A Bayesian model of information cascades

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Abstract. An information cascade is a circumstance where agents make decisions in a sequential fashion by following other agents. Bikhchandani *et al.* predict that once a cascade starts it continues, even if it is wrong, until agents receive an external input such as public information. In an information cascade, even if an agent has its own personal choice, it is always overridden by observation of previous agents' actions. This could mean agents end up in a situation where they may act without valuing their own information. As information cascades can have serious social consequences, it is important to have a good understanding of what causes them. We present a detailed Bayesian model of the information gained by agents when observing the choices of other agents and their own private information. Compared to prior work, we remove the high impact of the first observed agent's action by incorporating a prior probability distribution over the information of unobserved agents, and investigate an alternative model of choice to that considered in prior work: weighted random choice. Our results shows that, in contrast to Bikhchandani's assumption, cascades will not necessarily occur and adding prior agents' information will delay the effects of cascades.

Keywords: Information cascade, Coordination, Probabilistic graphical model · Bayesian inference

1 Introduction

Propagation of opinions in society have a significant impact. In daily life it is clear that people are affected by others' views. For example, in electoral and financial campaigns, the spreading of news, opinion and rumours can have an enormous effect on the behaviour of the crowd. When people look at others' actions, or listen to others they update their assessment of the value of those actions and imitate accordingly.

Information cascades are a social phenomenon in which all individuals from some point in a sequence onwards make the same decision. It occurs when other people's prior choices can strongly impact the choices of those who follow and it is the result of solely following others while discounting their own opinion. Bikhchandani et al. [3] say that, "An informational cascade occurs when it is optimal for an individual, having observed the actions of those ahead of him, to follow the behavior of the preceding individual without regard to his own information". This procedure can be found for example when people make a decision on a certain situation such as choosing a restaurant or school [2].

People get more attracted when they see that someone already adopted a trend or chose a specific fashion. New communication technology has become a habit like social media, helping to share new trends, fashion and ideas [6]. It passes very quickly across web technologies when someone shares a particular fashion he takes and their respective friends also share this [17, 12]. Re-sharing the posts develops an information cascade in social media and leads others to choose a product, movie, on a specific fashion. Furthermore, recent studies [8, 15] have shown that e-marketing is driven by information cascades. Customer ratings and reviews make a huge impact in online shopping. Thus, often people become more conscious and start to pursue other's opinions when they need to shop online, even though their intuition suggests the opposite is the case. It is evident that when shopping in Amazon or Ebay, people rely heavily on others' choices since the quality of the goods is still an issue [21].

In addition, an information cascade can aid society to unite its members for environmental collective action such as minimising emission of carbon and global warming as well as social and political collective action. Lohamann [16] says that cascades are often observed in the society in which individuals join and protest against the regime to address their political needs, even if they have the opposite thought.

It is obvious that people often avoid their own preference or personal signals and choose to pursue others' options as soon as cascade begins. However, even though they have strong qualities, people's own preferences are concealed once the cascade starts. For instance, the algorithm for discovering the real rating of an Amazon product was proposed by Wang et al. [21] as people demand plausible and accurate online reviews. Wang et al. attempted as much as possible to eliminate the herding effect occurring through online purchasing using his algorithm. But the actual rating was very difficult to obtain.

Bikhchandani et al. [3] have presented a probabilistic model to explain how an information cascade comes about, in the context of uptake of a fashion or fad. Their model predicts that cascades are inevitable, once it starts. A cascade can be either a correct cascade where eventually all individuals choose "adopt" or an incorrect cascade where individuals choose "reject" when V is equal to 1. They used deterministic choice, which means that the agents choose the action with the greatest evidence for its correctness, based on observations of other agents and their own private information. Bikhchandani et al. presented a model with high level analysis. It lacks mathematical details. We wish to investigate the impact of agents making choices non-deterministically, via probabilistic choice (random weighted choice). Indeed, probabilistic choice model could be seen as psychologically more realistic. We adopt the theory underlying Luce's Choice mechanism [18, 20].

In the model of Bikhchandani et al., the first agent in the chain of observed agents is assumed to be the first to make a decision to adopt or reject the fashion. This makes its choice highly influential, as its action is known to correspond directly to its private signal. Moreover, this is unrealistic in many settings. An agent may know that the fashion has been around for a while, so it could estimate a number of prior agents whose actions were unobserved. A probability distribution over the count of adopt decisions by these agents is used in our model. Even if this is a completely uninformative prior distribution, it weakens the dominance of the first agent.

