THE YONEDA LEMMA

SAUNDERS MAC LANE

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Noboru Yoneda is rightly honored for his well known lemma: if F is a functor from a category C to Sets, then the natural transformations from $\hom(c, -)$ to F corresponds by a bijection to the set $F(c)$.

$$\mathrm{Nat}(\mathrm{Hom}(c, -), F) \cong F(c)$$

Yoneda enjoyed relating the story of the origins of this lemma, as follows. He had guided Samuel Eilenberg during Eilenberg's visit to Japan, and in this process learned homological algebra. Soon Yoneda spent a year in France (apparently in 1954 or 1955). There he met Saunders Mac Lane. Mac Lane, then visiting Paris, was anxious to learn from Yoneda, and commenced an interview with Yoneda in a café at the Gare du Nord. The interview was continued on Yoneda's train until its departure. In its course, Mac Lane learned about the lemma and subsequently baptized it.

Yoneda made other important contributions to homological algebra. The functor Ext.(G, A) had been defined in terms of short exact sequence $A \to X - G$; In 1954, he showed that the related function $\mathrm{Ext}^n(G, A)$ could be defined by long exact sequences (in the J. Fac. Sci. Tokyo, Sec. 1,7 pp.193-227; subsequently he showed that the products here could be given by composition of such sequences. He was the first to formulate the notion of an "end" of a bifunctor, in a 1960 paper in the same journal (vol. 8, 507-526). This notion has been widely used, as by Day and Kelly and by Mac Lane. In Short, Yoneda has made decisive contributions to albegra.

We mourn his recent death.