Applied stochastic programming models and computation

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Outline

- Introduction to stochastic programming
- Scope of study
- Results

Milestones in stochastic programming

1955 – First models and algorithms (Dantzig; Beale)
1956 – Airline scheduling problem (Ferguson and Dantzig)
1959 – Chance-constrained programming (Charnes and Cooper)
1969 – L-shaped method (van Slyke and Wets)
1984 – Nested Benders decomposition (Birge)
1989 – Russell Yasuda Kasai model (Cariño et al.)
1995 – OSL/SE (IBM)
2003 – van der Vlerk's on-line bibliography on SP contains 3840 entries
2006 – EURO application prize (Zenios et al.)

Two-stage recourse model

$$
\begin{aligned}
\min\ & c^{\mathrm{T}} x \quad && + \mathrm{E}_{\omega} q(\omega)^{\mathrm{T}} y(\omega) \\
\text{s.t.}\ & Ax && \le b \\
& T(\omega)x + W(\omega)y(\omega) && \le h(\omega) \\
& x \ge 0, \quad y(\omega) \ge 0 &&
\end{aligned}
$$

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Multistage recourse model

$$\min_{x_0} \quad c_0^{\mathrm{T}} x_0 + \mathrm{E}_{\omega_1}\{\min_{x_1} c_1^{\mathrm{T}}(\omega_1)x_1 + \mathrm{E}_{\omega_2|\omega_1}[\min_{x_2} c_2^{\mathrm{T}}(\omega_2)x_2 + \ldots + \mathrm{E}_{\omega_T|\omega_{T-1}\ldots\omega_1}(\min_{x_T} c_T^{\mathrm{T}}(\omega_T)x_2)\ldots]\}$$

$$\text{s.t.} \quad \begin{aligned} A_{00}x_0 &\leq b_0 \\ A_{10}x_0 + A_{11}x_1 &\leq b_1(\omega_1) \\ \vdots \qquad\qquad \ddots \qquad &\quad \vdots \\ A_{T0}x_0 + A_{T1}x_1 \qquad + A_{TT}x_T &\leq b_T(\omega_T) \end{aligned}$$

$$l_0 \leq x_0 \leq u_0, l_t(\omega_t) \leq x_0 \leq u_t(\omega_t)$$

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Chance-constraint model

$$\begin{aligned} \min\ & c^{\mathrm{T}} x \\ \text{s.t.}\ & Ax \le b \\ & \Pr\{R_i(\omega)x \le r_i(\omega)\} \ge \alpha_i, i = 1, \ldots, I \\ & x \ge 0, \end{aligned}$$

Integrated chance-constraints

$$\begin{aligned}
\min\ & c^{\mathrm{T}} x \\
\text{s.t.}\ & Ax \leq b \\
& \mathrm{E}\{R_i(\omega)x \mid R_i(\omega)x > r_i(\omega)\} \leq d_i, i = 1, \ldots, I \\
& x \geq 0,
\end{aligned}$$

Scope of the study

- Included
  - Applications and novel models
  - Computational papers
  - All types of stochastic programs
  - Illustrative and “industrial-strength” models
- Not included
  - Theoretical papers
  - New algorithms for old models
  - Randomly generated problems

The sources

- Research bibliographies
  - Stancu-Minasian and Wets (1976)
  - P. Birge (1984)
  - Van der Vlerk (2003)
- Citation indexes
- Online searches

Result: 144 models

Papers by year

Decade	Number
1956-1965	2
1966-1975	13
1976-1985	26
1986-1995	31
1996-2005	72
Total	144

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Papers by application area

Application	Number of Problems
Finance	30
Energy planning	16
Scheduling	14
Water management	12
Capacity expansion	9
Agriculture/Fishery/Forestry	8
Transportation	8
Operations Management	8
Supply chain management	7
Blending problems	4
Telecommunications	3
Other	27

How published?

Source	Number of Problems
Operations Research	12
Management Science	11
Annals of Operations Research	6
European Journal of Operational Research	6
Mathematical Programming	4
Other journal	56
Book / collection	26
SPEPS	8
Other web source	4
Technical report	4

Features

Feature	Number of problems
Random RHS	94
Random Cost	52
Random Bounds	1
Random Subdiagonal Blocks	41
Random Blocks on Diagonal	39
Linking Constraints	2
Integer Variables	26
Chance Constraints	34
Integrated Chance Constraints	3

Features by decade

Feature	‘56-’65	‘66-’75	‘76-’85	‘86-’95	‘96-’05
Random RHS	2	9	22	17	44
Random Cost	1	2	3	11	35
Random Bounds	0	1	0	0	0
Random Subdiagonal Blocks	0	2	3	14	22
Random Blocks on Diagonal	0	4	3	12	20
Linking Constraints	1	5	16	23	46
Integer Variables	0	0	0	0	2
Chance Constraints	1	4	12	5	12
Integrated Chance Constraints	0	0	0	0	3

Number of stages

Number of stages	Number of problems
Single stage	29
Two-stage	41
Multistage	75
Staircase	51
Non-staircase	24

Number of stages per decade

Number of stages	‘56- ‘65	‘66- ’75	‘76- ‘85	‘86- ‘95	‘96- ‘05
Single stage	0	5	7	4	13
Two stages	1	5	4	10	21
Multiple stages	1	3	18	16	37
Staircase	0	1	12	12	26
Non-staircase	1	2	6	4	11

Scope of the application

Scope	Number of problems
Serious application	21
Academic model	55
Illustrative example	53
Toy problem	6

Conclusions

- SP applications are getting more numerous
- Finance applications dominate
- Considerable breadth of other applications