2 Prior Work

Besides Bikhchandani et al., many other researchers have highlighted the importance of information cascades. Benerji et al. [2] introduced a model to investigate herd behaviour to understand how people adopt others' actions while ignoring their own information. They showed that people observe others actions and tend to act according to that as they believe the previous person has better information than them. Easley et al. [9] explained the theoretical and experimental views of a herding behaviour. They presented a Bayesian model for sequential decision-making where people consider the counts of the previous actions and choose the highest one.

Vany et al. [7] developed an agent-based model to compare the theoretical aspects of Bikichandani's concept. Specially, they analysed how people are attracted to watch a specific movie as they have multiple choices. The key aspect of his model is how an agent will be highly influenced by the nearest neighbour's action as well as the popularity of different movies. In the same fashion, Lee et al. [11] examined how online reviews of a movie are socially influenced [19] by two different groups such as a general crowd and friends. They concluded that the reviews of the popular movies show a cascading behaviour. However, people tend to follow friend ratings regardless of popularity since, it has more impact than the general crowd rating. Similarly Liu et al. [14, 13] analysed how information cascades occur in e-book marketing. Particularly, for both paid and free e-books they experimented to find the effect of information cascades, and found that an information cascade has a strong impact when selecting paid e-books in comparison with free books.

Anderson et al. [1] designed a laboratory experiment to show how an information cascade occurs. It became apparent that people at some stage discard their own knowledge and continue to follow others. In addition, Huber et al. [10] conducted a neural medical experiment to research how a cascade can be stimulated by individual preference. They concluded from their findings that overweighting personal information can trigger and stop the information cascade.

Watts [23, 22] developed a model to show how agents' actions interact with neighbours' actions by setting a simple threshold rule. Cheng et al. [6] addressed the prediction of a cascade when sharing a picture post in Facebook. They found that the size of an information cascade can be predicted by its temporal (observed time) and structural (caption, language and content) features. Lu et al. [17] proposed a system to analyse the collective behaviour of information cascades by analysing a huge social media data set.

3 Bikhchandani *et al.*'s information cascade model

Bikhchandani *et al.* [3] considered in their models a sequence of individuals who will choose between accepting (A) or rejecting (R) a fashion or fad. The model assumes that there is some true value $V \in \{0, 1\}$ representing the benefit of following the fashion/fad. Every individual perceives a private signal $X_i \in \{H, L\}$, representing a "high" or "low" perception of V. As shown in Table 1 a correct perception of V has probability p for $V = 1$ and $1 - p$ for $V = 0$, and vice versa for incorrect perceptions.

In the "basic model" considered by Bikhchandani *et al.*, each agent considers the actions of earlier agents in the sequence, and takes those actions of adopt or reject as a proxy for what each of those agents perceived as a true value, which is unknown by observing agents. The agent can count all of the previous accepts or rejects, adds a count 0 or 1 (respectively) given his own information (L or H), and then chooses his action based on the action with the greatest count. Here we notice that the actions of agents earlier in the chain are repeatedly used as evidence by later agents, and their impact may therefore be exaggerated.

The "general model" of Bikhchandani *et al.* states that an agent will make its decision based on the expected value of V given the agent's private signal and observations of other agents' actions. Bikhchandani *et al.* gave the following definition: $V_{n+1}(x; A_n) \equiv E[V|X_{n+1} = x, X_i \in J_i(A_{i-1}, a_i)$, for all $i \leq n]$. Here x is the signal perceived by agent $n+1$, A_{i-1} is sequence of prior actions agent i has observed and $J_i(A_{i-1}, a_i)$ is a set of signals that lead individual i to choose action a_i. Individual $n+1$ adopts if $V_{n+1}(x; A_n) \geq C$, where C is the cost of adoption[3]. However, no computational account is given of how this expected value is determined by the agent.

Bikhchandani *et al.* found that "An informational cascade occurs if an individual's action does not depend on his private signal". Hence, he ignores his private signal and will adopt based on the prior agents' actions alone. This is true for all subsequent agents too, so they follow their predecessors and create a cascade. A cascade can be either a correct cascade where all adopt or incorrect cascade where all reject for V = 1 and vise versa for V = 0.

