Mutual Exclusion
  - Handshake Protocol
  - Brockett Annuli
- Liveness Properties
  - Initialization: stable within 200ps
  - Uncontested Requests: grant the client within 350ps
  - Contested Requests: metastability within hyper-rectangle
  - Reset: withdraw grants within 270ps
  - Fairness: grant the other client within 420ps

Metastability and Liveness

- Metastable Behaviours
  - Fail to show a client is eventually granted when both requests asserted concurrently
  - Trajectories cannot leave the metastable region M because of over-approximation
  - Cannot be solved by reachability analysis
- Almost-Surely Verification
  - Bound the metastable region M by COHO
  - Compute interval Jacobian matrix in M
  - Show divergence holds everywhere in M using dynamical system theory [Mitchell96]



Other Circuits

- The Yuan-Svensson Toggle Circuit
  - Revealed a leakage current bug
  - The period of output is twice that of the input
  - Verify that the output and input signals satisfy the same Brockett's annulus
- A Flip-Flop Circuit
  - Showed the output satisfies the specification
  - Modeled circuit with multiple inputs with timing constraints
- The Rambus Ring Oscillator
  - Real problem from industry
  - Space reduction to improve performance of reachability analysis
  - Combine reachability analysis with other methods (*e.g.,* static analysis) to solve practical problems
  - Find sufficient conditins that guarantee oscillation from all initial conditions except for a set of measure zero.

Conclusions

- Circuit Verification as Reachability Analysis
  - A systematic way of translating verification problems to reachability analysis problems
    - Modeling: modified nodal analysis, table-based models from simulations.
    - Specification: Brockett annulus, LTL and probability theory
  - Efficient reachability analysis
    - Projectagon-based reachability analysis
    - Improvements: robustness, efficiency, accuracy, *etc.*
- Correctness and Efficiency Demonstrated by Circuit Verification Examples
  - Synchronous: Flip-flop, toggle
  - Asynchronous: Arbiter
  - Analog: Rambus ring oscillator
- Verification of Practical Circuits
  - Stiffness: challenges of reachability analysis
  - Metastability: cannot be solved by reachability analysis alone
- Analog Properties of Practical Circuits Can be Formally Verified Based on Reachability Analysis.

Future Work

- Verify Larger, Practical AMS Circuits
  - Parameterized verification
  - Point verification: specialized tools for common circuit classes
- Combine Reachability Analysis with Other Methods
  - Small-signal analysis
  - Static invariant computations: HySAT, HSOLVER
  - Almost-surely verification
- Specification
  - Check properties automatically
  - Integrate with other verification tools
- Improve Performance of COHO
  - Parallel computation
  - More efficient approximation algorithms
- More applications
  - Apply to other hybrid system problems
  - Compare with other reachability analysis tools