Circuit-Level Verification of Practical Circuits Based on Reachability Analysis

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Formal verification of analog and mixed signal circuits using continuous models is a promising area. As a consequence, hybrid system techniques, especially *reachability analysis*, have been applied to verify AMS designs. However, reachability computation is extremely expensive for high-dimensional, nonlinear hybrid systems. Therefore, it is challenging to verify large, practical circuits using accurate models.

We presented a method to verify practical properties of large AMS designs and applied the method to two circuits from industry. Our approach combines reachability analysis and other methods, such as static analysis, decomposition, *etc.* Reachability analysis is used to compute accurate analog behaviors of critical parts of AMS designs, while other methods are used to speed up computation for non-critical parts. Our examples include a full-buffer circuit from ST Microsystem and a differential ring oscillator from Rambus Inc.

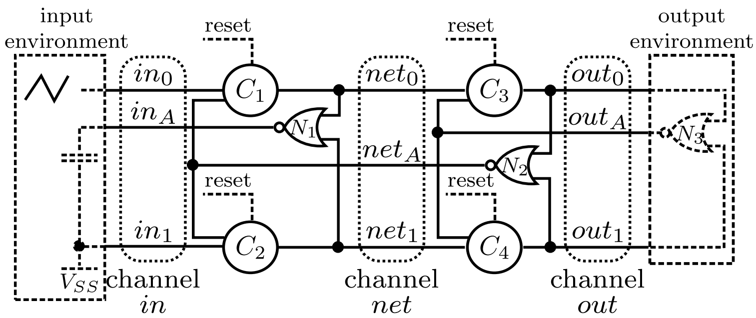

Figure 1: The Full-Buffer Circuit

Figure 1 shows the asynchronous micro-pipeline based circuit to be verified. It is designed to work as a buffer between input and output environments. However, it may fail to satisfy its designed handshake protocol when the input slope is very small, typically used in low power designs, or capacitances of internal nodes are not balanced. Therefore, it is demanding to verify a given design fully ensures the handshake protocol or not.

As the full-buffer circuit has tens of nodes and transistors, computing all possible states of the whole circuit is not practical. To solve the problem, we decomposed the whole circuit into individual C-elements and NOR gates. These basic elements are much smaller and simpler, thus reachability computations of these gates can be performed more efficiently. We have successfully computed reachable regions of two typical C-element designs: a weak-feedback C-element and a conventional C-element. To verify properties of a whole circuit based on verified properties of sub-systems, we applied Brockett annulus as a standard specification for analog signals between sub-systems. Firstly, we showed that the output signals of these circuit elements (*e.g.,* C-elements) satisfy the same Brockett annulus with the one used to specify their input signals. Based on this property, we demonstrated that all internal nodes of the full-buffer circuit satisfy the same specification, if all signals from input and output environments satisfied the Brockett annulus. This condition guarantees that the given design always supports the designed handshaking protocol. This method can be applied to other complex asynchronous circuits using C-elements.

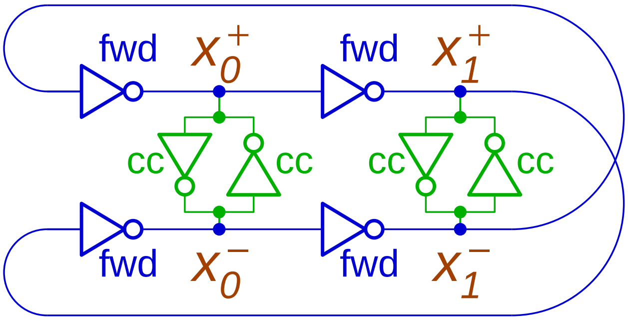

Figure 2: A Two-Stage Rambus Ring Oscillator

Figure 2 shows the ring oscillator proposed by researchers at Rambus Inc. They noted that some implementations of the circuit had failed in real, fabricated chips and posed the problem of showing that the oscillator starts from all initial conditions for a particular choice of transistor sizes.

We applied reachability analysis to demonstrated that there is a unique oscillation orbit for a two-state oscillator from all initial conditions. However, the number of reachability computations is huge to cover the entire state space. In addition, our reachability tool over-approximate reachable regions. Therefore, forward reachable regions alaways contain the metastable region because trajectories could stay in the stable manifold for arbitrary long time. To solve these problems, we applied static analysis. Our approach consists of three steps. First, we showed that nodes X_0^+ and X_0^- in the oscillator from Figure 2 form a "differential pair" and likewise for nodes X_1^+ and X_1^-. When the oscillator is operating properly, the common mode $X_0^+ + X_0^-$ are roughly constant and the oscillation is manifested in the differential components $X_0^+ - X_0^-$. Therefore, each of these differential pairs can be treated as a single signal. This step reduces the state space and simplifies the subsequent analysis. Second, we applied dynamical theory to show that any trajectory in the metastable region diverges and leaves the regions with probability one. The first two phases showed that most initial conditions lead to a fairly small subset of the full phase space. We divided the remaining space into small regions, and used existing reachability methods to show that the oscillator starts up properly from each such region. To improve performance further, we reduced the state space from four dimensions to two by construing differential inclusion circuit models. These methods enable us to verify the two-stage ring oscillator oscillates from all initial states.

In conclusion, reachability analysis tools are powerful enough to analyze kernels of some large AMS circuits. By employing other methods, such as decomposition strategy and static analysis, it is practical to verify significant properties of real designs.