Bikhchandani *et al.* showed that once the cascade starts it will last forever even if it is wrong. They also discussed the fragility of cascades. For instance, cascades can be broken if public information is revealed.

Our motivations to create a probabilistic choice model in terms of Bayesian approach are: firstly, to provide an executable model that provides a detailed mathematical account of agent choices. Secondly, to avoid the strong influence of the first agent's action on later agent's action through the ability to model a probability distribution over the information of unobserved prior agents. Bikhchandani *et al.*'s model uses a deterministic mode of choice in which agent ranks options and chooses the highest ranked one. This deterministic technique is commonly used in economic models [5]. Inspired by the work of Luce, [20, 18] we consider the effects on cascades if agents use weighted non-deterministic choice when choosing to adopt or reject. This means that choice is probabilistic and an agent may choose randomly from a set of weighted choices, in proportion to their weights. This has been claimed to be more psychologically plausible than deterministic choice [20].

4 Our Model and Approach

In this section we present our Bayesian model of information cascades.

[3] This cost isn't used in our model.

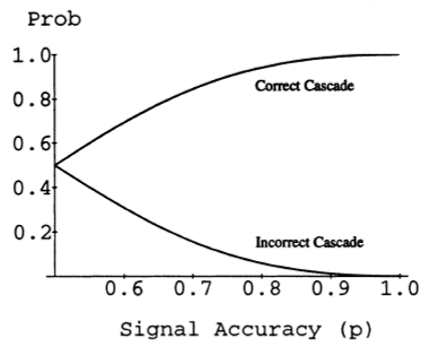


Fig. 1: Graph of probability of correct and incorrect cascade as a function of *p* in the simple model of Bikhchandani *et al.* [3]

Table 1: Signal probabilities [3]

Gain of adopting	$P(X_i = H \mid V)$	$P(X_i = L \mid V)$
$V = 1$	p	$1 - p$
$V = 0$	$1 - p$	p

We define the following variables

- $V \in \{0, 1\}$, is the true value of the fad/fashion.
- $X_i \in \{H, L\}$, is the private signal of the agent: high or low. This represents agent i's possibly incorrect perception of V.
- $A_i \in \{A, R\}$, is the action of the agent: adopt or reject. C_i is a count of how many times H appears in $\{X_1, \cdots, X_n\}$. This will be probabilistically inferred by each agent since it cannot be directly perceived.
- k is a number of prior agents who made choices that were not observed by any of the agents. C_0 is the count of H signals observed by these prior agents. As this is unknown, our model uses an estimated probability distribution for C_0.
- p is the signal accuracy, which is assumed to be the same for all agents.

The dependencies between these variables are shown using a probabilistic graphical model [4] in Figure 2, which is expressed from the view point of agent 4 . p is a constant, but the other nodes are random variables. Shaded boxes represent observed variables, and X_i is known by agent i. A_4 is shown as a rectangle, as this is a decision node. Agent 4 will choose A if $P(V = 1|A1, A2, A3) > 0.5$ and R otherwise.

The information cascade model is used as follows. V and C_i are conditionally independent given C_{i-1} and X_i, therefore:

$$P(V, C_i \mid C_{i-1}, X_i) = p(V \mid C_{i-1}, X_i)\, p(C_i \mid C_{i-1}, X_i) \tag{1}$$

Each agent i maintains a joint probability distribution over V and C that is conditional on the actions observed so far. This is $p(V, C_0) = p(V)\, p(C_0 \mid V)$ for first agent. We

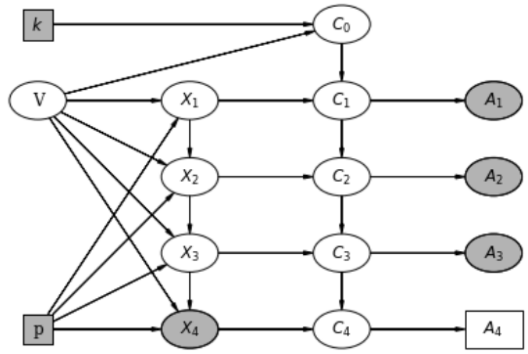


Fig. 2: Probabilistic graphical model of information cascade from agent 4's viewpoint

use a uniform prior over V and a binomial distribution for C_0 given V:

$$P(C_0 = c \mid V = 0) = (1-p)^c p^{k-c} \quad \text{and} \quad P(C_0 = c \mid V = 1) = p^c (1-p)^{k-c} \tag{2}$$

For $i > 1$, agent i will observe the actions of all prior agents $j < i$ and compute the joint conditional distribution $p(V, C_j \mid C_0, A_1, \cdots, A_j)$. Agents act and are observed sequentially, so it will already have computed $p(V, C_{j-1} \mid C_0, A_1, \cdots, A_{j-1})$.[4] Once it observes A_j, it first uses Bayes' Theorem to compute $P(X_j \mid A_1, \cdots, A_j)$:

$$P(X_j \mid A_1, \cdots, A_j) \propto P(X_j \mid A_1, \cdots, A_{j-1})\, P(A_j \mid X_j, A_1, \cdots, A_{j-1}) \tag{3}$$

where:

$$\begin{aligned} P(X_j \mid & A_1, \cdots, A_{j-1}) \\ &= \sum_{v \in \{0,1\}} P(X_j \mid V = v, A_1, \cdots, A_{j-1})\, P(V = v \mid A_1, \cdots, A_{j-1}) \\ &= \sum_{v \in \{0,1\}} P(X_j \mid V = v)\, P(V = v \mid A_1, \cdots, A_{j-1}) \end{aligned} \tag{4}$$

$$P(A_j \mid X_j, A_1, \cdots, A_{j-1}) = P(A_j \mid X_j, C_{j-1}) \tag{5}$$

$P(A_j \mid X_j, C_{j-1})$ is calculated as shown in Tables 2 and 3 for the deterministic and non-deterministic models of choice, respectively.
Agent i can compute $p(V, C_j \mid A_1, \cdots, A_j)$ as follows:

$$\begin{aligned} P(V = v, C_j = c \mid A_1, \cdots, A_j) = & \\ & P(V = v, C_{j-1} = c \mid A_1, \cdots, A_{j-1})\, P(X_j = L \mid A_1, \cdots, A_j) + \\ & P(V = v, C_{j-1} = c - 1 \mid A_1, \cdots, A_{j-1})\, P(X_j = H \mid A_1, \cdots, A_j) \end{aligned} \tag{6}$$

[4] C_0 appears as a condition whenever a sequence of action variables does. Henceforth, we omit it for brevity.

Table 2: $P(A_j \mid X_j, C_{j-1})$ for deterministic choice

X_j	Condition on C_{j-1}	$P(A_j\!=\!A)$	$P(A_j\!=\!R)$
H	$P(C_{j-1}+1 > \frac{j}{2}) > P(C_{j-1}+1 < \frac{j}{2})$	1	0
	$P(C_{j-1}+1 > \frac{j}{2}) < P(C_{j-1}+1 < \frac{j}{2})$	0	1
	$P(C_{j-1}+1 > \frac{j}{2}) = P(C_{j-1}+1 < \frac{j}{2})$	0.5	0.5
L	$P(C_{j-1} > \frac{j}{2}) > P(C_{j-1} < \frac{j}{2})$	1	0
	$P(C_{j-1} > \frac{j}{2}) < P(C_{j-1} < \frac{j}{2})$	0	1
	$P(C_{j-1} > \frac{j}{2}) = P(C_{j-1} < \frac{j}{2})$	0.5	0.5

Table 3: $P(A_j \mid X_j, C_{j-1})$ for non-deterministic choice

X_j	$P(A_j\!=\!A)$	$P(A_j\!=\!R)$
H	$P(C_{j-1}+1 > \frac{j}{2}) + \frac{1}{2}P(C_{j-1}+1 = \frac{j}{2})$	$P(C_{j-1}+1 < \frac{j}{2}) + \frac{1}{2}P(C_{j-1}+1 = \frac{j}{2})$
L	$P(C_{j-1} > \frac{j}{2}) + \frac{1}{2}P(C_{j-1} = \frac{j}{2})$	$P(C_{j-1} < \frac{j}{2}) + \frac{1}{2}P(C_{j-1} = \frac{j}{2})$

When it is agent i's turn to act, it will know $P(V = v, C_{i-1} = c \mid A_1, \cdots, A_{i-1})$ and will have its own signal X_i to calculate $P(V = v, C_i \mid C_{i-1}, X_i)$. Since V and C_i are conditionally independent given C_{i-1} and X_i, we have:

$$P(V = v, C_i \mid C_{i-1}, X_i) = P(V = v \mid C_{i-1}, X_i) P(C_i \mid C_{i-1}, X_i) \tag{7}$$

where

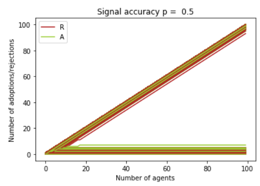
$$P(V = v \mid C_{i-1}, X_i) \propto P(V = v \mid C_{i-1}) P(X_i \mid V = v, C_{i-1}) \tag{8}$$

$$P(X_i\!=\!x \mid V\!=\!v, C_{i-1}) = P(X_i\!=\!x \mid V\!=\!v) = \begin{cases} p & \text{if } (v, x) \in \{(1, H), (0, L)\} \\ 1\!-\!p & \text{otherwise} \end{cases} \tag{9}$$

and

$$P(C_i = c \mid C_{i-1}, X_i = x) = \begin{cases} P(C_i = c\!-\!1) & \text{if } x = H \\ P(C_i = c) & \text{if } x = L \end{cases} \tag{10}$$

Finally, as agent i now has an up-to-date probability distribution over V from Equation 8, it can look up $P(V = 1 \mid \cdots)$ and $P(V = 0 \mid \cdots)$, and use these as the weights for the options A and R. For deterministic choice, if one weight is larger than the other, it chooses the corresponding option. If they are equal, it tosses an evenly weighted coin to choose between the options. For non-deterministic choice, a random weighted choice is made based on the probabilities for V = 0 and V = 1.



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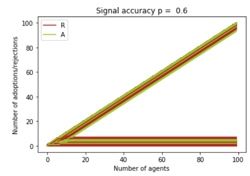

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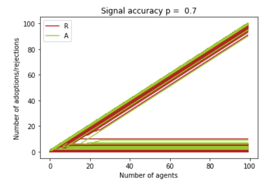

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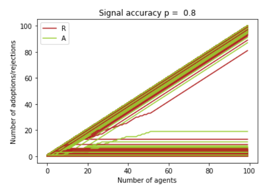

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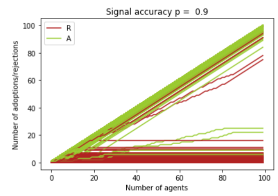

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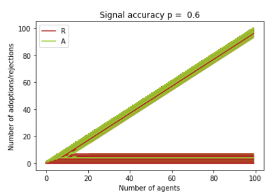
Fig. 3: Cumulative frequencies of adopts and rejects for 100 agents over 1000 runs with v = 1 for each p with 1 prior agents for deterministic choice (best viewed in colour)



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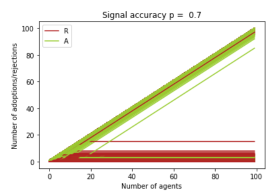

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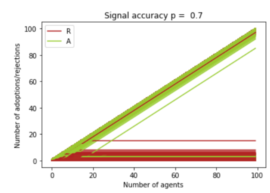

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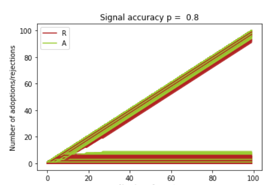

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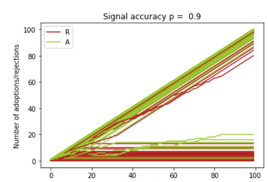

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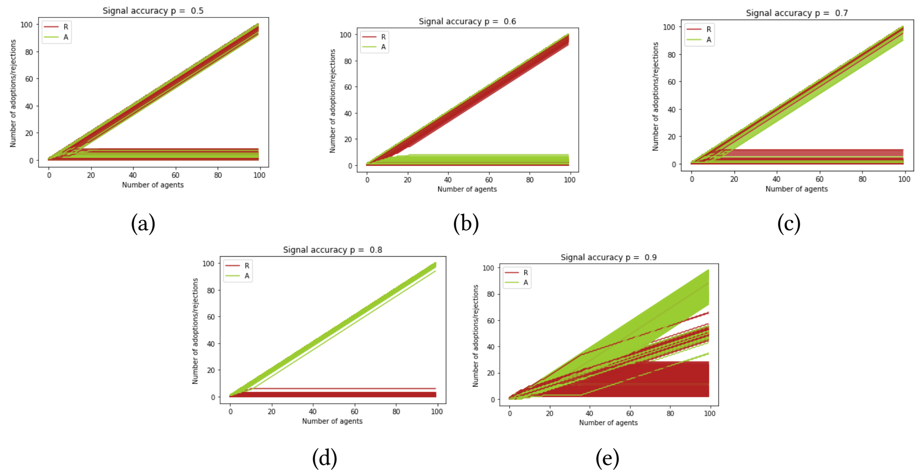
Fig. 4: Cumulative frequencies of adopts and rejects for 100 agents over 1000 runs with v = 1 for each p with 20 prior agents for deterministic choice (best viewed in colour)

5 Experiment and Results

5.1 Deterministic Model

As a first step we implemented[5] a deterministic choice model [3] using our Bayesian approach. Our model uses the probability distribution $P(V, C_i \mid C_{i-1}, X_i)$ to choose actions.

[5] The implementation of our model in Python can be found at https://github.com/ashalya86/Information-cascade-models.



(a) (b) (c) (d) (e)

Fig. 5: Cumulative frequencies of adopts and rejects for 100 agents over 1000 runs with v = 1 for each p with 40 prior agents for deterministic choice (best viewed in colour)

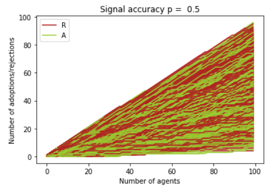
We ran the simulation 1000 times for 100 agents across the list of $p \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$ for values of $v \in \{0, 1\}$ while varying the number of prior agents. In a single graph we plot two lines, red and green, for each run. The green line indicates the progression of cumulative frequency of acceptances (A) and red line indicates the same for rejections (R). There are separate graphs for each value of p $p \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$. Figures 3, 4 and 5 show the outcome of cascades which start with prior agents 1, 20 and 40 respectively for V = 1. The results for V = 0 are similar to V = 1, except that the colours are swapped as R is dominant when V = 0. It is evident from the graphs that there is always a high chance of cascades where everyone adopts when V = 1 and rejects when V = 0. These graphs that we plotted are similar to the results obtained from Bikhchandani's [3] original model.

5.2 Non-deterministic Model

We wish to investigate how the deterministic model of choice in the model of Bikhchandani et al. impacts their results, given that non-deterministic choice has been described as more psychologically plausible. We modified our implementation to pick an action using random weighted choice. As a result, even if the most likely optimal choice is to adopt, the agent could still choose the less likely one and reject.

As for the deterministic choice model, in a single graph we plot cumulative frequency of acceptances (A) and rejections (R) for 1000 runs of 100 agents for each p. The plots of cascades which begin with 1, 20 and 40 prior agents for V = 1, are seen in Figures 6, 7, 8. The results for V = 0 are similar to V = 1, except that the colours are swapped as R is dominant when V = 0.

Simulation results suggest that for high accuracy perception of the true value of a choice ($p \in \{0.8 - 0.9\}$) (Figures 6d, 6e, 7d, 7e), cascades still occur, but for lower accu-



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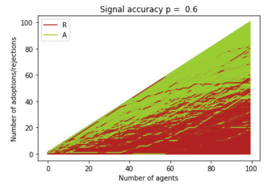

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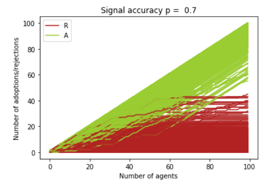

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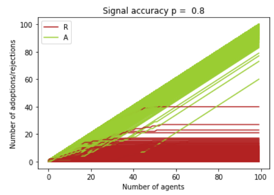

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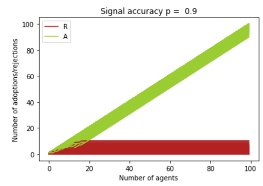

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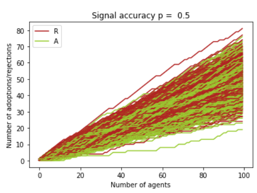
Fig. 6: Cumulative frequencies of adopts and rejects for 100 agents over 1000 runs with v = 1 for each p with 1 prior agents for non deterministic choice (best viewed in colour)



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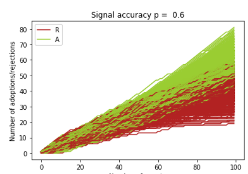
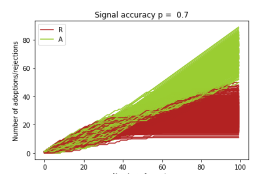
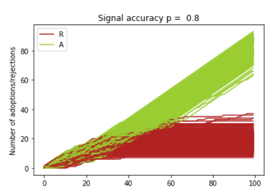


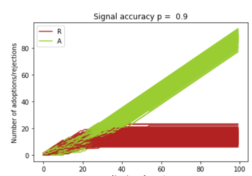
(b)



(c)



(d)



(e)

Fig. 7: Cumulative frequencies of adopts and rejects for 100 agents over 1000 runs with v = 1 for each p with 20 prior agents for non deterministic choice (best viewed in colour)

racy perception, cascades are replaced by a bias towards one choice that is greater than would be expected if only the odds of correct vs. incorrect perception were considered (Figures 6a, 6b, 7a, 7b). Although the ratio of choosing action As' and Rs' starts with 0.6 and 0.4 for p = 0.6, one choice becomes increasingly dominant and it still shows the possibility of no cascade. (Figures 6b, 7b).

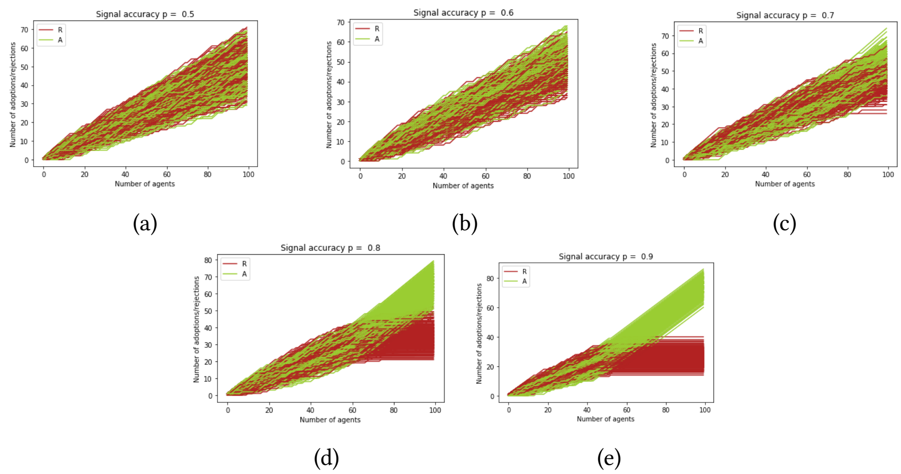


Fig. 8: Cumulative frequencies of adopts and rejects for 100 agents over 1000 runs with v = 1 for each p with 40 prior agents for non deterministic choice (best viewed in colour)

According to Bikhchandani's model, the first agent's private information can be uniquely determined from its action, which makes its choice highly influential. Therefore, our model starts by assuming there are some unobserved agents present and uses a prior distribution over C and V.

For instance, suppose there are assumed to be 20 unobserved prior agents. Then the model creates a binomial distribution over count (C) and V in terms of p for 20 prior agents which spreads out the probability across all choices. We then do the Bayesian inference. We obtained a significant change in cascades while plotting the cascade with different number of prior agents. We notice that, the prior agents delay the occurrence of cascades. While the proportions of two choices have wider deviation for 1 prior agent (Figures 6b, 6c, 6d, 6e), it gradually reduces for 20 prior agents (Figures 7b, 7c, 7d, 7e) and 30 (Figures 8b, 8c, 8d, 8e)

6 Conclusion

An information cascade happens when people observe the actions of their predecessors and try to follow these observations regardless of their own private information. We presented a full Bayesian account for the model of Bikhchandani *et al* [3]. We maintain a probability distribution over V and the (unknown) counts of High signals received by the prior agents. Rather than always choosing the most likely option, agents make a weighted choice between Adopt and Reject. We do not assume that the first agent that was observed was the first to consider the fad. Instead, we incorporate prior knowledge of unobserved agents. This means that first (observed) agent's choice is less dominant than in the prior model. Our findings show that prior agents delay the occurrence of cascades. Furthermore, in contrast to the predictions of Bikhchandani *et al.*, our results show that the cascades will not necessarily occur and there is a possibility for no cascade. The graphs obtained show that, for lower accuracy perception, cascades occur with much less probability. However, as p gets high, there is a high chance of cascades.

While people may not be good at Bayesian reasoning, as used in our model, we believe that appropriate software using our model could support users to assess the evidence from others' choices, including the possible presence of unobserved agents. This could help to reduce the likelihood of cascades.

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